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A GENERAL NUMERICAL MODEL FOR GROUNDING ANALYSIS IN LAYERED SOILS

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Abstract

The safety of the persons, the protection of the equipment and the continuity of the power supply are the main objectives of the grounding system of a large electrical installation. For its accurate design, it is essential to determine the potential distribution on earth surface and the equivalent resistance of the system. In this paper, we present a numerical approach based on the Boundary Element method for grounding analysis in layered soils. The feasibility of this formulation is discussed with its application to a real grounding system embedded in different kinds of layered soil models.

1. Introduction

Since the early days of the industrial use of the electricity, obtaining the potential distribution in large electrical installations produced when a fault current is derived into the soil (through a single conductor or a mesh of them: a grounding grid) has been a challenging problem. And the potential distribution on earth surface is without doubt the most important parameter that it is necessary to know in order to design a safe grounding system. A "grounding" or "earthing" system comprises all interconnected grounding facilities of an specific area of an electrical installation, being the "grounding grid" (or the "grounded electrode") the main element of these systems.

In most of real electrical substations, the grounding grid consists of a mesh of interconnected cylindrical conductors, horizontally buried and supplemented by ground rods vertically thrusted in certain places of the substation site. Its main objective is to carry and dissipate electrical currents produced during fault conditions into the ground, in order to ensure that a person in the vicinity of the grounded installation is not exposed to a critical electrical shock, and to assure the power supply continuity and the integrity of the equipment. To attain these goals, the apparent electrical resistance of the grounding system must be low enough to guarantee that fault currents dissipate mainly through the grounding electrode into the soil, while electrical potential values between close points on the earth surface that can be connected by a person must be kept under certain maximum safe limits (step, touch and mesh voltages), established in most of the guides and legal procedures of grounding system design[1, 2].

Since the sixties, several methods and procedures for the grounding analysis and design of electrical substations have been proposed. These methods, generally based on the professional practice, on semi-empirical works, on experimental data obtained from scale model assays, or on intuitive ideas, represented an important improvement in the grounding analysis area. However, some problems have been reported such as large computational requirements, unrealistic results when segmentation of conductors is increased, and uncertainty in the margin of error[1, 3].

Equations governing the physical phenomena of the electrical current dissipation into a soil are well-known and can be stated from Maxwell's Electromagnetic Theory. Nevertheless, their application and resolution for the computing of grounding grids of large installations in practical cases present some difficulties. Obviously, no analytical solutions can be obtained in a real problem. Furthermore, the characteristic geometry of the grounding systems (a mesh of interconnected bare conductors with a ratio diameter/length relatively small) makes very difficult the use of numerical techniques commonly used for the resolution of boundary value problems (such as finite elements or finite differences) since the discretization of the domain (the ground) is required. In these cases, obtaining sufficiently accurate results should imply unacceptable computing efforts in memory storage and CPU time.

For these reasons, in the last years the authors have developed a numerical formulation based on the Boundary Element Method for the analysis of grounding systems embedded in uniform soil models[4, 5]. This approach has been implemented in a CAD environment for earthing systems[6] which has allowed to analyze real grounding installations in real-time in personal computers. Next, we present a generalization of the boundary element formulation for grounding grids embedded in layered soils, and its application to several practical cases by using the geometry of a real earthing system.

2. Mathematical Model of the Problem

2.1. Governing Equations of the Problem

As we have exposed in the introduction, equations which govern the dissipation of electrical current into the ground through a grounded electrode can be stated by means of Maxwell's Electromagnetic Theory. Thus, restricting the analysis to the electrokinetic steady-state response and neglecting the inner resistivity of the earthing conductors (then, potential is assumed constant in every point of the grounding electrode surface), the 3D problem can be written as

$$\begin{gathered} \operatorname{div}(\boldsymbol{\sigma}) = 0, \quad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}(V) \text{ in } E \;; \\ \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \text{ in } \Gamma_E \;; \quad V = V_\Gamma \text{ in } \Gamma \;; \\ V \to 0, \text{ if } |\boldsymbol{x}| \to \infty; \end{gathered} \tag{1}$$

where E is the earth, $\boldsymbol{\gamma}$ is its conductivity tensor, Γ_E is the earth surface, $\boldsymbol{n}_E$ is its normal exterior unit field and Γ is the electrode surface [5].

Therefore, when the grounded electrode attains a voltage V_Γ (Ground Potential Rise, or GPR) relative to a remote earth, the solution to problem (1) gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$. Then, it is possible to obtain the potential distribution on earth surface (and consequently, the step, mesh and touch voltages of the earthing system), and the total surge current and the equivalent resistance by means of the current density $\boldsymbol{\sigma}$ on Γ[1, 2]. On the other hand, since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, it will be used the normalized boundary condition $V_\Gamma = 1$ from here on.

In many of the methods and theoretical procedures proposed for grounding analysis, the soil is commonly considered homogeneous and isotropic. Then, conductivity $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ that must be experimentally obtained[1]. Obviously, this hypothesis does not introduce significant errors if the soil is essentially uniform in all directions in the vicinity of the grounding grid[1], and this model can be used with loss of accuracy if the soil resistivity changes slightly with depth. Nevertheless, since safety parameters involved in the grounding design can significantly vary if the soil electrical properties change through the substation site (for example, due to changes of the material nature, or the humidity of the soil), it seems advisable to develop more advanced models that could take into account variations of the soil conductivity in the surroundings of the electrical installation.

