FREQUENCY DOMAIN FEEDFORWARD COMPENSATION

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Abstract: This work deals with some practical aspects of PID controllers regarding multivariable non-linear feedforward compensation. This contribution concerns the task of compensation for multivariable disturbances on the controlled variable. The strategy consists of establishing an adaptive function capable of compensatinge by means of a feedforward strategy all disturbance variables under any dynamic condition.

Keywords: QFT theory, frequency domain design, frequency domain identification, multivariable disturbances, multivariable feedforward compensation.

1. FEEDFORWARD CONTROL BACKGROUND.

Conventional feedforward control deals with the task of correcting the manipulated variable for disturbances on the controlled process. Most common industrial processes are disturbed by more than a variable. For instance, in heat exchangers, controlled temperature is disturbed from flow and temperature variations of heated fluid. Furthermore, they are disturbed also by variations of the operating point because dissipation heat may depend on the ambient and operating temperatures and by process parameters.

In conventional feedforward control (Shinskey 1980) an error must be detected in a controlled variable before the feedback controller can act to change the manipulated variable. Therefore, disturbances must upset the system before the feedback controller can do anything. It seems very reasonable that if a disturbance entering a process could be detected, a controller should begin to correct it before it upsets the process.

This is the basic idea of feedforward control. If disturbance can be measured, this result will be used to send a signal through a feedforward control algorithm that makes appropriate changes in the manipulated variable so as to keep the controlled variable near its desired value.

Classical industrial controllers offer the possibility of compensation for only a disturbance variable entering the process, if such a disturbance can be measured.

The real problem concerning industrial control, in which a good performance is needed, requires the compensation task for more than a single disturbance variable included disturbance model parameters. In such a case, conventional controllers are not efficient and proposed adaptive controller takes advantage. Furthermore, disturbance variables are associated by non-linear functions. Non-linear feedforward compensator can be designed for non-linear systems.

An alternative to implement feedforward control systems in order to compensate multivariable disturbances, may be implemented by means of a frequency analysis procedure based in disturbance model identification by FFT algorithm.

2. SPECTRAL IDENTIFICATION

In order to get information related to system frequency response, it is necessary to implement a procedure by means of FFT algorithms. This information will be used later to design the corresponding PID controller. The application of FFT algorithms to the system dynamics by exciting it with sinusoidal signals of different frequencies permits the achievement of the magnitude and phase angle at concrete frequencies, but also subarmonic components originated by external disturbances are to be found.

The identification task starts searching for the ultimate frequency (w_{relay}) with system phase angle response $-\pi$ rad performing the Relay Feedback Analysis (Åström.K.J et al., 1989). This frequency will be the key for the control frequencies selection used in the regulator design (Ferreiro et al., 1995). The feedback PID controller and the perturbation lead/lag feedforward compensator (Figure 1,2 and 3) will be designed in function of the performance specification indicated by the designer. The control frequency (w_{cp}) for the feedback controller and the time constants (τa, τr) for the lead/lag feedforward compensator will be obtained as fraction of the w_{relay} and the control system performance specifications. With the FFT algorithm the following information will be found about the plant, working in open-loop configuration:

- The magnitude $M = \left|G\left(jw_{cp}\right)\right|$ and phase $P = \angle G\left(jw_{cp}\right)$ at the control frequency (wcp) selected for the design of the feedback PID regulator.
- The central frequency of high frequency disturbances.

The performance specification includes datas like: phase margin (ϕ_M), settling time, overshoot and bandwidth. With such data proportional gain, integral and derivative parameters can be achieved deterministically (Phillips et al., 1995 and Åström.K.J et al., 1984).

The design expressions for three types of regulators are presented in Table1. The contribution angle θc corresponds with the regulator phase angle at the control frequency (w_{cp}) . Proportional gain, integral and derivative parameters are achieved deterministically (Table 1) for PI and PD regulators. Design criteria can be achieved by selecting frequencies at which the contribution angle achieves acceptable regulators in terms of relative stability. Then the control frequency (w_{cp}) can vary in order to verify the design restrictions.

From frequency analysis by means of a digital signal processor algorithm which implements the FFT (Decimation in Frequency) (Oppenheim et al., 1989), it is possible to introduce further computer-based calculations to identify salient characteristics, disturbance characteristics and system frequency response with some a priori knowledge.

