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DEVELOPMENT OF A COMPUTER-AIDED DESIGN SYSTEM BASED ON A BEM APPROACH FOR EARTHING GRIDS IN STRATIFIED SOILS

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ABSTRACT

The design of safe earthing systems is esential to assure the security of the persons as well as the protection of the equipment and the continuity of the power supply. For the attainment of these aims, it is necessary to compute the equivalent electrical resistance of the system and the potential distribution on earth surface when a fault condition occurs. In the last years, we have proposed a numerical approach based on the Boundary Element Method for the earthing analysis in uniform and homogeneous soils. This formulation has been succesfully applied to the analysis of several grounding grids in real electrical instalations. In this paper we present the generalization of this BE formulation for the grounding systems embedded in layered soils and the development of a CAD system based on it. The feasibility of this BEM approach is demonstrated and it is applied to a frequent practical case of an earthing system in layered soil.

KEYWORDS

Boundary element method, grounding analysis, nonuniform soil models

INTRODUCTION

A safe grounding system has to grant the integrity of the equipment and the continuity of the electrical supply, providing means to carry and dissipate electric currents into the ground, and to assure that a person in the vicinity of grounded installations is not exposed to the danger of suffering a critical electric shock. To achieve these goals, the equivalent electrical resistance of the system must be low enough to assure that fault currents dissipate mainly through the grounding grid into the earth, while maximum potential gradients between points that can be contacted by the human body must be kept under certain safe limits (ANSI/IEEE Std.80, 1986).

Since the sixties, several procedures and methods for substation grounding design and computation have been proposed, most of them founded on practice, on semiempirical works or on the basis of intuitive ideas (Heppe R.J., 1979). Although these techniques represented an important improvement in this area, some problems such as large computational requirements, unrealistic results when segmentation of conductors is increased, and uncertainty in the margin of error, were reported (Sverak *et al.*, 1981-1982; ANSI/IEEE, 1986; Garret & Pruitt, 1985).

(Navarrina *et al.*, 1992) and (Colominas *et al.*, 1999) have developed in the last years a general boundary element formulation for grounding analysis in uniform soils, in which these intuitive methods can be identified as the result of introducing suitable assumptions in the BEM approach in order to reduce computational cost for specific choices of the test and trial functions. Furthermore, starting from this BE numerical approach, more efficient and accurate formulations have been developed and succesfully applied (with a very reasonable computational cost) to the analysis of large grounding systems in electrical substations.

MATHEMATICAL MODEL OF THE PHYSICAL PROBLEM

Statement of the General Problem

Physical phenomena of fault currents dissipation into the earth can be modelled by means of Maxwell's Electromagnetic Theory(Durand, 1966). If the analysis is constrained to the obtention of the electrokinetic steady-state response and the inner resistivity of the earthing conductors is neglected —i.e.,potential can be assumed constant in every point of the electrodes surface—, the 3D problem can be written as

$$\mathbf{div}\boldsymbol{\sigma} = 0,\ \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}V \text{ in } E\ ;\ \boldsymbol{\sigma}^t\boldsymbol{n}_E = 0 \text{ in } \Gamma_E\ ;\ V = V_\Gamma \text{ in } \Gamma;\ V \to 0,\ \text{if } |\boldsymbol{x}| \to \infty; \tag{1}$$

being E the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the electrode surface (Navarrina *et al.*, 1992; Colominas *et al.*, 1999). Therefore, when electrode attains a voltage V_Γ (Ground Potential Rise, or GPR) relative to a distant grounding point, the solution to problem (1) gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$. Furthermore, the grounding design parameters such as the leakage current density at an arbitrary point of the electrode surface, the total surge current I_Γ that flows into the ground during a fault condition, and the equivalent resistance of the earthing system R_{eq} (apparent resistance of the earth-electrode circuit) can be easily obtained in terms of V and $\boldsymbol{\sigma}$ (Colominas *et al.*, 1999).

Statement of the Problem with a Multilayer Soil Model

Most of the methods proposed are founded on the hypothesis that soil can be considered homogeneous and isotropic. Therefore, $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ, that can be experimentally obtained (Sverak *et al.*, 1981-1982). Generally speaking, when the soil is essentially uniform, horizontally and vertically, in the surrondings of the grounding grid, this assumption does not introduce significant errors (ANSI/IEEE, 1986). However, when the conductivity of the soil varies, grounding design parameters can significantly change, being necessary to develop more accurate models that take into account the variation of soil conductivity in the surroundings of the substation site. Obviously, from a technical —and also economical— point of view, the development of models to describe all variations of the conductivity in the surroundings of a grounding system would be unaffordable. For these reasons, a more practical and quite realistic approach to situations where conductivity is not markedly uniform with depth is to consider the earth stratified in a number of horizontal layers, characterized with an appropriate thickness and apparent scalar conductivity. In fact, in most cases, an equivalent two-layer soil model (or a three-layer soil model) is enough to obtain safe designs of grounding systems (ANSI/IEEE, 1986).