It is obvious that to take into account all the variations of the soil conductivity would never be affordable, neither from the economical nor from the technical point of view. For this reason, a more practical soil model proposed consists of assuming the soil stratified in a number of horizontal or vertical layers, defined by an appropriate thickness and an apparent scalar conductivity that must be experimentally obtained. In fact, it is widely accepted that two-layer soil models should be sufficient to obtain good and safe designs of grounding systems in most practical cases[1]. This paper is devoted to studying the application of numerical methods for grounding analysis in stratified soil models, and specially in cases with two different horizontal or vertical layers (figures 1 and 2).

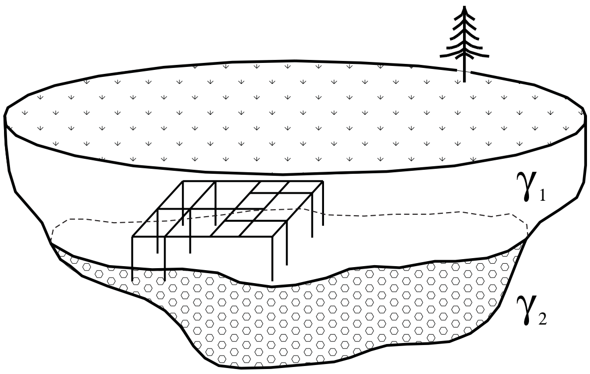

Figure 1. Scheme of a soil model with two horizontal layers.

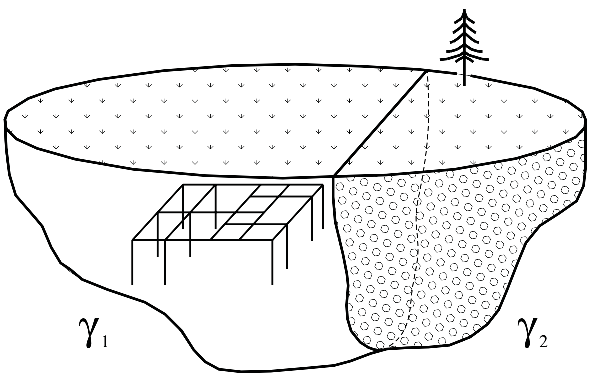

Figure 2. Scheme of a soil model with two vertical layers.

Therefore, in the hypothesis of a stratified soil model formed by C layers with different conductivities and the grounded electrode buried in the layer b, the mathematical problem (1) can be written in terms of the following Neumann exterior problem[7]

$$\begin{gathered} \operatorname{div}(\boldsymbol{\sigma}_c) = 0, \; \boldsymbol{\sigma}_c = -\gamma_c\,\mathbf{grad}(V_c) \text{ in } E_c, \; 1 \le c \le C; \\ \boldsymbol{\sigma}_1^t \boldsymbol{n}_E = 0 \text{ in } \Gamma_E, \; V_b = 1 \text{ in } \Gamma; \\ V_c \to 0 \text{ if } |\boldsymbol{x}| \to \infty, \; \boldsymbol{\sigma}_c^t \boldsymbol{n}_c = \boldsymbol{\sigma}_{c+1}^t \boldsymbol{n}_c \text{ in } \Gamma_c, \\ 1 \le c \le C-1; \end{gathered} \tag{2}$$

where E_c is each one of the soil layers, γ_c is its scalar conductivity, V_c is the potential at an arbitrary point of layer E_c, $\boldsymbol{\sigma}_c$ is its corresponding current density, Γ_c is the interface between layers E_c and E_{c+1}, and $\boldsymbol{n}_c$ is the normal field to Γ_c[7].

2.2. Integral Expression for Potential V

Most of grounding systems of real electrical substations consist of a grid of interconnected bare cylindrical conductors, horizontally buried and supplemented by rods, being the ratio between the diameter and the length of the conductors relatively small ($\sim 10^{-3}$). This apparently simple geometry implies serious difficulties in the modellization of the problem, since it seems obvious that no analytical solutions can be obtained in real cases, and the use of standard numerical techniques which are widespread used for solving boundary value problems (such as, FEM or FDM) should involve a completely out of range computing effort since it is required the discretization of the 3D domains E_c. For these reasons, we have turned our attention to other numerical techniques which require only the discretization of the boundaries of the problem. For this, first of all it is essential to derive an integral expression for potential V in terms of unknowns defined on the boundary[5].

During the construction process of the electrical installation, the surroundings of the substation site are levelled and regularized. Thus, earth surface Γ_E and interfaces Γ_c between layers can be assumed horizontal; consequently we will adopt an "horizontal layer soil model" in our mathematical model in order to catch the variations of the soil conductivity with depth (figure 1). In the case of horizontal variations of the soil conductivity near the grounding system, the simplest model we can state (a "vertical layer soil model") is to assume that the earth surface Γ_E is horizontal and that the interfaces Γ_c are perpendicular to Γ_E and parallel among them (figure 2).

