Preprint of the paper

"Numerical approaches for grounding installations in uniform and stratified soils"
I. Colominas, J. Gómez-Calviño, G. Mosqueira, F. Navarrina, M. Casteleiro (2002)
Proceedings del "JEE'02 Symposium: Numerical Models in Electrical Engineering Problems".
Toulouse, France.

Numerical analysis of grounding installations in uniform and stratified soils

I. Colominas, J. Gómez-Calviño, G. Mosqueira, F. Navarrina & M. Casteleiro
Civil Engineering School, Universidad de La Coruña, SPAIN
colominas@iccp.udc.es

Abstract

The paper summarizes the main numerical approaches based on the BEM developed by the authors in the last years for the grounding analysis of electrical installations in uniform and layered soils.

1 Introduction

The "grounding" or "earthing" system of an electrical substation comprises all interconnected grounding facilities of an specific area, being the "grounding grid" the main element of these systems. In general, it consists of a mesh of interconnected cylindrical conductors, horizontally buried and supplemented by ground rods vertically thrusted in specific places of the installation site [1].

The accurate design of a grounding system is essential to assure the safety of the persons, to protect the equipment and to avoid interruptions in the power supply. Thus, the apparent electrical resistance of the grounding system must be low enough to guarantee that fault currents dissipate mainly through the earthing electrode into the ground, while the values of electrical potentials between close points on the earth surface that can be connected by a person must be kept under certain maximum safe limits [1, 2].

Obtaining the potential distribution on the earth surface produced when a grounding grid is energized during a fault condition has been a challeging problem since the early days of the industrial use of the electricity. It is a well-known fact that the physical phenomena of fault currents dissipation into the earth can be modelled by means of Maxwell's Electromagnetic Theory. However, its application and resolution for the computing of grounding grids of large installations in practical cases present some severe difficulties. Obviously, no analytical solutions can be obtained in a real case. Moreover, the specific geometry of the grounding systems (a mesh of interconnected bare conductors in which ratio diameter/length is relatively small) precludes the use of standard numerical techniques (such as the FEM or the FDM), and obtaining sufficiently accurate results should imply unacceptable computing efforts in memory storage and CPU time [3, 4].

From a technical point of view, several methods for grounding design have been proposed. Most of them are founded on semiempirical works or on the basis of intuitive ideas, such as superposition of punctual current sources and error averaging [1]. Although these techniques represented a significant improvement in the earthing analysis area, also present some problems related with their large computational requirements, the unrealistic results obtained when segmentation of conductors is increased, and the uncertainty in the margin of error [1, 2, 5, 6].

Taking into account all these facts, the authors have proposed in the last years a general numerical formulation for grounding analysis based on the Boundary Element Method. This BEM approach has been successfully applied to the analysis and design of real grounding grids embedded in uniform [3, 4] and stratified soil models [7, 8].

In this paper we present the main highlights of this BEM numerical approach and we discuss its main characteristics, advantages and restrictions. Finally, an application example by using the real geometry of a grounding system is presented.

2 Equations of the Mathematical Model

Equations of Maxwell's Electromagnetic Theory are the starting point for studying the physical phenomena of the fault current dissipation into the ground [9]. Constraining the analysis to the obtention of the electrokinetic steady-state response (that is, the electrical part is the only considered problem) and one neglects the inner resistivity of the grounding electrode (therefore, potential can be assumed constant in every point of the surface of the electrode), the 3D problem can be stated as

$$\operatorname{div}(\boldsymbol{\sigma}) = 0, \quad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\operatorname{grad}(V) \text{ in } E\ ; \quad \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \text{ in } \Gamma_E\ ; \quad V = V_\Gamma \text{ in } \Gamma\ ; \quad V \to 0, \text{ if } |\boldsymbol{x}| \to \infty; \tag{1}$$

being E the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the electrode surface [4]. Therefore, when the earthing electrode attains a known voltage V_Γ (the so-called "Ground Potential Rise", or GPR) relative to a distant grounding point, the solution to (1) gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$. On the other hand, as well as obtaining the potential distribution, other important parameters for grounding analysis can be easily computed from (1), such as the leakage current density σ at an arbitrary point of the electrode surface, the total surge current I_Γ that flows into the ground, and the equivalent resistance of the earthing system R_{eq}:

$$\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}, \qquad I_\Gamma = \iint_\Gamma \sigma \, d\Gamma, \qquad R_{eq} = \frac{V_\Gamma}{I_\Gamma}, \tag{2}$$

where $\boldsymbol{n}$ is the normal exterior unit field to Γ. Finally, since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, it is generally used the non-restrictive normalized boundary condition $V_\Gamma = 1$ [4].

