Journal of Mathematical Economics 117 (2025) 103086

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Factor substitution and economic growth in a Romer-type model with monopolistic competition

Manuel A. Gómez[1]

ECOBAS, Universidade da Coruña, Spain

ARTICLE INFO

Manuscript handled by Editor A. Toda

JEL classification:
E23
L11
O41

Keywords:
Elasticity of substitution
Economic growth
Monopolistic competition

ABSTRACT

This paper studies the relationship between the elasticity of factor substitution (EOS) and steady-state values in a simplified Romer-type model with an expanding variety of products. Final goods production uses a nested CES technology, allowing substitution both among intermediate goods and between a composite intermediate good and labor. Consumption is a fixed fraction of output, and the cost of new intermediate production is constant and exogenous. The existence of an interior steady state requires an EOS between labor and the composite intermediate good above one, and an EOS among intermediates that exceeds the output–consumption ratio. Under these conditions, for a developing economy with an increasing variety of products, a higher EOS between labor and the intermediate composite good results in lower per capita output. This finding contrasts with results from infinite-horizon growth models, where a higher capital–labor EOS tends to boost per capita income (or the long-run growth rate) if baseline capital per capita is below its steady-state level. If the EOS between labor and the intermediate composite good is below unity and income per capita converges asymptotically to a constant as the number of intermediates continues to grow without bound, the usual positive relationship between the EOS and income per capita reappears.

1. Introduction

de La Grandville (1989) and Klump and de La Grandville (2000) uncovered the positive link between the elasticity of substitution (EOS) between production factors and economic growth in the Solow (1956) model. Shortly after, Klump (2001) showed that this positive relationship also holds in the Ramsey–Cass–Koopmans (RCK) model, even if labor supply is elastic (Gómez, 2018), for a developing economy with an increasing capital per capita. Miyagiwa and Papageorgiou (2003) examined this relationship in the Diamond (1965) OLG model. Xue and Yip (2012) presented a unified treatment of the relationship between the EOS and growth in the Solow, RCK and OLG models. Recently, this research has been extended to models of endogenous growth (e.g., Gómez, 2016, 2023, 2024).

A common assumption in the existing literature is that markets are perfectly competitive. This paper aims to explore the implications for the growth–EOS nexus when this assumption is relaxed. To this end, we employ a simplified version of the Romer (1990) model with an expanding variety of goods which incorporates monopolistic competition in the intermediate-goods sector. Given our focus on the effects of input substitutability, the final goods are produced using a nested CES technology, which allows for substitutability between intermediate goods, as well as between the intermediate composite good and labor. In this context, Etro (2023) demonstrates that long-run growth is not sustainable even with population growth due to the vanishing productivity of intermediate goods. Therefore, the model may converge to a steady-state under certain conditions. To simplify the analysis and derive clear-cut results, we follow Etro (2019) by assuming an exogenous consumption–output ratio à la Solow (1956), and an exogenous cost of producing new intermediates, as in Barro and Sala-i-Martin (2004, Ch. 6).

We first analyze the dynamics of the model. The existence of an interior steady state requires an EOS between labor and intermediates greater than one. In this case, in a developing economy with an increasing variety of products, a higher elasticity of substitution

E-mail address: mago@udc.es.
[1] Facultad de Economía y Empresa, Campus de Elviña, 15071 A Coruña, Spain

[2] In the OLG model, Miyagiwa and Papageorgiou (2003) show that the relationship can also be negative if the efficiency effect is smaller than the distribution effect (see also Xue and Yip, 2012).

https://doi.org/10.1016/j.jmateco.2025.103086
Received 3 November 2024; Received in revised form 19 December 2024; Accepted 19 January 2025
Available online 27 January 2025
0304-4068/

between labor and intermediates results in a lower steady-state income per capita. This finding contrasts sharply with the results obtained in previous studies considering infinite-horizon growth models, where a higher EOS unambiguously leads to a higher income per capita (or a higher long-run growth rate) if the baseline capital per capita is below its steady-state value (e.g., Klump and de La Grandville, 2000; Klump, 2001; Xue and Yip, 2012; Gómez, 2020).[2] However, if there is a long-run solution with an increasing number of intermediates where income per capita asymptotically approaches a constant, the standard positive relationship between EOS and income per capita re-emerges.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 solves for the equilibrium dynamics of the model. Section 4 determines the steady-state equilibrium. Section 5 analyzes the link between factor substitutability and the steady-state levels. Section 6 concludes.

2. The model

We consider a simplified Romer (1990) model with a constant consumption–output ratio à la Solow (1956), an exogenous cost of producing a new intermediate constant as in Barro and Sala-i-Martin (2004, Ch. 6), and a nested CES technology featuring substitutability between intermediate goods, and between the composite intermediate good and labor following Etro (2023). The economy is inhabited by a population of L identical infinitely-lived agents that grows at a rate $\gamma_L \geq 0$.