Thus, taking into account these assumptions, the application of the "method of images" and the Green's Identity to problem (2) yields the following integral expression[7] for potential $V_c(\boldsymbol{x}_c)$ at an arbitrary point $\boldsymbol{x}_c \in E_c$, in terms of the unknown leakage current density $\sigma(\boldsymbol{\xi})$ ($\sigma = \boldsymbol{\sigma}^t\boldsymbol{n}$, where $\boldsymbol{n}$ is the normal exterior unit field to Γ) at any point $\boldsymbol{\xi}$ of the electrode surface $\Gamma \subset E_b$:

$$V_c(\boldsymbol{x_c}) = \frac{1}{4\pi\gamma_b}\int\int_{\boldsymbol{\xi}\in\Gamma} k_{bc}(\boldsymbol{x}_c,\boldsymbol{\xi})\,\sigma(\boldsymbol{\xi})d\Gamma,\ \forall \boldsymbol{x}_c \in E_c, \quad (3)$$

where integral kernels $k_{bc}(\boldsymbol{x}_c,\boldsymbol{\xi})$ are formed by series of infinite terms corresponding to the resultant images obtained when Neumann exterior problem (2) is transformed into a Dirichlet one[7, 9]. Depending on the type of the soil model used, these series can have an infinite or a finite number of terms. In the case of uniform model ($C = 1$), they are reduced to only two summands since there is only one image of the original grid[4, 5]:

$$k_{11}(\boldsymbol{x}_1,\boldsymbol{\xi}) = \frac{1}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,\xi_z])} + \frac{1}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,-\xi_z])}, \quad (4)$$

where $r(\boldsymbol{x}_1,[\xi_x,\xi_y,\xi_z])$ indicates the distance from $\boldsymbol{x}_1$ to $\boldsymbol{\xi} \equiv [\xi_x,\xi_y,\xi_z]$ —and to its symmetric point with respect to the earth surface Γ_E—. It is assumed that the origin of the coordinates system is on the earth surface and the z-axis is perpendicular to Γ_E.

In the case of a two-layer vertical model (figure 2), the series of the kernels are also reduced to a finite number of terms:

$$\begin{aligned} k_{11}(\boldsymbol{x}_1,\boldsymbol{\xi}) &= \frac{1}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,\xi_z])} + \frac{1}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,-\xi_z])} \\ &+ \frac{\kappa}{r(\boldsymbol{x}_1,[\xi_x,-\xi_y,\xi_z])} + \frac{\kappa}{r(\boldsymbol{x}_1,[\xi_x,-\xi_y,-\xi_z])}, \\ k_{12}(\boldsymbol{x}_2,\boldsymbol{\xi}) &= \frac{1+\kappa}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,\xi_z])} + \frac{1+\kappa}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,-\xi_z])}, \end{aligned} \quad (5)$$

where $r(\boldsymbol{x},[\xi_x,\xi_y,\xi_z])$ indicates the distance from $\boldsymbol{x}$ to $\boldsymbol{\xi} \equiv [\xi_x,\xi_y,\xi_z]$ —and to its symmetric point with respect to the earth surface Γ_E, and the resultant images with respect to the vertical interface—. It is assumed that the origin of the coordinates system is on the earth surface and on the vertical interface, and the z-axis and y-axis are perpendicular to Γ_E and Γ_1 respectively. Coefficient κ is a ratio defined in terms of the layer conductivities

$$\kappa = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2}. \quad (6)$$

In the case of an horizontal two layer soil model (figure 1), the expressions of the integral kernels are given by

$$\begin{aligned} k_{11}(\boldsymbol{x}_1,\boldsymbol{\xi}) &= \sum_{i=0}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,2iH+\xi_z])} \\ &+ \sum_{i=0}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,2iH-\xi_z])} \\ &+ \sum_{i=1}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,-2iH+\xi_z])} \\ &+ \sum_{i=1}^{\infty} \frac{\kappa^i}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,-2iH-\xi_z])}; \\ k_{12}(\boldsymbol{x}_2,\boldsymbol{\xi}) &= \sum_{i=0}^{\infty} \frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,-2iH+\xi_z])} \\ &+ \sum_{i=0}^{\infty} \frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,-2iH-\xi_z])}; \\ k_{21}(\boldsymbol{x}_1,\boldsymbol{\xi}) &= \sum_{i=0}^{\infty} \frac{(1-\kappa)\kappa^i}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,-2iH+\xi_z])} \\ &+ \sum_{i=0}^{\infty} \frac{(1-\kappa)\kappa^i}{r(\boldsymbol{x}_1,[\xi_x,\xi_y,2iH-\xi_z])}; \\ k_{22}(\boldsymbol{x}_2,\boldsymbol{\xi}) &= \frac{1}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,\xi_z])} - \frac{\kappa}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,2H+\xi_z])} \\ &+ \sum_{i=0}^{\infty} \frac{(1-\kappa^2)\kappa^i}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,-2iH+\xi_z])}; \end{aligned} \quad (7)$$

In the above expressions, $r(\boldsymbol{x},[\xi_x,\xi_y,\xi_z])$ indicates the distance from $\boldsymbol{x}$ to $\boldsymbol{\xi} \equiv [\xi_x,\xi_y,\xi_z]$ —and to the symmetric points and images of $\boldsymbol{\xi}$ with respect to the earth surface Γ_E and to the interphace surface between layers—, H is the thickness of the upper layer, and κ is given by (6). It is assumed that the origin of the coordinates system is on the earth surface and the z-axis is perpendicular to Γ_E.

