etamm

3rd Emerging Trends in Applied Mathematics and Mechanics

COORDINACIÓN EDITORIAL
María Teresa Cao Rial; Ángel D. Arós Rodríguez

20-24 de maio

20
24

ETAMM 2024

UNIVERSIDADE DA CORUÑA

ETS Náutica e Máquinas
Universidade da Coruña

3^{rd} Emerging Trends in Applied Mathematics and Mechanics

ETAMM 2024

May, 20–24, 2024

Third Emerging Trends in Applied Mathematics and Mechanics. Book of Abstracts

Coordinación Editorial:

María Teresa Cao Rial; Ángel D. Arós Rodríguez

Idioma: Inglés

Colección: Cursos, congresos e simposios, CCS-160

Depósito legal: exento

ISBN: exento

DOI: https://doi.org/10.17979/spudc.000028

HANDLE: http://hdl.handle.net/2183/36337

Edita: Servizo de Publicacións. Universidade da Coruña.

N.º de páxinas: 83

Índice: p. 1

Como citar: *Third Emerging Trends in Applied Mathematics and Mechanics*. Book of abstracts. Universidade da Coruña. University Press, 2024.

Edición: Universidade da Coruña, Servizo de Publicacións

<http://www.udc.gal/publicacions>

Deseño e maquetación: María Teresa Cao Rial

Contents

Preface

Welcome to ETAMM2024!

This volume contains the abstracts of more than fifty oral presentations to be given at the International Conference *Emerging Trends in Applied Mathematics and Mechanics* (ETAMM 2024) hosted by the School of Nautical Studies and Marine Engineering of the University of A Coruña, Spain.

It is the third edition of a series of conferences in Applied Mathematics. Previous editions took place in June 2016 at the University of Perpignan Via Domitia, France (ETAMM 2016) and June 2018 at the Jagiellonian University in Krakow, Poland (ETAMM 2018). Unfortunately, the series was interrupted in 2020 by the eruption of the COVID-19 pandemic.

The main aim of this series of conferences is to identify new trends in the domains of Applied Mathematics and Mechanics, and their Applications in Physics, Engineering Sciences, Biology and Economy and provide an opportunity for the dissemination of important and recent results. The conference will bring together mathematicians, interdisciplinary scientists and engineers from all over the world. I hope that it will be a scientifically enriching and socially exciting event for all the participants.

The conference will consist of ten invited lectures, four specialized minisymposia and more than twenty contributed talks, focused on recent results and future challenges in the field. Selected and peer reviewed papers will be included in a special issue of the Springer international revue SeMA Journal, which is the official journal of the Spanish Society of Applied Mathematics (SeMA).

I would like to thank the invited speakers for their valuable presentations, and all the other participants for turning this conference into a great success. Furthermore, I wish to express my gratitude to the members of the International Scientific Committee and the organizers of minisymposia for their help in the scientific coordination and logistic assistance.

The conference is rendered possible by the generous financial and organizative support of the University of A Coruña, the Galician Centre for Mathematical Research and Technology (CITMAga), the Centre for Information and Communications Technology Research (CITIC), the Spanish Society of Applied Mathematics (SeMA), the A Coruña province administration (Deputación da Coruña) and the A Coruña council, through the Turism Consortium (Con-

sorcio de Turismo do Concello da Coruña). I am really grateful to all these organizations for their help in making this meeting a reality. This conference has also received funding from the European Union's Horizon 2020 Research and Innovation Programme, under the Marie Sklodowska-Curie Grant Agreement No 823731, CONMECH. Special mention deserves the School of Nautical Studies and Marine Engineering through its Managing Team for hosting the conference in this special and historical building, one of the oldest in our University, and for facilitating the logistics of the everyday organization with their availability to the Organizing Committee inquiries.

Last but not least, special thanks go to my colleagues in the Organizing Committee Teresa, Carmen and María for their tireless effort, dedication and hard work in the preparation of this meeting and to Luis Fraguela for creating and updating the web page.

Ángel Arós

Chair of the Organizing Committee of ETAMM2024.

Invited Lectures

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

On a thickness optimization of plates in dynamic contacts

Igor Bock [1]

[1] Institute of Computer Science and Mathematics FEI,
Slovak University of Technology Bratislava, Slovak Republic

igor.bock@stuba.sk

Abstract: We deal with optimal design problems governed by initial-boundary value problems for a hyperbolic variational inequality describing deflections of simply supported viscoelastic and elastic plates vibrating against an inner rigid obstacle. A variable thickness of each plate plays the role of a control variable. The deriving of *a priori* estimates of accelerations plays the crucial role in proofs of existence theorems. While in the viscoelastic case the acceleration term does not appears in a weak form of the problem, in the elastic case it has a form of a vector measure in the same way as in [1], where the constant thickness of the plate was considered. The set of admissible states for the design problem consists of solutions of a state problem gained as limits of the sequences of functions solving penalized problems. This restriction is due to overcoming the lack of admissible estimates of accelerations of all possible states. It enables us to receive an optimal thickness of a plate as a limit of a sequence of optimal thicknesses solving penalized optimal control problems. We assume the generalized penalized function $u \mapsto \eta^{-1}\beta\left(u - \frac{1}{2}e - \Phi\right)$ with a deflection u, a thickness function e and an obstacle function Φ. In the case of a differentiable function β and other more regular data it is possible to derive generalized optimality conditions starting with penalized problems. It is an addition to the considered control problems in [2] and [3].

References

[1] I. Bock, J. Jarušek, M. Šilhavý, Existence of solutions to a dynamic contact problem for a thermoelastic von Kármán plate. *Nonlinear Analysis RWA*, **32**, 111–136 (2016).

[2] I. Bock, M. Kečkemétyová, An optimal control problem for a viscoelastic plate in a dynamic contact with an obstacle. *Tatra Mountains Math. Pub.*, **71**, 27–37 (2018).

[3] I. Bock, An optimal design problem for an anisotropic elastic plate in a dynamic contact with an obstacle. *Proceedings of Applied Mathematics and Mechanics*, **23(4)**, 8p (2023).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

Periodic subdifferential evolution inclusions in reflexive Banach spaces

Zhenhai Liu[1], Nikolaos S. Papageorgiou

[1] Guangxi Minzu University, China

zhhliu@hotmail.com

Abstract: We consider a subdifferential evolution inclusion with a multivalued perturbation and periodic boundary condition. Using approximation techniques, fixed point theory and tools from multivalued analysis and from nonlinear analysis, we prove three existence theorems.

Invited Lecture

Quasi-variational-hemivariational inequalities for incompressible Navier-Stokes system with Bingham fluid

Stanislaw Migórski[1]

[1] Faculty of Mathematics and Computer Science
Jagiellonian University in Krakow, Poland

stanislaw.migorski@uj.edu.pl

Abstract: In the talk we consider the stationary incompressible Navier-Stokes equations with mixed multivalued boundary conditions which model a generalized Newtonian fluid of Bingham-type. The corresponding boundary value problem involves a nonmonotone version of the slip condition of frictional type described by the Clarke subgradient law of a locally Lipschitz potential and an implicit obstacle constraint set. The Bingham-type model in its weak form is expressed as a constrained elliptic quasi-variational-hemivariational inequality. For the latter, the nonemptiness and compactness of the solution set is proved. The upper semicontinuity property of the solution set with respect to a parameter appearing in the data is also established. Finally, within the framework of optimal control, a double minimization problem for the fluid model is examined and the existence of its solution is proved.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

Grounded shallow ice sheets melting as an obstacle problem

Paolo Piersanti[1] and Roger Temam

[1] Department of Mathematics and Institute for Scientific Computing and Applied Mathematics, Indiana University, USA

ppiersan@iu.edu

Abstract: In this talk, which is the result of a joint work of the speaker with Roger Temam (Indiana University), we will study a model describing the evolution of the thickness of a grounded shallow ice sheet. Since the thickness of the ice sheet is constrained to be nonnegative, the problem under consideration is an obstacle problem.
A rigorous modelling exercise shows that this model, which is time-dependent, is governed by a set of variational inequalities that involve nonlinearities in both the time derivative and in the elliptic term. In order to establish the existence of solutions for the time-dependent model we recovered, formally, upon completion of the aforementioned modelling exercise, we first depart from a penalized relaxation, and we show - by resorting to a discretization in time - that the corresponding relaxed problem admits at least one solution.
Secondly, by means of Dubinskii's lemma and other new results and new inequalities, we extract compactness for the family of solutions of the relaxed problems and we show that this family of solutions converges to a solution of a doubly nonlinear parabolic variational inequality akin to the one that was recovered formally.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

Proposal of homogenized masonry models. Application in the simulation of blast furnaces.