2.1. Final output

Final output Y is produced by means of the nested CES technology

$$Y = A\left[\alpha\left(\int_0^N X_j^\sigma dj\right)^{\psi/\sigma} + (1-\alpha)L^\psi\right]^{1/\psi},$$
$$A > 0,\ 0<\alpha<1,\ 0<\sigma<1,\ \psi<1, \tag{1}$$

where X_j is the jth intermediate good, N is the number of intermediates, α (resp., $1-\alpha$) represents the factor share of income from intermediate goods (resp., labor), σ parametrizes the elasticity of substitution between varieties of intermediate inputs, and $\xi = 1/(1-\psi)$ is the elasticity of substitution between labor and the composite intermediate input. The case $\psi = 0$ corresponds to the Cobb–Douglas production function

$$Y = A\left(\int_0^N X_j^\sigma dj\right)^{\frac{\alpha}{\sigma}} L^{1-\alpha},$$

and if we additionally set $\sigma = \alpha$ we get the function $Y = AL^{1-\alpha}\int_0^N X_j^\alpha dj$ used by Romer (1990) and most of the related literature. Denoting $y = Y/L$ and $x_j = X_j/L$, output per capita is given by

$$y = A\left[1-\alpha+\alpha\left(\int_0^N x_j^\sigma dj\right)^{\psi/\sigma}\right]^{1/\psi}.$$

The representative firm maximizes its profits

$$\Pi_Y = Y - wL - \int_0^N p_j X_j dj = L\left(y - w - \int_0^N p_j x_j dj\right), \tag{2}$$

in a perfectly competitive market, taking as given w, N, and the price of intermediates $\{p_j\}_{j=0}^N$. Here, the price of the final output is normalized to unity. The first order conditions for profit maximization entail that factors are paid their marginal products,

$$\frac{\partial \Pi_Y}{\partial L} = \frac{\partial Y}{\partial L} - w = (1-\alpha)A^\psi y^{1-\psi} - w = 0,$$
$$\frac{\partial \Pi_Y}{\partial x_i} = L\left(\frac{\partial y}{\partial x_i} - p_i\right) = 0,$$

and profits are zero. The inverse demand of the intermediate input i is

$$p_i = \frac{\partial y}{\partial x_i} = \alpha A x_i^{\sigma-1}\left[1-\alpha+\alpha\left(\int_0^N x_j^\sigma dj\right)^{\psi/\sigma}\right]^{(1-\psi)/\psi}\left[\int_0^N x_j^\sigma dj\right]^{(\psi-\sigma)/\sigma}. \tag{3}$$

2.2. Intermediate goods

Each producer of intermediates is a monopolist with a perpetual patent. One unit of the variety X_i is produced with one unit of the final good. Taking as given the aggregator of intermediate inputs in (3), the firm solves the profit's maximization problem

$$\max_{x_i}\ \pi_i = (p_i - 1)x_i L. \tag{4}$$

The first-order condition for profit maximization is

$$\frac{\partial \pi_i}{\partial x_i} = (\sigma p_i - 1)L = 0.$$

Hence, all intermediate producers charge the same price

$$p_i = p = \frac{1}{\sigma},$$

and produce the same amount

$$x_i = x = A^{\frac{1}{1-\psi}}\sigma^{\frac{1}{1-\psi}}\alpha^{\frac{1}{1-\psi}}N^{\frac{\psi-\sigma}{(1-\psi)\sigma}}\left[\frac{1-\alpha}{1-A^{\frac{\psi}{1-\psi}}\alpha^{\frac{1}{1-\psi}}\sigma^{\frac{\psi}{1-\psi}}N^{\frac{(1-\sigma)\psi}{(1-\psi)\sigma}}}\right]^{1/\psi}. \tag{5}$$

The symmetric profit of each intermediates' producer is

$$\pi_i = \pi = \frac{1-\sigma}{\sigma}Lx. \tag{6}$$

Replacing intermediates (5) in the production function, output per capita is given by

$$y = A\left[1-\alpha+\alpha(N^{1/\sigma}x)^\psi\right]^{1/\psi} = A\left[\frac{1-\alpha}{1-A^{\frac{\psi}{1-\psi}}\alpha^{\frac{1}{1-\psi}}\sigma^{\frac{\psi}{1-\psi}}N^{\frac{(1-\sigma)\psi}{\sigma(1-\psi)}}}\right]^{1/\psi}, \tag{7}$$

which, if $\psi = 0$, reduces to

$$y = A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}\sigma^{\frac{\alpha}{1-\alpha}}N^{\frac{\alpha(1-\sigma)}{(1-\alpha)\sigma}},$$

and if, additionally, we have that $\sigma = \alpha$ as in Romer (1990), then

$$y = A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}N.$$

Output per capita y is increasing in N given that

$$\frac{dy}{dN} = \frac{(1-\sigma)}{(1-\alpha)(1-\psi)}\alpha^{\frac{1}{1-\psi}}A^{\frac{\psi^2}{1-\psi}}\sigma^{\frac{2\psi-1}{1-\psi}}N^{\frac{\psi-\sigma}{\sigma(1-\psi)}}y^{1+\psi} > 0. \tag{8}$$

