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Size effects on the tensile strength and fracture toughness of granitic rock in different tests

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A B S T R A C T

This study investigates the tensile failure mechanisms in granitic rock samples at different scales by means of different types of tests. To do that, we have selected a granitic rock type and obtained samples of different sizes with the diameter ranging from 30 mm to 84 mm. The samples have been subjected to direct tensile strength (DTS) tests, indirect Brazilian tensile strength (BTS) tests and to two fracture toughness testing approaches. Whereas DTS and fracture toughness were found to consistently grow with sample size, this trend was not clearly identified for BTS, where after an initial grow, a plateau of results was observed. This is a rather complete database of tensile related properties of a single rock type. Even if similar databases are rare, the obtained trends are generally consistent with previous scatter and partial experimental programs. However, different observations apply to different types of rocks and experimental approaches. The differences in variability and mean values of the measured parameters at different scales are critically analysed based on the heterogeneity, granular structure and fracture mechanics approaches. Some potential relations between parameters are revised and an indication is given on potential sample sizes for obtaining reliable results. Extending this database with different types of rocks is thought to be convenient to advance towards a better understanding of the tensile strength of rock materials.

1. Introduction

Applied rock mechanics focuses the study of underground excavations, slopes, or foundations on rocks, in all of which materials are typically subjected to compressive stresses. It is therefore not surprising that rock mechanics studies have mainly addressed the behaviour of rocks and rock masses under compressive stresses. However, in some circumstances (hydraulic fracturing, excavations in bedded materials or upper part of slopes), tensile stresses do occur in some parts of the rock structures under scrutiny. Both the intact rock (due to weaker nature of the covalent bonds of rock-forming minerals such as silicate) and rock masses (due to the occurrence of pre-existing discontinuities) have limited capability of withstanding tensile stresses, in relation to sustaining compressive ones (Martin, 1993). This is why a number of problems may arise in rock engineering design associated with tensile-driven fracturing and forward instability and a large part of the discontinuities observed in rock masses are formed under tensile stresses in association with large pore pressures in the rock formations (Cosgrove and Hudson, 2016).

The estimation of tensile strength of intact rocks is usually based on two types of tests, i.e. direct tensile strength (DTS) tests performed on cylindrical specimens and indirect or Brazilian tensile strength (BTS) tests, where tensile stresses are induced through compression. Results of both tests tend to differ significantly (Perras and Diederichs, 2014).

Tensile strength of intact rocks and their compressive strength at low confinements are at least partially controlled by propagation of micro-cracks within rocks, a fact observed and formalized in a

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roughly accurate manner by Griffith (1921). Indeed, different types of micro-crack growth (Kranz, 1983) ultimately contribute to the failure of intact rock even under compressive loads, which makes the Griffith approach of limited practical applicability. However, Diederichs (2000) showed how even with some confinement, micro-crack/micro-fracture propagation associated with local tensional stresses is one of the most relevant mechanisms controlling the compressive strength of an intact rock sample.

Fracture propagation in rock and other materials is controlled by fracture toughness and it is relevant to tensile strength of rocks, even if their relations have been seldom studied in due detail in the past. Fracture toughness is the critical stress intensity factor at the tip of a sharp ellipsoidal crack when propagation of the crack suddenly proceeds. A component's thickness affects the constraint conditions at the tip of a crack, but the energy components of the form of the element where the crack is growing also affects results. For thick components having plane strain conditions, the lowest fracture toughness value is obtained, which is considered a material property. Under these plane strain conditions, the critical value of stress intensity factor measured in Mode I (pure tension) loading is known as the plane strain fracture toughness, denoted as K_{IC}.

It should also be noted that the heterogeneity and discontinuity inherent of rocks and rock masses cause their mechanical properties to be scale-dependent, i.e. dependent on the size of the sample under study. At larger scales, rock masses contain a larger proportion of discontinuities of different sizes and nature, and consequently properties such as rock strength are expected to reduce. Considering that on-site testing is generally more expensive and difficult to carry out than laboratory testing, a good understanding of size effects is the key for upscaling of the laboratory-measured data to field scale.

Scale effects were noticed by some researchers at the early stages of rock mechanics development (Mogi, 1962; Bieniawski, 1968; Hoskins and Horino, 1969; Koifman, 1969; Pratt et al., 1972) while studying the compressive properties of different rocks. Later on, Hoek and Brown (1980, 1997) tried to understand scale effects in rocks from an experimental point of view. It was during the 1990s when the study of scale effects in rock mechanics started to become a relevant topic, with the celebration of two international conferences on scale effects in rock masses (Pinto da Cunha, 1990, 1993).

Nevertheless, the studies related to this topic have been mainly focused so far on the size effect of uniaxial and triaxial compressive strengths (Quiñones et al., 2017; Wang et al., 2020; González-Fernández et al., 2021), and limited research was developed on the effect of scale on other mechanical parameters like tensile strength (Obert et al., 1946; Wijk et al., 1978; Butenuth, 1997; Thuro et al., 2001; Coviello et al., 2005) or fracture toughness (Iqbal and Mohanty, 2006; Ueno et al., 2013; Muñoz-Ibáñez et al., 2021). Taking into account that the tensile strength of rocks is considerably lower than their compressive strength, and that therefore tensile failure is one of the main failure modes in geomaterials, the accurate assessment of rock tensile strength can sometimes be a key factor for the safe design of projects in areas such as mining, slope stability or tunnelling.

Moreover, in Mode I (opening or tensile) loading, crack initiation and propagation occur under tensile conditions, thus it is expected that Mode I fracture toughness and tensile strength would be related. In this regard, several scholars (e.g. Gunsallus and Kulhawy, 1984; Whittaker et al., 1992; Zhang et al., 1998; Zhang, 2002) have proved that different empirical relations can be established between the two parameters.

The thorough literature review carried out in this paper brings to light the limited available knowledge regarding the scale effects on the tensile strength and fracture toughness of rocks. In this line, we have considered conducting a multi-laboratory experimental program involving direct and indirect tensile strength tests complemented with results of fracture toughness. This last fracture mechanics parameter has been obtained from a recently developed method (pseudo-compact tension (pCT) test (Muñoz-Ibáñez et al., 2020)) and from semi-circular bending (SCB) tests to estimate the rock fracture toughness, K_{IC}, at various scales. The operational features and loading conditions of these methods are illustrated in Fig. 1. A good number of tests on specimens of different diameters, ranging from 30 mm to 100 mm, were carried out and critically analysed.

The main objectives of this study are, first, to have available tensile strength and fracture toughness values obtained from different types of tests performed in granitic rock samples at different scales, with the aim of exploring potential trends of scale effects on tensile parameters. In this line, tensile strengths obtained from DTS and BTS tests were analysed with already developed statistical models (Brook, 1985; Wang et al., 2020) relating rock strength to specimen diameter. Moreover, based on previous empirical relationships obtained for Mode I fracture toughness and tensile strength of rocks, an attempt to relate these two parameters has been made. Finally, a discussion section is opened to assess the experimental results from a statistical point of view, and to explore possible mechanisms and relationships between test results that can be of help for better understanding the tensile strength behaviour of rocks.

2. A state-of-the-art review

2.1. Scale effects on DTS

Very limited research investigating size effects on DTS or uniaxial tensile strength of rocks has been reported so far. In this line, it is worth mentioning a seminal study developed by Obert et al. (1946) representing an early attempt to standardise the uniaxial tensile strength test, besides encompassing the study of the effect of specimen diameter on tensile strength results.

Wijk et al. (1978) studied the effect of the sample size on the uniaxial tensile strength of Bohus granite, concluding that for the studied cases (cylindrical specimens with diameters (d) varying from 8 mm to 62 mm and different lengths (l), covering $l = d$, $2d$ and $4d$), the tensile strength showed almost negligible dependence on the scale.

Van Vliet and van Mier (2000), while testing concrete and sandstone flat (two-dimensional) dog-bone shaped specimens of different sizes, found an increasing trend of the tensile strength with the size of the studied rock specimens. Coviello et al. (2005) performed a broad study on the tensile strength behaviour of soft rocks through different test approaches and including different specimen sizes; from results obtained after DTS tests carried out with a dry calcarenite and four specimen diameters (25 mm, 38 mm, 50 mm and 60 mm), no scale effect trend could be discerned.

Potential size effects in other sedimentary rocks were analysed by Jensen (2016), who performed DTS tests in Castlegate sandstone and Mons chalk, observing an increasing trend of the tensile strength with the increase of specimen diameter for the sandstone, whereas an inverse trend was found for the chalk. Nevertheless, it has to be pointed out that only two diameters were considered in their study.