The two main problems that appear when ones tries to solve problem (1) in the grounding analysis field are the definition of the kind of soil and the geometry of the earthing electrode.

With regard to the soil, the development of soil models describing all variations of the soil conductivity around the grounding system would be obviously unaffordable, neither from the economical nor from the technical point of view. For these reasons, in most of methods for grounding analysis proposed up to this moment, the soil is considered homogeneous and isotropic. Therefore, the conductivity tensor $\boldsymbol{\gamma}$ is substituted by an "apparent" scalar conductivity γ that can be experimentally obtained [1, 2]. This hypothesis can be accepted and does not introduce significant errors if the soil is basically uniform (in the horizontal and the vertical direction) up to a distance of approximately 3 to 5 times the diagonal dimension of the grounding grid, measured from its edge. And it can also be used but with less accuracy, if the resistivity varies slightly with depth [1].

Nevertheless, since design parameters involved in grounding analysis can significantly change as soil conductivity varies, more accurate models have been proposed to take into account its variations in the vicinity of the grounding site. Hence, a more practical and quite realistic approach when conductivity is not markedly uniform with depth consists of considering the soil stratified in a number of L horizontal layers (normally, two or three), each one defined by an appropriate thickness and an apparent scalar conductivity [1]. With this soil model, if the grounding electrode is buried in layer b, the mathematical problem (1) can be written in terms of the following Neumann exterior problem

$$\begin{gathered}
\Delta V_1 = 0 \ \text{ in } \ E_1; \ \ldots\ ; \Delta V_L = 0 \ \text{ in } \ E_L; \\
V_1 = V_2, \ \text{ in } \ \Gamma_{(1,2)}; \ \ldots\ ; V_{L-1} = V_L, \ \text{ in } \ \Gamma_{(L-1,L)}; \\
\gamma_1 \frac{dV_1}{dn} = \gamma_2 \frac{dV_2}{dn} \ \text{ in } \ \Gamma_{(1,2)}; \ \ldots\ ; \gamma_{L-1} \frac{dV_{L-1}}{dn} = \gamma_L \frac{dV_L}{dn} \ \text{ in } \ \Gamma_{(L-1,L)}; \\
\frac{dV_1}{dn} = 0 \ \text{ in } \ \Gamma_E; \quad V_b = 1 \ \text{ in } \ \Gamma; \quad V_1 \longrightarrow 0, \ \ldots, V_L \longrightarrow 0, \ \text{ if } \ |\boldsymbol{x}| \to \infty;
\end{gathered} \tag{3}$$

being E_c each one of the soil layers ($c = 1, \ldots, L$), $\Gamma_{(c-1,c)}$ the interphase between two layers ($c-1$ and c), γ_c the scalar conductivity of layer c, and V_c the potential at every point of layer c [8, 10]. Obviously, the grounding analysis with a uniform soil model is a particular case of (3) with one layer [4].

Regarding the geometry of the earthing systems, in most of real electrical substations the grounding grid consists of a mesh of hundreds of interconnected cylindrical conductors horizontally buried and supplemented by ground rods, with a ratio between the diameter and the length of the electrodes very small ($\sim 10^{-3}$). Evidently, no analytical solutions exist to problem (3) with this geometry, and the use of widespread numerical techniques (i.e., finite differences or finite elements) should imply a completely out of range computing effort due to the cost of meshing (with a 3D discretization) in the vicinity of the electrodes in an effective way to impose the essential boundary condition $V_b = 1$ on their surfaces [4].

For these reasons, we work to achieve an equivalent expression to problem (3) in terms of the unknown leakage current density σ defined on the boundary Γ. In this way, a boundary element approach for this equivalent problem would only require the discretization of the grounding surface Γ, avoiding the discretization of the domains E_l [4].

