OPTIMUM DESIGN OF CABLE-STAYED BRIDGES CONSIDERING CABLE FAILURE

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ABSTRACT
A methodology to obtain the minimum weight of cables in cable-stayed bridges when a cable fails has been developed. To this end, a multi-model strategy is proposed that takes into account design constraints in both the intact and damaged models. The dynamic effect of the cable breakage is considered by the application of impact loads at the tower and deck anchorages. The methodology is applied to the Queensferry Crossing Bridge, a multi-span cable-stayed bridge with cross stay cables in the central section of each main span. The number of cables, anchorage position on the deck, cable areas and prestressing forces are considered as design variables into the optimization process simultaneously. The fail-safe optimum design results in a different cable layout than the optimized design of the intact structure, with minimum volume increase.
Keywords: cable-stayed bridge, optimum design, fail-safe, cable rupture, cable breakage, cable system, cable arrangement, cable layout.

1 INTRODUCTION

Optimization techniques applied to cable-stayed bridges have gained prominence in the research community. While several papers focus on optimizing the shape or thicknesses of the deck and cable areas [1]–[8], other researchers have concentrated their efforts on minimizing the weight and arrangement of the cable system. The reason is that a reduction in the steel volume of the cable system can lead to considerable savings, since the cable system represents approximately 10% of the total cost of the bridge, as presented in Sun et al. [9]. In this sense, the determination of the optimum cable forces distribution has been thoroughly studied [10]–[13]. Among these works, Baldomir et al. [12] obtained the cable areas for a long span bridge by minimizing the cables volume through a gradient-based optimization algorithm. Then, Baldomir et al. [14] considered a multi-model optimization technique to minimize the cable weight with crossing cables and fixed anchor positions. Cid et al. [15] proposed a methodology to define the optimum cable system in multi-span cable-stayed bridges, allowing crossed cables in the main spans, different number of cables at each side of the towers and different cable areas. Martins et al. [16] presented a comprehensive summary of the state-of-the-art through an extensive literature survey, with 90 articles studied for a detailed review.

The previous approaches were applied to the intact configuration of the bridge. Therefore, a weakness of those optimum designs is that they do not contemplate a cable break scenario. As several dramatic events have occurred throughout history associated with this kind of accidents, it seems appropriate to propose a strategy to optimize the cable system that takes into account a cable failure. Cross sectional areas, cable anchor positions and post-tensioning cable forces will be the design variables of the problem. A MATLAB code [17] has been programmed and combined the structural analysis software ABAQUS [18] in order to solve the proposed fail-safe optimization problem.

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2 OPTIMIZATION STRATEGY

2.1 Structural analysis considering a cable loss

The existing codes and regulations in civil engineering field establish that bridges must resist a single-cable breakage. The structural response derived from this accidental event can be contemplated by non-linear dynamic analyses or by a quasi-static approach. The quasi-static approach assumes that two impact forces must be applied in the opposite direction of the broken cable. These static forces correspond to the cable tensile strength multiplied by an amplification factor, denoted as DAF, which can be understood as the ratio between the dynamic response and the static response [19]. Eurocodes and the PTI recommendations establish a value the DAF between 1.5 and 2.0. Fig. 1 shows the impact load due to the loss of a cable.

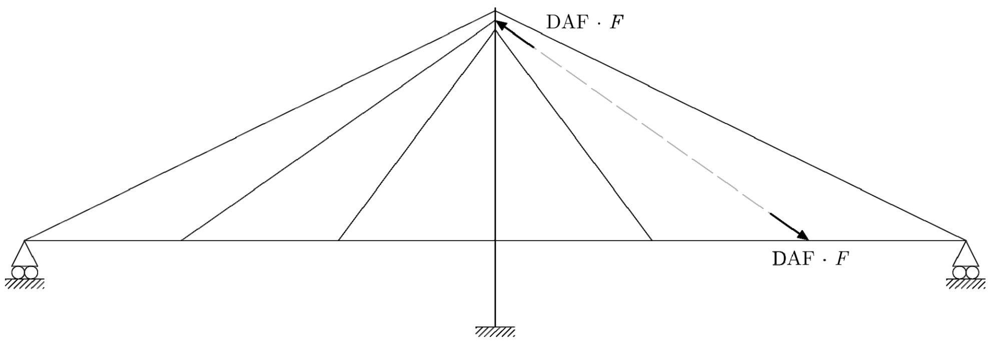

Figure 1: Impact load due to the loss of a cable.

