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Factor substitution, long-run equilibrium, and convergence speed in the Lucas model

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A B S T R A C T

We study the effect of factor substitution on long-run equilibrium in the Lucas model with CES production. The long-run growth rate does not depend on the elasticity of substitution. However, there is a negative (positive) relationship between the elasticity of factor substitution and the convergence speed if the baseline ratio of physical capital to effective labor is below (above) its steady-state value.

1. Introduction

After de La Grandville (1989) and Klump and de La Grandville (2000) uncovered the positive link between the Elasticity of Substitution (EOS) and economic growth in the Solow model, this relationship has been examined in a variety of models including the one-sector endogenous growth model (Klump and Preissler, 2000), the Ramsey–Cass–Koopmans (RCK) model (Klump, 2001), the Diamond model (Miyagiwa and Papageorgiou, 2003), the Barro model (Gómez, 2016), and the Lucas model with leisure (Gómez, 2017). The steady-state analysis has often been combined with the study of the effect of factor substitution on the speed of convergence. This is important because it determines the relevance of transitional dynamics relative to the steady state, which may have significant consequences, e.g., for evaluating the consequences of public policies or shocks. However, the effect of factor substitution on the long-run equilibrium and the convergence speed has not been examined yet in some of the most prominent endogenous growth models; in particular, in the original Lucas (1988) model. The purpose of this paper is to fill this gap.

This paper studies the relationship between factor substitution and the long-run equilibrium in the Lucas model. As human capital accumulation depends only on effective time devoted to education, we can focus on the effect of factor substitutability in the goods production sector. The long-run growth rate does not depend on the EOS. However, we show that a higher EOS entails a lower (higher) convergence speed if the baseline ratio of physical capital to effective labor is below (above) its stationary value. This is also the finding of Klump and Preissler (2000) in the Solow model. However, it differs somewhat with the RCK model, in which the relationship is ambiguous if the baseline capital per capita is above its stationary value (Klump, 2001; Gómez, 2018).

This paper is related to Ortigueira and Santos (1997), who study the speed of convergence in the Lucas model. However, they assume a Cobb–Douglas technology in the goods-production sector, so the effect of factor substitutability cannot be studied. It is also related to Gómez (2017), who examines the effect of factor substitutability in the Lucas model. However, its focus is on the effect on long-run growth so, to ensure that the EOS does indeed affect it, leisure as raw time is introduced in the model. In this paper we consider instead the original Lucas model with inelastic labor supply. Furthermore, Gómez (2017) does not analyze the effect of factor substitution either on the stationary values or the convergence speed.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the link between factor substitutability and the long-run equilibrium. Section 4 concludes.

2. The Lucas model with CES production

The economy is populated by a large number of identical infinitely-lived agents which, for simplicity, is normalized to

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unity. At each moment of time, the agent is endowed with a unit of time which can be devoted to goods production, u, or studying, $1-u$.

2.1. Firms

Output Y is produced using physical capital K and effective labor uH, where H denotes human capital, with the CES technology

$$Y = F(K, uH) = A[\alpha K^{\psi} + (1-\alpha)(uH)^{\psi}]^{1/\psi},$$
$$A > 0, \quad 0 < \alpha < 1, \quad \psi < 1,$$

where $\sigma = 1/(1-\psi)$ is the elasticity of substitution. Denoting $y = Y/(uH)$ and $k = K/(uH)$, the production function in intensive form is

$$y = f(k) = F(k, 1) = A\left(1-\alpha+\alpha k^{\psi}\right)^{1/\psi}.$$

Profit maximization entails that

$$r = f'(k) = \alpha A^{\psi}[f(k)/k]^{1-\psi} = \alpha A k^{\psi-1}\left(1-\alpha+\alpha k^{\psi}\right)^{(1-\psi)/\psi}, \tag{1}$$

$$w = f(k) - kf'(k) = (1-\alpha)A^{\psi}f(k)^{1-\psi} = (1-\alpha)A\left(1-\alpha+\alpha k^{\psi}\right)^{(1-\psi)/\psi}, \tag{2}$$

where r is the interest rate and w is the wage rate.