Patricia Barral, Luis J. Pérez-Pérez and Peregrina Quintela[1]

[1] University of Santiago de Compostela (USC) and CITMAga, Spain

peregrina.quintela@usc.es

Abstract: In this lecture, two new homogenization methodologies proposed by the authors for linear elastic masonry type structures will be presented [1]. These methodologies are based on different numerical tests inspired by basic tensile and compression experimental tests. It will be analysed under which hypothesis the proposed methodologies lead to equivalent results to those deduced for homogenized models in the literature using a rigorous mathematical analysis [2]. The advantage of the proposed methodologies is that they eliminate the typical periodicity and zero-mean conditions on the unit cell required by the classical homogenized models, so that the homogenized material characteristics, of special interest in engineering, can be obtained using standard commercial software. The application of these models to the homogenization of the ceramic cup of the blast furnace will be presented. The numerical results will also be compared with other homogenization methodologies commonly used in engineering.

References

[1] L.J. Pérez-Pérez, *Contributions to the mathematical modelling and numerical simulation of some problems related to blast furnace ironmaking.* Ph.D., MINERVA (USC) repository. http://hdl.handle.net/10347/30356

[2] D. Cioranescu, P. Donato, An introduction to homogenization. *Oxford Lecture Series in Mathematics and Its Applications*, **17**, Oxford University Press, (1999).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

The new Damped Normal Compliance condition, and detachment waves in friction

Meir Shillor[1]

[1] Oakland University, Rochester, MI, USA

shillor@oakland.edu

Abstract: The talk introduces the new Damped Normal Compliance (DNC) contact condition and describes its merits. It consists of the usual normal compliance condition with an added damping term. Some of its analysis is based on a 'simple' system in which a mass contacts a reactive obstacle with the DNC. Some of the results are somewhat surprising. We apply it to the contact of an elastic rod with a reactive foundation, too. It is found that by adding damping, the natural oscillations die out. Also, it is found that the mechanical concept of 'restitution coefficient' is not well defined, since it is not a constant, but it depends on the applied force and initial velocity. We also present the results of some computer simulations of both cases.
Then, the talk describes some initial results on the generation of detachment waves, in which a system in frictional contact overcomes the frictional resistance and starts to slide.This is exemplified in a system with two or three masses that move on a rail in the presence of friction. The long term interest in this project is to correlate the process of sliding with the system settings and parameters.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

Well-posedness concepts and convergence results

Mircea Sofonea[1]

[1] University of Perpignan Via Domitia, France

sofonea@univ-perp.fr

Abstract: We start by recalling the concepts of Tykhonov and Levitin-Polyak well-posedness for minimization and variational inequality problems. These concepts are based on two main ingredients: the existence of a unique solution to the considered problem and the convergence to it of a special class of sequences, the so-called approximating sequences. Inspired by these properties, we define a new mathematical object, the so-called Tykhonov triple, denoted by $\mathcal{T}$. Then, we introduce a new concept of well-posedness for abstract problems in metric spaces, the $\mathcal{T}$-well-posedness concept, which extends both the Tykhonov and Levitin-Polyak well-posedness concepts, among others. The theory of $\mathcal{T}$-well-posedness problems we construct gives necessary and sufficient conditions which guarantee the convergence to the solution of a nonlinear problem, unifies different convergence results and provides a framework in which the link between different problems can been established. It can be used in the study of a large class of nonlinear problems like fixed point problems, minimization problems, inequality problems and various inclusions, for instance. Details can be found in [1]. We illustrate the theory in the study of elliptic variational inequalities and history-dependent inclusions in Hilbert spaces. For these problems we state and prove $\mathcal{T}$-well-posedness results and convergence criteria, as well. Finally, we apply our abstract results in the study of two mathematical models which describe the equilibrium of an elastic body with an obstacle. We also present numerical simulations which validate the corresponding convergence results. In this way we illustrate the cross fertilization between the abstract mathematical concepts, on one hand, and their applications in Contact Mechanics, on the other hand.

References

[1] M. Sofonea, *Well-posed Nonlinear Problems. A Study of Mathematical Models of Contact.* Birkhäuser, Cham (2023).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

Optimal control problems for elliptic variational and hemivariational inequalities and their asymptotic behaviors

Domingo A. Tarzia[1]

[1] Austral University and CONICET, Argentine

DTarzia@austral.edu.ar

Abstract:
We consider a bounded domain Ω in $\mathbb{R}^d$ whose regular boundary Γ consist of the union of three disjoint portions Γ_i, $i = 1, 2, 3$ with $meas(\Gamma_i) > 0$. We formulate the following nonlinear elliptic problem with mixed boundary conditions:

$$-\Delta u = g \text{ in } \Omega, \quad u\big|_{\Gamma_1} = 0, \quad -\frac{\partial u}{\partial n}\big|_{\Gamma_2} = q, \quad -\frac{\partial u}{\partial n}\big|_{\Gamma_3} \in \alpha\, \partial j(u), \tag{1}$$

where α is a positive constant, $g \in L^2(\Omega)$, $q \in L^2(\Gamma_2)$ and the function $j \colon \Gamma_3 \times \mathbb{R} \to \mathbb{R}$, called a superpotential (nonconvex potential), is such that $j(x, \cdot)$ is localy Lipschitz for a.e. $x \in \Gamma_3$ and not necessary differentiable. Such multivalued condition on Γ_3 is denoted for a nonmonotone relation expressed by the generalized gradient of Clarke. The weak formulation of (1) is given by the elliptic hemivariational inequality:

$$\text{find } u \in V_0 \text{ such that } a(u, v) + \alpha \int_{\Gamma_3} j^0(u; v)\, d\Gamma \geq L(v), \quad \forall v \in V_0, \tag{2}$$

where j^0 represent the generalized (Clarke) directional derivative, $a(u, v) = \int_\Omega \nabla u \nabla v\, dx$, $L(v) = \int_\Omega gv\, dx - \int_{\Gamma_2} qv\, d\gamma$ and $V_0 = \{v \in H^1(\Omega) : v = 0 \text{ on } \Gamma_1\}$.
We formulate for each $\alpha > 0$, different optimal control problems (C_α), on the internal energy g and the heat flux q, for quadratic cost functional and we prove existence results for the optimal solutions. We also consider a problem as (1), with a Dirichlet condition on Γ_3 and we formulate similar optimal control problems (C), on control variables g and q. We obtain, convergence results for optimal controls and system states (C_α) to the corresponding optimal control and system state (C), when the parameter α goes to infinity. Moreover, we study simultaneous distributed and Neumann boundary optimal control problems for elliptic hemivariational inequalities and their asymptotic behaviors.

Invited Lecture

Penalization of the state system in topology optimization problems

Dan Tiba[1]

[1] Institute of Mathematics, Romanian Academy, Bucharest, Romania

dan.tiba@imar.ro

Abstract: The penalization approach has a long history and enters the family of fictitious domain methods in the approximation of differential equations. Initially motivated by the application of finite differences or finite element techniques in domains with a complicated geometry, it was extended in the 80's to the solution of free boundary problems that also involve finding geometric unknowns like the coincidence set for the obstacle problem. More recently, such ideas found their way in the treatment of shape and topology optimization problems, where the main unknowns are connected open subsets (not necessarily simply connected) from a bounded holdall domain. This fixed domain methodology has clear advantages in allowing topology changes during the optimization process, in the convergence analysis and in devising new algorithms or at the computational level, since it avoids remeshing and recomputing the mass matrix in each iteration, etc. It also has many applications: to second order elliptic equations with various boundary conditions, to higher order equations, to variational inequalities, to other nonlinear boundary value problems like the Navier-Stokes system. However, there are as well cases when the penalization of the state system is not possible or difficult to use. In our talk, we shall discuss many such situations and indicate new results and arguments in each case.
We underline that this approach uses essentially various properties of the state system, adapted to each situation and should be distinguished from the level set or the phase field methods, that are also very popular in free boundary problems and in optimal design, as fixed domain techniques. In the last cases, one also applies penalization, but only for the corresponding cost functionals, while the state system is just perturbed by the introduction of certain control parameters.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Invited Lecture

Review of my contributions to modeling and computing in solid mechanics

Juan M. Viaño[1]

[1] University of Santiago de Compostela and CITMAga, Spain

juan.viano@usc.es

Abstract: I take a tour of my most important contributions in modeling and computing in solid mechanics made throughout my research career, dividing them into the following two groups:
1.*Asymptotic methods*. Obtention of mathematical models for elastic, viscoelastic or piezoelectric beams using asymptotic methods as the diameter of the cross section tends to zero. Straight and curved beams (arches), solid and thin-walled sections (open and closed profiles) will be distinguished. The best-known classic bending-extension-torsion models in elasticity (Bernoulli, St. Venaint, Timoshenko and Vlasov) are rigorously justified and generalized. I will also show the use of this technique to obtain contact models with another solid for elastic and viscoelastic beams and shells.
2. *Numerical methods*. Analysis of finite elements method with penalty-duality algorithms for solving the contact problems of an elasto-visco-plastic solid with another rigid or also deformable solid, including some examples of application in orthodontics. I will also show numerical simulation for a real application in design of steering wheels and another for human bone remodelling.