The load combination used for the cable loss can be found in the PTI recommendations [20] and it is presented in eqn (1), being *DC* and *DW* the dead load of structural and non-structural components, respectively, *LL* the live load, and *CLDF* the cable loss dynamic forces.

$$1.1 \cdot DC + 1.35 \cdot DW + 0.75 \cdot LL + 1.1 \cdot CLDF. \quad (1)$$

Thus, the proposed methodology will integrate the cable loss effect in the fail-safe optimization approach presented below.

2.2 Formulation of the optimization problem

The objective is to minimize the total steel volume of material of the cable system when a cable breaks. The design variables are the anchorage position on the deck (x_k^P), the cross-sectional area of the cables (x_k^A), the number of cables and their prestressing forces (x_k^F).

Since there could be as many damaged bridge configurations as there are cables, a multi-model optimization should be considered. This idea was proposed by Baldomir et al. [21] to achieve safe designs with minimum weight while fulfilling all limit-state requirements for the intact model and a set of partial collapses. It is therefore an optimization problem with a high computational cost since the design constraints must be evaluated by carrying out structural analyses in the intact and all possible damaged configurations.

A general formulation of the optimization problem is presented as follows:

$$\min V = 2 \cdot \sum_{k=1}^{N} \left(x_k^A \cdot L_k(x_k^P) \right) \tag{2a}$$

s.t.

$$g_j^{M_i}(x_k^P, x_k^A, x_k^F) \leqslant 0 \quad k = 1, \dots, N \quad j = 1, \dots, m^{M_i} \quad i = 0, \dots, D \tag{2b}$$

where V is the total volume of steel in the cable system; L_k is the total length of the cable k; and N is the number of cables. The whole set of design constraints is represented by the expressions $g_j^{Mi,l} \leq 0$, where M_i refers to the structural configuration i. If $i = 0$ the constraint refers to the intact model, while if it is non-zero, it refers to a damaged model.

As can be seen, the number of cables has not been explicitly considered as a design variable. In fact, it should be considered as a binary variable, whose value would be 0 if the cable did not exist and 1 if it did. Such an approach would entail the use of optimization algorithms with discrete variables that have proven to be inefficient when the number of variables is high. In this research, the existence or non-existence of the cable is considered as a function of its area, i.e., by means of a continuous variable. A lower limit of the cable area is defined with a very low value and if the variable tends to this value, the cable will be considered not to exist. By doing so, all the design variables of the optimization problem are continuous and a gradient-based optimization algorithm could be used to solve the problem presented in eqn (2).

The optimization code was implemented in MATLAB. After defining the mechanical properties, geometry, mesh of FEM, design variables and optimization parameters, the Python Script generates D+1 finite element models of the bridge. The sequential quadratic programming (SQP) algorithm implemented in the MATLAB function *fmincon* was used as optimizer. The Python Script is externally run through Abaqus at each iteration of the optimization process to obtain the structural responses and evaluate the design constraints. The intact model has to be analyzed first in order to obtain the internal forces of the cables. Then, the impact forces are applied to the damaged models, which are launched in parallel to evaluate their design constraints. The process is repeated until the objective function converges and the design constraints are satisfied.

3 APPLICATION EXAMPLE

3.1 Bridge description

The previous optimization strategy will be applied to the Queensferry Crossing Bridge, also known as Forth Replacement Crossing. A view of the bridge is shown in Fig. 2. The cable-stayed bridge has three towers around 200 m high, with a deck 1,950 m long, divided into two main spans of 650 m and two lateral spans of 325 m, being the latter composed of a back span of 221 m and one approach viaduct of 104 m. A scheme of the bridge is presented in Fig. 3.

The number of cables in the bridge is N = 144. As the FEM used is 2D, the cables of the model represent the combined capacity of the two cable planes of the real bridge. The mechanical properties of the deck and towers are summarized in Cid et al. [15]. The permanent loads applied to the FE model are the structural deck weight (DC = 146 kN/m), the weight of the non-structural elements of the deck (DW = 54 kN/m) and the cable prestressing forces (PS). It also was considered a live load on spans 1 and 3 ($LL1$ = 102.5 kN/m) and their symmetrical case on the spans 2 and 4 ($LL2$). Finally, cable loss dynamic forces ($CLDF$) are applied to damaged models. Load combinations considered in the optimization problem are shown in eqn (3).

Figure 2: Queensferry Crossing Bridge.