The limited number of studies already listed suggests either a light increase in uniaxial tensile strength with sample size or slightly size-independent uniaxial tensile strength of rocks.

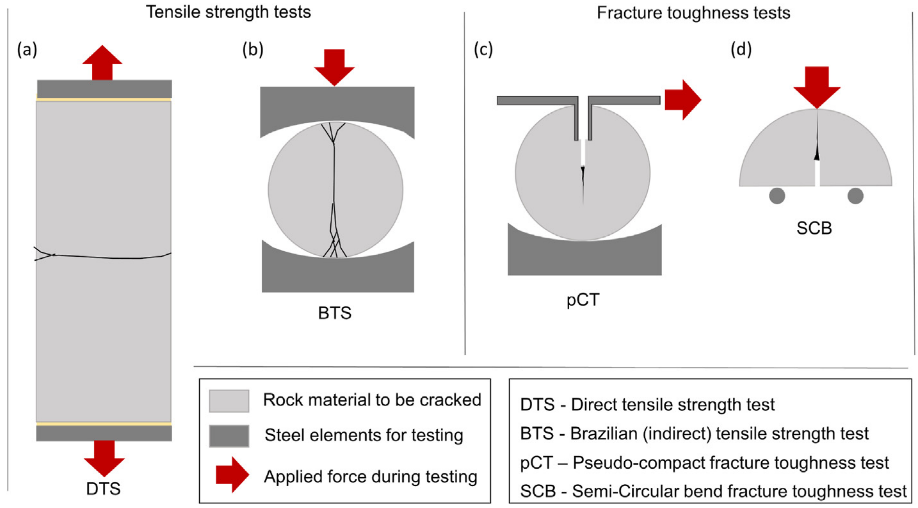

Fig. 1. Illustrative operational features and loading conditions in (a) DTS tests with cylindrical specimens, (b) BTS tests with disk-shaped specimens, (c) pCT tests, and (d) SCB tests.

2.2. Scale effects on indirect tensile strength

The probably most widely used indirect tensile strength test, known as Brazilian test (Akazawa, 1943; Carneiro, 1943) and yielding the so-called BTS, represents an alternative method to estimate tensile strength of rocks, overcoming certain typical difficulties associated with the direct tensile testing, particularly, the sample preparation and the attainment of a strictly uniaxial loading condition. A relatively scarce but scattered body of research has been reviewed, regarding the scale effect on indirect tensile strength testing.

Lundborg (1967) studied the tensile behaviour of granite through Brazilian tests with specimens with 2, 3, 4 and 6-cm diameters. A decaying trend of the tensile strength was observed with the increase of diameter, and fitted to a Weibull-type ('weakest link theory') expression (Weibull, 1939), as shown by

$$6\log_{10}\left(\frac{\tau_1}{\tau_2}\right)=\log_{10}\left(\frac{\nu_1}{\nu_2}\right) \tag{1}$$

where τ_1 and τ_2 are the indirect tensile strengths of a sample with volumes ν_1 and ν_2, respectively.

Sundae (1974) developed a study to assess the effect of specimen diameter on the tensile strength indirectly estimated with a sort of Brazilian test ('apparent tensile strength') carried out with a point load test machine. From this study, a decreasing trend of the tensile strength with increasing rock specimen volume was derived, for two granite types and a granodiorite.

Newman and Bennett (1990) concluded that the influence of the specimen size on the tensile strength results estimated from Brazilian tests cannot be disregarded. This was concluded through analysis of variance (ANOVA) tests and orthogonal linear contrasts (OLC) performed on results obtained from specimens with three different length-to-diameter ratios (L/d) of 0.5, 0.75 and 1, for a given d about 50.5 mm. In line with other previous studies, Kramadibrata and Jones (1993) also reported decreasing trends with the increase of diameter for the tensile strength as estimated from Brazilian tests carried out on basalt-mafic, porphyry and dolerite rock samples. A power law in the form $\sigma_t = ad^b$ (a and b are the fitting coefficients) was fitted to tensile strength results obtained from rock specimens with d ranging from 36 mm to 150 mm.

Rocco et al. (1999) performed an experimental study on the size effect in Brazilian test with cylindrical granite specimens presenting 30 mm in thickness and four diameters (30 mm, 60 mm, 120 mm and 240 mm). They found a decrease in the tensile strength with increasing specimen size with a difference up to 30%.

Thuro et al. (2001) performed an experimental study encompassing indirect tensile strength (Brazilian) tests carried out on three rock types (two-mica granite, kerstantite and limestone). Rock specimens presented diameters from 45 mm to 80 mm, by keeping a length-to-diameter ratio equal to 1. From this study and in contrast to the already mentioned works, they found a negligible relationship between the sample size (specimen diameter) and derived BTS.

As part of a technical report carried out to characterize the stress-strain behaviour of Cobourg sandstone, Jaczkowski et al. (2017) explored the influence of specimen size on BTS test results using cylindrical rock specimens with four diameters (50 mm, 76 mm, 101 mm and 126 mm). A strength decrease was found as the diameter of the specimen increased, observing a particularly drop (about 25%) of the tensile strength between the two lowest diameters, and from 76 mm to 126 mm, a plateau was observed.

A broad study was performed by Masoumi et al. (2018), encompassing 40 Brazilian tests carried out on Gosford sandstone, with specimens presenting eight different diameters in the range of 19 mm–145 mm. A descending trend was observed with the increase of the specimen diameter, even though the results from specimens with 145 mm diameter lied above the rest, something the authors attributed to a different fracture pattern in this case.

More recent studies, i.e. that carried out by Li et al. (2020) analysing both the size and anisotropic effects on indirect tensile strength, showed a decrease on the tensile strength with the increase of specimen diameter, independently on the loading direction with respect to the rock foliation. In that study, six diameters encompassing 25 mm, 38 mm, 50 mm, 63 mm, 75 mm and 100 mm were used. Delgado-Martín et al. (2021) conducted a series of Brazilian tests on two igneous rocks of the Forsmark site. Their results did not show evidence of scale effects for the four specimen sizes considered (42-, 54-, 63- and 100-mm diameter). In addition, the variation of the dip angle had no apparent influence on the tensile strength obtained.

Through three-dimensional (3D) finite element modelling (FEM), Yu et al. (2006) studied the effects of size and shape on tensile strength results in the numerically simulated Brazilian tests. The obtained values for the tensile strength showed a decreasing trend with the increase of volume (thickness-to-diameter ratio). A similar decreasing trend of BTS with increasing size was reported by Xu et al. (2016) by simulating tests carried out by means of 3D particle flow distinct element modelling.

The documents reviewed concerning scale effects on BTS results, unlike the case of those reporting DTS, often show the generalised size effect concept where strength (BTS) decreases with an increase in size, particularly for larger samples. There must be some reasons behind these diverging trends that have not been deeply investigated so far because, among other reasons, there are not many studies considering both types of tests on the same rock type. It is relevant to remark that under indirect tensile testing, the rock cracks under no pure tension. Moreover, as pointed out by Masoumi et al. (2018), with an increase in size, the failure of rock changes from pure tension to a combination of shearing and tension, which may be behind the divergence between scale trends of DTS and BTS results.

This study presents both types of tests at different scales combined with fracture toughness results with the aim of having available a database of multiple type results and helping to understand the reasons behind these apparently paradoxical observations.

2.3. Scale effects on mode I fracture toughness

The application of fracture mechanics to rock engineering and particularly to rock stability, hydraulic fracturing or geothermal energy issues appears as a potential interesting approach around 1980s (Ingraffea, 1979; Rummel, 1987; Takahashi and Abé, 1987). Some progresses were made at this time including the proposal of methods to compute fracture toughness and fracture propagation energy. Considering fracture toughness as an intrinsic material property, it should be independent of factors such as the specimen size. However, different authors have reported a consistent inverse size effect on K_{IC} in quasi-brittle materials such as rocks, i.e. increasing values of K_{IC} for larger specimens, in line with DTS results. From the early developments of laboratory measuring techniques, this scale effect was a matter of concern (Matsuki et al., 1991; Scavia, 1996). At the laboratory-scale, there is indeed a general trend for K_{IC} to increase as the size of the specimen increases (Asem et al., 2021; Zhang et al., 2021). In this sense, size effects have been observed even for the four suggested methods (i.e. chevron bend (CB), short rod (SR), cracked chevron notched Brazilian disc (CCNBD), and SCB) endorsed by the International Society for Rock Mechanics and Rock Engineering (ISRM) to measure Mode I fracture toughness in rocks (ISRM, 1988; Fowell, 1995; Kuruppu et al., 2014).