As we can observe in expressions (4), (5) and (7), weakly singular kernel $k_{bc}(\boldsymbol{x}_c,\boldsymbol{\xi})$ depends on the conductivity of the layers, and on the inverse of the distances from the point $\boldsymbol{x}_c$ to the point $\boldsymbol{\xi}$ and to all the images of $\boldsymbol{\xi}$ with respect to the earth surface Γ_E and to the interfaces Γ_c between layers[9, 7]. For uniform and two-layer soil models, these kernels can be expressed in the general form:

$$k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi}) = \sum_{l=0}^{l_k} k_{bc}^l(\boldsymbol{x}_c, \boldsymbol{\xi}), \quad k_{bc}^l(\boldsymbol{x}_c, \boldsymbol{\xi}) = \frac{\psi^l(\kappa)}{r(\boldsymbol{x}_c, \boldsymbol{\xi}^l(\boldsymbol{\xi}))}, \quad (8)$$

where ψ^l is a weighting coefficient that only depends on ratio κ defined by (6), and $r(\boldsymbol{x}_c, \boldsymbol{\xi}^l(\boldsymbol{\xi}))$ is the Euclidean distance between the points $\boldsymbol{x}_c$ and $\boldsymbol{\xi}^l$, being $\boldsymbol{\xi}^0$ the point $\boldsymbol{\xi}$ on the electrode surface ($\boldsymbol{\xi}^0(\boldsymbol{\xi}) = \boldsymbol{\xi}$), where $\boldsymbol{\xi}^l$ ($l \neq 0$) are the images of $\boldsymbol{\xi}$ with respect to the earth surface and to the interfaces between layers[7]. l_k is the number of summands in the series of the integral kernels, and it depends on the case analyzed (see (4), (5) and (7)).

2.3. Variational Form of the Problem

The integral expression for the potential (3) is also satisfied on electrode surface Γ where potential value is known by the boundary condition $V_b(\boldsymbol{\chi}) = 1$, $\forall \boldsymbol{\chi} \in \Gamma$. Consequently, leakage current density σ must verify the following Fredholm integral equation of the first kind on Γ

$$\frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi} \in \Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi})\, d\Gamma = 1, \qquad \forall \boldsymbol{\chi} \in \Gamma. \quad (9)$$

It is important to remark that obtaining the leakage current density from 9 is the key of the problem, since the potential distribution produced when a fault current is derived through the grounding grid can be easily obtained by using σ and expression (3). Other safety and design parameters (such as the equivalent resistance and the total surge current) can be directly computed from σ[5].

The above equation can be written in a variational form if one imposes that (9) is verified in the sense of weighted residuals, i.e., the following integral identity

$$\iint_{\boldsymbol{\chi} \in \Gamma} w(\boldsymbol{\chi}) \left(\frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi} \in \Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi})\, d\Gamma - 1 \right) d\Gamma = 0, \quad (10)$$

must hold for all members $w(\boldsymbol{\chi})$ of a suitable class of test functions defined on Γ[4, 5]. Now, it is clear that a numerical approach based on the Boundary Element Method seems to be the best choice to solve equation (10) and to obtain the leakage current density σ, since its resolution requires the discretization of the domain (i.e., the surface of the cylindrical conductors).

3. BEM Numerical Formulation

3.1. General 2D approach

The leakage current density σ and the electrode surface Γ can be discretized as

$$\sigma(\boldsymbol{\xi}) = \sum_{i=1}^{\mathcal{N}} \sigma_i\, N_i(\boldsymbol{\xi}), \qquad \Gamma = \bigcup_{\alpha=1}^{\mathcal{M}} \Gamma^\alpha, \quad (11)$$

for given sets of $\mathcal{N}$ trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on Γ, and $\mathcal{M}$ two dimensional boundary elements $\{\Gamma^\alpha\}$. Next, integral expression (3) for potential $V_c(\boldsymbol{x}_c)$ can also be discretized as

$$V_c(\boldsymbol{x}_c) = \sum_{i=1}^{\mathcal{N}} \sigma_i V_{c,i}(\boldsymbol{x}_c); \;\; V_{c,i}(\boldsymbol{x}_c) = \sum_{\alpha=1}^{\mathcal{M}} \sum_{l=0}^{l_V} V_{c,i}^{\alpha l}(\boldsymbol{x}_c); \quad (12)$$

$$V_{c,i}^{\alpha l}(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi} \in \Gamma^\alpha} k_{bc}^l(\boldsymbol{x}_c, \boldsymbol{\xi})\, N_i(\boldsymbol{\xi})\, d\Gamma^\alpha; \quad (13)$$

where l_V represents the number of summands considered in the evaluation of the series of the kernels, being $l_V = l_k$ if this number is finite, or the necessary number of terms to consider until convergence is achieved, if l_k is infinite.