Let us denote

$$\tilde{N} = A^{-\frac{\sigma}{1-\sigma}}\alpha^{-\frac{\sigma}{(1-\sigma)\psi}}\sigma^{-\frac{\sigma}{1-\sigma}}, \tag{9}$$

and

$$\tilde{y} = A(1-\alpha)^{\frac{1}{\psi}}. \tag{10}$$

Table 1 summarizes the domain and the range of output per capita (7) depending on the values of the EOS.

Table 1
Domain and range of income per capita, y.

EOS	Domain, N	Range, y
$\psi < 0$	$[\tilde{N}, +\infty)$	$[0, \tilde{y})$
$\psi = 0$	$[0, +\infty)$	$[0, +\infty)$
$0 < \psi < 1$	$[0, \tilde{N})$	$[\tilde{y}, +\infty)$

2.3. R&D

Let P represent the stock price of the firm producing the intermediate good i, which, under symmetry, satisfies the no-arbitrage condition[3]

$$rP = \pi + \dot{P}, \tag{11}$$

where r is the interest rate. The cost of creating a new intermediate is η units of the final good. There is free entry into the R&D sector, and so $P(t) = \eta$ at all points in time. Since η is constant we have that $\dot{P} = 0$ and, hence, the condition (11) entails that

$$P = \frac{\pi}{r} = \eta. \tag{12}$$

2.4. Agents

Assets equal the market value of firms $PN = \eta N$, and so, per capita assets are $a = \eta N/L$. The evolution of per capita assets, a, is governed by the dynamic equation

$$\dot{a} = (r - \gamma_L)a + w - c - \gamma_L a. \tag{13}$$

Taking into account (12) and that $\pi = (p-1)xL$ from (4), we have that

$$\dot{a} = r\eta\frac{N}{L} + w - c - \gamma_L a = \frac{N\pi}{L} + w - c - \gamma_L a$$
$$= Npx + w - c - Nx - \gamma_L a = y(N) - c - Nx(N) - \gamma_L a.$$

Here, the last equality follows from the zero-profit condition in the final sector, so that $y = Npx + w$.

To close the model, we have to describe how the agents choose consumption. To keep things simple and obtain clear-cut analytical results, we follow Etro (2019), as well as the seminal papers of de La Grandville (1989) and Klump and de La Grandville (2000), by assuming an exogenous consumption–output ratio à la Solow (1956): $c = (1-s)y$, with $0 < s < 1$. Hence, we get that

$$\dot{a} = sy(N) - Nx(N) - \gamma_L a. \tag{14}$$

3. Dynamics

Let us first derive the equation that drives the dynamics of the number of intermediates N. Given that per capita assets are $a = \eta N/L$, we have that

$$\frac{\dot{a}}{a} = \frac{\dot{N}}{N} - \frac{\dot{L}}{L} = \frac{\dot{N}}{N} - \gamma_L.$$

Using (14), we get that

$$\dot{N} = \frac{L}{\eta}\left[y(N) - c - Nx(N)\right] = \frac{L}{\eta}\left[sy(N) - Nx(N)\right],$$

where $x(N)$ and $y(N)$ are given by (5) and (7), respectively. Using (10), this equation can be rewritten as

$$\dot{N} = \frac{L}{\eta}\left[\sigma\tilde{y}^{\psi}y(N)^{1-\psi} - (\sigma - s)y(N)\right]. \tag{15}$$

The dynamics of per capita income can be obtained as follows. Using (9) and (10), we can express dy/dN from (7) as

$$\frac{dy}{dN} = \frac{(1-\sigma)\tilde{y}^{-\psi}}{\sigma(1-\psi)\tilde{N}}\left(y - \tilde{y}^{\psi}y^{1-\psi}\right)^{\frac{\psi-\sigma}{(1-\sigma)\psi}} y^{\psi+\frac{\sigma(1-\psi)}{\psi(1-\sigma)}} > 0. \tag{16}$$