In the case of the SR test, the ISRM proposed using a sample with a minimum diameter of 50 mm to reduce size effects. Experimental results reported by Yi et al. (1991) for Kallax gabbro support this recommendation, as they suggest that the influence of specimen size is minimised for SR specimens with a diameter larger than 51.5 mm, in line with the suggestion by Matsuki et al. (1991), but not with observations by Scavia (1996). Similarly, Wei et al. (2017) also reported a low size effect for SR specimens ($d = 50-74$ mm) of a weak sandstone. However, for the same rock type, a more significant dependency was found for CCNBD samples. These results are in agreement with those obtained by Sangsefidi et al. (2021) for CCNBD marble specimens with diameters in the range of 50–390 mm.

For the CB test, a positive relation between K_{IC} and specimen diameter was also found for three different types of granite (Iqbal and Mohanty, 2006). In the case of SCB test, a number of experimental and numerical studies highlight the sensitivity of K_{IC} to specimen size for this methodology, even for samples larger than the dimensions proposed by the ISRM ($d > 76$ mm). Kataoka and Obara (2015) reported that a minimum diameter of 140 mm would be required to obtain consistent K_{IC} values when using the SCB method. Similarly, Ueno et al. (2013) found that K_{IC} increased with increasing SCB specimen diameter ($d = 50-100$ mm) for two types of sandstone, as also reported by Muñoz-Ibáñez et al. (2020) on four different rock types (three sandstones and one granite) and three specimen diameters (i.e. 38 mm, 50 mm and 100 mm). These authors also observed a more pronounced influence of the specimen size in SCB tests than in pCT tests, and their results suggest that lithology may play an important role when setting the minimum sample size that provides consistent K_{IC} values in fracture toughness testing.

Size effects in fracture toughness measurements could be related to the development of a fracture (nonlinear) process zone (FPZ) ahead of the crack tip during testing (Wei et al., 2016). Considering that the concept of fracture toughness is derived from linear elastic fracture mechanics (LEFM), and to guarantee a linear elastic behaviour of the material, the size of the FPZ should be small enough compared to the dimensions of the sample to render consistent results (Wei et al., 2022). That is, a sufficiently small FPZ would be required for the fracture toughness to be independent of specimen size (Brevik, 2016). A number of authors have previously reported an impact of the FPZ in K_{IC} measurements (Labuz et al., 1987; Hu and Duan, 2008). In fact, the increase in fracture toughness with specimen size reported by Ueno et al. (2013) for SCB specimens would be related to a greater influence of the FPZ on smaller specimens. This effect could be minimised by introducing a chevron notch instead of a straight notch in SCB specimens, as suggested by Wei et al. (2016), so that smaller specimens could be used to obtain good consistency in K_{IC} results.

Another aspect that should be accounted for in the application of LEFM is related to the stress intensity factor and the high-order stress terms of the infinite series that describe the elastic stress field at the crack tip. T-stress, which represents the first non-singular stress term, is considered to significantly influence the stress and strain fields, and thus, affects the fracture behaviour (Aliha and Ayatollahi, 2013). Therefore, disregarding T-stress could provide less reliable results and inaccurate estimations of fracture toughness, which is more notable for brittle materials. However, it has been recently discussed that considering the role of the FPZ or the T-stress alone would not be sufficient, and that a combined approach between them would be required to properly characterize the fracture toughness of rocks (Wei et al., 2021).

In addition, the presence of flaws within rocks could also be a possible reason for smaller specimens yielding lower K_{IC} values. For smaller samples, the size of the defects would be larger, reducing the resistance to crack propagation. If the specimens are large enough to minimise this effect, K_{IC} would converge to a constant value (Kataoka and Obara, 2015).

2.4. Relationships between tensile strength and fracture toughness

In Mode I (i.e. opening or tensile mode), the crack faces open perpendicularly to the crack plane. Consequently, it is expected that Mode I fracture toughness (K_{IC}) would be related to the tensile strength (σ_t) of the material. In this regard, Whittaker et al. (1992) argued that fracture toughness is controlled by the minimum principal stress (i.e. tensile strength), which implies that there is an inherent relation between the two parameters. In the same way, Zhang (2002) suggested that the fracture patterns occurring in tensile strength and fracture toughness tests would be related due to the following reasons: (1) failure occurs due to the extension of a single crack, and (2) the meso-fracture characteristics of the fractured surfaces of the specimen are similar. The connection between K_{IC} and tensile strength has also been studied by Wang and Hu (2017) through experiments on notched 3-point-bend (3 PB) granitic samples. Hu et al. (2022) proposed a simple and straightforward relationship for K_{IC} and tensile strength through the material microstructure (grain size) and the width of the FPZ (FPZ_W), which does not depend on any empirical parameter, as shown by

$$K_{IC} = 2f_t\sqrt{3C_{ch}} \quad (2)$$

where f_t corresponds to the tensile strength of the rock, and C_{ch} is the average grain size.

In past years, several authors derived empirical relations between K_{IC} and σ_t for different rock types and soils, as listed in Table 1.

Although the previous studies agree in that K_{IC} is proportional to the tensile strength, the empirical formulae proposed differ from each other. This could be due to the fact that different rock types were tested in each study. In addition, data were obtained using a variety of test methods. In this sense, it had been reported previously that the tensile strength of rocks varied considerably when measured by different methods (Hudson et al., 1972), and the same occurred in the case of fracture toughness (Iqbal and Mohanty, 2006; Kataoka et al., 2015; Erarslan, 2018). Many studies have reported that the tensile strength and the fracture toughness vary considerably with factors such as specimen size and loading rate, which must be considered when comparing results. To avoid discrepancies due to these features, Zhang (2002) suggested selecting a single testing method or converting the experimental data recorded using different methodologies.

3. Methodology and results

3.1. Tested rock type

The rock samples tested are the Blanco Mera granite, a rock type widely studied in the rock mechanics literature. This is a Variscan leucogranite from NW Spain containing plagioclase with signs of sericitization (35%), K-feldspar (27%), quartz (20%), and with relevant mica components including muscovite (~7%) and biotite (~5%), the last commonly chloritized (~4%). The grain size and shape vary: 1–6 mm allotriomorphic quartz; <6 and up to ~30 mm subidiomorphic plagioclase and K-feldspar; and 1.5–2.5 mm idiomorphic biotite and muscovite. Locally, groups of biotite crystals may cluster together. While it is generally considered a moderately homogeneous granite, local heterogeneities are observed (Fig. 2).

Table 1
Relationships between tensile strength and Mode I fracture toughness from some selected references.

Source	Relation	R^2	Material	Tensile strength	K_{IC}
Gunsallus and Kulhawy (1984)	$K_{IC} = 0.0736\sigma_t + 0.76$		Three rock types	BTS	SR
Whittaker et al. (1992)	$K_{IC} = 0.107\sigma_t + 0.271$	0.83	Rock	Compilation of data obtained with different testing methods	
Zhang et al. (1998)	$\sigma_t = 8.88K_{IC}^{0.62}$	0.94	Rock	Compilation of data obtained with different testing methods	
Zhang (2002) [a]	$\sigma_t = 6.88K_{IC}^{0.62}$	0.94	Rock	Compilation of data obtained with different testing methods	
Muñoz-Ibáñez et al. (2020)	$K_{IC} = 0.11\sigma_t$		Four rock types	BTS	pCT
Backers (2004)	$\sigma_t = 4K_{IC}$		Six rock types	BTS	CB
Wang et al. (2007)	$K_{IC} = 0.3546\sigma_t$		Clay	DTS	3 PB

Note.
[a] This correlation is built upon the results from more than 50 different rock types, including coal, shale and sandstone.

3.2. DTS tests

The testing frame used for performing the DTS tests was a conventional universal 500-kN testing apparatus developed by Mecánica Científica (Spain) that allows either the performance of compressive strength or DTS tests (Fig. 3).

The tensile load is applied to the rock specimen through two metal caps adhered to and linked to the system by two roller chains mounted perpendicular one to another. This prevents the application of any undesired moment and ensures a uniaxial load is only applied (ASTM D2936-20, 2020). Cylindrical specimens were used for DTS tests. The dimensions of the specimens were set by following a length-to-diameter ratio equal to 2.5, considering four diameters (i.e. 30 mm, 38 mm, 54 mm and 84 mm). All specimens had to be cemented to metal caps, to be able to transmit the axial tensile load. For this purpose, a bi-component epoxy resin Loctite EA 9483 with a nominal tensile strength of 13 MPa was used. Four 30-mm length strain gauges were adhered at the middle point of each rock specimen, with two being glued vertically and two glued horizontally, except for the case of 30-mm diameter specimens, where only two vertical bands could be attached (Fig. 4a).