Table 1:Design Equations for a concrete Phase Margin

$$\theta_c = \angle G_c\left(jw_c\right) = 180 + \phi_M - \angle G_p\left(jw_c\right)$$

	Design Equations	Restrictions
PD	$Kp = \frac{\cos(\theta_c)}{\lvert G_p(jwc_i)\rvert}$ $T_d wc_i = \tan(\theta_c)$	$0 < \theta_c < \frac{\pi}{2}$
PI	$Kp = \frac{\cos(\theta_c)}{\lvert G_p(jwc_i)\rvert}$ $\frac{-1}{T_i wc_i} = \tan(\theta_c)$	$0 > \theta_c > \frac{3\pi}{2}$
PID	$Kp = \frac{\cos(\theta_c)}{\lvert G_p(jwc_i)\rvert}$ $T_d = \frac{\tan(\theta_c) + \sqrt{\tan(\theta_c)^2 + \frac{4}{K_t}}}{2w_c}$ $T_i = K_t T_d \quad 2 \le K_t \le 8$	$\frac{\pi}{2} > \theta_c > \frac{3\pi}{2}$

3. DESIGN ALGORITHM.

As we can see in Figure 1 ,2 and 3 the controller will be composed by a PID controller and a Lead/Lag Feedforward compensator.

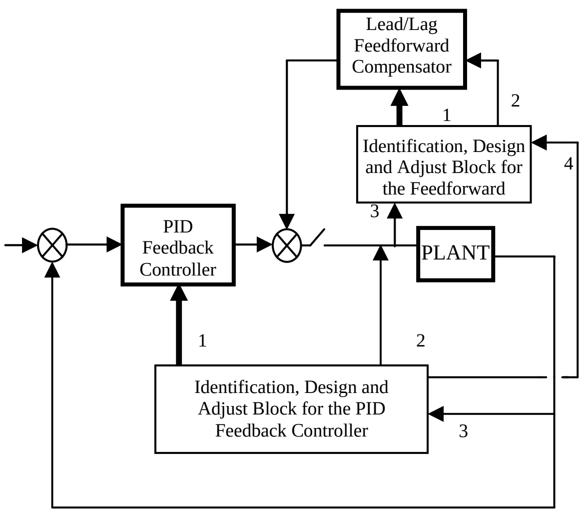

Figure 1. General configuration

The feedback one will be working all the time and will be adapted to give response to changes in operating conditions. Its input will be the error obtained as the difference between the set point and the controlled variable.

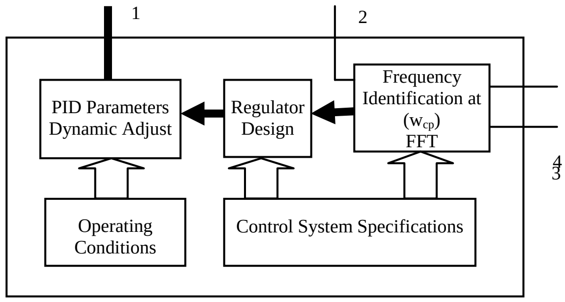

Figure 2. Identificación, Design and Adjust Block for the PID Controller

The lead/lag feedforward compensator will give response to any disturbance. In the identification process the manipulated variable measured under different operating conditions will be stored in order to estimate the manipulated variable during the control system performance.

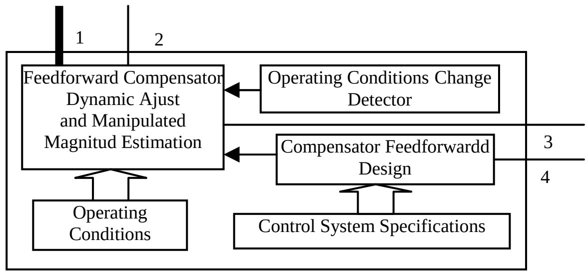

Figure 3. Identificación, Design and Adjust Block for the Feedforward Compensator

Its input (output 2 Figure 3) will be the error between the manipulated variable estimation when a variation in the disturbance variable is detected and the real system manipulated variable before the perturbation . Then the design algorithm will be related to the development of both elements. The design objective is to obtain two crisp sets look-up tables where we will map a complete set of PID parameters and the lead/lag time constants for any combination of operating conditions. This set of parameters will be obtained trying to give optimum responses depending on the design criteria specified for every concrete controller. As explained in section 2 the fuzzy adaptive design procedure is based in the plant identification by frequency techniques obtaining w_{relay} and subsequentally w_{cp} ,τa and τr.

It is important to mention that it is necessary to obtain these frequencies for any combination of operating conditions if we want to map the system nonliniarities.

The Figure 4 shows a PID design block diagram (Åström.K.J et al., 1984) as important part of the fuzzy adaptive procedure. Input data is divided into two types:

- Data concerning the definition of the performance specification.
- Data concerning the dynamic system behaviour.