2.2. Agents

The representative agent maximizes the utility derived from consumption C,

$$U = \int_0^{\infty} \frac{C^{1-\epsilon}}{1-\epsilon} e^{-\rho t} dt, \quad \epsilon > 0, \ \rho > 0, \tag{3}$$

subject to the budget constraint

$$\dot{K} = rK + wuH - C - \delta_K K, \quad \delta_K > 0, \tag{4}$$

and the constraint on human capital accumulation

$$\dot{H} = \xi(1-u)H - \delta_H H, \quad \xi > 0, \ \delta_H > 0. \tag{5}$$

Here, δ_K and δ_H are the rates of depreciation of physical and human capital, respectively. The current-value Hamiltonian of the problem is

$$\mathcal{H} = \frac{C^{1-\epsilon}}{1-\epsilon} + \lambda(rK + wuH - C - \delta_K K) + \mu\left[\xi(1-u)H - \delta_H H\right].$$

The first-order conditions for an interior solution are

$$\frac{\partial \mathcal{H}}{\partial C} = C^{-\epsilon} - \lambda = 0, \tag{6}$$

$$\frac{\partial \mathcal{H}}{\partial u} = (\lambda w - \mu\xi)H = 0, \tag{7}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial \mathcal{H}}{\partial K} = (\rho + \delta_K - r)\lambda, \tag{8}$$

$$\dot{\mu} = \rho\mu - \frac{\partial \mathcal{H}}{\partial H} = \left[\rho + \delta_H - \xi(1-u)\right]\mu - \lambda w u = (\rho + \delta_H - \xi)\mu, \tag{9}$$

where we have used (7) to get the last equality in (9), together with the transversality condition

$$\lim_{t\to\infty} e^{-\rho t}\lambda K = \lim_{t\to\infty} e^{-\rho t}\mu H = 0. \tag{10}$$

2.3. Equilibrium

Log-differentiating (6), using (1) and (8), we get the growth rate of consumption,

$$\frac{\dot{C}}{C} = \frac{1}{\epsilon}\left[f'(k) - \delta_K - \rho\right]. \tag{11}$$

Using (1) and (2), the budget constraint (4) can be expressed as

$$\frac{\dot{K}}{K} = \frac{f(k)}{k} - \frac{C}{K} - \delta_K. \tag{12}$$

Log-differentiating (7), using (8) and (9), we get

$$\frac{\dot{w}}{w} = \frac{\dot{\mu}}{\mu} - \frac{\dot{\lambda}}{\lambda} = f'(k) - \delta_K - \xi + \delta_H. \tag{13}$$

Log-differentiating (2) we obtain that

$$\frac{\dot{w}}{w} = (1-\psi)\frac{f'(k)}{f(k)}\dot{k}. \tag{14}$$

The dynamics of the economy in terms of the variables $k = K/(uH)$, $q = C/K$ and u —which are constant at the balanced growth path— is driven by the dynamic system

$$\frac{\dot{k}}{k} = \frac{f(k)}{(1-\psi)kf'(k)}\left[f'(k) - \delta_K - \xi + \delta_H\right], \tag{15}$$

$$\frac{\dot{q}}{q} = \frac{\dot{C}}{C} - \frac{\dot{K}}{K} = \frac{1}{\epsilon}\left[f'(k) - \delta_K - \rho\right] - \left[\frac{f(k)}{k} - q - \delta_K\right], \tag{16}$$

$$\frac{\dot{u}}{u} = \frac{f(k)}{k} - q - \delta_K - \frac{f(k)}{(1-\psi)kf'(k)}\left[f'(k) - \delta_K - \xi + \delta_H\right] - \xi(1-u) + \delta_H. \tag{17}$$

Eq. (15) results from (13) and (14). Eq. (16) is obtained from (11) and (12). Finally, (17) is obtained from $\dot{u}/u = \dot{K}/K - \dot{H}/H - \dot{k}/k$, using (12), (5) and (15).