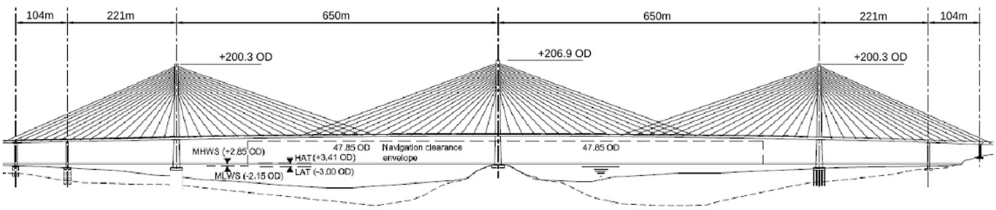

Figure 3: Scheme of the Queensferry Crossing Bridge [22].

<u>Intact model:</u>

Load Case 0 ($l = 0$) SLS: $1.00 \cdot DC + 1.00 \cdot DW + 1.00 \cdot PS$ (3a)

ULS: $1.25 \cdot DC + 1.50 \cdot DW + 1.25 \cdot PS$ (3b)

Load Case 1 ($l = 1$) SLS: $1.00 \cdot DC + 1.00 \cdot DW + 1.00 \cdot LL1 + 1.00 \cdot PS$ (3c)

ULS: $1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL1 + 1.25 \cdot PS$ (3d)

<u>Damaged models:</u>

Load Case 1 ($l = 1$) EELS: $1.1 \cdot DC + 1.35 \cdot DW + 0.75 \cdot LL1 + 1.1 \cdot CLDF$ (3e)

Load Case 2 ($l = 2$) EELS: $1.1 \cdot DC + 1.35 \cdot DW + 0.75 \cdot LL2 + 1.1 \cdot CLDF$ (3f)

The volume of the cable-system in the real bridge corresponds to 759 m^3. In a previous work [15], the optimization of the intact model was performed, resulting in a volume of 634.15 m^3, which corresponds to a reduction of 16.45%.

3.2 Fail-safe optimization of the Queensferry Crossing Bridge

The formulation of the fail-safe optimization problem is presented in eqn (4):

$$\min V = 2 \cdot \sum_{k=1}^{N} \left(x_k^A \cdot L_k(x_k^P) \right) \quad (4a)$$

s.t

$$|w_j^{M_{0,l}}| \leq w_{\max}^{M_{0,l}} \qquad j = 1, \ldots, N_D \qquad l = 0,1 \quad (4b)$$

$$|u_{\text{tower},p}^{M_{0,l}}| \leq u_{\max}^{M_{0,l}} \qquad p = 1,2,3 \qquad l = 0,1 \quad (4c)$$

$$0 < \sigma_{\text{cable},k}^{M_{i,l}} \leq \sigma_{\text{cable,max}} \qquad k = 1, \ldots, N \;/\; k \neq i \qquad l = 1,2 \qquad i = 0, \ldots, D \quad (4d)$$

$$\sigma_{C,\text{deck}} \leq \sigma_{\text{top,deck},j}^{M_{i,l}} \leq \sigma_{T,\text{deck}} \qquad j = 1, \ldots, E_D \qquad l = 1,2 \qquad i = 0, \ldots, D \quad (4e)$$

$$\sigma_{C,\text{deck}} \leq \sigma_{\text{bottom,deck},j}^{M_{i},l} \leq \sigma_{T,\text{deck}} \qquad j = 1, \ldots, E_D \qquad l = 1,2 \qquad i = 0, \ldots, D \quad (4f)$$

$$|x_{k+1} - x_k| \geq d_{\min} \qquad k = 1, \ldots, N_P - 1 \quad (4g)$$

where:

$w_j^{M0,l}$ (m)		Deflection of the node j in the deck
$w_{\max}^{M0,l}$ (m)	$w_{\max}^{M0,0} = L/7500$ $w_{\max}^{M0,1} = L/500$	Maximum allowable deflection in the deck (side span: L = 325 m, Main span: L = 650 m)
N_D	= 244	Total number of deck nodes in which the displacements are checked
$\sigma^{Mi,l}_{\text{cable},k}$ (MPa)		Tensile stress in the cable k
$\sigma_{\text{cable max}}$ (MPa)	= 837	Maximum allowable tensile stress in cables
$\sigma^{Mi,l}_{\text{bottom,deck},j}$		Normal stress in the bottom fiber of the deck in jth element
$\sigma^{Mi,l}_{\text{top,deck},j}$		Normal stress in the top fiber of the deck in jth element
$\sigma_{C,\text{ deck}}$ (MPa)	= –200	Minimum allowable compression stress in deck
$\sigma_{T,\text{ deck}}$ (MPa)	= 300	Maximum allowable tensile stress in deck
E_D	= 243	Number of elements of the deck in which stresses are checked
$d_{\min}$ (m)	= 5	Minimum distance between the anchor position of two consecutive cables