On the other hand, variational form (10) is reduced to a linear system of equations, for a given set of $\mathcal{N}$ test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ:

$$\sum_{i=1}^{\mathcal{N}} R_{ji}\sigma_i = \nu_j \qquad (j = 1, \ldots, \mathcal{N})$$
$$R_{ji} = \sum_{\beta=1}^{\mathcal{M}} \sum_{\alpha=1}^{\mathcal{M}} \sum_{l=0}^{l_R} R_{ji}^{\beta\alpha l}, \qquad \nu_j = \sum_{\beta=1}^{\mathcal{M}} \nu_j^\beta, \quad (14)$$

being coefficients $R_{ji}^{\beta\alpha l}$ and ν_j^β:

$$R_{ji}^{\beta\alpha l} = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\chi} \in \Gamma^\beta} w_j(\boldsymbol{\chi}) \iint_{\boldsymbol{\xi} \in \Gamma^\alpha} k_{bb}^l(\boldsymbol{\chi}, \boldsymbol{\xi}) N_i(\boldsymbol{\xi}) d\Gamma^\alpha d\Gamma^\beta, \quad (15)$$

$$\nu_j^\beta = \iint_{\boldsymbol{\chi} \in \Gamma^\beta} w_j(\boldsymbol{\chi}) d\Gamma^\beta, \quad (16)$$

where l_R represents the number of summands considered in the evaluation of the series of the kernels, being $l_R = l_k$ if this number is finite, or the necessary number of terms to consider until convergence is achieved, if l_k is infinite.

Solution of linear system (14) provides the values of the current densities σ_i ($i = 1, \ldots, \mathcal{N}$) leaking from the nodes of the grid. However, the statement of this system requires the discretization of a 2D domain (the whole surface Γ of the grounded electrodes), its matrix is full and the computation of its coefficients requires to perform double integration on 2D domains. These facts imply a large number of degrees of freedom and an unaffordable computing effort in CPU time in the analysis of practical cases. For all these reasons, it is necessary to introduce some additional hypotheses in order to decrease the computational cost[5].

3.2. Approximated BE approach and efficiency of the numerical scheme

As we have presented, grounding grids of most electrical installations are formed by a mesh of conductors with ratio diameter/length very small ($\sim 10^{-3}$). Due to this specific geometry, the hypothesis of circumferential uniformity can be assumed producing a notable fall of the computational cost[1, 5]. In this way, the leakage current density σ is constant around the cross section of the cylindrical conductors of the grounding grid, and discretizations (11) and (14) become much simpler, since the classes of test and trial functions are restricted to those with circumferential uniformity while only the axial lines of the grounding electrodes have to be discretized[5].

In comparison with the general 2D boundary element formulation, the number of elemental contributions $R_{ji}^{\beta\alpha}$ and ν_j^{β} needed to state the system of linear equations (14) and the number of unknowns σ_i are now significantly smaller for a given level of mesh refinement.

In spite of the important reduction in the computation cost that implies to discretize only the axial lines of the electrodes, extensive computing is still necessary mainly because of the circumferential integration on the perimeter of the electrodes that are involved in the integral kernels. Nevertheless, in previous works we have proposed the evaluation of these circumferential integrals in an approximated form by using specific quadratures if suitable simplifications in the general approach are introduced[5]. The final result is an approximated 1D approach similar to the presented in section 3.1., where the computation of coefficients of the equations system requires integration on 1D domains, i.e. the axial lines of the electrodes[7].

Now, specific numerical approaches can be derived for different selection of sets of trial and test functions in the numerical scheme (e.g., a Point Collocation scheme or a Galerkin one). Further discussion in this paper is restricted to the case of a Galerkin type approach, since the matrix of coefficients is symmetric and positive definite[5]. In the analysis of grounding systems in uniform soil models, for Point Collocation and Galerkin type weighting numerical formulation, authors have derived a highly efficient analytical technique to evaluate the coefficients of the linear system of equations[5]. Now, since the 1D approximated expressions for terms $V_{c,i}^{\alpha l}$ and $R_{ji}^{\beta\alpha l}$ in (13) and (15) are formally equivalent to those obtained in the case of uniform soil models, their computation can also be performed analytically[7].

An important aspect of the numerical formulation proposed is its total computational cost. Thus, for specific discretization ($\mathcal{M}$ elements of p nodes each, and a total number of $\mathcal{N}$ degrees of freedom), a linear system (14) of order $\mathcal{N}$ must be generated and solved. Matrix generation process requires $O(\mathcal{M}^2p^2/2)$ operations, since p^2 series of contributions of type (10) have to be computed for every pair of elements, and approximately half of them are discarded because of symmetry. In uniform soil models these series are formed by only two terms, while in two-layer models the series can have an infinite number of them, that will be numerically added up until a tolerance is fulfilled or an upper limit of summands is achieved. Consequently, matrix generation will be much more expensive in two-layer models.

Table 1. Balaidos II Grounding System: General Characteristics

Balaidos II Characteristics	
Max. Grid Dimensions:	61.1 m $\times$ 79.1 m
Grid Depth:	0.80 m
Number of Grid Electrodes:	107
Number of Vertical Rods:	67
Electrode Diameter:	11.28 mm
Vertical Rod Diameter:	14.00 mm
Vertical Rod Length:	1.5 m
Ground Potential Rise (GPR):	10 kV

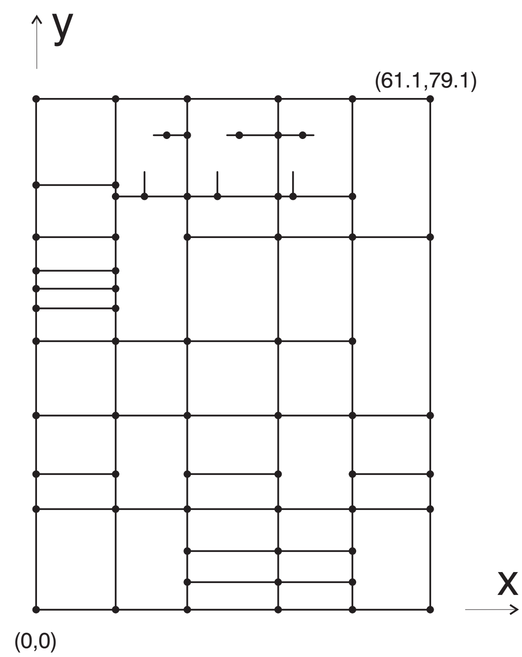

Figure 3. Balaidos II Grounding System: Plan of the grounding grid (vertical rods are marked with black points)

On the other hand, since the matrix is symmetric but not sparse, system solving process requires $O(\mathcal{N}^3/3)$ operations if the resolution is carried out with a direct method.