2.4. Steady state

Let $\bar{\gamma}$ be the (common) steady-state growth rate of income, consumption, physical capital and human capital. In the steady state we have that

$$\bar{\gamma} = \frac{f(\bar{k})}{\bar{k}} - \bar{q} - \delta_K, \tag{18}$$

$$\bar{\gamma} = \xi(1-\bar{u}) - \delta_H, \tag{19}$$

$$\bar{\gamma} = \frac{1}{\epsilon}\left[f'(\bar{k}) - \delta_K - \rho\right], \tag{20}$$

$$\bar{\gamma} = \frac{1}{\epsilon}(\xi - \delta_H - \rho). \tag{21}$$

Eq. (18), (19) and (20) result from (12), (5) and (11), respectively. Eq. (21) results from equating (15) to zero, which entails that $f'(\bar{k}) = \xi - \delta_H + \delta_K$, and substituting this result in (11). Thus, the steady-state values of u, q and k are

$$\bar{u} = 1 - \frac{\bar{\gamma} + \delta_H}{\xi}, \tag{22}$$

$$\bar{q} = \left(\frac{\xi - \delta_H + \delta_K}{\alpha A^{\psi}}\right)^{1/(1-\psi)} - \bar{\gamma} - \delta_K, \tag{23}$$

$$\bar{k} = (1-\alpha)^{1/\psi}\left[\left(\frac{\xi - \delta_H + \delta_K}{\alpha A}\right)^{\psi/(1-\psi)} - \alpha\right]^{-1/\psi}. \tag{24}$$

The existence of a steady state requires that

$$\lim_{k\to 0} f'(k) > \xi - \delta_H + \delta_K > \lim_{k\to\infty} f'(k),$$

where

$$\lim_{k\to 0} f'(k) = \infty, \quad \lim_{k\to\infty} f'(k) = A\alpha^{\frac{1}{\psi}}, \quad \text{if } 0 < \psi < 1,$$
$$\lim_{k\to 0} f'(k) = A\alpha^{\frac{1}{\psi}}, \quad \lim_{k\to\infty} f'(k) = 0, \quad \text{if } \psi < 0.$$

The transversality condition (10) is equivalent to

$$-\rho + \frac{\dot{\bar{\lambda}}}{\lambda} + \frac{\dot{\bar{K}}}{K} = -\rho + \frac{\dot{\bar{\mu}}}{\mu} + \frac{\dot{\bar{H}}}{H} = \delta_K - f'(\bar{k}) + \bar{\gamma}$$
$$= \delta_H - \xi + \bar{\gamma} = -(\epsilon - 1)\bar{\gamma} - \rho < 0. \quad (25)$$

We assume that parameter values are such that there exists a balanced growth path, the transversality condition is met and long-run growth is positive:

Assumption 1. Parameter values are such that $\lim_{k\to 0} f'(k) > \xi - \delta_H + \delta_K > \lim_{k\to\infty} f'(k)$, and $\max\{(1-\epsilon)(\xi - \delta_H), 0\} < \rho < \xi - \delta_H$.

Eq. (19) entails that

$$\bar{u} = \frac{\xi - \delta_H - \bar{\gamma}}{\xi} = \frac{(\epsilon - 1)\bar{\gamma} + \rho}{\xi}.$$

We have that $1 - \bar{u} = (\delta_H + \bar{\gamma})/\xi > 0$, so the feasibility condition $\bar{u} < 1$ is satisfied if long-run growth is positive, and the condition $\bar{u} > 0$ is fulfilled if the transversality condition (25) is met. Note that $\bar{q} > 0$ because $\bar{q} = f(\bar{k})/\bar{k} - \bar{\gamma} - \delta_K > f'(\bar{k}) - \bar{\gamma} - \delta_K > 0$.