References

[1] L. Trabucho, J.M. Viaño, Mathematical Modelling of Rods. In *Handbook of Numerical Analysis* Vol. IV. Ciarlet, P.G.-Lions, J. L. Edts. North-Holland. pp. 487-974 (1996).

[2] A.D. Rodríguez-Arós, M. Sofonea, J.M. Viaño, Numerical analysis of a frictional contact problem for viscoelastic materials with long-term memory. *Numer. Math.* **108**, pp. 327–358 (2007).

Minisymposia

Minisymposium 1: Recent trends in differential equations and inequalities

Organized by

Leszek Gasiński
Institute of Mathematics, University of the National Education Commission, Krakow, Poland

and

Anna Ochal
Chair of Optimization and Control, Jagiellonian University, Krakow, Poland

Description: The minisymposium is poised to provide a comprehensive overview of the latest advancements in the field of partial differential equations, boundary value problems and differential inequalities. This minisymposium aims to bring together experts, researchers, and practitioners to discuss emerging trends, innovative techniques, and applications in these fundamental areas of mathematics. Within this minisymposium, attendees will delve into the pure mathematics aspects, where discussions will center around novel analytical and topological approaches to differential equations and inequalities, shedding light on their underlying structures and properties. In parallel, the minisymposium will spotlight the practical and applied dimensions, revealing how these mathematical tools are harnessed to address complex problems across fields such as physics, engineering, mechanics or biology.

List of speakers:

- Eleonora Amoroso, University of Messina, Italy
- Piotr Bartman-Szwarc, Jagiellonian University, Poland
- Krzysztof Bartosz, Jagiellonian University, Poland
- Pasquale Candito, Mediterranean University of Reggio Calabria, Italy
- Antonia Chinnì, University of Messina, Italy

MS1: Recent trends in differential equations and inequalities.

Existence results for discrete boundary value problems

Eleonora Amoroso[1]

[1] University of Messina, Italy

eleonora.amoroso@unime.it

Abstract: In this talk, we present some results on the existence of a priori bounded solutions for nonlinear difference equations, obtained combining variational methods, truncation techniques and a priori bounds theory. Moreover, we offer a link between the discrete problems and the continuous ones, solving second order differential problems with the results obtained for the discrete case. In particular, we construct the solution as the limit of a sequence of linear interpolations of the discrete solutions and we give a priori bounds of the solution u and its derivatives u', u''.

References

[1] E. Amoroso, P. Candito, J. Mawhin, Existence of a priori bounded solutions for discrete two-point boundary value problems, *J. Math. Anal. Appl.*, **519 (2)**, 18 p. (2023)

MS1: Recent trends in differential equations and inequalities.

Advancements in nonsmooth optimization for solving hemivariational inequalities.

Piotr Bartman-Szwarc[1], Adil Bagirov and Anna Ochal

[1] Jagiellonian University, Krakow, Poland

piotr.bartman@uj.edu.pl

Abstract: In this talk, we will review optimization methods for solving hemivariational inequality problems. As is commonly known, these problems often require the application of nonsmooth optimization methods. We will particularly focus on the aggregate subgradient method which consists of null and serious steps. In each null step, only two subgradients - aggregate subgradient and the subgradient computed at the current iteration point - are used to determine search directions.

MS1: Recent trends in differential equations and inequalities.

Rothe method for a class of evolutionary second order variational-hemivariational inequalities

Krzysztof Bartosz[1]

[1] Jagiellonian University, Krakow, Poland

krzysztof.bartosz@uj.edu.pl

Abstract: We consider a second order variational-hemivariational inequality modeling a dynamic visco-elastic problem of contact mechanics. We assume that the contact process is governed by a normal damped response condition with a unilateral constraint and that the body is non-clamped. The existence and uniqueness result for this problem was obtained in [1] by means of the Rothe method. Now we discuss several possible generalizations of the above mentioned result. The first idea is to replace the linear viscosity operator by a nonlinear one. The second modification relies on removing the viscosity term which leads to a noncoercive problem. Simultaneously, we replace the normal damped contact condition related to velocity by the one related to the displacement, which is more reasonable from the physical point of view. Finally, we consider a supplementation of the contact condition by an additional adhesive term. All these ideas can open new perspectives in the study of variational hemivariationa inequalities.

References

[1] K. Bartosz, Convergence of Rothe scheme for a class of dynamic variational inequalities involving Clarke subdifferential, *Applicable Analysis*, **97 (13)**, pp. 2189-2209 (2018).

MS1: Recent trends in differential equations and inequalities.

Multiple solutions for a nonlocal Dirichlet problem involving the p-Laplacian

Pasquale Candito[1]

[1] Mediterranean University of Reggio Calabria, Italy

pasquale.candito@unirc.it

Abstract: The aim of this talk is to present a multiplicity result concerning a nonlinear boundary problem with a degenerate nonlocal term depending on the L^q norm of a solution and on the p-Laplacian operator. In particular, it is highlighted that the number of positive solutions doubles the number of "positive bumps"of the degenerate term. The solutions are also ordered according to the L^q norms.

References

[1] P. Candito, L. Gasiński, R. Livrea, J.R. Santos Júnior, Multiplicity of positive solutions for a degenerate nonlocal problem with p-Laplacian, *Adv. Nonlinear Anal.*, **11 (1)**, 357–368 (2022).

MS1: Recent trends in differential equations and inequalities.

On existence and multiplicity of solutions for some problems involving variable exponent

Antonia Chinnì[1]

[1] University of Messina, Italy

achinni@unime.it

Abstract: We present recent results concerning the existence and multiplicity of solutions related to differential problems involving the functions

$$h_p : \Omega \times \mathbb{R} \to \mathbb{R} \quad \text{e} \quad \tilde{h}_p : \Omega \times \mathbb{R}^N \to \mathbb{R}^N$$

defined respectively by

$$h_p(x,\xi) := |\xi|^{p(x)-2}\xi, \;\; \forall (x,\xi) \in \Omega \times \mathbb{R} \quad \text{and} \quad \tilde{h}_p(x,\xi) := |\xi|^{p(x)-2}\xi, \;\; \forall (x,\xi) \in \Omega \times \mathbb{R}^N$$

with

- Ω bounded and open subset of $\mathbb{R}^N$, $N \geq 1$,
- $p \in \{l \in C(\bar{\Omega}) : l(x) \geq 1, \; \forall x \in \bar{\Omega}\}$.

In particular, we will address the study of some problems in which the involved differential operators are included in the following typologies

- $\Delta^2_{p(x)} u := \Delta(h_p(x, \Delta u(x))$ called $p(\cdot)$–biharmonic operator
- $\Delta_{w,p(x)}(u) := div(w(x)\tilde{h}_p(x, \nabla u(x)))$ called degenerate $p(\cdot)$-Laplacian operator.

Minisymposium 2: Nonlinear analysis methods for differential problems

Organized by

Eleonora Amoroso,
University of Messina, Italy

and

Pasquale Candito
Mediterranean University of Reggio Calabria, Italy

Description: The main purpose of this minisymposia is to present some very recent topics and results concerning nonlinear differential problems, with a special attentions to variational-hemivariational inequalities, differential inclusions and some meaningful related issues, such as existence, multiplicity, qualitative properties of solutions and so on. Emphasis is placed on investigations that exploit the synergy between nonlinear analysis methods, like critical point theory, in both smooth and non-smooth frameworks, fixed point theorems and set-valued analysis. Moreover, differential problems arising in the modelling of important mechanical and engineering phenomena are considered.

List of speakers:

- Anna Ochal, Chair of Optimization and Control, Jagiellonian University, Krakow
- Leszek Gasiński, Institute of Mathematics, University of the National Education Commission, Krakow
- Giuseppe Failla, University of Messina, Italy
- Valeria Morabito, University of Messina, Italy
- Bruno Vassallo, University of Messina, Italy

MS2: Nonlinear analysis methods for differential problems.