The design procedure has to follow the next sequence of actions:

1. Take the system to a steady state under a concrete operating conditions
2. Store the value of the manipulated variable (MV).
3. Apply the Relay Feedback Analysis.
4. Select the control frequencie
5. Identify frequency system response (magnitude and phase) applying FFT at the control frequencie
6. Verification of the design restrictions. If not come back to the step 3.
7. Application of the design expressions, obtaining the controllers parameters.
8. Choice of lead/lag time constants for the lead/lag feedforward compensator.

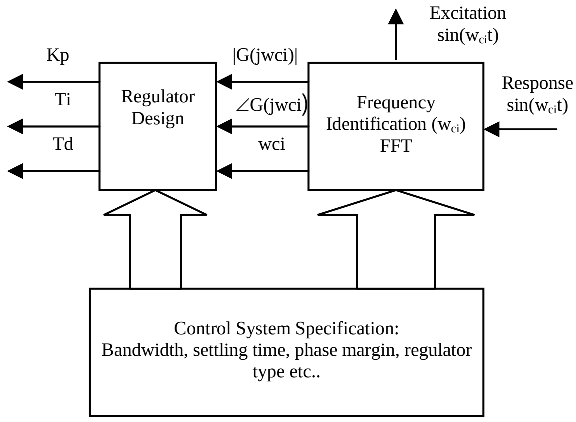

Figure 4. PID design block diagram

During the design process we can find that for some control frequencies to obtain acceptable regulators it is not possible. In these cases it is necessary to restart the design, searching control frequencies that will generate stable controllers.

Applying the above method for different system operating conditions we will built a crisp set look-up table with a complete set of controller parameters and time constants. The defuzzyfication method will be performed by least square regression procedure (Johansson et al., 1993, Brown et al., 1994) obtaining polynomials expressions for every PID parameter, lead/lag time constants and manipulated variable estimation as function of the operating conditions. These expressions permit to adapt in real time during the system performance the regulator parameters as soon as the plant mathematical models change due to its implicit nonliniarities.

3. CASE STUDY.

A tank system is used to check the performance of the algorithm due to its nonlinear characteristics. The Tank system model is given by the expression (1)

$$\frac{d}{dt}\left[A(h)h\right] = qi - qo = qi - a\sqrt{2gh} \quad (1)$$

A(h)	tank section (m2)
h	tank level (m)
qi, qo	input, output liquid flow (m3/sec.)
a	outlet pipe section (m2)
g	gravity (9.8 m/sec.2)

A tank with the following characteristics was used to verify experimentally the controller.

Height = 10m
Base cross section diameter = 1m
Top cross section diameter = 4m
qimax=3.514 m^3//sec.
amax=0.251 m^2

The tank section is a function of the h variable. Taking it into account the equation (1) is converted in equation (2) which represents the mathematical model of our tank system

$$\pi \left[\frac{dh}{dt} + \frac{d}{dt}\left(\frac{h^3}{100}\right) + \frac{d}{dt}\left(\frac{h^2}{5}\right)\right] = qi - a\sqrt{2gh} \quad (2)$$

The design procedure starts identifying the system by frequency techniques. First of all, it will be applied the relay Feedback Analysis (Figure 5) and secondly working with the open loop configuration we introduce sinusoidal stimulus to our plant, processing its responses by the FFT algorithm (Figure 6)

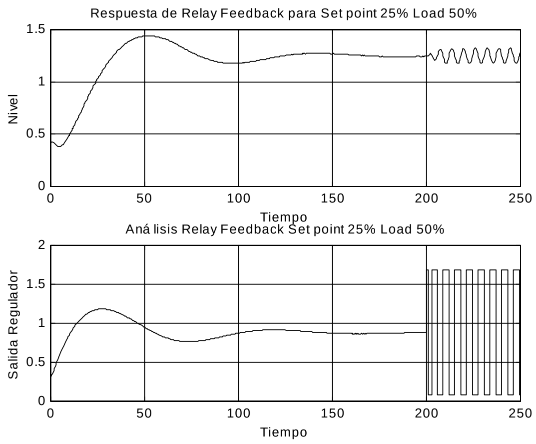

Figure 5. Relay Feedback Analysis

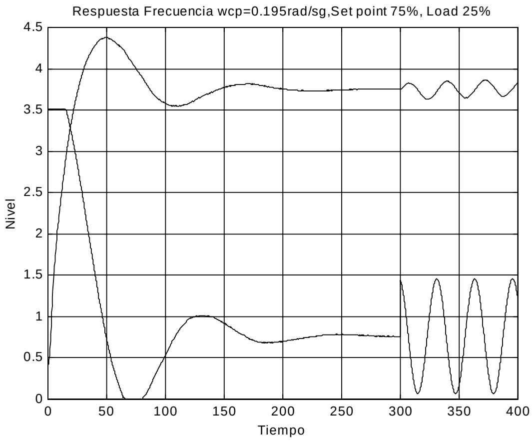

Figure 6. Frequency Response Analysis

The objective is to find, for every combination of operating conditions (set point and load) the frequency (w_{relay}) with system response phase –180 deg. It has been specified a set of performance specifications (time response, bandwidth and phase margin) initially. A slow time response trying to avoid great overshoots and a phase margin of 65 deg are some of these specifications.