On the other hand, since the substation site is levelled and regularized during its construction, the earth surface Γ_E and the interphase between soil layers $\Gamma_{(c-1,c)}$ can be assumed horizontal [4]. Now, the subsequent application of the "method of images" and the Green's Identity to (3) allows to obtain the following expression for potential $V_c(\boldsymbol{x}_c)$ at an arbitrary point $\boldsymbol{x}_c \in E_c$ $(c = 1, \ldots, L)$, in terms of the leakage current density $\sigma(\boldsymbol{\xi})$ at every point $\boldsymbol{\xi}$ of the electrode surface Γ, which is buried in the layer b:

$$V_c(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi}\in\Gamma} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \qquad \forall \boldsymbol{x}_c \in E_c; \quad (c = 1, \ldots, L); \tag{4}$$

where the integral kernel $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$ is formed by an infinite series of terms corresponding to the resultant images [10, 11]. This weakly singular kernel depends on the inverse of the distances from the point $\boldsymbol{x}_c$ to the point $\boldsymbol{\xi}$ and to all the symmetric points of $\boldsymbol{\xi}$ (its "images") with respect to the earth surface Γ_E and to the interphases $\Gamma_{(c-1,c)}$ between layers. Therefore, this kernel depends on the thickness and conductivity of each layer [11]. It is important to remark that, in a general case, the expression of the integral kernels $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$ can be very complicated, despite the generation of electrical images is a conceptually simple well-known process [12], and its evaluation in practice may require a high computing effort (explicit expressions of the integral kernels for a two-layer soil model can be found in references [8, 10, 11]).

Expression (4) is very important for the solution of the problem, because if the leakage current density σ is known, then it is possible to obtain the value of the electrical potential at an arbitrary point $\boldsymbol{x}_c$, and by using (2) it is also possible to compute the total surge current that flows from the grounding system, its equivalent resistance and most of the remaining safety and design parameters of a grounding grid [4].

The leakage current density σ can be obtained by solving a Fredholm integral equation of the first kind defined on Γ [4]. This integral equation results by imposing the boundary condition $V_b(\boldsymbol{\chi}) = 1$ $\forall \boldsymbol{\chi} \in \Gamma$ in the expression for potential (4), which also holds on the electrode surface Γ. Finally, we can obtain a variational form of this integral expression imposing that it is verified in the sense of weighted residuals, that is, the following integral identity

$$\iint_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi}) \left(1 - \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi}\in\Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi})\, d\Gamma\right) d\Gamma = 0, \tag{5}$$

which must hold for all members $w(\boldsymbol{\chi})$ of a suitable class of weight functions defined on the surface Γ [4]. Now, since the unknown of the problem (that is, the leakage current density σ) is only defined on the electrode surface Γ, a boundary element formulation seems to be the right choice to solve (5) [4].

3 Numerical Models based on the Boundary Element Method

The development of a general numerical approach based on the BEM for solving the equation (5) requires the discretization of the unknown leakage current density σ and the electrode surface Γ, in terms of a given set of N trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on Γ and a given set of M 2D boundary elements $\{\Gamma^\alpha\}$:

$$\sigma(\boldsymbol{\xi}) \approx \sigma^h(\boldsymbol{\xi}) = \sum_{i=1}^{N} N_i(\boldsymbol{\xi})\, \sigma_i^h, \qquad \Gamma = \bigcup_{\alpha=1}^{M} \Gamma^\alpha. \tag{6}$$

Likewise, it is possible to obtain a discretized version of the integral expression (4) for potential $V_c(\boldsymbol{x}_c)$

$$V_c(\boldsymbol{x}_c) = \sum_{i=1}^{N} V_{c_i}(\boldsymbol{x}_c)\, \sigma_i^h, \qquad \forall \boldsymbol{x}_c \in E_c; \qquad (c = 1, \ldots, L); \tag{7}$$

being

$$V_{c_i}(\boldsymbol{x}_c) = \sum_{\alpha=1}^{M} V_{c_i}^\alpha(\boldsymbol{x}_c), \qquad V_{c_i}^\alpha(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi}\in\Gamma^\alpha} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\, N_i(\boldsymbol{\xi})\; d\Gamma^\alpha. \tag{8}$$

Finally, for a given set of N test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ, the variational form (5) is reduced to the following system of linear equations

$$\sum_{i=1}^{N} [R_{ji}]\; \sigma_i^h = \nu_j, \qquad j = 1, \ldots, N, \tag{9}$$

being

$$R_{ji} = \sum_{\beta=1}^{M}\sum_{\alpha=1}^{M} R_{ji}^{\beta\alpha}, \qquad R_{ji}^{\beta\alpha} = \frac{1}{4\pi\gamma_b}\iint_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi}) \iint_{\boldsymbol{\xi}\in\Gamma^\alpha} k_{bb}(\boldsymbol{\chi},\boldsymbol{\xi}) N_i(\boldsymbol{\xi}) d\Gamma^\alpha d\Gamma^\beta,$$

$$\nu_j = \sum_{\beta=1}^{M} \nu_j^\beta, \qquad \nu_j^\beta = \iint_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi}) d\Gamma^\beta. \tag{10}$$