Parameters estimate for p-Laplacian sub-critical singular problem

Giuseppe Failla[1]

[1] University of Messina, Italy

giuseppe.failla@studenti.unime.it

Abstract: We consider the following singular elliptic problem

$$\begin{cases} -\Delta_p u = \lambda f(x,u) + \mu g(x,u) & \text{in} \quad \Omega, \\ u > 0 & \text{in} \quad \Omega, \\ u = 0 & \text{on} \ \partial\Omega, \end{cases} \tag{3}$$

where $\Omega \subseteq \mathbb{R}^N$ $(N \geq 3)$ is a bounded domain, $p \in (1, N)$, λ, μ are real positive parameters, $\Delta_p u := \text{div}(|\nabla u|^{p-2}\nabla u)$ is the classical p-Laplacian operator, $f : \Omega \times (0, +\infty) \to [0, +\infty)$ and $g : \Omega \times [0, +\infty) \to [0, +\infty)$ are nonnegative Carathéodory functions satisfying suitable growth conditions. In particular, f is mildy singular near zero and g possesses a subcritical growth. Combining variational with truncation techniques, a computable estimate of the parameters λ, μ, for which the problem (3) admits a positive solution, is established. Furthermore, if g has a p-superlinear growth and, in addition, it fulfills a classical unilateral Ambrosetti-Rabinowitz condition, a second positive solution is obtained by using the Mountain Pass Theorem, in the same range of parameters. This type of elliptic problem has a wide range of applications in the so-called non-Newton fluids. Particularly, media with $p > 2$ are called dilatant fluids, and media with $p < 2$ are called pseudoplastics, while for $p = 2$ they are called Newtonian fluids.

References

[1] P. Candito, G. Failla, R. Livrea, Positive solutions for a p-Laplacian equation with sub-critical singular parametric reaction term, *submitted*

MS2: Nonlinear analysis methods for differential problems.

Least energy sign-changing solutions for Kirchhoff double phase problems

Leszek Gasiński[1], Ángel Crespo-Blanco and Patrick Winkert

[1] University of the National Education Commission, Krakow, Poland

leszek.gasinski@up.krakow.pl

Abstract: Presentation concenrs the nonlocal Dirichlet equation of double phase type with the right hand side function of superlinear and subcritical growth. The existence of two constant sign solutions (one positive, the other one negative) and of a sign-changing solution which has exactly two nodal domains and which turns out to be the least energy sign-changing solution is presented. The proof is based on variational tools in combination with the quantitative deformation lemma and the Poincare-Miranda existence theorem.

MS2: Nonlinear analysis methods for differential problems.

Robin double phase problems with variable exponents

Valeria Morabito[1]

[1] University of Messina, Italy

valeria.morabito2@studenti.unime.it

Abstract:
In this talk we present existence results for nonlinear double phase problems with variable exponents under critical Robin boundary conditions. In the framework of Musielak-Orlicz Sobolev spaces, we obtain the existence of two nontrivial weak solutions with opposite energy sign through critical point theory. Moreover, we present some cases in which the solutions turn out to be nonnegative.

References

[1] E. Amoroso, V. Morabito, Nonlinear Robin problems with double phase variable exponent operator, *preprint.*

MS2: Nonlinear analysis methods for differential problems.

On a new penalty method for quasivariational inequalities

Anna Ochal[1]

[1] Jagiellonian University, Krakow, Poland

anna.ochal@uj.edu.pl

Abstract: The aim of this talk is to present a convergence result concerning an elliptic quasivariational inequality in a reflexive Banach space. Considering a sequence of unconstrained variational-hemivariational inequalities, we show that a sequence of their unique solutions converges to the solution of the quasivariational inequality. Finally, we apply the abstract results in the study of elastic frictional contact problem with unilateral constraints.

It is a joint work with P. Bartman-Szwarc, M. Sofonea and D. Tarzia.

MS2: Nonlinear analysis methods for differential problems.

Three weak solutions to a periodic second order boundary value problem with highly discontinuous nonlinearity

Bruno Vassallo[1] and **Roberto Livrea**

[1] University of Messina, Italy

bruno.vassallo@studenti.unime.it

Abstract: We consider the following periodic boundary value problem with the Sturm-Liouville equation

$$\begin{cases} -(pu')' + qu = \lambda f(x, u(x)) & \text{in }]0, T[, \\ u(0) = u(T),\ u'(0) = u'(T), \end{cases} \qquad (P_\lambda)$$

where $p, q \in L^\infty([0, T])$ satisfying $p(0) = p(T)$, $q_0 := \operatorname{essinf}_{[0,T]} q > 0$, $p_0 := \operatorname{essinf}_{[0,T]} p > 0$ and $f : [0, T] \times \mathbb{R} \to \mathbb{R}$ is an almost everywhere continuous function satisfying suitable growth conditions. Several fields of theoretical and applied mathematics give rise to ordinary differential problems with discontinuous nonlinearities, in particular, here, the set of the points of discontinuity of the nonlinear term f may also be uncountable. The existence of three weak solutions to problem (P_λ) is established, for an appropriate range of the parameter λ, combining variational methods with the critical point theory for non-differentiable functions.

References

[1] Livrea R., Vassallo B., Three weak solutions to a periodic boundary Sturm-Liouville problem with discontinuous reaction, *preprint*, (2023).

Minisymposium 3: Numerical methods for coupled problems in porous media

Organized by

Kundan Kumar,
University of Bergen, Norway

Description: Most of the applications in porous media involve coupling of flow, transport, mechanical, and chemical effects. Examples include geoengineering applications such as carbon sequestration, geothermal energy, and energy storage; biological applications dealing with brains, bones, and cells; and technical applications such as fuel cells. These multiphysics processes are often described by a set of coupled partial differential equations with highly heterogeneous coefficients posed in a possibly mixed dimensional setting. Efficient, stable, and convergent numerical schemes for such problems require careful design of suitable iterative and discretization schemes. The mathematical modelling and numerical analysis of these schemes involve several topics of the conference including variational inequalities, fixed point methods, nonlinear boundary value problems, and applications involving contact mechanics. This minisymposium will gather talks that focus on numerical methods for porous media applications.

List of speakers:

- Laura Portero, Public University of Navarre, Spain
- Florin A. Radu, University of Bergen, Norway
- Kundan Kumar, University of Bergen, Norway
- Francisco J. Gaspar, University of Zaragoza, Spain
- Nadia S. Taki, University of Bergen, Norway
- Nicolae Suciu, Romanian Academy of Sciences, Romania

MS3: Numerical methods for coupled problems in porous media.

A novel oscillation-free numerical scheme for Biot's model

Francisco J. Gaspar[1], Alvaro Pé de la Riva, Carmen Rodrigo, James Adler, Xiaozhe Hu and Ludmil Zikatanov

[1] University of Zaragoza, Spain.

fjgaspar@unizar.es

Abstract: Flow problems on deformable porous media are usually modelled by the Biot's model. In this work we propose a new stabilized finite element scheme that provides numerical solutions that are free of non-physical oscillations, and that, at the same time, allows us to iterate the fluid and mechanic problems in a convergent way to obtain the solution of the whole coupled system. We present numerical results illustrating the robust behavior of both the stabilization and iterative solver with respect to the physical and discretization parameters of the model.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

MS3: Numerical methods for coupled problems in porous media.

Iterative methods for coupled flow and geomechanics including friction in fractured porous media

Kundan Kumar[1], Tameem Almani, Mircea Sofonea and Maarten de Hoop

[1] University of Bergen, Norway

kundan.kumar@uib.no

Abstract: Coupling of flow and mechanical deformations in a subsurface is relevant for a wide range of applications including hydrocarbon recovery, geothermal energy production, and geological hazard assessment. Faults and fractures are mechanically the most vulnerable regions for mechanical failures. Moreover, they have a strong influence on the flow profiles. Understanding the mechanics of fractures and their interaction with fluids is therefore a topic of great importance in geosciences.
We consider a poromechanics model describing the coupling of flow and geomechanics in a fractured porous medium. The ingredients of the model include: linear elasticity equation for porous matrix; flow equation both in the porous matrix (3D) and along the fracture surface (2D) – leading to a mixed dimensional PDE model; transmission conditions across the fracture and matrix interface including the description of normal and frictional contact forces – leading to variational inequality. The resulting model is the extended Biot equations in a fractured porous domain with contact boundary conditions leading to a variational inequality coupled to a system of linear parabolic PDEs. Iterative methods are suitable for such problems as they decompose the problem in their individual constituents of flow and mechanical problems. The individual equations can be accordingly solved using well-developed solvers and pre-conditioners. Moreover, the splitting methods allow flexibility in terms of time stepping for each of these equations. However, design of such iterative schemes require careful considerations as poorly designed schemes may lead to unstable schemes. We propose a fully discrete iterative scheme for solving this model, extending the widely used fixed-stress splitting scheme for the Biot equations. We show that the fixed stress split scheme is a contraction. We show this for both conformal and mixed finite elements in space and a backward Euler discretization in time.

MS3: Numerical methods for coupled problems in porous media.