The control frequency (w_{cp}) has been selected as fraction of w_{relay} (0.2 w_{relay} in this case) and the lead/lag unit pole and zero just the same ($w_{ccf}=0.5w_{relay}=1/\tau a$ $w_{cpf}=w_{relay}=1/\tau r$). The results of the application of the algorithm is presented in Table 2. The Feedback controller is a PI and Feedforward compensator a lead/lag unit.

Table 2. Design Algorithm Results

Frequency Identification		LOAD 25%	LOAD 50%	LOAD 75%
SET OINT	25%	$W_{relay11}$=1.005 **M.V.**=0.4395 W_{cp11}=0.201 W_{cpf11}=1 0.37 W_{ccf11}=0.5 -94.7º	$W_{relay12}$=1.04 **M.V.**=0.8781 W_{cp12}=0.208 W_{cpf12}=1 0.32 W_{ccf12}=0.5 -90º	$W_{relay13}$=1.08 **M.V.**=1.318 W_{cp13}=0.216 W_{cpf13}=1 0.36 W_{ccf13}=0.5 -106º
	50%	$W_{relay21}$=0.94 **M.V.**=0.6226 W_{cp21}=0.188 W_{cpf21}=0.94 0.244 W_{ccf21}=0.47 -108º	$W_{relay22}$=0.63 **M.V.**=1.237 W_{cp22}=0.126 W_{cpf22}=0.63 0.32 W_{ccf22}=0.31 -90º	$W_{relay23}$=0.65 **M.V.**=1.863 W_{cp23}=0.13 W_{cpf23}=0.65 0.62 W_{ccf23}=0.32 -78.3º
	75%	$W_{relay31}$=0.973 **M.V.**=0.7594 W_{cp11}=0.1946 W_{cpf31}=0.97 0.155 W_{ccf31}=0.48 -110º	$W_{relay32}$=0.973 **M.V.**=1.525 W_{cp32}=0.1946 W_{cpf32}=0.97 0.155 W_{ccf32}=0.48 -94.7º	$W_{relay33}$=0.973 **M.V.**=2.284 W_{cp33}=0.1946 W_{cpf33}=0.97 0.157 W_{ccf33}=0.48 -94.74º

Parameters PID		LOAD 25%	LOAD 50%	LOAD 75%
SET POINT	25%	$K_{PP11=2.5\ 35}T_{IP11=13.45}$	$K_{PP12=2.83}$ $T_{IP12=10.31}$	$K_{PP13=2.74}$ $T_{IP13=29.23}$
	50%	$K_{PP21=4.07}$ $T_{IP21=43.32}$	$K_{PP22=2.83}$ $T_{IP22=17.02}$	$K_{PP23=1.29}$ $T_{IP23=10.32}$
	75%	$K_{PP31=6.42}$ $T_{IP31=58.74}$	$K_{PP32=6.05}$ $T_{IP32=13.9}$	$K_{PP33=5.97}$ $T_{IP33=13.92}$

With the information of Table2 it is possible, applying numerical analysis perform a deffuzyfication procedure by means of polynomial expressions. These expressions have as independent variables the set point and the load and as dependent variables the controller parameters. Then the defuzzyfication process will be performed by least square regression procedure, obtaining the polynomial expressions. The expressions (3) represents the functions that relates the operating conditions to the manipulated variable estimation (MV), the PID parameters and the limit cycle frequency.

$$Kpp = -2.385 + 19.13x + -8.68x^2 + 16.73y + -66.14xy + 52.72x^2 y + 0.28y^2 + -21.52xy^2 + 32.32x^2 y^2$$

$$1 / Tip = -0.1637 + 0.7329x + -0.8513x^2 + 1.9757y + -7.2847xy + 7.1781x^2 y + -2.5498y^2 + 9.3053xy^2 + 32.32x^2 y^2$$

$$w_{relay} = -0.842 + 9.692x + -9.696x^2 + 9.99y + -7.80xy + -52.62x^2 y + 52.4y^2 + 41.84xy^2 + -41.92x^2 y^2$$

$$MV = -0.0656+0.3716x + -0.4016x^2 + 1.1778y + 2.13xy + 0.5808x^2 y + -0.3y^2 + 1.6976xy^2 + -1.824x^2 y^2 \quad (3)$$

The variables x and y represent the operating conditions set_point and load. The universe of discourse for both variables is [0,1].