Hence, using (15), the dynamics of per capita output is driven by the equation

$$\dot{y} = \frac{dy}{dN}\dot{N} = \frac{(1-\sigma)\tilde{y}^{-\psi}L}{\eta\sigma(1-\psi)\tilde{N}}\left(1 - \tilde{y}^{\psi}y^{-\psi}\right)^{\frac{\psi-\sigma}{(1-\sigma)\psi}}\left[\sigma\tilde{y}^{\psi}y^{-\psi} - (\sigma - s)\right]y^{2+\psi}, \tag{17}$$

and the number of intermediates N can be obtained as a function of y from (7) as

$$N = \tilde{N}\left(1 - \tilde{y}^{\psi}y^{-\psi}\right)^{\frac{(1-\psi)\sigma}{(1-\sigma)\psi}}. \tag{18}$$

4. Steady state

4.1. The case $\psi < 0$

If $\psi < 0$, as Table 1 shows, the range of per capita output is $[0, \tilde{y})$. Let us first consider the case $\sigma \le s$, which can be rewritten as $1/(1-\sigma) \le 1/(1-s)$, so it is equivalent to the EOS between intermediates being lower than or equal to the output–consumption ratio. Eq. (17) shows that $\dot{y} > 0$ for $y \in (0, \tilde{y})$. Hence, per capital income tends asymptotically to the (unfeasible) steady state $\tilde{y}$, whereas N in (18) diverges to $+\infty$. Using (15), the dynamics of the number of intermediates per capita, $n = N/L$, is determined by

$$\gamma_n = \frac{\dot{n}}{n} = \frac{1}{\eta n}\left[\sigma\tilde{y}^{\psi}y^{1-\psi} - (\sigma - s)y\right] - \gamma_L. \tag{19}$$

When $y \to \tilde{y}$, if we assume that $\lim_{t\to\infty}\gamma_n > 0$ then $n \to \infty$, and so $\gamma_n \to -\gamma_L$, contradicting the assumption. If we assume that $\lim_{t\to\infty}\gamma_n < 0$ then $n \to 0$, and so $\gamma_n \to +\infty$, contradicting the assumption. Hence, it must be that $\gamma_n \to 0$, and so $\gamma_N \to \gamma_L$, and the per capita number of intermediates tends to the constant[4]

$$\tilde{n} = \frac{s\tilde{y}}{\eta\gamma_L}. \tag{20}$$

Let us now consider the case $\sigma > s$, which can be rewritten as $1/(1-\sigma) > 1/(1-s)$, so it is equivalent to the EOS between intermediates being greater than the output–consumption ratio. Now, there is an interior steady-state of per capita output, $\bar{y}$, which is given by

$$\bar{y} = \left(1 - \frac{s}{\sigma}\right)^{-\frac{1}{\psi}}\tilde{y} < \tilde{y}. \tag{21}$$

From (18), the corresponding steady-state value of N is

$$\bar{N} = \left(\frac{s}{\sigma}\right)^{\frac{(1-\psi)\sigma}{(1-\sigma)\psi}}\tilde{N} > \tilde{N}. \tag{22}$$

The share of labor on income, $\omega = wL/Y = w/y$, is

$$\bar{\omega} = (1-\alpha)A^{\psi}\bar{y}^{-\psi} = \tilde{y}^{\psi}\bar{y}^{-\psi} = 1 - \frac{s}{\sigma}.$$

In this case, we have that $\dot{y} > 0$ if $\bar{y} < y < \tilde{y}$, and $\dot{y} < 0$ if $0 < y < \bar{y}$, which entails that the steady state $y = \bar{y}$ is unstable. Hence, if $y_0 < \bar{y}$, per capita income converges asymptotically to 0 and N tends to $\tilde{N}$, and if $y_0 > \bar{y}$, then y tends to $\tilde{y}$ and N goes to $+\infty$. As discussed above, in this case the growth rate of the number of intermediates tends to the population growth rate, $\gamma_N \to \gamma_L$, and the per capita number of intermediates tends to $\tilde{n}$ in (20).

4.2. The case $0 < \psi < 1$

As Table 1 shows, the range of per capita output is $[\tilde{y}, +\infty)$. If $\sigma \le s$, Eq. (17) shows that $\dot{y} > 0$ for $y > \tilde{y}$. Hence, per capital income y goes to $+\infty$, whereas N in (18) tends to $\tilde{N}$. In this case, growth is explosive as $\dot{y}/y \to +\infty$ as $y \to +\infty$. If $s < \sigma$ there is an interior feasible steady state $y = \bar{y}$ from (21), which satisfies that $\bar{y} > \tilde{y}$, and the corresponding

[3] I thank an anonymous referee for highlighting a potential connection to asset price bubbles and suggesting the consideration of the stock price rather than the fundamental value — the discounted sum of profits — as in Barro and Sala-i-Martin (2004, Ch. 6). As summarized in Table 2, the dynamics of this model may result in the absence of a steady-state equilibrium, with the economy is perpetually on a transition path of unbalanced growth. In such cases, as demonstrated by Hirano and Toda (2025), there is a possibility that asset price bubbles emerge (see also the review by Hirano and Toda, 2024). While this paper does not focus on asset pricing, this observation suggests an interesting avenue for future research.