Solution of this linear system provides the values of the current densities σ_i^h $(i = 1, \ldots, N)$ leaking from the nodes of the grid. This general BEM approach can be applied in cases of grounding grid with a few number of electrodes since the statement of linear system (9) requires the discretization of the whole surface Γ of the grounding electrodes. In practical cases, with hundreds of cylindrical electrodes, this fact implies a large number of degrees of freedom. Besides, the coefficient matrix is full and the computation of terms (10) requires double integration on a 2D domain [4]. Apart from these numerical drawbacks, in the case of kernels $k_{bb}(\cdot,\cdot)$ defined by infinite series (i.e., in the two-layer soil models), the computation of coefficients $R_{ji}^{\beta\alpha}$ may require the evaluation of an extremely high number of terms of the series [8].

Consequently, the authors have developed an approximated approach to this general BEM formulation by introducing some additional hypotheses in order to reduce the computational effort [4]. Basically, this approximated approach lies in taking into account the real geometry of grounding systems in practice, by assuming that the leakage current density σ is constant around the cross section of the conductors of the grid (now, only the axial lines of the electrodes have to be discretized). Besides, it is possible to develop a highly efficient analytical integration technique to compute the coefficient terms of the linear system of equations so that they can be computed by means of explicit formulae [4].

The resulting numerical approach reminds the so-called "computer methods" for grounding analysis [6], where R_{ji} coefficients correspond to "mutual and self resistances" between segments of conductors. In fact, some particular cases of our BEM approach (e.g., a Point Collocation scheme using constant leakage current elements) can be identified with any of the very early intuitive methods that were proposed in the sixties on the basis of replacing each segment of electrode by an 'imaginary sphere'. In the case of a Galerkin type weighting with constant leakage current elements, the numerical approach can be identified with a kind of more recent methods, like the APM [5], in which each segment of electrode is substituted by 'a line of point sources over the length of the conductor' [4, 6]. Now, our BEM approach allows to explain from a mathematical point of view [4] the problems encountered by other authors with the application of these widespread methods [6], while new more efficient and accurate numerical formulations can be derived [4].

The example presented in this paper corresponds to a case of a grounding grid of a real substation computed by using two different soil models: a uniform and a two-layer soil model. Obviously, this BEM formulation can be applied to any other case with a higher number of layers. However, CPU time may increase exponentially due to the poor rate of convergence of the underlying series expansions. We have dealt this problem by two different ways: the improvement of the convergence by using new extrapolation techniques [11], and the development of a massively parallel numerical approach based on the previous ones for grounding analysis in layered soils [13]. In this way, the proposed BEM technique has been implemented in a CAD system developed by the authors for grounding substation design, extending the original capabilities of this tool that was initially developed for uniform soil models [4].

Recently, we have developed a new numerical approach from the previous one in order to tackle a common and very important engineering problem in the grounding field: the analysis of transferred potentials. In this case, the objective is to compute the potentials can be transferred to other grounded conductors in the vicinity of the earthing installation, and subsequently they could affect distant points through communication or signal circuits, neutral wires, pipes, rails, or metallic fences. This effect could produce serious safety problems that should be estimated somehow [1]. Our BEM numerical approach allows the analysis of transferred potentials in grounding installations embedded in uniform and layered soils, being possible to check the safety parameters of the grounding system and its surroundings.

4 Application Example and Conclusions

The proposed BEM approach has been successfully applied to the computational design of grounding grids of real electrical substations by considering different types of soil models [3, 4, 7, 8, 11, 13]. In this paper, we present a comparison of the grounding analysis with two different soil models by using the geometry of a real earthing system: the Barberá grounding grid, close to the city of Barcelona. Figure 1 and Table I show the plan of the earthing grid and its general characteristics. Table II summarizes the numerical results obtained for two different soil models: a uniform and a two-layer model, and Figure 2 and 3 show the potential distribution on earth surface obtained for each case.

As it is shown in this example, the results obtained by using a multiple-layer soil model can be noticeably different from those obtained by using a uniform soil model, and the computed design parameters of the grounding system do significantly vary. Therefore, it could be advisable or even mandatory to use efficient multi-layer soil formulations (such as the proposed approach) to analyze grounding systems as a general rule, in spite of the increase in the computational cost, specially when the conductivity of the soil changes markedly with depth.