All tests were carried out at a tensile stress rate of 0.04 MPa/s, and a test was considered valid when the failure of the specimen was produced through the rock material, ideally about the middle plane (Fig. 4b), and not trough the resin.

The available rocks allowed to perform the following valid tests: 5 DTS tests on 84-mm diameter specimens, 4 DTS tests on 54-mm diameter specimens, 7 DTS tests on 38-mm diameter specimens, and 7 tests in 30-mm diameter specimens, whose DTS values are presented in Table 2.

Results rather scattered are plotted in Fig. 5 showing a typical irregular increasing trend as typically reported in the literature for some of this rock type. The observed scattering is attributed to the heterogeneous grain size and structure of this rock, as observed in Fig. 2. The increasing value of strength is initially considered associated with the occurrence of minor size grain cracks that can be more easily opened into smaller samples, where they may encompass a large part of the generated tensile crack. In larger samples, these minor cracks will play a less relevant role, thus larger tensile stresses are needed to open the final tensile macrocrack. In non-granular rocks, these effects may be not so relevant.

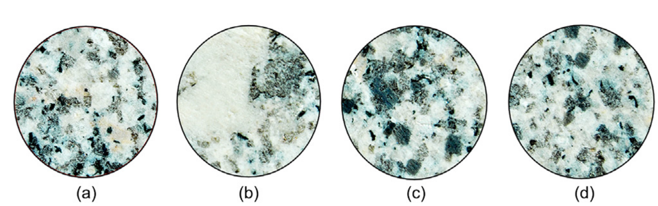

Fig. 2. Four pictures of circular 54 mm diameter surfaces of Blanco Mera granite showing its variability and heterogeneity. While units (a) and (d) are rather regular, unit (b) shows a large plagioclase crystal and (c) a cluster of micas.

Fig. 3. General view of the testing equipment: (a) Testing frame, (b) Control unit and gauge meter, (c) Double-action loading piston, and (d) Loading cell.

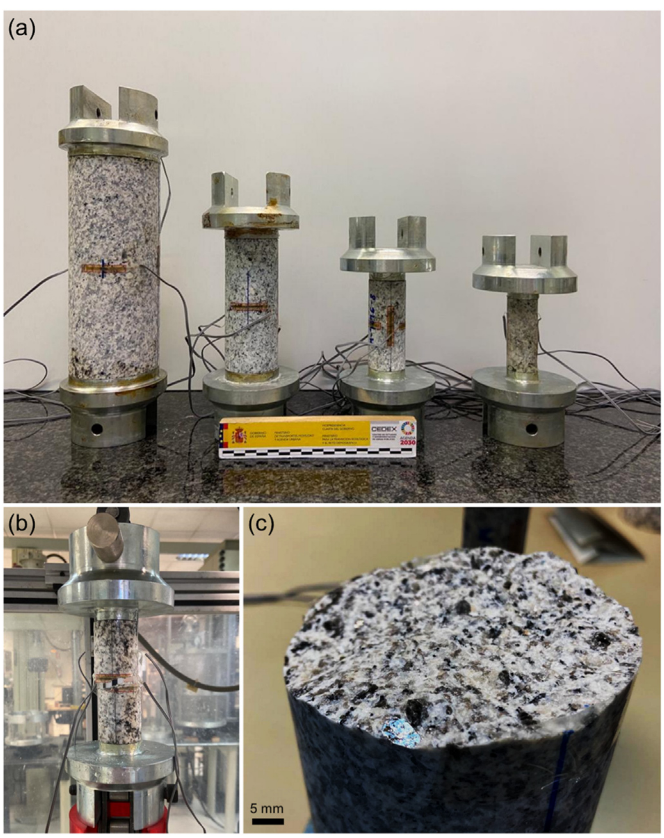

Fig. 4. (a) Rock specimens for DTS testing (from left to right: 84 mm, 54 mm, 38 mm and 30 mm in diameter) with metal caps adhered; (b) Specimen after a DTS test showing a valid failure; and (c) Detailed view of a fracture surface.

Table 2
Mean results and standard error of mean per group (30 mm, 38 mm, 54 mm and 84 mm in diameter), corresponding to DTS tests.

d (mm)	Statistic parameter	*DTS* (MPa)
30	Mean (7 tests)	3.69
	Standard error	0.32
38	Mean (7 tests)	5.56
	Standard error	0.33
54	Mean (4 tests)	4.45
	Standard error	0.16
84	Mean (5 tests)	6.06
	Standard error	0.11

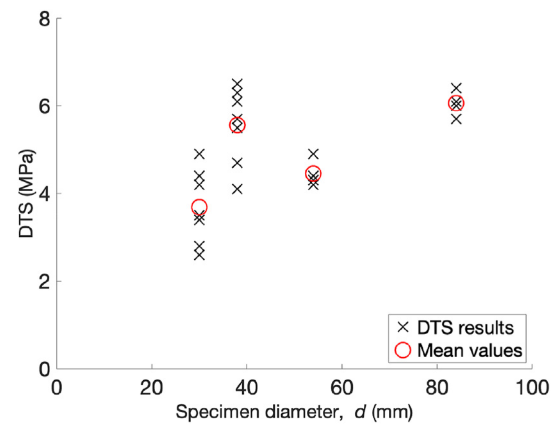

Fig. 5. Individual and average results of DTS tests in relation to sample size.

3.3. Indirect (Brazilian) tensile strength tests

To perform the indirect tensile strength (BTS) tests, four set of samples (Fig. 3a) were obtained from Blanco Mera granite blocks. Cores of 30 mm, 38 mm, 54 mm and 84 mm in diameter were extracted with a core driller (WEKA, model DK22). In order to obtain the discs for the tests, they were cut to the proposed length. In this case, the length-to-diameter ratio was $L = d/2$. Using a disc saw (CEDIMA model CTS-265, 400-mm diameter), specimens with an approximate thickness equal to the radius of the cylindrical sample (15 mm, 19 mm, 27 mm and 42 mm, respectively) for each set were obtained. Ten specimens were prepared for each size (Fig. 6a) but only 9 valid tests could be performed for 38-mm diameter specimens.

The tests were carried out in a system consisting of a movable loading platform, loading frame, hydraulic pumps for confinement control, test controller, test processor and a PC. The press is operated by a hydraulic pump that pushes the lower platen towards the upper platen. The servo-control system is installed to manage the load or deformation rate. The rotational speed of the servomotor is automatically adjusted to provide the speed and pressure commanded by the control.

For these tests, two types of jaws of different sizes were used depending on the size of the specimen, to allow a contact surface with the jaws of about 15°. For specimens of 30 mm, 38 mm and 54 mm in diameter, jaws with end arc radii of 75 mm were used, while for 84-mm diameter specimens, jaws with end arc radius of 112.5 mm were used. For the tests, the specimens were placed centred in the jaws, and the contact surface between the specimen and the jaws should be cushioned with cardboard, having a thickness equivalent to 0.01 times the diameter of the specimen. The measurement of the deformation modulus perpendicularly to the vertical loading axis (E_s, as defined by Ye et al. (2009)) was carried out through two strain gauges attached to both flat sides of the specimen in the way shown by Fig. 6b and c.

To carry out the tests, the first step was to determine the duration of the tests for better data collection and analysis.

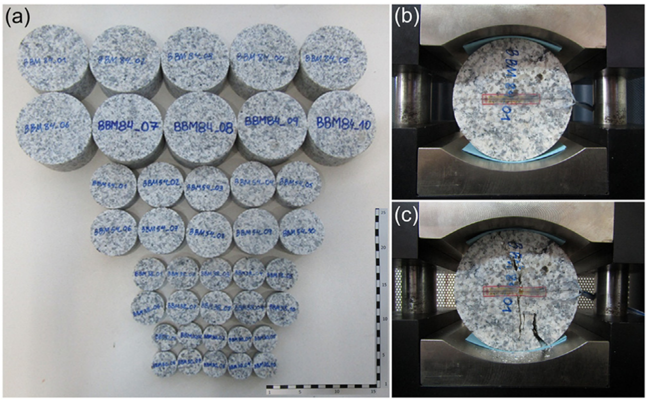

Fig. 6. (a) Specimens for BTS testing, (b) Specimen BBM84_01 before test, and (c) Specimen BBM84_01 after test.