Space-time parallel solvers for the solution of parabolic problems

Laura Portero[1], Iñigo Jimenez, Andrés Arrarás and Francisco J. Gaspar

[1] Public University of Navarre, Pamplona, Spain

laura.portero@unavarra.es

Abstract: Many phenomena in science and engineering are governed by evolutionary partial differential equations. In some cases, as when considering three-dimensional models and/or long-time simulations, the amount of computational work can be a bottleneck to obtain fast and accurate approximations to the solution. In order to take advantage of the computational capabilities of modern parallel clusters, significant research is being conducted in the field of space-time parallel methods. In this work, we propose a combination of the parallel-in-time parareal algorithm (cf. [2]) and several time-splitting schemes that allow for spatial parallelization (cf. [1]). Both dimensional and domain decomposition partitioning strategies can be considered for the solution of parabolic problems. The main theoretical results will be illustrated by a collection of numerical experiments. Finally, some applications to porous media flow problems will be commented.

References

[1] W. Hundsdorfer, J. Verwer Numerical solution of time-dependent advection-diffusion-reaction equations, *In Springer Ser. Comput. Math.*, **33**, Springer-Verlag, Berlin/Heidelberg (2003).

[2] J.L. Lions, Y. Maday, G. Turinici, Résolution d'EDP par un schéma en temps "pararéel". *C. R. Acad. Sci. (1) - Math.*, **337(7)**, pp. 661-668 (2001).

MS3: Numerical methods for coupled problems in porous media.

On splitting schemes for poromechanics

Florin A. Radu[1]

[1] University of Bergen, Norway

florin.radu@uib.no

Abstract: Poromechanics, i.e. flow in deformable porous media, plays an important role in many relevant practical applications like e.g. geothermal energy extraction, biomedical research or CO2 storage. The typical mathematical models behind poromechanics are consisting on coupled, linear or nonlinear partial differential equations. This talk will focus on the design and analysis of efficient and robust splitting schemes, mainly based on the fixed-stress splitt, for numerically solving these equations. The L-scheme will be used for linearizations. Stabilization and optimization will be discussed as well [2]. Further, a new family of splitting schemes based on approximate Schur complement will be presented [1].

References

[1] R. Nuca, E. Storvik, F.A. Radu, M. Icardi, Splitting schemes for coupled differential equations: block Schur-based approaches and partial Jacobi approximation. *ArHiv 2212.11111* (2023).

[2] E. Storvik, J. W. Both, K. Kumar, J.M. Nordbotten, F.A. Radu, On the optimization of the fixed-stress splitting for Biot's equations. *Int. J. Numer. Methods Eng.*, **120**, pp. 179–194 (2019).

MS3: Numerical methods for coupled problems in porous media.

From systems of particles to continuous fields: a space-time upscaling

Nicolae Suciu[1]

[1] Tiberiu Popoviciu Institute of Numerical Analysis,
Romanian Academy - Cluj-Napoca Branch, Romania

nsuciu@ictp.acad.ro

Abstract: Discrete systems of particles with piece-wise analytical trajectories are described macroscopically by almost everywhere continuous fields. The macroscopic description is achieved through space-time averages on d-dimensional cubes and symmetric time intervals of a kinematic description available at the discrete level. The latter consists of piece-wise analytic time functions modeling physical properties of the particles. The fields defined by these averages verify unclosed relations having the structure of the hydrodynamic balance equations. One obtains in this way a versatile tool which enables, for instance, derivations of the Liouville and Schrödinger equations, verifications of material laws for granular materials, hydrodynamic analysis of financial time-series, continuous modeling of lipidic bilayers, with applications in cell biology. A practical application that will be presented in more detail is the space-time upscaling of the reactive transport in variably saturated porous media. The kinematic description is provided by an ensemble of as many random walkers as molecules involved in chemical reactions. The space-time scales of the averaging procedure may be chosen as representative of the macroscopic observations and experimental measurements.

MS3: Numerical methods for coupled problems in porous media.

Existence and uniqueness of a frictional adhesion contact problem

Nadia Skoglund Taki[1]

[1] University of Bergen, Norway

Nadia.Taki@UiB.no

Abstract: Frictional contact problems including adhesion processes have high importance in both industry and geophysical applications. The intensity of the adhesion β is modelled by a nonlinear ODE and has its physical meaning in the interval $[0, 1]$; $\beta = 1$ when the bonds are active, $0 < \beta < 1$ when the bonds are partially active, and $\beta = 0$ when the bonds are severed. The ODE is coupled with a nonlinear variational inequality. The difficulty is to prove that β is in $[0, 1]$ without making restrictions on the physical system. In this talk, I will present a new existence and uniqueness result for a coupled nonlinear frictional adhesion model that verifies these physical properties.

Minisymposium 4: New results in asymptotic analysis of elastic and viscoelastic shells

Organized by

Ángel Arós,
University of A Coruña and CITMAga, Spain

and

Paolo Piersanti
Indiana University, USA

Description: In solid mechanics, shells are three-dimensional structures of small thickness compared to the extension they cover. Such structures are abundant in nature (eggs, snails, turtles, blood vessels,...) but also in industry (ship hulls, plane fuselage, roofs, glasses, tires...). One of the reasons for this popularity is because of their ability to sustain applied loads in a very effective way, with the minimum amount of material they require, and the lightness and economy that this represents. In this session, the participants will show some of their most recent results in the mathematical theory of elastic and viscoelastic shells. By using asymptotic analysis, the authors will show convergence results, error estimates and numerical simulations for a variety of new models for shells, some of them involving contact conditions.

List of speakers:

- Ángel Arós, University of A Coruña and CITMAga, Spain
- Gonzalo Castiñeira, University of Santiago de Compostela, Spain
- Célio Fernandes, CMAT, University of Minho and University of Porto, Portugal
- Paolo Piersanti, University of Indiana, USA
- Lucas Venturato, Austral University and CONICET, Argentine

MS4: New results in asymptotic analysis of elastic and viscoelastic shells.

Asymptotic analysis of Koiter shells in normal compliance contact

Ángel Arós[1]

[1]University of A Coruña and CITMAga, Spain.

angel.aros@udc.es

Abstract: We consider a model for Koiter linear elastic shells in contact with a deformable obstacle. Following [1], where the case without contact was analysed, here we study the convergence of the solution of this model towards the solutions of the corresponding contact problems for elastic elliptic membrane shells and elastic flexural shells when the small parameter of the model (thickness) tends to zero, all depending on the type of boundary conditions, curvature of the middle surface and order of the applied forces.

References

[1] P.G. Ciarlet, *Mathematical elasticity. Vol. III: Theory of shells*, Studies in Mathematics and its Applications, **vol. 29**. North-Holland Publishing Co., Amsterdam (2000).

MS4: New results in asymptotic analysis of elastic and viscoelastic shells.

Error estimates in asymptotic analysis of linearly viscoelastic shells for the membrane case

Gonzalo Castiñeira[1], **Ángel Arós and Lucas D. Venturato**

[1] University of Santiago de Compostela, Spain

gonzalo.castineira.veiga@usc.es

Abstract: We consider a family of linearly viscoelastic elliptic membrane shells, all sharing the same middle surface and with their lateral face clamped. Under these conditions, when the thickness tends to zero, the solution of the three-dimensional problem converges to the solution of a two-dimensional reduced problem for a viscoelastic membrane shell (see [1]–[2]). In this communication we will show error estimates for this convergence by using a corrector method.

References

[1] G. Castiñeira, Á. Rodríguez-Arós, Linear viscoelastic shells: an asymptotic approach. *Asymptotic Analysis*, **107**, pp. 169-–201 (2018).

[2] G. Castiñeira, Á. Rodríguez-Arós, On the justification of the viscoelastic elliptic membrane shell equations. *Journal of Elasticity*, **130**, pp. 85-–113 (2018).

MS4: New results in asymptotic analysis of elastic and viscoelastic shells.

Exploring wear phenomena in elastic elliptic membrane shells

Ángel Arós, Célio Fernandes[1] and Sabrina D. Roscani

[1]Center of Mathematics (CMAT), University of Minho and Transport Phenomena Research Center (CEFT), University of Porto, Portugal

`cbpf@fe.up.pt`

Abstract:
We consider a family of linearly elastic elliptic membrane shells, all sharing the same middle surface, with thickness 2ε, clamped along their entire lateral face. Moreover, tractions may act on the upper face of a shell and it may become in frictional contact along its lower face with a moving deformable foundation. Friction is modeled taking into account the effect of material wear on contact surfaces [1]. By using asymptotic analysis, we show that when the small parameter ε tends to zero, the solution of the three-dimensional variational contact problem converges to a limit that is independent of the transverse variable and which can be identified with the solution of a two-dimensional variational problem describing deformations and wear on the common middle surface.

References

[1] M. Shillor, M. Sofonea, J.J. Telega Elastic contact. In: *Models and Analysis of Quasistatic Contact.* **7** Springer, Berlin, Heidelberg, pp. 101–115 (2004).

MS4: New results in asymptotic analysis of elastic and viscoelastic shells.

On the justification of Koiter's model for linearly elastic shells confined in a half-space. Theory and numerical analysis.