According to the design regulations, the vertical displacement on the deck must be evaluated in SLS, i.e., only in the intact model for the Load Cases 0 and 1 (eqns (4b) and (4c)). On the other hand, cables and deck stresses are evaluated in ULS for the intact model and in EELS for damaged configurations (eqns (4d), (4e), (4f)). It is important to note that for the intact model, it is not necessary to check the stress constraints in the Load Case 0, as these values are always more unfavourable in the Load Case 1. In addition, a minimum distance between two consecutive cables was imposed in order to reduce the chances of a vehicle hitting more than one cable (eqn (4g)). A total number of 21,779 design constraints were considered in the fail-safe optimization problem.

3.3 Numerical results

The layout of the initial design is presented in Fig. 4. The cables are equally spaced along each span and a cable area of 0.03 m^2 has been considered, with different prestressing forces.

Fig. 5 shows the final cable arrangement obtained and Fig. 6 shows the evolution of the objective function.

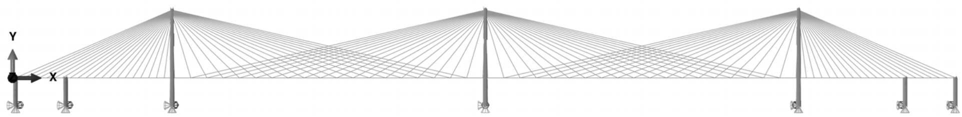

Figure 4: Cable arrangement of the initial design.

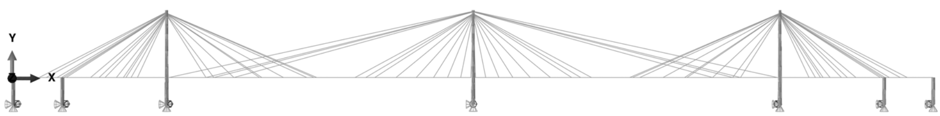

Figure 5: Optimum cable area distribution of the fail-safe optimization.

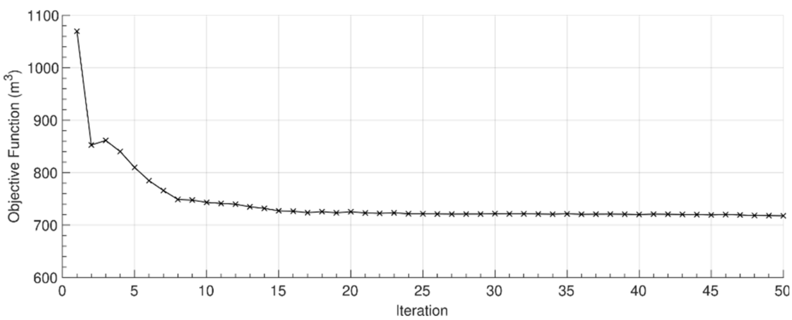

Figure 6: Evolution of the objective function in the fail-safe optimization.

The final steel volume of the cable system corresponds to 719.76 m^3, this is, a penalty in steel volume of 13.50% with respect to the optimization of the intact model but this value is still lower than the volume of material of the real bridge. This leads to eight additional cables in the side span and six additional cables in the main span. The values of the design variables appears in Fig. 7. Grey color corresponds to the cables anchored at tower 1, whereas green color is associated with cables anchored at tower 2.

The area of the cables in the lateral spans can be divided into two distinct groups. Cables anchored between deck coordinates (50–120) m and cables located between deck coordinate 150 m and the first tower. In the first group, the cables have significant areas with values between 0.04 m^2 and 0.06 m^2. In addition, most of these cables are located around the intermediate pile of the lateral span. In the second group, the area of the cables is lower, with the aim of providing vertical support to the deck. In the main spans, there is a group of four cables of great length and similar areas around 0.04 m^2 which are anchored in the central tower and in the deck near the side towers. These cables control the horizontal displacement of the central tower tip. The remaining cables are arranged in a fan-shaped distribution as in conventional cable-stayed bridges. The areas of these cables grow from the towers towards the center of the span with areas between 0.02 m^2 and 0.055 m^2.