Now the identification and controller design procedures are finished. The control system behaviour under different operating conditions and disturbances will be tested.

4. RESULTS AND CONCLUSIONS.

Initially the system time response is tested modifying by steps the set point and the load at the same time during the experiment. Three combinations of set point and load have been tested (0.25,0.25), (0.5,0.5) and (0.75,0.75).

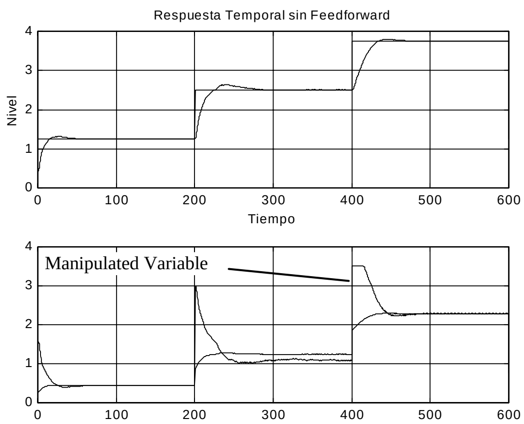

Figure7. System Time Response

In this first experiment we just try to evaluate the feedback controller element without using the perturbation feedforward compensator. Trying to observe how we can obtain quick time responses a control frequency $w_{cp}=0.2w_{relay}$ is selected and designed the corresponding set of controllers. The results are represented in Figure 7.

The second experiment was designed to analyze at the same time the controller performance under different operating conditions and disturbances. These conditions are set point (25%), load (25%) and a load perturbation.

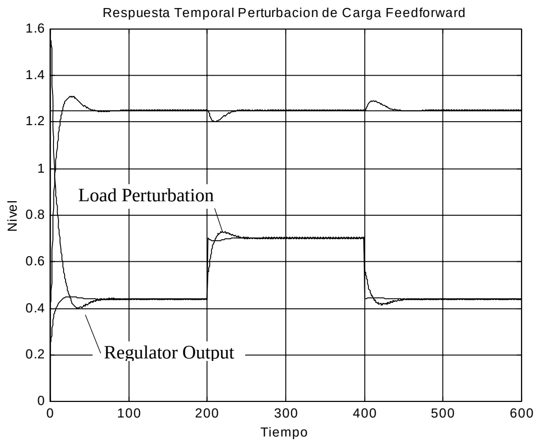

Figure 8. Feedback Controller Time Response under Disturbances

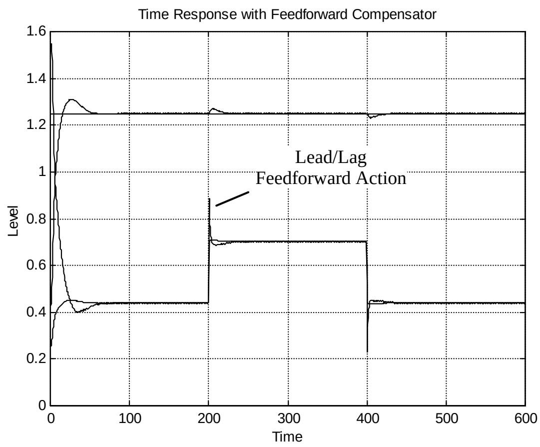

Figure 9. Feedforward Compensator Time Response

In Figure 8 and 9 the effect of the load perturbation with and without lead/lag feedforward compensator is represented. Its clear the correction effect of the lead/lag unit avoiding system long transient periods out of the operating point under load disturbance conditions.

The results show how valve modulation activity is correct in the three conditions where energy demanded for rapid following is achieved under good valve modulation, depending on the required tracking speed. It is possible to achieve performance while keeping robustness of a controlled system under load changes in acceptable limits as per time response of results, where feedforward compensation of loads and set points with inherent modelling errors do not distort too much the response and avoids the limitation of the integral action.

The adaptive frequency method has been revealed as effective in feedforward dynamic compensation where uncertainties from environmental conditions are met, and some points are to be raised as follows:

- Low man machine interaction is needed for the adjustment task
- Acceptable time response to disturbances
- Robustness in both cases, that is under parameter variations and relative stability

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