[4] If $\gamma_L = 0$ then $n \to \infty$.

Table 2
Dynamics of the model.

EOS	$s < \sigma$		$s \geq \sigma$
$\psi < 0$	Unstable interior steady state, $\bar{y}$		$N \to \infty,\ y \to \tilde{y}$
	$y_0 < \bar{y}\ (N_0 < \bar{N})$ $N \to \tilde{N},\ y \to 0$	$y_0 > \bar{y}\ (N_0 > \bar{N})$ $N \to \infty,\ y \to \tilde{y}$ $n \to s\tilde{y}/(\eta\gamma_L)$	$n \to s\tilde{y}/(\eta\gamma_L)$
$0 < \psi < 1$	Stable interior steady state $N \to \bar{N},\ y \to \bar{y}$		Explosive growth $N \to \tilde{N},\ y \to \infty$

value of N is $\bar{N} < \tilde{N}$ from (22). In this case, we have that $\dot{y} > 0$ if $\tilde{y} < y < \bar{y}$, and $\dot{y} < 0$ if $y > \bar{y}$, which entails that the steady state $y = \bar{y}$ is stable. Hence, per capital income y converges to $\bar{y}$ and the number of intermediates N converges to $\bar{N}$. The condition $s < \sigma$ can be rewritten as $1/(1-\sigma) > 1/(1-s)$, so it is equivalent to the EOS between intermediates being higher than the output–consumption ratio.

Evaluating (17) at $y = \bar{y}$ in (21) we get that

$$\frac{d\dot{y}}{dy}(\bar{y}) = -\frac{\psi(1-\sigma)L}{\eta(1-\psi)\bar{N}}\,(1-\tilde{y}^{\psi}\bar{y}^{-\psi})^{\frac{\psi-\sigma}{(1-\sigma)\psi}}\,\bar{y}\begin{cases} <0, & \text{if } 0<\psi<1,\\ >0, & \text{if } \psi<0,\end{cases}$$

which confirms the previous analysis that the steady state $y = \bar{y}$ is stable if $0 < \psi < 1$, and unstable if $\psi < 0$.

4.3. Summary

Table 2 summarizes the dynamics of the model.

5. Factor substitution and economic growth

de La Grandville (1989) and Klump and de La Grandville (2000) have uncovered the importance of normalizing the underlying CES production function to analyze in a meaningful way the effect of factor substitution on economic variables. Normalization is to consider a specific family of CES functions that are tangent at the same baseline point, which differ uniquely in the elasticity of substitution. To this end, the efficiency parameter A and the distribution parameter α are made functions of the baseline point.

5.1. Normalization

The normalized CES production function is

$$Y = A(\xi)\left[\alpha(\xi)\left(\int_0^N X_j^{\sigma}dj\right)^{\psi/\sigma} + (1-\alpha(\xi))L^{\psi}\right]^{1/\psi}, \tag{23}$$

where $\xi = 1/(1-\psi)$ is the elasticity of substitution between intermediates and labor. The productivity and distribution parameters are computed by considering the baseline values of N_0, $y_0 = f(N_0, \xi)$ and, for simplicity, the wages share of income

$$\omega_0 = \frac{L}{Y}\frac{\partial Y}{\partial L}\bigg|_{N=N_0} = (1-\alpha)A^{\psi}y_0^{-\psi}.$$

Here, y is given by (7), so we have to solve the system

$$y_0 = A\left[\frac{1-\alpha}{1-\alpha^{\frac{1}{1-\psi}}A^{\frac{\psi}{1-\psi}}\sigma^{\frac{\psi}{1-\psi}}N_0^{\frac{(1-\sigma)\psi}{(1-\psi)\sigma}}}\right]^{1/\psi},$$
$$\omega_0 = (1-\alpha)A^{\psi}y_0^{-\psi}.$$

The solution of the former system is

$$A(\xi) = \frac{1}{\sigma}N_0^{-(1-\sigma)/\sigma}(1-\omega_0)^{(1-\psi)/\psi}\left[1+\omega_0\sigma^{\psi}(1-\omega_0)^{\psi-1}y_0^{\psi}N_0^{\psi(1-\sigma)/\sigma}\right]^{1/\psi}, \tag{24}$$

$$\alpha(\xi) = \left[1+\omega_0\sigma^{\psi}(1-\omega_0)^{\psi-1}y_0^{\psi}N_0^{\psi(1-\sigma)/\sigma}\right]^{-1}, \tag{25}$$

so that output in intensive form becomes

$$y = y_0\omega_0^{1/\psi}\left[1-(1-\omega_0)\left(\frac{N}{N_0}\right)^{\frac{(1-\sigma)\psi}{(1-\psi)\sigma}}\right]^{-1/\psi}.$$