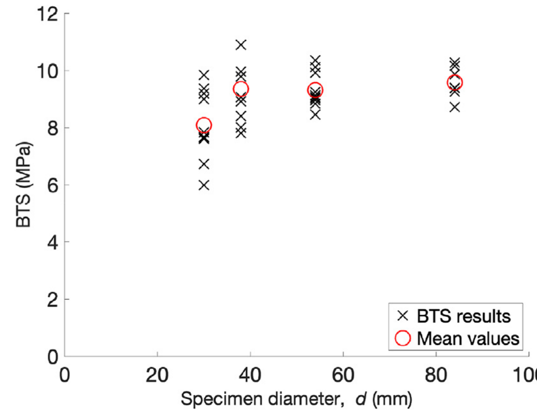

Fig. 7. Individual and average results of BTS tests in relation to specimen diameter.

Following ISRM guidelines (ISRM, 2007), it was decided that the duration of the test was about 1 min. Therefore, the loading rate was adjusted for each scenario according to the specimen diameter, to be able to keep the same stress rate (0.18 MPa/s) in all tests. Each test was stopped manually after the tensile failure occurred (Fig. 6c).

Results are summarised in Table 3 and graphed in Fig. 7. Results for each size are moderately scattered, which is attributed to the small compressive stress normal to the tensile strength generated, making the role of pre-existing cracks less relevant. While the BTS obtained for the smaller (30-mm diameter specimens) is lower, the mean values for the rest of specimen diameters (38 mm, 54 mm and 84 mm) are sensibly similar. The presence of macro-cracks associated with mineral grains may be relevant for the smaller samples, but not so much relevant for larger samples, which justifies the observed plateau.

In a few studies on BTS dependence with scale as mentioned above, a decreasing trend of BTS was observed for larger samples, usually attributed to the weakest link chain theory. This decrease in strength has not been observed in a clear way for our rock type in the range of the analysed diameters.

3.4. pCT tests

The pCT tests were performed using Blanco Mera granite specimens obtained from rock cores of 30 mm, 38 mm, 54 mm and 84 mm in diameter (Fig. 8a). The test procedure and interpretation are described in Muñoz-Ibáñez et al. (2020). To prepare the samples, the cores were first sliced into discs, and then a U-shape groove and a straight notch were cut along their generatrix (Figs. 1 and 8b). The groove was carved in several parallel saw passes using a 2 mm-thick diamond disc whose vertical position (which determines the depth of the groove) can be set with a vertical spindle.

Table 3
Strength results corresponding to BTS tests carried out with specimens of four diameters.

d (mm)	Statistic parameter	*BTS* (MPa)
30	Mean (10 tests)	8.09
	Standard error	0.39
38	Mean (9 tests)	9.2
	Standard error	0.34
54	Mean (10 tests)	9.32
	Standard error	0.19
84	Mean (10 tests)	9.18
	Standard error	0.28

Similarly, the notch was cut using a thinner diamond disc (1 mm) in one single pass. Once prepared, the specimens were oven-dried for 24 h.

The pCT specimens were loaded under pure tensile conditions using a specially designed testing device equipped with a 50 kN load cell (Muñoz-Ibáñez et al., 2020). The sample is attached to a couple of steel jaws, one of which is pulled away at a constant displacement rate of 0.1 mm/min (Fig. 8b and c). Load point displacement (LPD) was measured using two linear variable differential transformers (LVDTs) placed on both sides of the specimen. Tests were performed at room conditions, and load and LPD were recorded continuously during the experiments.

Mode I fracture toughness for pCT specimens (K_{IC}^{pCT}) is estimated by

$$K_{IC}^{pCT} = Y'_{pCT} \frac{P_{max}}{bB} \sqrt{\pi a} \tag{3}$$

where P_{max} is the peak load; b is the distance from the base of the groove to the bottom of the specimen; B is the specimen thickness; a is the length of the straight notch; and Y'_{pCT} is the dimensionless intensity factor, which is given by

$$Y'_{pCT} = C_0 + C_1\left(\frac{a}{b}\right) + C_2\left(\frac{a}{b}\right)^2 + C_3\left(\frac{a}{b}\right)^3 + C_4\left(\frac{a}{b}\right)^4 \tag{4}$$

In this work, values of the coefficients C_0–C_4 for the specimens of 54 mm and 84 mm in diameter were obtained by interpolating the values given in Muñoz-Ibáñez et al. (2020) for the specimens of 38 mm, 50 mm and 100 mm in diameter. However, for 30 mm-

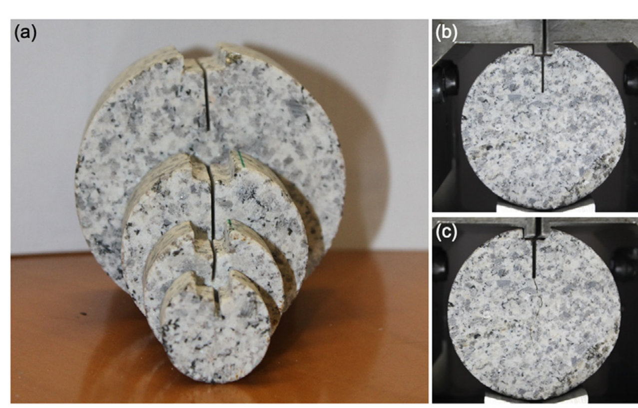

Fig. 8. (a) pCT specimens, (b) pCT specimen before testing, and (c) pCT specimen after testing.

diameter samples, it was necessary to derive the corresponding coefficients using numerical methods. To this aim, and following the procedure described in Muñoz-Ibáñez et al. (2020), Abaqus 6.14 was used to compute the Mode I stress intensity factor (K_I) using the J-integral method around the notch tip. Once K_I was obtained for each case, Y'_{pCT} was derived and values were fitted with a fourth-order polynomial function. The coefficients obtained are listed in Table 4, and K_{IC} values obtained in this study for each specimen size are listed in Table 5.

Mode I fracture toughness results of this study are graphed in Fig. 9, together with previously obtained results (Muñoz-Ibáñez et al., 2021) in 38-, 50- and 100-mm diameter Blanco Mera granite samples. Results for each size are moderately scattered, which demonstrate the rather good repeatability of the testing method even if a moderately heterogeneous rock type is tested. A growing trend of fracture toughness is clearly depicted in the corresponding graph, in line with all known relevant studies on scale effects of K_{IC} with scale mentioned above.

Apparently, the value of K_{IC}^{pCT} for the specimens of 84 mm and 100 mm in diameter is similar, which does not necessarily imply a plateau, due to the small diameter difference. Interestingly, the average toughness of the 54-mm diameter specimens is smaller than that of the 50-mm diameter samples, attributed to different origin of the samples used for this study and those recovered from a previous study (Muñoz-Ibáñez et al., 2021). Anyway, this difference is below the standard deviation observed for both groups of samples.

The growing of K_{IC}^{pCT} with scale apparently follows a similar trend to that of DTS results, which suggests a more relevant relation to this parameter than to BTS. The reasons behind this observed trend are associated with the larger quantity of energy needed to propagate a fracture starting from a notch for larger samples. It will be analysed in further details in the discussion section.

3.5. SCB tests

The SCB test is one of the suggested methods of the ISRM to compute Mode I fracture toughness of rocks due to its simplicity in terms of the specimen geometry, sample preparation, loading configuration, and testing procedure (Kuruppu et al., 2014). In this approach, semi-circular samples containing a straight notch are loaded under 3 PB. The SCB tests performed on Blanco Mera granite specimens obtained from rock cores of 38 mm, 50 mm and 100 mm in diameter were recovered from the literature (see Muñoz-Ibáñez et al., 2021 for further details) to complement this study and for comparative purposes (Fig. 10).

