

# **A Numerical Approach based on the BEM for Computing Transferred Earth Potentials in Grounding Analysis**

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## **Abstract**

In this paper we present a numerical approach based on the Boundary Element Method for the analysis of a very common problem in electrical engineering practice: the existence of transferred earth potentials in a grounding installation [1]. We propose a numerical approach to analyze this phenomenon. We demonstrate its feasibility by means of an application example with the geometry of a real grounding system.

## 1 Mathematical and Numerical Model of the Electrical Current Dissipation into a Soil

The main goals of a grounding system are to guarantee the integrity of equipment and the continuity of the service under fault conditions and to safeguard that persons working or walking in the surroundings of the grounded installation are not exposed to dangerous electrical shocks. Consequently, the apparent electrical resistance of the earthing system must be low enough to assure that fault currents dissipate mainly through the grounded electrode, while maximum potential differences between close points on the earth surface must be kept under certain tolerances (step, touch and mesh voltages) [1].

In the last four decades, the operation of grounding systems has been extensively analyzed, and several methods for analysis and design have been proposed. Most of these methods are based on the professional experience, on semi-empirical works, on experimental data obtained from scale model assays and laboratory tests, or on intuitive ideas. These contributions represented an important improvement in the grounding analysis area; however, some problems have been systematically reported, such as the large computational costs required in the analysis of real cases, the unrealistic results obtained when segmentation of conductors is increased, and the uncertainty in the margin of error [1, 2, 3].

The electrical current dissipation into the soil is a well-known phenomenon which equations can be stated from Maxwell's Electromagnetic Theory. Thus, restricting the analysis to the electrokinetic steady-state response and neglecting the inner resistivity of the earthing conductors, the 3D problem is

$$\operatorname{div}(\boldsymbol{\sigma}) = 0; \quad \boldsymbol{\sigma} = -\boldsymbol{\gamma} \operatorname{grad}(V) \text{ in } E; \quad \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \text{ in } \Gamma_E; \quad V = V_\Gamma \text{ in } \Gamma; \quad V \rightarrow 0, \text{ if } |\boldsymbol{x}| \rightarrow \infty \quad (1)$$

where  $E$  is the earth,  $\boldsymbol{\gamma}$  is its conductivity tensor,  $\Gamma_E$  is the earth surface,  $\boldsymbol{n}_E$  is its normal exterior unit field and  $\Gamma$  is the electrode surface [4]. Therefore, the solution to (1) gives the potential  $V$  and the current density  $\boldsymbol{\sigma}$  at an arbitrary point  $\boldsymbol{x}$  when the electrode attains a voltage  $V_\Gamma$  (Ground Potential Rise, or GPR) with respect to remote earth. Next, for known values of  $V$  on  $\Gamma_E$  and  $\boldsymbol{\sigma}$  on  $\Gamma$ , it is straightforward to obtain the design and safety parameters of the grounding system [4].

In the last years, the authors have proposed a numerical approach based on the BEM to solve problem (1) in the case of grounding grids of large electrical installations (generally, a mesh of interconnected bare conductors with a relatively small ratio diameter-length). These numerical formulations were originally developed for grounding grids embedded in uniform soil models [4], which have been recently generalized for layered soil models [5]. From this general approach, it has been possible to derive specific numerical formulations of high accuracy [4], and to explain from a mathematical point of view the anomalous asymptotic behaviour of the classical methods proposed for grounding analysis, identifying rigorously the sources of error [3].

The starting point of these numerical approaches is the transformation of the 3D problem given by (1) onto an integral problem which unknown function is defined in the boundaries of the domain, i.e., the electrode surface  $\Gamma$ . The techniques presented in this paper can be extended to layered soil models, and our objective is to analyze the problem of the transferred potentials in grounding systems. Thus, we will consider the simplest soil model, that is, the homogeneous and isotropic soil model. Then, the conductivity tensor  $\boldsymbol{\gamma}$  is substituted by an scalar conductivity  $\gamma$ . Thus, the application of the “method of images” and Green’s Identity yields the following integral expression [4] for the potential  $V$  at an arbitrary point  $\boldsymbol{x} \in E$ , in terms of the unknown leakage current density  $\sigma(\boldsymbol{\xi})$  at any point  $\boldsymbol{\xi}$  of the electrode surface  $\Gamma \subset E$  ( $\boldsymbol{\sigma} = \sigma^t \boldsymbol{n}$  being  $\boldsymbol{n}$  the normal exterior unit field to  $\Gamma$ ):