If $\sigma > s$ and $0 < \psi < 1$, the interior steady-state values of N and y are then given by the following expressions:

$$\bar{N} = s^{\frac{\sigma(1-\psi)}{(1-\sigma)\psi}}\sigma^{-\frac{\sigma(1-\psi)}{(1-\sigma)\psi}}(1-\omega_0)^{-\frac{\sigma(1-\psi)}{(1-\sigma)\psi}}N_0 = \left(\frac{1-\bar{\omega}}{1-\omega_0}\right)^{\frac{\sigma(1-\psi)}{(1-\sigma)\psi}}N_0, \tag{26}$$

$$\bar{y} = \omega_0^{1/\psi}\left(1-\frac{s}{\sigma}\right)^{-1/\psi}y_0 = \left(\frac{\omega_0}{\bar{\omega}}\right)^{1/\psi}y_0. \tag{27}$$

The effects of the elasticity of substitution and the initial labor income share on income is given by

$$\frac{\partial y}{\partial \psi} = -\frac{y}{\psi}\ln\left(\frac{y}{y_0}\right),$$

so that

$$\operatorname{sign}\frac{\partial y}{\partial \psi} = \begin{cases} -\operatorname{sign}\psi, & \text{if } y > y_0,\\ \operatorname{sign}\psi, & \text{if } y < y_0,\end{cases}$$

and

$$\frac{\partial y}{\partial \omega_0} = -\frac{y}{\omega_0(1-\omega_0)}\frac{1}{\psi}\left[1-\left(\frac{y}{y_0}\right)^{\psi}\right],$$

so that

$$\operatorname{sign}\frac{\partial y}{\partial \omega_0} = \begin{cases} \operatorname{sign}\psi, & \text{if } y > y_0,\\ -\operatorname{sign}\psi, & \text{if } y < y_0.\end{cases}$$

The EOS has no effect on the labor income share, $\omega = w/y = (1-\alpha)A^{\psi}y^{-\psi} = \omega_0(y_0/y)^{\psi}$, as

$$\frac{\partial \omega}{\partial \psi} = \omega\left[\ln\left(\frac{y_0}{y}\right) - \frac{\psi}{y}\frac{\partial y}{\partial \psi}\right] = 0.$$

5.2. The interior steady state $\bar{y}$

Let us now determine the effect of the elasticity of substitution between intermediates and between the composite intermediate input and labor. We start with the case in which the economy converges to the interior steady state (26) and (27), which happens when $0 < \psi < 1$ and $s < \sigma$. Differentiating (27) with respect to σ and ψ, the effect of the EOS between intermediates on per capita income is

$$\frac{\partial \bar{y}}{\partial \sigma}(\sigma,\psi) = -\frac{s\bar{y}}{(\sigma-s)\sigma\psi} < 0, \tag{28}$$

so per capita income decreases in σ due to a lower marginal productivity of innovation. The effect of the EOS between capital and labor is

$$\frac{\partial \bar{y}}{\partial \psi}(\sigma,\psi) = \frac{\bar{y}}{\psi^2}\ln\left(\frac{\bar{\omega}}{\omega_0}\right)\begin{cases} <0, & \text{if } 0<\bar{\omega}<\omega_0 \iff y_0<\bar{y}\ (\iff N_0<\bar{N}),\\ =0, & \text{if } 0<\bar{\omega}=\omega_0 \iff y_0=\bar{y}\ (\iff N_0=\bar{N}),\\ >0, & \text{if } 0<\omega_0<\bar{\omega} \iff y_0>\bar{y}\ (\iff N_0>\bar{N}).\end{cases} \tag{29}$$

Differentiating (26) with respect to σ, the effect of the EOS between intermediates on the number of intermediates is

$$\frac{\partial \bar{N}}{\partial \sigma}(\sigma,\psi) = -\frac{(1-\psi)\bar{N}}{(1-\sigma)^2\psi}\left[1-\sigma+\ln\left(\frac{1-\omega_0}{1-\bar{\omega}}\right)\right]\begin{cases} >0, & \text{if } \sigma(1-\omega_0)e^{1-\sigma}<s<\sigma,\\ =0, & \text{if } \sigma(1-\omega_0)e^{1-\sigma}=s<\sigma,\\ <0, & \text{if } s<\sigma(1-\omega_0)e^{1-\sigma}.\end{cases}$$