At the present moment, the study of large installations with multi-layer soil models still requires an important computing effort. The new techniques and numerical formulations which are currently been developed for the authors in order to deal with this problem may represent a significant progress since the proposed multi-layer BEM formulation could be used shortly as a real-time design tool.

Acknowledgments

This work has been partially financed by projects #1FD97-0108 and #DPI2001-0556 of the "Subdirección General de Proyectos de Investigación del Ministerio de Ciencia y Tecnología", and by research fellowships of the "Secretaría General de I+D de la Xunta de Galicia" and the "Universidad de La Coruña".

References

[1] ANSI/IEEE Std.80, IEEE Guide for safety in AC Substation Grounding, New York, (2000).

[2] J.G. Sverak et al., Safe substation grounding, Part I: IEEE T. on Power Apparatus and Systems, 100, 4281-4290, (1981); Part II: IEEE T. on Power Apparatus and Systems, 101, 4006-4023, (1982)

[3] F. Navarrina, I. Colominas, M. Casteleiro, Analytical Integration Techniques for Earthing Grid Computation by BEM, Num. Met. in Eng. and Appl. Sci., 1197–1206, CIMNE, Barcelona, (1992).

[4] I. Colominas, F. Navarrina, M. Casteleiro, A boundary element numerical approach for grounding grid computation, Comput. Methods Appl. Mech. Engrg., 174, 73-90, (1999).

[5] R.J. Heppe, Computation of potential at surface above an energized grid or other electrode, allowing for non-uniform current distribution, IEEE T. Power Apparatus and Systems, 98, 1978-1988, (1979).

[6] D.L. Garrett and J.G. Pruitt, Problems encountered with the Average Potential Method of analyzing substation grounding systems, IEEE T. on Power Apparatus and Systems, 104, 3586-3596, (1985).

[7] I. Colominas, J. Gómez-Calviño, F. Navarrina, M. Casteleiro, Computer analysis of earthing systems in horizontally or vertically layered soils, Electric Power Systems Research, 59, 149-156, (2001).

[8] I. Colominas, F. Navarrina, M. Casteleiro, A numerical formulation for grounding analysis in stratified soils, IEEE Trans. on Power Delivery, (2001). [In press].

[9] E. Durand, Électrostatique, Masson, Paris, (1966).

[10] G.F. Tagg, Earth Resistances, Pitman Pub. Co., New York, (1964).

[11] J.M. Aneiros, A numerical formulation for computing design of grounding grids of electrical substations by using two-layer soil models. Research Rep., Civil Engrg. Sch., Univ. de La Coruña, (1996).

[12] O.D. Kellog, Foundations of potential theory, Springer Verlag, Berlín, (1967).

[13] I. Colominas, J. Gómez-Calviño, F. Navarrina, M. Casteleiro A massively parallel BEM numerical model for grounding analysis in layered soils, In "II ECCM: Solids, Structures and Coupled Problems in Engineering" (CDROM), Cracow University of Technology, Cracow, (2001).