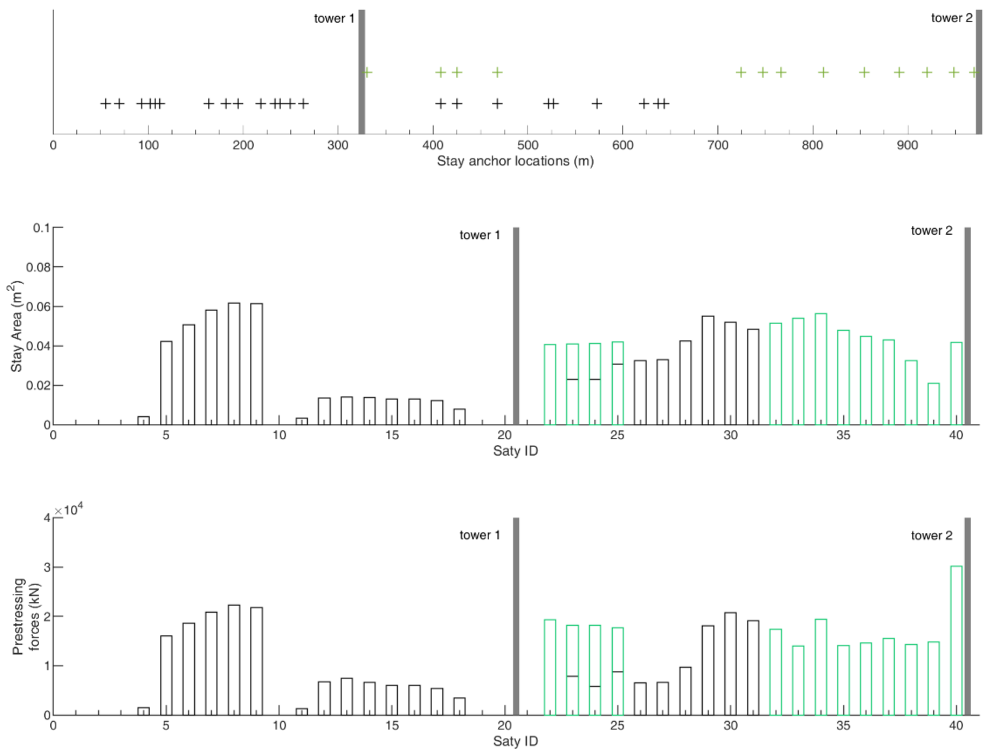

Figure 7: Values of design variables at the optimum design (only half a bridge is shown).

Regarding the active constraints, displacement constraints in the deck are active in the intact model for the Load Case 0 and 1, while displacement constraints of the tower head are active only in Load Case 1, as can be seen in Figs 8 and 9. As for the damaged models, it can be observed that most of the active stress constraints in cables occur in the vicinity of the damaged cable, in some cases activating up to eight cables simultaneously. Regarding the deck stress constraints in damaged models, they occur when cables break in the main span.

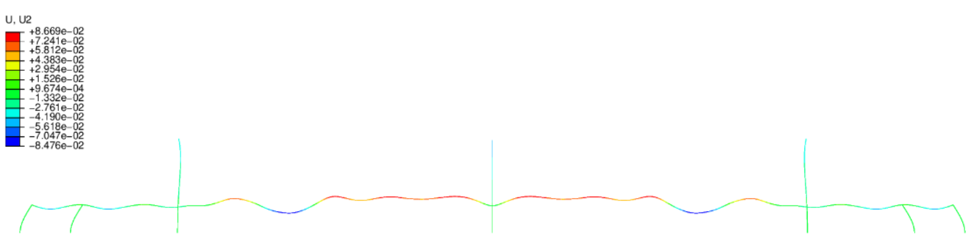

Figure 8: Displacements in the intact model (M_0) for the Load Case 0 (scale factor = 200).

(a)

(b)

Figure 9: Displacements in the intact model (M_0) for the Load Case 1. (a) Active horizontal displacement of the central tower head (scale factor = 40); and (b) Active vertical displacement of the deck (scale factor = 40).

4 CONCLUSIONS

Several conclusions can be drawn from this work:

1. Cable breakage has been successfully incorporated into the optimization of cable-stayed bridges, with satisfactory results.
2. Apart from the cable removal, the dynamic effect of the rupture on the remaining structure is taken into account by the application of two impact forces at the anchorage locations of the broken cable.
3. The consideration of a cable rupture into the optimization process greatly influences the volume of the optimum cable system, reaching a penalty volume of 13.5%.
4. There are active constraints in both the intact and damaged models, demonstrating the importance of the application of fail-safe optimization strategies, leading to a minimum steel volume increase.

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