Differentiating (26) with respect to ψ, the effect of the EOS between the composite intermediate input and labor is

$$\frac{\partial \bar{N}}{\partial \psi}(\sigma,\psi) =$$

$$\frac{\sigma \bar{N}}{(1-\sigma)\psi^2}\ln\left(\frac{1-\omega_0}{1-\bar{\omega}}\right)\begin{cases} <0, & \text{if } 0<\bar{\omega}<\omega_0 \iff y_0<\bar{y}\ (\iff N_0<\bar{N}),\\ =0, & \text{if } 0<\bar{\omega}=\omega_0 \iff y_0=\bar{y}\ (\iff N_0=\bar{N}),\\ >0, & \text{if } 0<\omega_0<\bar{\omega} \iff y_0>\bar{y}\ (\iff N_0>\bar{N}).\end{cases}$$

Hence, in a developing economy that evolves with an expanding variety of products, so that the initial number of intermediates is below its steady state value, $N_0 < \bar{N}$, an increase in the EOS decreases the steady-state value of income per capita. This is in sharp contrast with what happens in the Solow or Ramsey models in which the higher the EOS the higher the steady-state income per capita (Klump and de La Grandville, 2000; Klump, 2001; Xue and Yip, 2012), and in the one sector endogenous growth model in which the higher EOS the higher the long-run growth rate (e.g., Gómez, 2020).

Intuitively, in a Romer-type model, productivity gains stem from introducing new intermediate goods, which complement existing intermediates and labor, boosting overall productivity through specialization. When substitutability between the composite intermediate good and labor is high, the value of adding new varieties declines, as labor can more easily replace these specialized inputs. This weakens the productivity impact of expanding intermediate varieties, so that when substitutability is high, increasing the number of intermediates contributes less to per capita output. In contrast, in the Solow or Ramsey infinite-horizon models, growth is driven by the accumulation of capital over time. When capital is more substitutable with labor, it becomes easier for an economy in which capital is relatively scarce to substitute it with labor which results in higher output per capita (or long-run growth).

5.3. The asymptotic steady state $\tilde{y}$

If $\psi < 0$ and (i) $\sigma \le s$ or (ii) $\sigma > s$ and $N_0 > \bar{N}$, per capita income converges asymptotically to

$$\tilde{y} = A(1-\alpha)^{1/\psi} = y_0\omega_0^{1/\psi}.$$

Differentiating with respect to σ and ψ, we can derive the effect of the EOS as

$$\frac{\partial \tilde{y}}{\partial \sigma}(\sigma,\psi) = 0,$$
$$\frac{\partial \tilde{y}}{\partial \psi}(\sigma,\psi) = -\frac{\tilde{y}}{\psi^2}\ln \omega_0 > 0.$$

In this case, the number of intermediates tends to infinity, but the number of per capita intermediates tends to $\tilde{n}$ in (20). The effect of the EOS on the number of intermediates per capita is given by

$$\frac{\partial \tilde{n}}{\partial \sigma}(\sigma,\psi) = \frac{s}{\eta\gamma_L}\frac{\partial \tilde{y}}{\partial \sigma}(\sigma,\psi) = 0,$$
$$\frac{\partial \tilde{n}}{\partial \psi}(\sigma,\psi) = \frac{s}{\eta\gamma_L}\frac{\partial \tilde{y}}{\partial \psi}(\sigma,\psi) > 0.$$

Hence, an increase in the EOS between the composite intermediate input and labor increases per capita income and the per capita number of intermediates, which agrees with Klump and de La Grandville (2000), but the EOS between intermediates has no effect.[5]

One may ask which of the above could be the most realistic case. With an investment–output ratio below 30%, and a markup $1/\sigma$ lower than 2 (e.g., Conlon et al., 2023), the case $\sigma > s$ seems to be the most plausible. Considering the EOS between capital (as a proxy of the composite intermediate good) and labor, the meta-analyses in Chirinko (2008) and Knoblach et al. (2020) show that most estimates are below one. this would provide support to the case that $\psi < 0$, and so, the EOS would have a positive growth effect. However, some prominent studies as, e.g., Duffy and Papageorgiou (2000) and Karabarbounis and Neiman (2014), report estimates of the EOS above one. As Piketty and Zucman (2014) argue, this would be the case for high-tech economies in which capital has a wider range of different uses and labor can be more readily substituted. This would provide support to the case that $0 < \psi < 1$ and, therefore, a higher EOS would have a negative growth effect.

[5] If $\sigma > s$ and $\tilde{N} < N_0 < \bar{N}$, per capita income converges to zero, so the elasticities have no effect.