Paolo Piersanti[1]

[1] Department of Mathematics and Institute for Scientific Computing and Applied Mathematics, Indiana University, USA

ppiersan@iu.edu

Abstract: In this talk, we will review some recent results on the modelling, analysis and numerical analysis of Koiter's model for linearly elastic shells in the case where the shell under consideration is constrained to remain confined in a prescribed half-space. To begin with, we will see how to recover the constrained Koiter's model starting from the classical three-dimensional equations of linearized elasticity in the case where the shell under consideration is an elliptic membrane or a flexural shell.
Then, we will propose a Finite Element scheme for approximating the solution of the problem under consideration and we will present a series of numerical experiments corroborating the results previously obtained.

MS4: New results in asymptotic analysis of elastic and viscoelastic shells.

Error estimates for the elliptic membrane case of a linearly elastic shell in normal compliance contact

Lucas D. Venturato[1], María Teresa Cao-Rial and Sabrina D. Roscani

[1] Austral University and CONICET, Argentine

LVenturato@austral.edu.ar

Abstract: In mechanics, models for elastic shells, where one dimension (the thickness) is significantly smaller than the others, have been extensively studied. Different boundary conditions are considered to model distinct phenomena. Among these, the contact condition has received recent attention. This work focuses on a family of linearly elastic shells in normal compliance contact with a deformable foundation. A formal asymptotic analysis and a convergence result to an associated two-dimensional problem, as the thickness tends to zero, were obtained in [1]. We present error estimates for the approximation of three-dimensional solution with their associated two-dimensional counterpart. The proof employs a corrector method, assuming a certain regularity for the normal component in the solution of the two-dimensional limit problem.

References

[1] Á. Rodríguez-Arós and M.T. Cao-Rial, Asymptotic analysis of linearly elastic shells in normal compliance contact: convergence for the elliptic membrane case, *Z. Angew. Math. Phys.*. **69**, pp. 115 (2018).

Contributed Presentations

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Stochastic modelling of rough surfaces in tribology

Iñigo Arregui[1] **and Carlos Vázquez**

[1] Departament of Mathematics and CITIC, University of A Coruña, Spain

inigo.arregui@udc.es

Abstract: Lubricated journal bearing devices take part in many engines and mechanical processes in industry, so that they have been the object of many researches in modelling, mathematical analysis and numerical simulation. Most of models are based on Reynolds equation for the lubricant pressure, which can be understood as a limit model for thin films of fluid. As some random small manufacturing defects can be introduced in the surfaces of the journal or the bearing, the models need to incorporate a random roughness on the expression of the gap between them, which enters into Reynolds equation. The presence of this random component gives rise to the so called stochastic Reynolds equation, the solution of which is a stochastic process. Following [1], the gap is expressed as a Karhunen–Loève expansion. Additionally, cavitation phenomena arise due to the convergent divergent geometry of the gap, which are not considered in [1]. An original aspect of the present work is the consideration of a cavitation model in the context of the stochastic Reynolds equation. More precisely, we incorporate a linear complementarity formulation, which can be equivalently written as a variational inequality [2]. The cavitation region corresponds to zero pressure value (coincidence set). For the numerical solution, we combine a Monte Carlo simulation technique with an augmented Lagrangian active set algorithm [3] for the complementarity formulation and a finite element technique for the spatial discretization.

References

[1] T. Gustafsson, H. Hakula, M. Leinonen, Stochastic Galerkin approximation of the Reynolds equation with irregular film thickness. *Comput. Math. Appl.*, **74**, pp. 1590–1606, (2017).

[2] G. Bayada,M. Chambat, Sur quelques modélisations de la zone de cavitation en lubrification hydrodynamique. *J. Méch. Théor. Appl.*, **5**, pp. 703–729, (1986).

[3] T. Kärkkäinen K. Kunisch, P. Tarvainen, Augmented Lagrangian active set methods for obstacle problems. *J. Optim. Th. Appl.*, **119**, pp. 499–533, (2003).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

A nonlinear PDE model for the pricing of Renewable Energy Certificates

María A. Baamonde-Seoane[1], María del Carmen Calvo-Garrido and Carlos Vázquez

[1] Departament of Mathematics and CITIC, University of A Coruña, Spain

maria.baamonde1@udc.es

Abstract: In this work we present two valuation methods for Renewable Energy Certificates (RECs) or green certificates [3]. For this purpose, starting from a system of FBSDEs and using Ito lemma, we propose a mathematical model based on a semilinear PDE with two stochastic factors: the accumulated green certificates sold by an authorized generator and the natural logarithm of the renewable electricity generation rate. One main novelty of the work comes from the numerical treatment of the non-linear convective term in the PDE. Thus, we propose the use of the Bermúdez-Moreno algorithm [2] to deal with this nonlinearity. This duality algorithm is based on the Yosida regularization of non-linear maximal monotone operators. The resulting linear problem is discretized by using two methods. Firstly, we use a characteristics scheme to discretize the material derivative operator in one of the spatial directions combined with the use of a second order implicit finite differences scheme in the other spatial direction. Secondly, we use a Lagrange-Galerkin method which mainly consists of Crank-Nicolson characteristics scheme for time discretization combined with finite elements for the discretization in the accumulated green certificates and the natural logarithm of the renewable generation rate directions [1]. Finally, we show some numerical results that illustrate the performance of the proposed model and the numerical methods.

References

[1] M. A. Baamonde-Seoane, M. C. Calvo-Garrido, C. Vázquez, Pricing renewable energy certificates with a Crank-Nicolson Lagrange-Galerkin numerical method, *Journal of Computational and Applied Mathematics*, **422**, pp. 114891 (2023).

[2] A. Bermúdez, C. Moreno, Duality methods for solving variational inequalities, *Computers & Mathematics with Applications*, **7**, pp. 43–58, (1981).

[3] M. Coulon, J. Khazaei, W. B. Powell, SMART-SREC: A stochastic model of the New Jersey solar renewable energy certificate market, *Journal of environmental Economics and Management*, **73**, pp. 13–31, (2015).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Existence and uniqueness of solutions to rate independent systems with history variable of integral type

Livia Betz[1]

[1] University of Würzburg, Germany

livia.betz@uni-wuerzburg.de

Abstract: This talk focuses on rate independent systems (RIS) where the dissipation functional depends not only on the rate but also on the *history* of the state. The latter is expressed in terms of a Volterra integral operator. We establish an existence result for the original problem and for the control thereof. Under a smoothness condition, we prove the uniqueness of solutions to a certain class of history dependent RIS with unbounded dissipation potentials. In this context, we derive an essential estimate that opens the door to future research on the topic of optimization.

An energy-consistent discretization of hyper-viscoelastic contact models for soft tissues

Francesco Bonaldi[1], **Mikaël Barboteu, Serge Dumont and Christina Mahmoud**

[1] University of Perpignan Via Domitia, France

francesco.bonaldi@univ-perp.fr

Abstract: We propose a mathematical model of hyper-viscoelastic problems applied to soft biological tissues, along with an energy-consistent numerical approximation. We first present the general problem in a dynamic regime, with certain types of dissipative constitutive assumptions. We then provide a numerical approximation of this problem, with the main objective of respecting energy consistency during contact in adequacy with the continuous framework. Given the presence of friction or viscosity, a dissipation of mechanical energy is expected. Moreover, we are interested in the numerical simulation of the non-smooth and non-linear problem considered, and more particularly in the optimization of Newton's semi-smooth method and Primal Dual Active Set (PDAS) approaches. Finally, we test such numerical schemes on academic and real-life scenarios, the latter representing the contact deployment of a stainless-steel stent in an arterial tissue.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Generalized systems with variable exponents

Maria-Magdalena Boureanu[1]

[1] Department of Applied Mathematics, University of Craiova, Romania

mmboureanu@yahoo.com

Abstract: We rely on the critical point theory to study the weak solvability of a general class of elliptic systems involving Leray-Lions type operators. The preference for this class of nonhomogeneous differential operators is provided by their versatility, as they can produce, in particular, Laplace operators, orthotropic operators, mean curvature operators, and Laplace-like operators which are connected to capillary phenomena. Our work is conducted in the framework of the Lebesgue and Sobolev spaces with variable exponents and we explore the anisotropic case in which we have a different behavior on different space directions.

Optimal Shape Design Problem of Nonlinear Gao Beam

Jana Burkotová[1] and Jitka Machalová

[1] Faculty of Science, Palacký University Olomouc, Czech Republic

jana.burkotova@upol.cz

Abstract: The contribution is devoted to a shape optimization problem of a nonlinear elastic beam where the state problem equation is governed by a one-dimensional static Gao beam model. The aim is to find the optimal thickness distribution of a stepped beam in various optimization aspects. Primary, the focus is on maximizing the stiffness of the beam represented by compliance cost functional under an imposed condition preserving the beam volume. The question of stability in terms of maximal axial load leading to convex problems and avoiding the buckling phenomenon is also discussed. The corresponding existence analysis and subsequent convergence analysis for the problem discretization by finite elements are provided. The theory is illustrated by numerical examples.