Table 4
Coefficients (C_i) of the dimensionless stress intensity factor (Y'_{pCT}) expression (Eq. (3)) derived for 30-, 38-, 54-, and 84-mm diameter pCT specimens.

d (mm)	C_0	C_1	C_2	C_3	C_4
30	7.661	0.029	−39.998	148.442	−116.802
38	10.278	−24.069	82.329	−136.67	127.89
54	12.866	−49.254	166.825	−259.758	192.258
84	14.48	−65.752	227.611	−354.168	245.04

Table 5
Mode I fracture toughness results corresponding to pCT $\left(K_{IC}^{pCT}\right)$ tests carried out with four diameters.

d (mm)	Statistic parameter	K_{IC}^{pCT} (MPa m$^{1/2}$)
30	Mean (6 tests)	0.81
	Standard error	0.06
38	Mean (6 tests)	1.02
	Standard error	0.04
54	Mean (5 tests)	1.09
	Standard error	0.03
84	Mean (5 tests)	1.32
	Standard error	0.04

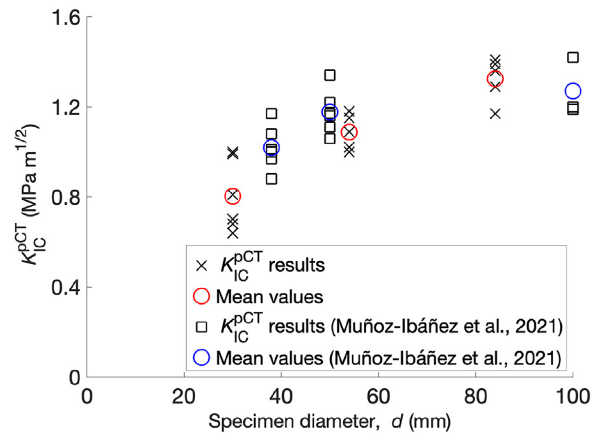

Fig. 9. Individual and average results of fracture toughness (K_{IC}^{pCT}) tests obtained specifically for this study and recovered from Muñoz-Ibáñez et al. (2021).

Mode-I fracture toughness results (K_{IC}^{SCB}) are summarised in Table 6 and graphed in Fig. 11. Results concerning mean and standard error of the mean are in the same range with those obtained with the pCT tests, which suggests that both methods are reliable to obtain this parameter, even if SCB is supported by an ISRM suggested method (Kuruppu et al., 2014). This justifies the application of the previous approach, which is more versatile in that it can better control fracture evolution and subsequently fracture energy. The same trend of growing fracture toughness with scale is also derived from these SCB test results.

4. Discussion

In the present section, the effects of testing method and specimen size are first discussed. The variability of tensile strength and fracture toughness results is studied from a statistical perspective. Additionally, a former statistical model, proposed by Brook (1985), relating strength behaviour with specimen size (diameter), is evaluated using the results obtained in this paper. The performance of several empirical relationships linking fracture toughness with tensile strength is finally analysed.

4.1. Effects of testing method and sample size

In this study, the DTS-to-BTS ratio ranges from ∼0.4 to ∼0.7 for the smallest and largest samples, respectively. For the standard size sample, this ratio is less than 0.5, therefore even smaller than 0.65 proposed by Perras and Diederichs (2014) for igneous rocks. This can be attributed to the moderate heterogeneity and interlocking of the granite under study due to the varied sizes and shapes of grains (Fig. 2), which under some compressive stress (normal to the tensile one in the Brazilian tests) is able to withstand larger tensile stresses. These growing DTS-to-BTS ratios also suggest that the influence of the testing method on tensile strength results would be magnified as the sample diameter is reduced.

A number of authors have previously reported ratios of ∼0.8–0.9 for different rock types, including samples of Lac du Bonnet granite (Martin, 1993) and Ufalei marble (Efimov, 2009). The larger strength observed in Brazilian test has been attributed to the presence of pre-existing micro-cracks (Erarslan and Williams, 2012) or to the biaxial stress state of the BTS test (Li and Wong,

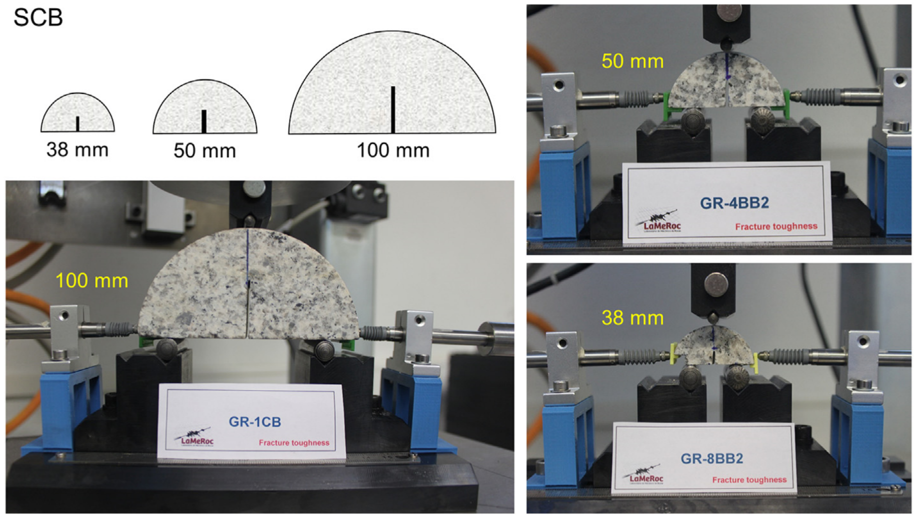

Fig. 10. Images of SCB tests at different scales.

Table 6
Mode I fracture toughness results corresponding to SCB $\left(K_{IC}^{SCB}\right)$ tests carried out with three different diameters.

d (mm)	Statistic parameter	K_{IC} (MPa m$^{1/2}$)
38	Mean (3 tests)	0.8
	Standard error	0.05
50	Mean (7 tests)	1.27
	Standard error	0.06
100	Mean (5 tests)	1.32
	Standard error	0.04

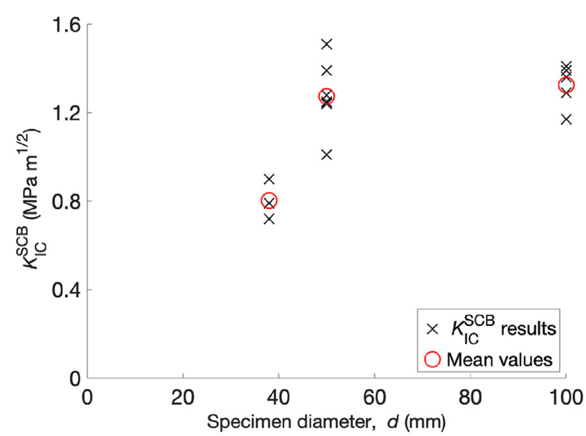

Fig. 11. Individual and average results of fracture toughness based on SCB tests (K_{IC}^{SCB}) recovered from a previous study (Muñoz-Ibáñez et al., 2021).

2013). Probably both these effects are behind these ratios tending to 1 for very large samples. In this regard, DTS specimens are loaded under pure tensile conditions, which generate a uniform tensile stress. On the contrary, BTS tests are performed by applying a compressive stress along the centre of the sample, i.e. under indirect tensile conditions.

Similarly, in the case of fracture toughness experiments, the pCT tests rely on the application of a direct tensile stress to the sample, while the SCB test relies on the application of an indirect tensile stress generated by 3 PB. In this study, the ratio of K_{IC} obtained from pCT and SCB tests is higher (~0.8–1.1), which indicates that fracture toughness would be less affected by testing conditions when compared to tensile strength for the sample sizes considered. These results are in line with those obtained previously by Wei et al. (2016), who reported a lower discrepancy for the fracture toughness values obtained using SR (i.e. pure tensile loading) and CB (i.e. 3 PB) specimens, and those obtained from CCBNB (i.e. indirect tensile loading generated by compression) tests. The different results obtained between DTS and BTS, and pCT and SCB, may be also related with the properties of the material. In a heterogeneous media such as rocks, particularly the one under study with different types and sizes of mineral grains, the internal structure and the presence of defects such as pores or micro-cracks significantly affect the mechanical response. In this sense, the transfer of loads through the medium, and therefore the stress distribution, would be dependent, for instance, on the contacts between the grain boundaries. In fact, numerical results reported by Qi et al. (2020) for Lac du Bonnet granite suggest that the large difference between shear and normal stiffnesses of the grain contacts would explain the discrepancies in the tensile strength values obtained from DTS and BTS tests.

Regarding size effects, it is usually assumed that the strength of rock decreases with sample size due to the greater probability of including micro-defects that would trigger unstable crack propagation. This is the case in several studies on Brazilian tests reported above. However, our experimental results suggest the opposite, i.e. increases in tensile strength and fracture toughness with specimen size, especially for smaller samples. As mentioned before, the mechanical properties depend on the mineralogical and textural characteristics of the rocks being tested. For DTS tests, the numerical results provided by Peng et al. (2017) show a slight increase in tensile strength with increasing heterogeneity index. This effect would be related with the orientations of the micro-cracks produced under direct tensile stress. In the case of fracture toughness, it would be expected that tests performed under 3 PB (e.g. CB and SCB) or pure tensile conditions (e.g. SR and pCT) would be less affected by size effects than those performed under compressive loading (e.g. CCNBD), as this configuration is more sensitive to the development of the FPZ, yielding more conservative (i.e. lower) results (Wei et al., 2016, 2017). However, Ghouli et al. (2021) also reported size-dependent K_{IC} values for three different rock types tested following the SCB testing approach, and they attributed this effect to the formation of a large FPZ ahead of the crack tip. As depicted in Fig. 12, the size effects would be more significant in the case of Blanco Mera granite than those reported by Ghouli et al. (2021) for a similar rock type.