Hence, most of computing effort is devoted to matrix generation in small/medium problems, while linear system resolution would prevail in medium/large ones. In these cases, the use of direct methods for the linear system resolution is out of range. Therefore iterative or semiiterative techniques will be preferable. The best results have been obtained by a diagonal preconditioned conjugate gradient algorithm with assembly of the global matrix[5]. This technique has turned out to be extremely efficient for solving large scale problems, with a very low computational cost in comparison with matrix generation. So the cost of the system resolution should never prevail.

On the other hand, once the leakage current has been obtained, the cost of computing the equivalent resistance is negligible. The additional cost of computing potential at any given point (normally at the earth surface) by means of (12) only requires $O(\mathcal{M}p)$ operations, since p contributions of type (13) have to be computed for every element. However, computing time may be important if it is necessary to compute potentials at a large number of points, for example to draw contour lines.

The example presented in the next section corresponds to the analysis of a grounding system for different kind of soil models. We compare and discuss results obtained by using an uniform and two-layer soil models.

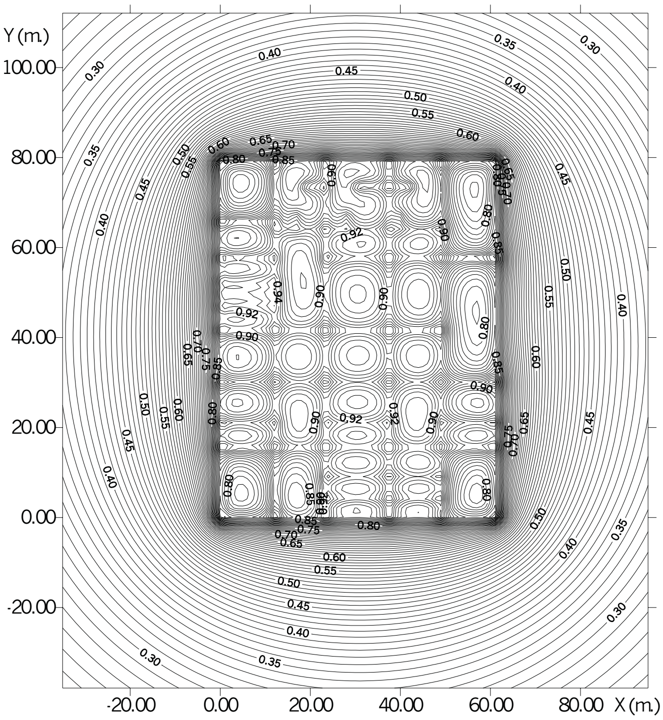

Figure 4. Balaidos II Grounding System: Potential distribution (×10 kV) on the earth surface obtained by using an uniform soil model (model A).

4. Application of the numerical approach to practical cases

4.1. Description of the earthing system

The proposed numerical approach based on the Boundary Element Method has been integrated in a Computer Aided Design system for earthing analysis. In this section we will present results obtained by using different kind of soil models in the analysis of a real grounding grid: the Balaidos II substation, close to the city of Vigo in Spain.

The earthing system of this substation is a grid of 107 cylindrical conductors (diameter: 11.28 mm) buried to a depth of 80 cm, supplemented with 67 vertical rods (each one has a length of 1.5 m and a diameter of 14.0 mm). The total surface protected is up to 4800 m^2. The total area studied is a rectangle of 150 m by 140 m, which implies a surface up to 21000 m^2. The Ground Potential Rise considered in this study has been 10 kV. The plan of the grounding grid is presented in figure 3 and a summary of the characteristics of the system is given in table 1.

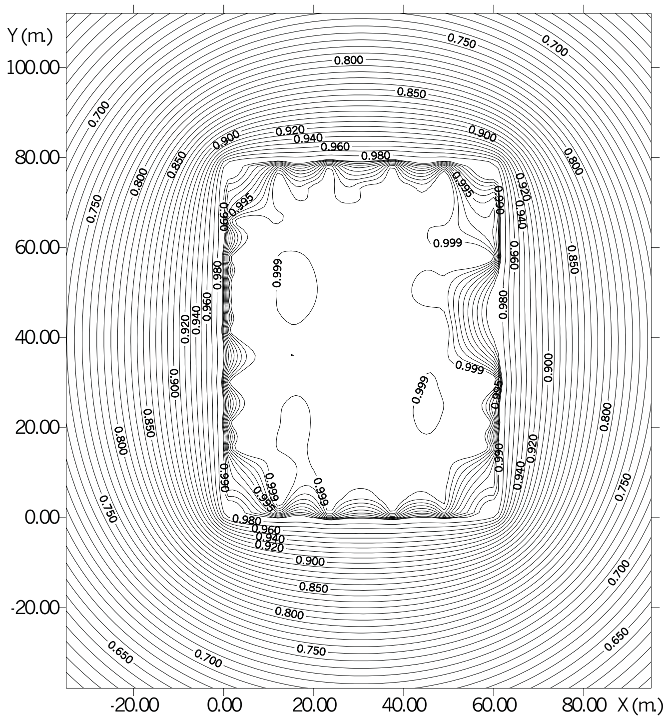

Figure 5. Balaidos II Grounding System: Potential distribution (×10 kV) on the earth surface obtained by using an horizontal two-layer soil model (model B).