$$V(\boldsymbol{x}) = \frac{1}{4\pi\gamma} \int \int_{\boldsymbol{\xi} \in \Gamma} k(\boldsymbol{x}, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi}) d\Gamma, \quad k(\boldsymbol{x}, \boldsymbol{\xi}) = \frac{1}{|\boldsymbol{x} - \boldsymbol{\xi}|} + \frac{1}{|\boldsymbol{x} - \boldsymbol{\xi}'|} \quad (2)$$

where  $\boldsymbol{\xi}'$  is the symmetric of  $\boldsymbol{\xi}$  with respect to the earth surface [4]. Now, since expression (2) also holds on  $\Gamma$ , where the potential is known ( $V(\boldsymbol{\chi}) = V_\Gamma, \forall \boldsymbol{\chi} \in \Gamma$ ), the leakage current density  $\sigma$  must satisfy an integral equation, which variational form is

$$\int \int_{\boldsymbol{\chi} \in \Gamma} w(\boldsymbol{\chi}) \left[ V_\Gamma - \frac{1}{4\pi\gamma} \int \int_{\boldsymbol{\xi} \in \Gamma} k(\boldsymbol{\chi}, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi}) d\Gamma \right] d\Gamma = 0, \quad (3)$$

for all members  $w(\cdot)$  of a class of test functions defined on  $\Gamma$  [4].

Next, for given sets of  $N$  trial functions  $\{N_i(\boldsymbol{\xi})\}$  defined on  $\Gamma$  and  $M$  boundary elements  $\{\Gamma^\alpha\}$ , we can discretize the leakage current density  $\sigma$  and the electrode surface  $\Gamma$ :

$$\sigma(\boldsymbol{\xi}) \approx \sigma^h(\boldsymbol{\xi}) = \sum_{i=1}^{\mathcal{N}} N_i(\boldsymbol{\xi}) \sigma_i, \quad \Gamma = \bigcup_{\alpha=1}^M \Gamma^\alpha, \quad (4)$$

and it is also possible to obtain a discretized version of expression (2). On the other hand, for a given set of  $N$  test functions  $\{w_j(\boldsymbol{\chi})\}$  defined on  $\Gamma$ , the variational form (3) can be written in terms of a linear system of equations:

$$\sum_{i=1}^{\mathcal{N}} R_{ji} \sigma_i = \nu_j \quad j = 1, \dots, \mathcal{N}, \quad (5)$$

which solution provides the values of the unknowns  $\sigma_i$  that are necessary to compute the potential  $V$  at any point  $\boldsymbol{x}$ . In the references [4, 5], the whole development of the numerical formulation based on the BEM for uniform and layered soil models can be found, as well as a fully explicit discussion about the main numerical aspects of the BEM numerical approaches. The result is a numerical technique mathematically and numerically well-founded, and highly efficient from a computational point of view. Next, we propose to solve the problem of transferred earth potentials in grounding systems by means of these BE techniques.

## 2 Analysis of Transferred Earth Potentials and Application Example

Transferred earth potentials refer to the phenomenon of the earth potential of one location appearing at another location where there is a certain earth potential. Specifically, during a fault condition the grounding grid of an electrical substation attains a voltage (GPR) which can be in the order of thousands of volts. This voltage (or a fraction of it) may be transferred out to a non-fault site by a ground conductor (such as metal pipes, rails, metallic fences, etc.), and it may produce serious hazards to the personnel, equipment and, in general, to the living beings at the non-faulted end.

Generally, there are two main cases of transferred potentials: **I**) the transference of the GPR to distant points by a conductor directly linked to the earthing system; and **II**), the transference of a fraction of the GPR to distant points by means of conductors close to the earthing grid but not directly connected to it. In both cases, the potential distribution on the earth surface will be significantly modified.

The analysis of transferred potentials in case **I**) does not imply a significantly change in the numerical approach, since the extra-conductors are formally part of the grounding grid, and they must be included in the earthing analysis as part of the grid.

Case **II**) is more difficult to deal with, because the extra-conductors attain an unknown voltage (i.e., a fraction of the GPR) due to their closeness to the grounding grid when a fault condition occurs. Then, our main objectives are to obtain this voltage and the potential distribution on the earth surface. Thus, if we call as “active grid” the electrodes which form the grounding grid (energized to the GPR) and “passive grid” the extra-conductors (which attain a fraction of the GPR) not connected to the grounding grid, the analysis of transferred potentials from an “active grid” (e.g., the left grid of Figure 1) to a “passive grid” (e.g., the right one of Figure 1) can be performed by means of a superposition of elementary states.