The growing trend of K_{IC} with sample size can be also interpreted from a fracture mechanics perspective. Accordingly, the theoretical approaches based on fracture mechanics were developed, which can be of help for understanding the observed scale-dependent rock response. Blunt fracture (Bažant, 1984) and fractal approaches (Carpinteri, 1994) were initially tried with not so reliable results. For instance, Scavia (1996) justifies his increasing K_{IC} values with scale using the fractal approach requiring a larger amount of energy to generate rougher tensile cracks in larger specimens. Later on, the theory of critical distances has served some authors to justify the evolution of the apparent fracture toughness in notched specimens (Justo et al., 2017), but the extension of this theory to analyse the scale effects is so far unclear.

Relatively recent approaches based on finite fracture mechanics (Leguillon, 2002; Cornetti et al., 2006; Chao Correas et al., 2021) could represent a more suitable way to understand the results, once a wider and more reliable database is available. According to this approach to propagate a fracture, a threshold stress level in the crack tip should be attained (typically scale-independent), but at the same time, sufficient energy should be input into the system (which depends on the size of the sample). The fracture will propagate when the induced force (P) is sufficient to initiate the crack and input sufficient energy into the system as illustrated in Fig. 13, which shows an illustrative graph based on finite fracture mechanics principles suggesting how larger size samples may produce larger fracture stiffness values. Therefore, the values of K_{IC} will grow with the size of the sample as observed but these increments will diminish for larger sample sizes. This, in combination with the granular nature of granitic samples with relative smaller grains for larger sample sizes (Fig. 14), can help to understand the trends derived from the performed tests.

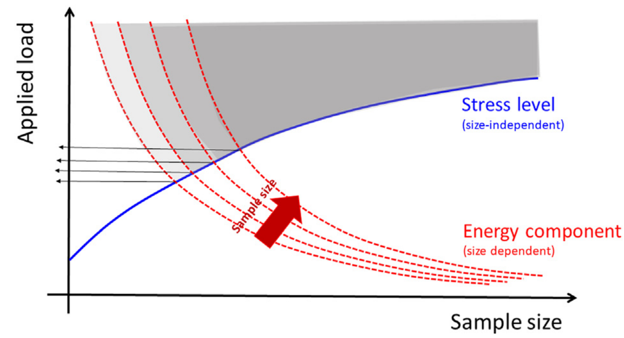

Fig. 13. Conceptual approach of increasing values of K_{IC} with sample size based on the principles of finite fracture toughness.

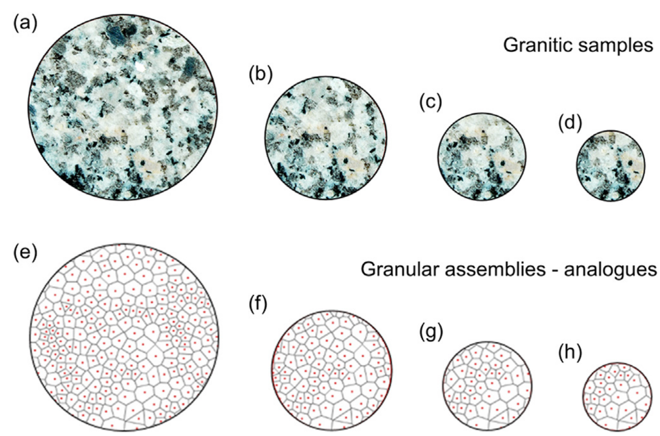

Fig. 14. Different size granitic sample surfaces for (a) 84 mm, (b) 54 mm, (c) 28 mm and (d) 30 mm in diameter and equivalent granular assemblies (e) to (h).

4.2. Variability of tensile strength and K_{IC} results

Experimental data were statistically analysed using the free software Past 3.0 (Hammer et al., 2001) with the aim of determining representativeness, repeatability and comparability of the results obtained with the four specimen sizes. This analysis is complementary to the graphical approach based on the box and whiskers plots, which can be useful to visually illustrate variability but limited in terms of significance.

To this aim, first we separated in groups the experimental data associated with each testing method (DTS, BTS and K_{IC}) and for each specimen size (30-, 38-, 54- and 84-mm diameter). Then, we performed within-group and between-group analyses to check repeatability and reproducibility, respectively.

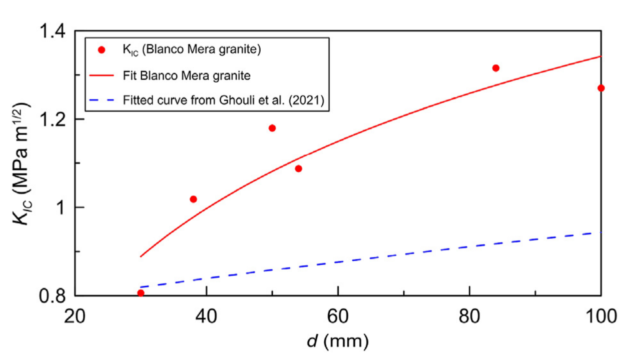

Fig. 12. Mode I fracture toughness (K_{IC}) as a function of specimen diameter (d) for Blanco Mera granite. Data from Ghouli et al. (2021) for granite is also plotted for comparison.

The within-group analysis consisted of the assessment of normality for each group of samples using a Shapiro-Wilk test, which is suitable for small sample populations. This test returns a test statistic ($W = 0$–1) and a probability value, p. For the analysis conducted in this study, we used a significance level of 95% ($a = 0.05$) so that the rejection of the null hypothesis (H_0) can be verified if the p-level is below this significance. Considering the experimental data obtained from valid tests, it was found that all the groups analysed conformed to normal distributions, with the only exception of the largest samples of DTS tests. The corresponding mean values were computed for each group of specimens, and the results are listed in Table 7.

Then, we performed a between-group analysis to compare the means and assess size effects for each testing method. This approach was based on a one-way analysis of variance (ANOVA) and the non-parametric Mann-Whitney pairwise test. From a statistical perspective, for both the DTS and BTS tests performed in specimens with $d \geq 38$ mm, relatively low significant influence of the sample size is observed. Nevertheless, the variability within each group and the number of tests performed may influence this analysis. Something similar can be observed for Mode I fracture toughness results for diameters in the range of 38–54 mm.

4.3. Comparison of results with existing statistical models

The results obtained from DTS and BTS tests were used to analyse the performance of already developed statistical models, to

Table 7
Mean and standard error of the mean derived from the statistical analysis for each specimen diameter (d) and testing method (DTS, BTS and K_{IC}).

d (mm)	DTS (MPa)	BTS (MPa)	K_{IC} (MPa m$^{1/2}$)
84	6.06 ± 0.11	9.18 ± 0.28	1.32 ± 0.04
54	4.45 ± 0.16	9.32 ± 0.19	1.09 ± 0.03
38	5.56 ± 0.33	9.2 ± 0.34	1.02 ± 0.03
30	3.69 ± 0.32	8.09 ± 0.39	0.81 ± 0.06

capture the typically observed descending trend of the strength with the increase of the specimen size (Weibull, 1939; Brook, 1985; Wang et al., 2020). The proposed model, particularly applied for tensile strength (Masoumi et al., 2018), is expressed by Eq. (5), as a modification of the equation proposed by Brook (1985), wherein k_2 is a positive constant exponent controlling the statistical decay of the strength with an increase in size.

$$\frac{\sigma_t}{\sigma_{t50}} = \left(\frac{50}{d}\right)^{k_2} \tag{5}$$

For the samples used in this study, the diameter corresponding to 54 mm can be set as the characteristic size, according to Wang et al. (2020). These features imply a slight modification of Eq. (5), which can be expressed by

$$\frac{\sigma_t}{\sigma_{t54}} = \left(\frac{54}{d}\right)^{k_2} \tag{6}$$

The model described with Eq. (6) can be fitted to the results through nonlinear least squares obtained both for DTS and BTS, in the way shown by Fig. 15, by considering $\sigma_{t54} = 4.45$ MPa (for DTS tests) and $\sigma_{t54} = 9.32$ MPa (for BTS tests).