Table 2. Balaidos II Grounding System: Characteristics of the Numerical Model

BEM Numerical Model	
Type of Approach:	Galerkin
Type of Current Density Element:	Linear
Number of Elements:	241
Degrees of Freedom:	208

4.2. Results by using different soil models

The numerical model used in the resolution of this problem is based on a Galerkin type weighting. Each bar is discretized in one single linear leakage current density element, which implies a total of 208 degrees of freedom (for this case, the use of one single constant density element per electrode would imply a total of 241, while the use of one single parabolic element would imply 449).

We have analyzed three cases in the Balaidos II grounding system for different kind of soil models. "Model A" corresponds to an uniform soil model with $\gamma = 50^{-1}$ $(\Omega\text{m})^{-1}$; "model B" corresponds to an horizontal two-layer soil model (as figure 1) with $\gamma_1 = 50^{-1}$ $(\Omega\text{m})^{-1}$, $\gamma_2 = 5000^{-1}$ $(\Omega\text{m})^{-1}$ and $H = 1.5$ m; and "model C" corresponds to a vertical two-layer soil model (as figure 2) with

Table 3. Balaidos II Grounding System: Numerical results for the different soil models

	Soil Models		
	A	B	C
Equiv. Resistance (Ω)	0.336	8.98	0.444
Total Current (kA)	29.7	1.11	22.5
CPU Time (s)	2.6	1093.5	4.2

$\gamma_1 = 50^{-1}$ $(\Omega m)^{-1}$, $\gamma_2 = 5000^{-1}$ $(\Omega m)^{-1}$ and the vertical interphase is to a distance of 1 m of the grounding grid (y = 80.1 m ∀x in the plan of figure 3).

Figure 4 shows the potential distribution on the earth surface obtained by using the uniform soil model A. Figures 5 and 6 show the potential distribution on the earth surface obtained by using the two-layer soil models B (horizontal) and C (vertical). Figure 7 is a comparison of the potential distribution on the earth surface obtained with models A and C in the earthing site close to the zone with variation in the conductivity of the ground.

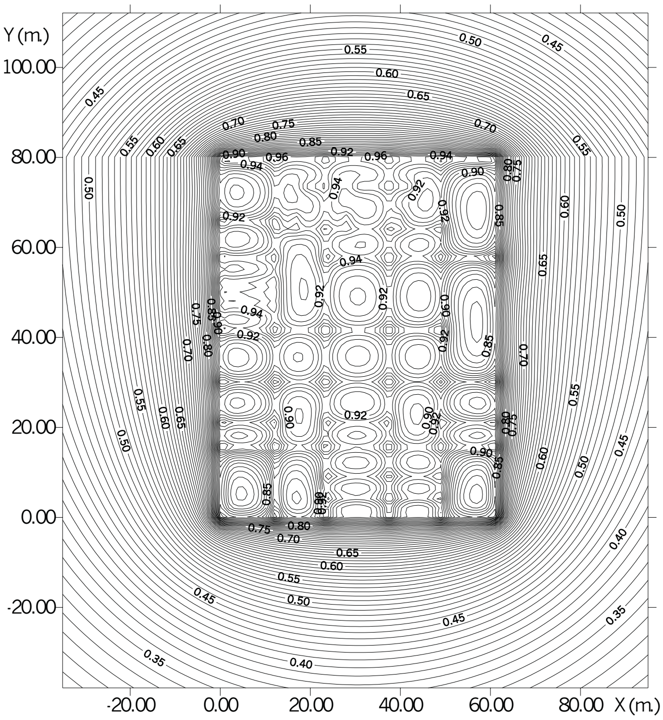

Figure 6. Balaidos II Grounding System: Potential distribution (×10 kV) on the earth surface obtained by using a vertical two- layer soil model (model C).

Table 2 summarizes the characteristics of the numerical formulation used in the solution of the different cases, and Table 3 shows some results obtained for each case analyzed: the equivalent resistance of the grounding system, the total current derived to the soil, and the CPU time.

The numerical approach has been implemented on a CAD system, which has been compiled and run onto an Origin 2000 Silicon Graphics computer at the European Center for Parallelism of Barcelona, CEPBA. Although the O2000 is a high-performance computer with 64 MIPS R10000 processors at 250 MHz, all the examples have been solved in sequential mode for the uniform and the two-layer models.