The two elementary states we can consider are the following: **state 1**) the “active grid” energized to 1 V and the “passive grid” to 0 V; and **state 2**) the “active grid” energized to 0 V and the “passive grid” to 1 V. With these values of unitary GPR, we can apply the BEM numerical approach presented to each elementary state, and to obtain the total electrical current by unit of voltage which flows from each grid. On the other hand, the final state is the “active grid” energized to the GPR (e.g., 10 kV) and the “passive grid” energized to a fraction  $\lambda$  of the GPR (e.g.,  $10\lambda$  kV). Due to the linear character of the problem to solve, this final state can be obtained by linear combination of the elementary states (state **1**) weighted by the GPR of the “active grid” (10 kV) and state **2**) weighted by the fraction of the GPR ( $10\lambda$  kV)). Finally, coefficient  $\lambda$  and the total current leaked to the soil ( $I_G$ ) can be easily obtained by imposing that the fault condition is produced only in the “active grid” and not in the passive grid [6].

Once the fraction  $\lambda$  of the GPR is known in the “passive grid”, it is possible to compute the potential distribution on the earth surface, and consequently, obtaining the touch and step voltages in all points of the substation site and in its surroundings. In Figure 2, it is shown the potential distribution on the earth surface if the “passive grid” is not considered, and in Figure 3 it is shown the potential distribution if this grid is considered. Note the important differences between both results and the appearance of potential gradients in (at first) non-expected areas.

### 3 Conclusions

In this paper, we have summarize the main highlights of the BEM numerical approach for grounding analysis developed by the authors in the last years. Furthermore we have presented a new approach to analyze the problem of transferred earth potentials in grounding grids. The results show its importance for the safety of these electrical installations.

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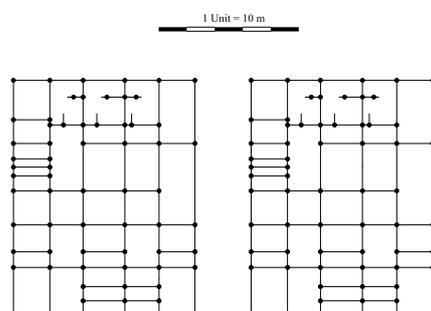


Figure 1: Grounded grids considered in this study (the left one is the “active grid”).

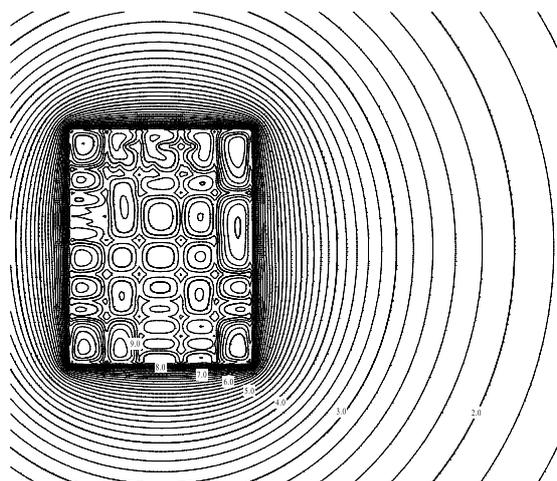


Figure 2: Potential distribution on earth surface if the passive grid is not considered in the BEM analysis.

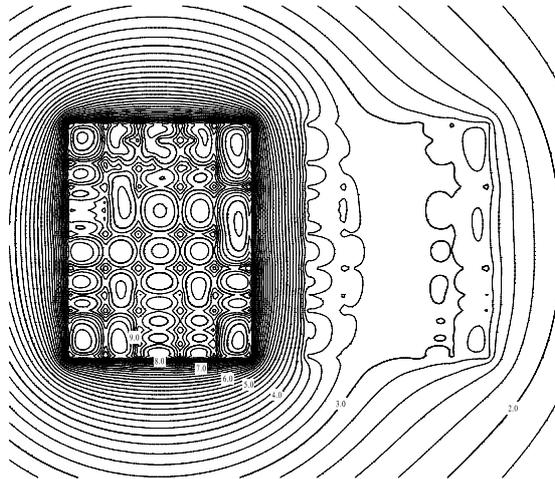


Figure 3: Potential distribution on earth surface if the passive grid is considered in the BEM analysis.