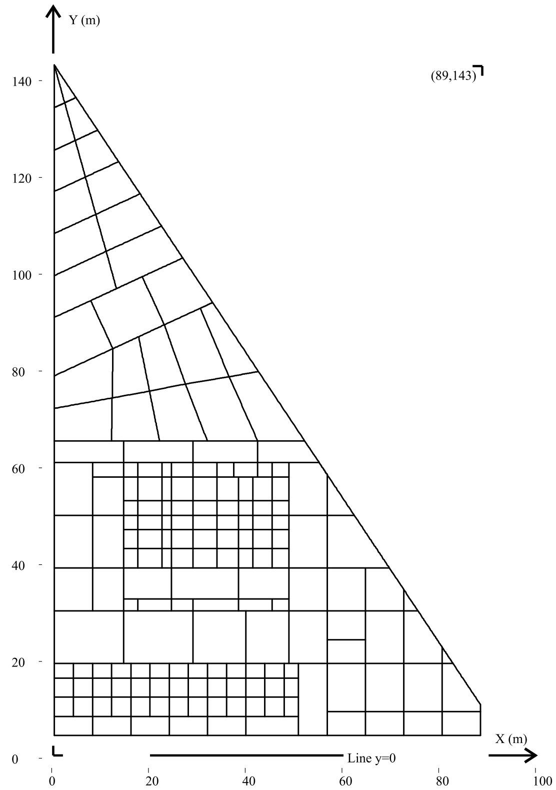


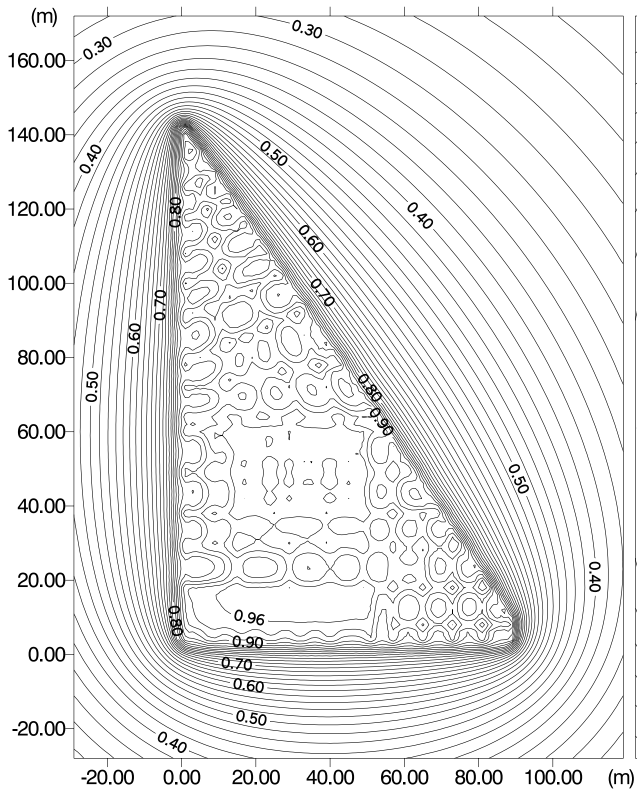
Figure 1: Plan of Barberá Grounding System

Data of the Grounding Grid	
Number of electrodes:	408
Diameter of electrodes:	12.85 mm
Depth of the grid:	0.80 m
Max. dimensions of grid:	89x143 m
GPR:	10 kV
BEM Numerical Model	
Type of approach:	Galerkin
Type of 1D element:	Linear
Number of elements:	408
Degrees of freedom:	238

Table I

Uniform Soil Model	
Earth resistivity:	60 Ωm
Total current:	31.97 kA
Equivalent resistance:	0.3128 Ω
CPU time (O2000 SG):	10 s
Two-layer Soil Model	
Upper layer resistivity:	200 Ωm
Lower layer resistivity:	60 Ωm
Thickness of upper layer:	1 m
Total current:	26.99 kA
Equivalent resistance:	0.3704 Ω
CPU time (O2000 SG):	1724 s

Table II



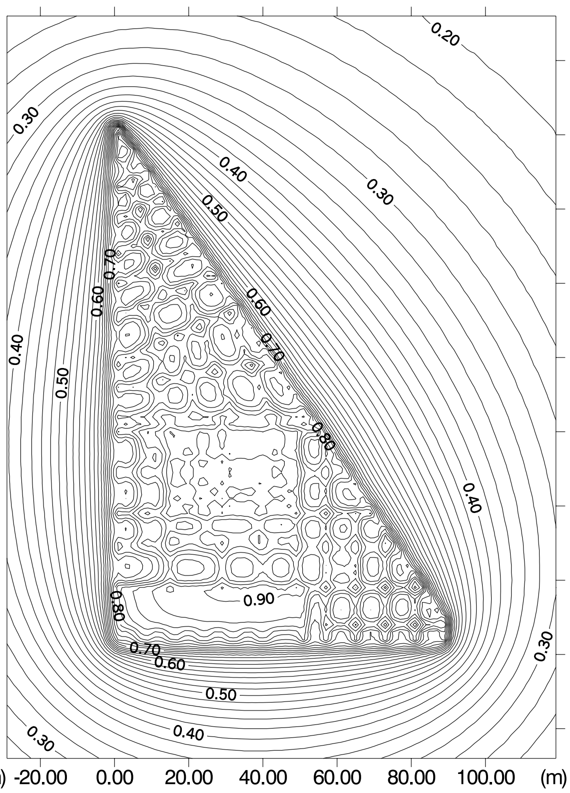


Figure 2: Potential distribution on earth surface (x 10 kV) at Barberá grounding system using the uniform soil model

Figure 3: Potential distribution on earth surface (x 10 kV) at Barberá grounding system using the two-layer soil model