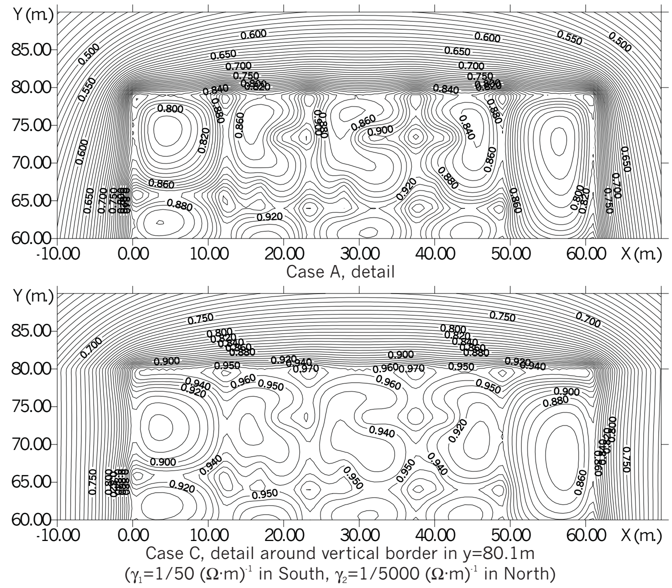

Figure 7. Balaidos II Grounding System: Comparison between the potential distribution (×10 kV) on the earth surface obtained by using soil models A and C (detail of the grounding site close to the zone with change in the conductivity).

As we can see in these examples, the selection of the suitable soil model is essential to guarantee the safety in the grounding installations, since results obtained by using a multiple-layer soil model can be noticeably different from those obtained by using a single layer (or uniform) soil model (figures 4,5,6,7,8 and table 3). Therefore, it could be advisable to use multi-layer soil formulations to analyze grounding systems as a general rule, in spite of the increase of the computational effort. In fact, the use of this kind of advanced models should be mandatory in cases where the conductivity of the soil changes markedly with depth or in the vicinity of the substation site.

Obviously, this boundary element formulation can be applied to any other case with a higher number of layers. However, CPU time may increase up to unadmisible levels, mainly due to the poor rate of convergence of the underlying series expansions, and the need to evaluate double series (in three-layer models), triple series (in four-layer models), and so on.

Nowadays, while single-layer models run in real time in conventional computers for the analysis of medium/big size grounding grids[5, 6], multiple-layer models require in general an out of order computing time. For this reason, we are studying the improvement of the convergence speed of the series of the kernels by using extrapolation

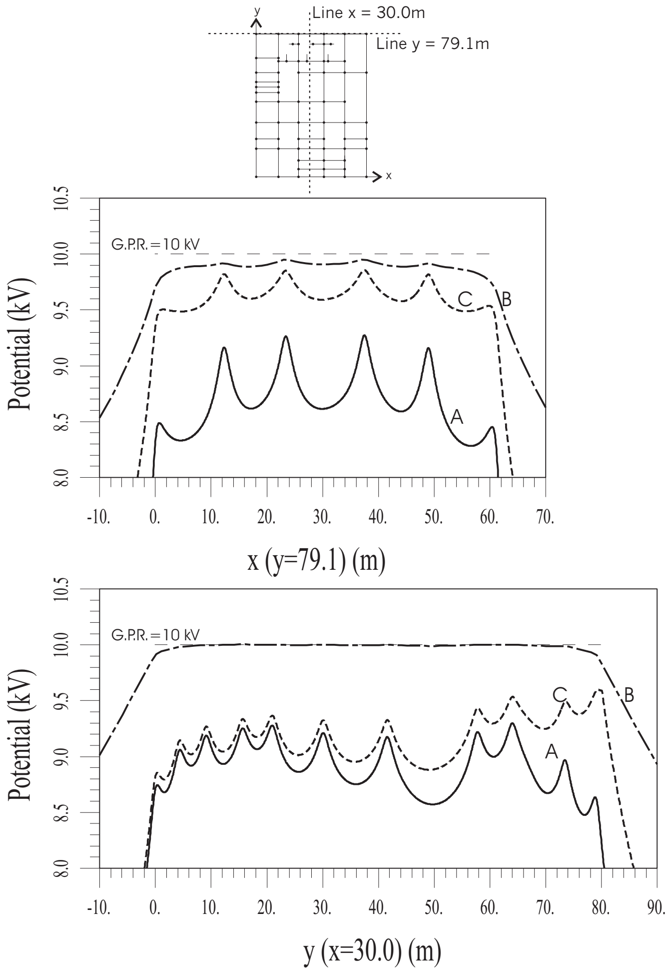

Figure 8. Balaidos II Grounding System: Potential profiles (kV) on earth surface along two lines by using the four types of soil models. Notice the important differences in the results depending on the kind of soil model selected.

techniques[10], and, on the other hand, the parallelization of the multi-layer boundary element numerical approach which could become a real-time design tool for grounding analysis[8].

5. Conclusions

In the last years, the authors have developed a high efficient numerical formulation based on the Boundary Element method for earthing analysis in uniform soil models, which has been succesfully applied to several real cases.

In this paper we have presented a generalization of this boundary element approach for grounding grids embedded in stratified soils. The proposed approach has been applied to the analysis of a real grid, considering different kind of soil models: a single and two layer models (horizontal and vertical) which allows to analyze the influence of the variations in the soil conductivity with depth and in the surroundings of the grounding system.

The suitable selection of the soil model is a key point in grounding analysis. We have shown that it is possible to obtain highly accurate results with the proposed boundary element numerical approach, and these results can be noticeably different depending on the kind of soil model considered in the study. Consequently, since the grounding safety parameters may significantly change, it could be advisable to use the multi-layer soil formulation as a general rule in spite of the increase in the computational cost.

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