In these cases, the obtained fits, with particularly low coefficients of determination (R^2) for DTS test results, were ascending, giving therefore negative k_2 exponents, i.e. $k_2 = -0.258$ (for DTS tests) and $k_2 = -0.151$ (for BTS tests). These coefficients were obtained with 95% confidence bounds.

4.4. Performance of tensile strength–K_{IC} correlations

As described in Section 2.4, several correlations between K_{IC} and tensile strength have been proposed for different materials (Gunsallus and Kulhawy, 1984; Whittaker et al., 1992; Hanson et al., 1994; Zhang et al., 1998; Zhang, 2002; Backers, 2004; Wang et al., 2007). The mean K_{IC}, as obtained from the pCT tests, was used to evaluate the performance of four empirical correlations related to rocks (Table 8), for each specimen diameter.

As can be observed in Table 7, all the empirical relationships tend to overestimate tensile strength results, if compared with those obtained from DTS tests. This fact is even more obvious if tensile strength (empirically obtained DTS) is plotted against specimen diameter (Fig. 16).

As it can also be noted in Table 7, Hu et al. (2022)'s approach yielded K_{IC} results through the mean DTS values, with similar trends as those obtained with empirical correlations (fitting models).

In general, it must be pointed out that some of the relationships indicated in Section 2.4 may not be sufficiently accurate, given that they were derived from a compilation of data from different tests.

5. Conclusions

In this work, we have investigated the potential effect of the specimen size on the tensile strength and fracture toughness of a given rock type (Blanco Mera granite). From a good number of tests, it can be concluded that the tensile strength and fracture toughness do depend on the specimen size, in such a way that an increase in size entails an increase in these properties. This size effect was marked in the case of DTS and fracture toughness results.

A relatively high variability of results has generally been found in general terms that could be caused by the heterogeneous nature of the rock (presence of mica clusters or large plagioclase crystals).

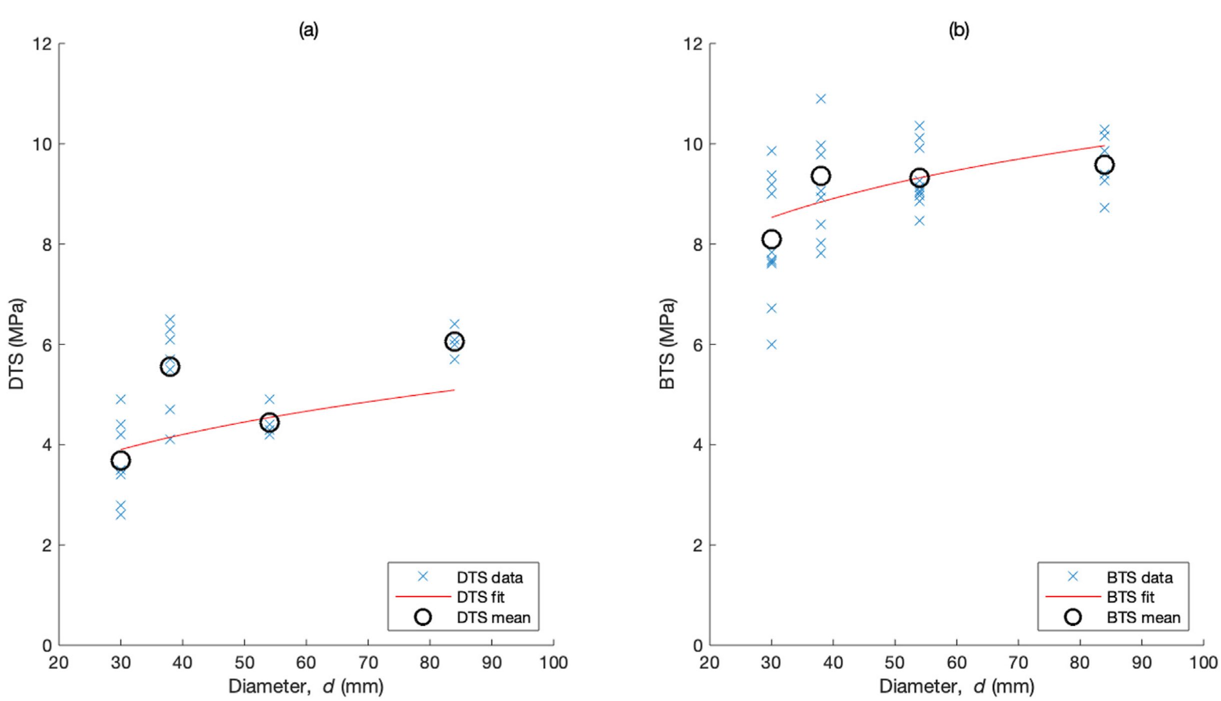

Fig. 15. DTS (a) and BTS (b) results with corresponding mean values and best fits according to Eq. (6) ($R^2 = 0.06$ for (a), and $R^2 = 0.2$ for (b)).

Table 8
Estimated values of the tensile strength (σ_t) from different empirical relationships with K_{IC} for the diameters considered in this study.

d (mm)	Mean K_{IC} (MPa $m^{1/2}$)	Eq. (2) (C_{ch} = 3 mm)	σ_t (MPa) (estimated from empirical correlation)				Mean DTS (MPa)
			Gunsallus and Kulhawy (1984)	Whittaker et al. (1992)	Zhang et al. (1998)	Zhang (2002)	
84	1.32	1.15	9.81	10.55	8.17	7.61	6.06
54	1.09	0.84	7.66	9.37	7.26	4.48	4.45
38	1.02	1.05	7.01	8.99	6.96	3.53	5.56
30	0.81	0.7	5.04	7.79	6.04	0.68	3.69

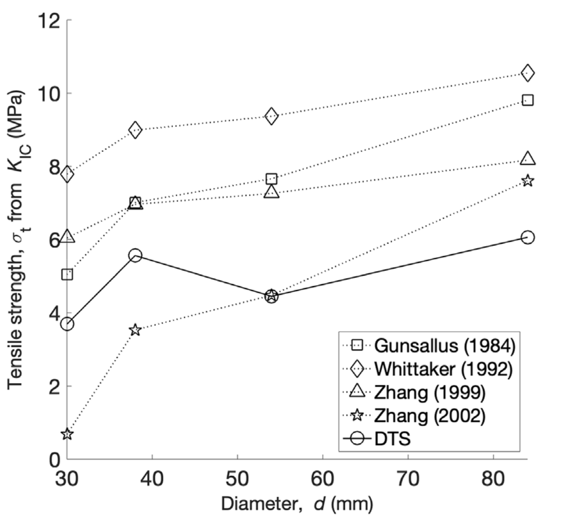

Fig. 16. DTS tests results compared with σ_t as obtained from empirical K_{IC}-σ_t relationships.

This variability has been particularly marked for results obtained for those smaller samples, where appropriate loading conditions may not be kept.

DTS-to-BTS ratios were observed to be smaller than those commonly reported in the literature for igneous rocks, and in the range of 0.4–0.7, for the smallest and largest specimens, respectively. This suggests that the influence of the testing method on tensile strength results becomes more relevant for smaller specimen sizes. In addition to the inherent heterogeneity, it must be noted that for a given diameter, the failure surface for DTS is $\pi/2$ greater than that for BTS tests, which may influence the onset of the specimen failure. Moreover, the presence of compressive stresses in BTS tests could also increase the apparent tensile strength obtained.

Some recent approaches on finite fracture mechanics can help understand the increasing K_{IC} trends with size, as derived from fracture toughness tests, taking into account the stress level necessary to propagate the fracture (typically size-independent) as well as the required energy to be input to the system (size-dependent). Regarding K_{IC} ratios from pCT and SCB tests, they were in the range of 0.8–1.1, indicating less influence of the type of test on results.

The present work also contributes to the rock mechanics database with a relatively large body of results concerning DTS and BTS tests, as well as fracture toughness values for different specimen sizes.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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List of symbols

d	Diameter
E_s	Splitting tensile elastic modulus
E_t	Tensile elastic modulus
K_{IC}	Mode I fracture toughness
K_{IC}^{pCT}	Mode I fracture toughness based on pCT tests
K_{IC}^{SCB}	Mode I fracture toughness based on SCB tests
σ_t	Tensile strength
σ_{t50}	Specific tensile strength for a sample with characteristic size of 50 mm
σ_{t54}	Specific tensile strength for a sample with characteristic size of 54 mm

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