2.5. Stability

The Jacobian matrix of system (15)–(17) evaluated at the steady state is

$$\bar{J} = \begin{pmatrix} \frac{\partial \dot{k}}{\partial k}(\bar{k},\bar{q},\bar{u}) & \frac{\partial \dot{k}}{\partial q}(\bar{k},\bar{q},\bar{u}) & \frac{\partial \dot{k}}{\partial u}(\bar{k},\bar{q},\bar{u}) \\ \frac{\partial \dot{q}}{\partial k}(\bar{k},\bar{q},\bar{u}) & \frac{\partial \dot{q}}{\partial q}(\bar{k},\bar{q},\bar{u}) & \frac{\partial \dot{q}}{\partial u}(\bar{k},\bar{q},\bar{u}) \\ \frac{\partial \dot{u}}{\partial k}(\bar{k},\bar{q},\bar{u}) & \frac{\partial \dot{u}}{\partial q}(\bar{k},\bar{q},\bar{u}) & \frac{\partial \dot{u}}{\partial u}(\bar{k},\bar{q},\bar{u}) \end{pmatrix}$$
$$= \begin{pmatrix} \frac{f(\bar{k})f''(\bar{k})}{(1-\psi)f'(\bar{k})} & 0 & 0 \\ \cdot & \bar{q} & 0 \\ \cdot & \cdot & \xi\bar{u} \end{pmatrix},$$

where a dot denotes an element that is irrelevant for the subsequent analysis.

Given that the Jacobian matrix is triangular, its eigenvalues are its diagonal elements. The first diagonal element is negative and the other two are positive, so the steady state is locally saddle-path stable. The asymptotic convergence speed is the absolute value of the negative eigenvalue; i.e.,

$$\bar{\lambda} = -\frac{f(\bar{k})f''(\bar{k})}{(1-\psi)f'(\bar{k})} = \frac{f(\bar{k})}{\bar{k}} - f'(\bar{k}), \quad (26)$$

where $f'(\bar{k}) = \xi - \delta_H + \delta_K$. Here we have used that differentiating (1) we get that

$$f''(k) = (1-\psi)\alpha A^{\psi}\left[\frac{f(k)}{k}\right]^{-\psi}\left[\frac{f'(k)k - f(k)}{k^2}\right]$$
$$= (1-\psi)f'(k)\left[\frac{f'(k)k - f(k)}{kf(k)}\right].$$

3. Factor substitution and long-run equilibrium

3.1. Normalized CES production function

The influential works of de La Grandville (1989) and Klump and de La Grandville (2000) have revealed the importance of normalization to perform a meaningful analysis of the effect of factor substitution. Normalization is to consider a specific family of CES functions that are tangent at the same baseline point and differ uniquely in the EOS, σ. For given baseline values of k_0, $y_0 = f(k_0, \sigma)$, and the marginal rate of substitution

$$m_0 = \frac{f(k_0,\sigma) - k_0\frac{\partial f}{\partial k}(k_0,\sigma)}{\frac{\partial f}{\partial k}(k_0,\sigma)},$$

the normalized CES production function in intensive form is (Klump and de La Grandville, 2000):

$$y = f(k,\sigma) = A(\sigma)\left[1 - \alpha(\sigma) + \alpha(\sigma)k^{\psi}\right]^{1/\psi}, \quad (27)$$

where the productivity and distribution parameters are

$$A(\sigma) = y_0\left(\frac{k_0^{1-\psi} + m_0}{k_0 + m_0}\right)^{1/\psi}, \quad (28)$$

$$\alpha(\sigma) = \frac{k_0^{1-\psi}}{k_0^{1-\psi} + m_0}. \quad (29)$$

The capital income share, π, is

$$\pi(k,\sigma) = \frac{k}{f(k,\sigma)}\frac{\partial f}{\partial k}(k,\sigma) = \frac{k^{\psi}k_0^{1-\psi}}{k^{\psi}k_0^{1-\psi} + m_0},$$

and so, at the baseline value k_0 we have

$$\pi_0 = \pi(k_0,\sigma) = \frac{k_0}{k_0 + m_0}.$$

Klump and de La Grandville (2000) show that the normalized CES function is increasing in the EOS:

$$\frac{\partial f}{\partial \sigma}(k,\sigma) = \frac{1}{\sigma^2}\frac{1}{\psi^2}f(k,\sigma)\Phi_{\sigma}(k,\sigma) > 0,$$