Differential and Integral Operators in the Study of Viscoelastic Fluids

Luís L. Ferrás[1]

[1] University of Porto, Portugal and CMAT - University of Minho, Portugal

lferras@fe.up.pt

Abstract: This work investigates the connection between classical and fractional viscoelastic models, focusing on the core mathematical principles that form the basis of these constitutive equations [1]. To overcome the inherent viscoelastic-solid behaviour observed in fractional viscoelastic models employing a springpot element, we investigate the fractional-dashpot element [2], which demonstrates a behaviour lying between the Maxwell model and a standard dashpot. We then introduce a generalised model comprising two elements.

References

[1] R.T. Leiva, L.L. Ferrás, A. Castelo, M.L. Morgado, M. Rebelo, J. Bertoco, A.M. Afonso, A generalisation of the integral Maxwell model: the gK-BKZ model—frame invariance and analytical solutions. *Meccanica* (2024).

[2] D. Yao, A fractional dashpot for nonlinear viscoelastic fluids. *Journal of Rheology* **62.2**, pp. 619-629 (2018).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

A posteriori error estimates for the Darcy-Forchheimer model

María González Taboada[1] and Hiram Varela Rodríguez

[1] Departament of Mathematics and CITIC, University of A Coruña, Spain

maria.gonzalez.taboada@udc.es

Abstract: The Darcy-Forchheimer model describes the fluid flow through porous media for moderate to high velocities. We consider here the mixed formulation introduced by Pan and Rui [1]. We present an adaptive mixed finite element method based on a posteriori error estimates derived for this formulation. We show several numerical experiments that support the theoretical results.

References

[1] H. Pan, H. Rui,: Mixed element method for two-dimensional Darcy-Forchheimer model. *J. Sci. Comput.* **52**, pp. 563-587 (2012).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Parameter identification in contact problems for nonlinear Gao beam and obstacle.

Jitka Machalová[1] and Jana Radová

[1] Faculty of Science, Palacký University Olomouc, Czech Republic

jitka.machalova@upol.cz

Abstract: This study focuses on identifying material parameters in contact problems using the nonlinear beam model developed by D.Y. Gao in 1996. Governed by a nonlinear fourth-order ordinary differential equation, the model describes a beam supported unilaterally by a perfectly rigid or elastic foundation. Determining the beam's material coefficients and the modulus of an elastic foundation is formulated as an optimal control problem, employing a least squares cost functional. This contribution addresses theoretical aspects, examining the existence of a solution to this identification problem and its finite element approximation. Several numerical examples will be presented to illustrate the practical application.

Quasi-geostrophic equation with fractional dissipation

Elżbieta Motyl[1]

[1] University of Lodz, Poland

elzbieta.motyl@wmii.uni.lodz.pl

Abstract: The two-dimensional surface quasi-geostrophic equation

$$\frac{\partial \theta}{\partial t} + (v \cdot \nabla)\theta \; = \; 0$$

is important in modeling geophysical fluid dynamics. Here $\theta : \mathbb{R}^2 \to \mathbb{R}$ represents the temperature and $v : \mathbb{R}^2 \to \mathbb{R}^2$ is the velocity of the fluid. In addition, various generalizations of this equation are studied. We consider stochastic fractionally dissipative QGs equation with a multiplicative Gaussian noise

$$\begin{cases} d\theta(t) + [(\mathcal{R}\theta(t) \cdot \nabla)\theta(t) + \kappa(-\Delta)^{\alpha}\theta(t)]\, dt = G(t, \theta(t))\, dW(t), \\ \theta(0) = \theta_0. \end{cases}$$

Here $\alpha \in (0, 1)$. The velocity is expressed in terms of the temperature by the relation $v = \mathcal{R}\theta$. A typical example is $\mathcal{R} = (-R_2\theta, R_1\theta)$, where R_j denotes the j-th Riesz transform $(j = 1, 2)$. Using appropriate approximations and compactness method we prove the existence of a weak solution. In the sub-critical case, i.e. $\alpha \in (\frac{1}{2}, 1)$, we also prove uniqueness of the solution.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Post-buckling analysis of the static 1D Gao beam

Horymír Netuka[1] and Jitka Machalová

[1] Faculty of Science, Palacký University Olomouc, Czech Republic

horymir.netuka@upol.cz

Abstract: If a beam is subjected to an axial compressive load, its potential energy becomes nonconvex as soon as this load reaches a certain limit value. This phenomenon is called buckling. In a mathematical sense, buckling is a bifurcation in the solution of static equilibrium equations.
The nonlinear model developed by D.Y. Gao in 1996 is intended for solution to buckling problems and post-buckling analysis. The problems are not easy to investigate, however, using data analysis on a large number of calculated examples can help to obtain interesting results.
We want to provide an overview of our research, focusing on the latest results from the analysis of the original static Gao beam model and taking into account our latest works relating to the buckling phenomenon caused by both axial and transverse loading.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Rigorous derivation of weakly dispersive shallow water models with large amplitude topography variations

Martin Oen Paulsen[1] and Louis Emerald

[1] University of Bergen, Norway

Martin.Paulsen@UiB.no

Abstract: The water wave equations are deduced from the Euler equations under the assumption of irrotational flow and are widely used to model the propagation of waves in a fluid. However, there are serval difficulties related to the complexity of these equations, both from an analytical perspective and from the perspective of applications. To overcome some of these issues, one typically considers simplified models characterized by dimensionless parameters that describe the main mechanisms involved.

In this talk, I will explain the mian steps involved in deriving weakly dispersive models with bathymetry from the water waves equations. The goal is to understand the difficulties related to having large amplitude variations of the bottom, where we derive asymptotic models that capture both the "weakly nonlinear regime" and the "shallow water regime."

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

A method to detect contact between a small ellipsoid and other quadrics

María José Pereira-Sáez[1], Miguel Brozos-Vázquez, María José Souto-Salorio and Ana Dorotea Tarrío Tobar

[1] University of A Coruña, Spain

maria.jose.pereira@udc.es

Abstract: A recurrent problem in very different contexts as computer graphics, computer animation, robotics or industrial manufacturing is to detect contact between bodies. The first step to solve these problems is to select geometric shapes which are easy to handle and that allow to bound the target objects. Then, we need to search for contact detection algorithms between the chosen geometric objects.
Quadric surfaces, specially ellipsoids, allow to approximate very accurately a large variety of shapes. We use the roots of the characteristic polynomial between an ellipsoid and another quadric to detect contact between them. This result was applied to give a low cost computational method to avoid collisions between an UAV and a rigid surface. Also, we find that if the ellipsoid is small compared with the other quadric, the contact detection algorithm becomes very simple. Indeed, in many cases the evaluation of the discriminant suffices to detect contact. As an application, we give a method to detect contact between a small ellipsoid and a combination of quadrics.

Towards Efficient Simulations: An Arlequin-based Multi-Scale Method for Direct Energy Deposition Addition Manufacturing

Miguel Picos[1], **Patricia Barral, Peregrina Quintela and Jerónimo Rodríguez**

[1] AIMEN, CITMAga and University of Santiago de Compostela, Spain

miguel.picos.maiztegui@usc.es

Abstract: Additive manufacturing (AM) is an emerging production method centered on material deposition rather than traditional subtraction, mitigating material wastage and energy consumption. Within the AM domain, Direct Energy Deposition (DED), based on simultaneously adding metallic material as a heat source melts it, stands out as a promising process. DED AM introduces new possibilities such as material gradients within pieces, reconditioning capabilities, and a wider range of geometry designs.

The high heat requirements of DED to melt metallic materials result in significant thermal gradients around the heat source in short time spans, while the global behaviour of the piece is captured over a much longer period. Simulation is a key to understand this technique, however, its embedded multi-scale nature renders it computationally expensive.

To address the cost of simulations, this study proposes a multi-scale Arlequin-based method, combining two meshes with different refinement levels to capture localized critical thermal behavior without necessitating a dense mesh over the entire piece.