6. Conclusions

This paper has analyzed the link between factor substitutability and steady-state values in a simplified version of the Romer (1990) model. We have shown that when intermediates and labor are substitutes and the EOS between intermediates is greater than the output–consumption ratio, a higher EOS between intermediate inputs and labor entails a lower output per capita in a developing economy with an increasing variety of inputs. This result is surprising because the previous literature considering infinite-horizon models found a positive link between the EOS and output per capita in exogenous-growth models and between the EOS and long-run growth in endogenous-growth models. Intuitively, in a Romer-type model productivity gains stem from introducing new intermediate goods through specialization. Thus, when the elasticity of substitution between intermediates and labor is higher, the productivity gain of adding new varieties declines, as labor can more easily replace these specialized inputs. When the economy evolves with an ever increasing variety of inputs and income per capita tends asymptotically to a constant, the standard positive link between the EOS and output per capita comes up again.

This work can be extended in several ways. A natural extension would be to endogenize the consumption–savings decision, thereby introducing an additional channel through which the elasticity of substitution could influence the steady-state income per capita. Moreover, in this paper, following Barro and Sala-i-Martin (2004, Ch. 6), we have assumed that the invention of a new intermediate requires a fixed quantity of final output. Another interesting extension would be to explore other cost structures proposed in the literature (e.g., Romer, 1990; and Jones, 1995).

Acknowledgments

I gratefully acknowledge the valuable comments of the editor and two anonymous reviewers. I also thank Tiago Sequeira for his comments on a previous draft. This work was supported by grant PID2021-127599NB-I00 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe".

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Manuel A. Gomez reports financial support was provided by Spain Ministry of Science and Innovation. Manuel A. Gomez reports financial support was provided by European Regional Development Fund. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

Barro, R.J., Sala-i-Martin, X., 2004. Economic Growth, second ed. The MIT Press.
Chirinko, R.S., 2008. σ: The long and short of it. J. Macroecon. 30, 671–686.
Conlon, C., Miller, N.H., Otgon, T., Yao, Y., 2023. Rising markups, rising prices? AEA Pap. Proc. 113, 279–283.
de La Grandville, O., 1989. In quest of the Slutsky diamond. Amer. Econ. Rev. 79, 468–481.

Diamond, P.A., 1965. National debt in a neoclassical growth model. Amer. Econ. Rev. 55, 1126–1150.

Duffy, J., Papageorgiou, C., 2000. A cross-country empirical investigation of the aggregate production function specification. J. Econ. Growth 5, 87–120.

Etro, F., 2019. The Romer model with monopolistic competition and general technologies. Econom. Lett. 181, 1–6.

Etro, F., 2023. Technologies for endogenous growth. J. Math. Econom. 105, 102808.

Gómez, M.A., 2016. Factor substitution is an engine of growth in a model with productive public expenditure. J. Econ. 117, 37–48.

Gómez, M.A., 2018. Factor substitution and convergence speed in the neoclassical model with elastic labor supply. Econom. Lett. 172, 89–92.

Gómez, M.A., 2020. Factor substitution, long-run growth, and speed of convergence in the one-sector convex endogenous-growth model. Metroeconomica 71, 2–21.

Gómez, M.A., 2023. Factor substitution, long-run equilibrium, and convergence speed in the lucas model. Econom. Lett. 232, 111358.

Gómez, M.A., 2024. Elasticity of substitution, long-run growth, and convergence speed: A general framework. Int. J. Econ. Theory 20, 352–370.

Hirano, T., Toda, A.A., 2024. Bubble economics. J. Math. Econom. 111, 102944.

Hirano, T., Toda, A.A., 2025. Bubble necessity theorem. J. Polit. Econ. 133, 111–145.

Jones, C.I., 1995. R&D-based models of economic growth. J. Polit. Econ. 103, 759–784.

Karabarbounis, L., Neiman, B., 2014. The global decline of the labor share. Q. J. Econ. 129, 61–103.

Klump, R., 2001. Trade, money and employment in intertemporal optimizing models of growth. J. Int. Trade Econ. Dev. 10, 411–428.

Klump, R., de La Grandville, O., 2000. Economic growth and the elasticity of substitution: Two theorems and some suggestions. Amer. Econ. Rev. 90, 282–291.

Knoblach, M., Roessler, M., Zwerschke, P., 2020. The elasticity of substitution between capital and labour in the US economy: A meta-regression analysis. Oxf. Bull. Econ. Stat. 82, 62–82.

Miyagiwa, K., Papageorgiou, C., 2003. Elasticity of substitution and growth: normalized CES in the diamond model. Econom. Theory 21, 155–165.

Piketty, T., Zucman, G., 2014. Capital is back: Wealth-income ratios in rich countries 1700–2010. Q. J. Econ. 129, 1255–1310.

Romer, P.M., 1990. Endogenous technological change. J. Polit. Econ. 98, S71–S102.

Solow, R.M., 1956. A contribution to the theory of economic growth. Q. J. Econ. 70, 65–94.

Xue, J., Yip, C.K., 2012. Factor substitution and economic growth: a unified approach. Macroecon. Dyn. 16, 625–656.