where, following Irmen and Klump (2009), the term

$$\Phi_{\sigma}(k,\sigma) = -\left\{\pi(k,\sigma)\ln\left[\frac{\pi_0}{\pi(k,\sigma)}\right] + [1-\pi(k,\sigma)]\ln\left[\frac{1-\pi_0}{1-\pi(k,\sigma)}\right]\right\} > 0,$$

is the efficiency effect. Furthermore, Klump and de La Grandville (2000) show that

$$\frac{\partial \pi}{\partial k}(k,\sigma) = \frac{\psi}{k}\pi(k,\sigma)[1-\pi(k,\sigma)],$$
$$\frac{\partial \pi}{\partial \sigma}(k,\sigma) = \frac{1}{\sigma^2}\pi(k,\sigma)[1-\pi(k,\sigma)]\ln\left(\frac{k}{k_0}\right)$$
$$= \frac{\pi(k,\sigma)[1-\pi(k,\sigma)]}{(\sigma-1)^2}\Phi_k(k,\sigma),$$

where, following (Irmen and Klump, 2009), the term

$$\Phi_k(k,\sigma) = \psi^2\ln\left(\frac{k}{k_0}\right)\begin{cases} > 0, & \text{if } k > k_0, \\ < 0, & \text{if } k < k_0, \end{cases}$$

is the distribution effect.

3.2. Factor substitution and the steady state

Proceeding as in Xue and Yip (2012), differentiating the steady-state condition

$$\frac{\partial f}{\partial k}(\bar{k}(\sigma),\sigma) = \frac{f(\bar{k}(\sigma),\sigma)}{\bar{k}(\sigma)}\pi(\bar{k}(\sigma),\sigma) = \xi - \delta_H + \delta_K,$$

we get that

$$\frac{d\bar{k}}{d\sigma}(\sigma) = \frac{\bar{k}(\sigma)}{\sigma\psi^2}\left[\frac{\Phi_{\sigma}(\bar{k}(\sigma),\sigma)}{1-\pi(\bar{k}(\sigma),\sigma)} + \Phi_k(\bar{k}(\sigma),\sigma)\right].$$

When $k_0 < \bar{k}$, so the efficiency and the distribution effects are both positive, a higher elasticity of substitution leads to a higher steady-state ratio of physical capital to effective labor, and also of income given that

$$\frac{d\bar{y}}{d\sigma}(\sigma) = \frac{\partial f}{\partial \sigma}\left(\bar{k}(\sigma), \sigma\right) + \frac{\partial f}{\partial k}\left(\bar{k}(\sigma), \sigma\right)\frac{d\bar{k}}{d\sigma}(\sigma) > 0.$$

Using (1) we have that

$$\frac{f(\bar{k}(\sigma), \sigma)}{\bar{k}(\sigma)} = \left[\frac{f'(\bar{k}(\sigma), \sigma)}{\alpha A^{\psi}}\right]^{1/(1-\psi)} = \left(\frac{\bar{r}}{\alpha A^{\psi}}\right)^{1/(1-\psi)},$$

where $\bar{r} = \xi - \delta_H + \delta_K$. Substituting (28) and (29), after simplification, we get

$$\frac{f(\bar{k}(\sigma), \sigma)}{\bar{k}(\sigma)} = \frac{y_0}{k_0}\left[\frac{(k_0 + m_0)\bar{r}}{y_0}\right]^{1/(1-\psi)} = \frac{y_0}{k_0}\left(\frac{\bar{r}}{r_0}\right)^{\sigma}, \quad (30)$$

where $r_0 = f'(k_0) = y_0/(k_0 + m_0)$. Differentiating the former expression we get

$$\frac{d}{d\sigma}\left[\frac{f(\bar{k}(\sigma), \sigma)}{\bar{k}(\sigma)}\right] = \frac{y_0}{k_0}\left(\frac{\bar{r}}{r_0}\right)^{\sigma}\ln\left(\frac{\bar{r}}{r_0}\right). \quad (31)$$