Finally, validation is performed through a series of comprehensive analytical tests and experimental data.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

Numerical solution of obstacle problems arising in equilibrium models with heterogeneous agents and two productive sectors

Jonatan Ráfales[1] and **Carlos Vázquez**

[1]Departament of Mathematics and CITIC, University of A Coruña, Spain

jonatan.rafales.perez@udc.es

In this talk, we assume rational expectations to pose general equilibrium models with heterogeneous firms, which can enter or exit the industry. We consider a general Itô process for two different productivity dynamics corresponding to two sectors, thus extending our previous work [2]. Obstacle-type problems associated with Hamilton-Jacobi-Bellman (HJB) partial differential equations (PDEs) model the endogenous decision of firms to remain or leave each productive sector. Moreover, the probability density function of firms in each productive sector satisfies a Kolmogorov-Fokker-Planck (KFP) PDE with a source term. Equilibrium models are completed with household problem formulations and feasibility conditions. For the numerical solution, we propose a Crank-Nicolson method for time discretization. Moreover, we use augmented Lagrangian active set (ALAS) methods for solving the unilateral and bilateral obstacle problems [1], jointly with finite difference discretizations for the HJB formulations. Also, appropriate finite difference discretizations for the KFP problems are considered. For the global nonlinear equilibrium problem, we propose a Steffensen algorithm. Finally, numerical examples illustrate the performance of proposed numerical methodologies as well as the expected behaviour of economic variables.

References

[1] T. Kärkkäinen, K. Kunisch and P. Tarvainen, Augmented Lagrangian active set methods for obstacle problems. *Journal of Optimization Theory and Applications*, **119(3)**, pp. 499—533, (2003).

[2] J. Ráfales and C. Vázquez, Equilibrium models with heterogeneous agents under rational expectations and its numerical solution. *Communications in Nonlinear Science and Numerical Simulation*, **96**, pp. 105673, (2021).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

On an enthalpy formulation for a sharp-interface memory-flux Stefan problem

Sabrina D. Roscani[1] **and Vaughan R. Voller**

[1]Austral University and CONICET, Argentine

sroscani@austral.edu.ar

Abstract: Starting from an appropriate thermodynamic balance statement, a new sharp interface time fractional Stefan problem is developed. With a mathematical analysis we prove that this formulation exhibits a natural regularization in that, unlike the classic Stefan problem, the flux is continuous across the melt interface. This interesting feature is also visualized via a close solution obtained in terms of special functions for a limit case. Finally the relation between this new model and previous models presented in literature is analyzed.

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

A two-phase Stefan problem with temperature-dependent thermal conductivity

José A. Semitiel[1], Julieta Bollati, María F. Natale and Domingo A. Tarzia

[1] Austral University, Argentine

jsemitiel@austral.edu.ar

Abstract: In the classical formulation of the Stefan problem, the physical coefficients involved are assumed to be constant in order to simplify the description of the phase change process. However, thermophysical properties of materials such as density, specific heat and thermal conductivity can vary with temperature and are usually represented via polynomials. Motivated by this fact, we consider a one dimensional two-phase Stefan problem for the melting of a semi-infinite material $x \geq 0$ with a non-negative potential type temperature-dependent thermal conductivity [1]. Imposing a Dirichlet type condition at the fixed face $x = 0$, the existence of a non-negative C^0 solution is guaranteed by using the Banach fixed point theorem. Moreover, approximate analytical solutions are found, applying a modified balance integral method assuming a quadratic temperature profile in space. In addition, numerical approximate solutions to the Stefan problem are derived by using Tau method based on shifted Chebyshev polynomials [2]. For a particular case, we use the exact solution available in the literature to check the accuracy of the approximate solutions.

References

[1] J. Bollati, M. Natale, J. Semitiel and D. Tarzia, Exact solution for non-classical one-phase Stefan problem with variable thermal coefficients and two different heat source terms, *Computational and Applied Mathematics*, **41:375**, pp. 1–11 (2022).

[2] A. Kumar, A. K. Singh and R. Rajeev, A Stefan problem with temperature and time dependent thermal conductivity, *Journal of King Saud University - Science*, **32**, pp. 97–101 (2020).

Modeling interacting systems describing human diseases

Ana Jacinta Soares[1]

[1]Center of Mathematics (CMAT), University of Minho, Portugal

ajsoares@math.uminho.pt

Abstract: The dynamics of a high number of human cells can be described by the kinetic theory of active particles. A kinetic system of partial-integral-differential equations can be derived to describe interactions among cells and the movement of immune cells stimulated by certain substances present in the organism. In this work, we utilize these tools to construct a model for the development of human diseases [1, 2]. An advantage that results from this approach is the multi-scale description in terms of the kinetic (cellular) system and its macroscopic (global) counterpart derived in the hydrodynamic limit of the kinetic system. We study the mathematical consistency of the model, investigate the equilibrium states, stability properties, and the occurrence of bifurcations. Additionally, we examine the formation of spatial patterns.

References

[1] R. Della Marca, M. P. Machado Ramos, C. Ribeiro, A. J. Soares,. Mathematical modelling of oscillating patterns for chronic autoimmune diseases. *Math. Meth. Appl. Sci.*, **45**, pp. 7144–7161 (2022).

[2] J. Oliveira, A. J. Soares, R. Travaglini, Kinetic models leading to pattern formation in the response of the immune system. *Rivista di Matematica dell'Università di Parma.* (accepted in 2023 for publication).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

A Two-Dimensional Flow Model for Viscous Fluids Between Moving Surfaces: Asymptotic Behavior

Raquel Taboada-Vázquez[1] **and José M. Rodríguez**

[1]University of A Coruña and CITMAga, Spain.

raquel.taboada@udc.es

Abstract:
In this work, we study the behavior of an incompressible viscous fluid moving between two closely spaced surfaces, both in motion as well. Using the asymptotic expansion method we formally justify a two-dimensional flow model. We show that its asymptotic behavior, as the distance between the two surfaces tends to zero, is the same as that of the the Navier-Stokes equations.
The solutions of the new model and Navier-Stokes equations converge to the same limit problem, which depend on the boundary conditions. If slip velocity boundary conditions are imposed on the upper and lower bound surfaces, the limit is a solution of a lubrication model. However, if tractions and friction forces are known on both bound surfaces, the limit is a solution of a thin fluid layer model.
The proposed model has been found to be a valuable tool for computing viscous fluid flow between two nearby moving surfaces, without the need to decide a priori whether the flow is typical of a lubrication or a thin fluid layer problem; and, furthermore, avoiding the substantial computational effort required to solve the Navier-Stokes equations in such a thin domain.

References

[1] J.M. Rodríguez, R. Taboada-Vázquez, Asymptotic analysis of a thin fluid layer flow between two moving surfaces, *J. Math. Anal. Appl.*, **507 (1)** pp. 125735, (2022).

[2] J.M. Rodríguez, R. Taboada-Vázquez, A new thin layer model for viscous flow between two nearby non-static surfaces, *Z Angew Math Mech.* **103**, e202200571. (2023).

3rd Emerging Trends in Applied Mathematics and Mechanics
A Coruña, Spain, May 20-24, 2024

On the dimension reduction for compressible nonlinearly viscous fluids

Rostislav Vodák[1] **and Richard Andrášik**

[1] Faculty of Science, Palacký University Olomouc, Czech Republic

rostislav.vodak@upol.cz

Abstract: In this presentation, we introduce an asymptotic analysis of a three- dimensional model for compressible, isentropic, and nonlinearly viscous fluids. This model is grounded in the Navier-Stokes equation and our primary objective is the derivation of lower-dimensional models. Our analysis encompasses scenarios involving both straight and curved domains. We also focus on the techniques employed to address problems posed by the nonlinear terms, which represent the most formidable aspect of this analysis. This approach can enhance understanding of fluid dynamics in varied geometrical contexts and suggest a lower-dimensional approximation of the original three-dimensional model.

Randomized Neural Networks with Petrov-Galerkin Methods for Solving Linear Elasticity and Navier-Stokes Equations

Fei Wang[1] **and Yong Shang**

[1] Xi'an Jiaotong University, China

`feiwang.xjtu@xjtu.edu.cn`

Abstract: We develop the Randomized Neural Networks with Petrov-Galerkin Methods (RNN-PG methods) to solve linear elasticity and Navier-Stokes equations. RNN-PG methods use the Petrov-Galerkin variational framework, where the solution is approximated by randomized neural networks and the test functions are piecewise polynomials. Unlike conventional neural networks, the parameters of the hidden layers of the randomized neural networks are fixed randomly, while the parameters of the output layer are determined by the least squares method, which can effectively approximate the solution. We also develop mixed RNN-PG (M-RNN-PG) methods for linear elasticity problems, which ensure the symmetry of the stress tensor and avoid locking effects. For the Stokes problem, we present various M-RNN-PG methods that enforce the divergence-free constraint by different techniques. For the Navier-Stokes equations, we propose a space-time M-RNN-PG method that uses Picard or Newton iteration methods to deal with the nonlinear term. We compare RNN-PG methods with the finite element method, the mixed discontinuous Galerkin method, and the physics-informed neural network on several examples, and the numerical results demonstrate that RNN-PG methods achieve higher accuracy and efficiency.

Author Index

ETAMM 2024

CITMAga

citic

Horizon 2020 Programme

Deputación DA CORUÑA

Concello da Coruña

SEMA
Sociedad Española de Matemática Aplicada

TURISMO CORUÑA
A Coruña CULTURA DE VIVIR