Differentiating (23), using (31), we obtain that

$$\frac{d\bar{q}}{d\sigma}(\sigma) = \frac{y_0}{k_0}\left(\frac{\bar{r}}{r_0}\right)^{\sigma}\ln\left(\frac{\bar{r}}{r_0}\right),$$

and

$$\frac{d(\overline{c/y})}{d\sigma}(\sigma) = -(\bar{\gamma} + \delta_K)\frac{d}{d\sigma}\left[\frac{\bar{k}(\sigma)}{f(\bar{k}(\sigma), \sigma)}\right] = (\bar{\gamma} + \delta_K)\frac{k_0}{y_0}\left(\frac{r_0}{\bar{r}}\right)^{\sigma}\ln\left(\frac{\bar{r}}{r_0}\right).$$

Given that $\partial f/\partial k$ is decreasing with respect to k, it is immediate that

$$\text{sign}\,\frac{d\bar{q}}{d\sigma}(\sigma) = \text{sign}\,\frac{d(\overline{c/y})}{d\sigma}(\sigma) = \text{sign}(\bar{r} - r_0) = -\,\text{sign}(\bar{k}(\sigma) - k_0).$$

Eqs. (21) and (22) entail that the long-run growth rate and labor supply do not depend on the elasticity of substitution,

$$\frac{d\bar{u}}{d\sigma}(\sigma) = \frac{d\bar{\gamma}}{d\sigma}(\sigma) = 0.$$

In summary, we can state the following result:

Proposition 1. *In the Lucas model with CES technology in the goods-production sector,*

(i) the long-run growth rate and the shares of time devoted to goods production and education do not depend on the EOS,
(ii) if the baseline ratio of physical capital to effective labor is below its steady-state value, a higher EOS entails higher steady-state ratios of physical capital and output to effective labor,
(iii) if the baseline ratio of physical capital to effective labor is below (above) its steady-state value, a higher EOS entails lower (higher) ratios of consumption to output and to physical capital.

3.3. Factor substitution and convergence speed

Substituting (28) and (29) into (26), using (30), we have that

$$\bar{\lambda}(\sigma) = \frac{y_0}{k_0}\left(\frac{\bar{r}}{r_0}\right)^{\sigma} - \bar{r}.$$

Differentiating with respect to σ, we have that

$$\frac{d\bar{\lambda}}{d\sigma}(\sigma) = \frac{y_0}{k_0}\left(\frac{\bar{r}}{r_0}\right)^{\sigma}\ln\left(\frac{\bar{r}}{r_0}\right) = \left[\bar{\lambda}(\sigma) + \bar{r}\right]\ln\left(\frac{\bar{r}}{r_0}\right),$$

and, therefore,

$$\text{sign}\,\frac{d\bar{\lambda}}{d\sigma}(\sigma) = \text{sign}(\bar{r} - r_0) = -\,\text{sign}(\bar{k}(\sigma) - k_0).$$

Therefore, we can state the following result.

Proposition 2. *If the baseline ratio of physical capital to effective labor is below (above) its steady-state value, a higher EOS entails a lower (higher) speed of convergence in the Lucas model with CES technology in the goods-production sector.*

4. Conclusions

This paper has analyzed the effect of factor substitution in the Lucas (1988) model. The long-run growth rate does not depend on the elasticity of substitution. However, if the baseline ratio of physical capital to effective labor is below its stationary value, the more substitutable are factor inputs the lower is the speed of convergence. This result also holds in the Solow and the RCK models, so this paper contributes to establish a robust theoretical link between the elasticity of substitution and the convergence speed. Thus, it would be of interest to test empirically this link. This result may also have implications for understanding real-world growth experiences and for the effect of economic policy. One such implication is that capital-poor economies would experience the slower convergence the more substitutable are factor inputs. In this model the elasticity of substitution is a parameter but, within the context of an endogenous elasticity of substitution (e.g., Gómez, 2020, 2021), it suggests that such economies could benefit from policies that at least temporarily reduce such elasticity. This is an issue that deserves more research. Other interesting extension would be to study the effect of factor substitution in other prominent endogenous growth models as, e.g., the Romer (1990) model. This will be the subject of future research.

Data availability

No data was used for the research described in the article.

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