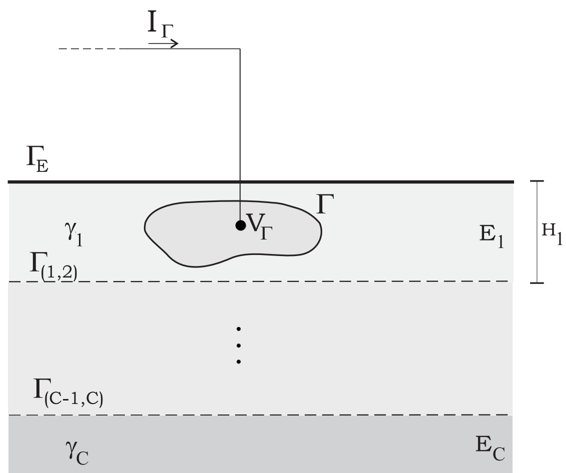

Fig. 1.—Scheme of the current disipation into the earth through an electrode in a multi layer soil.

In a general case, if we consider the earth formed by C horizontal layers with a different conductivity, and the grounding electrode buried in the upper layer, problem (1) can be written in terms of the following Neumann Exterior Problem:

$$\Delta V_1 = 0 \text{ in } E_1; \ldots ; \Delta V_C = 0 \text{ in } E_C;\ V_1 = V_2, \text{ in } \Gamma_{(1,2)}; \ldots ; V_{C-1} = V_C, \text{ in } \Gamma_{(C-1,C)};$$
$$\gamma_1 \frac{dV_1}{dn} = \gamma_2 \frac{dV_2}{dn} \text{ in } \Gamma_{(1,2)}; \ldots ; \gamma_{C-1} \frac{dV_{C-1}}{dn} = \gamma_C \frac{dV_C}{dn} \quad \text{in} \quad \Gamma_{(C-1,C)};$$
$$\frac{dV_1}{dn} = 0 \quad \text{in} \quad \Gamma_E; \quad V_1 = V_\Gamma \quad \text{in} \quad \Gamma;\ V_1 \to 0,\ \ldots\ , V_C \to 0, \text{ if } \quad |\boldsymbol{x}| \to \infty; \tag{2}$$

where E_c is each of the layers of the earth ($c = 1, C$), $\Gamma_{(c-1,c)}$ is the interface between every $c-1$ and c, γ_c is the apparent scalar conductivity of layer c, and V_c is its potential (Tagg, 1964; Aneiros, 1996), (figure 1). Obviously, if the grounding electrode is buried in other layer ($V_c = V_\Gamma$ in Γ), the statement of the exterior problem is analogous to (2).

Integral Equation of the Exterior Problem

As a general rule, grounding systems in most of real electrical substations consist of a grid of interconnected bare cylindrical conductors, horizontally buried and supplemented by a number of vertical rods, which ratio diameter/lenght uses to be relatively small ($\sim 10^{-3}$). Obviously, it is not possible to obtain analytical solutions, and the use of widespread numerical techniques, such as FD or FE that requires the discretization of the 3D domains E_c, implies an out of range computational effort. Other numerical techniques based on meshless methods have been recently proposed for grounding analysis (Colominas *et al.*, 1998); at present, these works are being developing and their application are still restricted to the solution of academic problems and numerical tests.

At this point, since computation of potential is only required on Γ_E and and the equivalent resistance can be obtained in terms of the leakage current density $\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}$ on Γ, being $\boldsymbol{n}$ the normal exterior unit field to Γ, we turn our attention to a boundary element approach, which will only require the discretization of grounding surface Γ (Colominas *et al.*, 1999), and will reduce the 3D problem to a 2D one. On the other hand, if one takes into account that surroundings of the substation site are levelled during its construction —i.e. the interface between the two soil layers can be assumed horizontal— (Aneiros, 1996), symmetry (method of images) allows to rewrite (2) in terms of a Dirichlet Exterior Problem (Colominas *et al.*, 1999).

The application of Green's Identity (Colominas *et al.*, 1999) to this Dirichlet Exterior Problem yields to the following integral expressions for potential $V_c(\boldsymbol{x}_c)$ at an arbitrary point $\boldsymbol{x}_c \in E_c$ $(c = 1, C)$, in terms of the leakage current density $\sigma(\boldsymbol{\xi})$ in a point $\boldsymbol{\xi}$ —with coordinates $[\xi_x, \xi_y, \xi_z]$— on electrode surface Γ buried in upper layer b :

$$V_c(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \quad \forall \boldsymbol{x}_c \in E_c; \quad (c = 1, C); \tag{3}$$

where integral kernel $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$ is formed by an infinite series of terms corresponding to resultant images obtained when Neumann Exterior Problems is transformed to a Diriclet one (Tagg, 1964). This weakly singular kernel depend on the inverse of distances from point $\boldsymbol{x}_c$ to point $\boldsymbol{\xi}$ —and to all symmetric points of $\boldsymbol{\xi}$ with respect to the earth surface Γ_E and interfaces $\Gamma_{(c-1,c)}$ between layers—, and the thickness and conductivities of each layer(Aneiros, 1996).

Although the generation of electrical images is a well-known process and it is conceptually simple, in a general case the final expression of these integral kernels can be very complicated, and its evaluation in practice may require a high computing effort. In this paper, we will present examples with two-layer soil models, and the integral expressions of kernels can be found in (Aneiros, 1996).

Variational Statement of the Problem

The integral expression for potential (3) also holds on the earthing electrode Γ where potential is known by the boundary condition on the Ground Potential Rise (since V and $\boldsymbol{\sigma}$ are proportional to GPR value, the normalized boundary condition $V_b(\boldsymbol{\chi}) = 1$, $\boldsymbol{\chi} \in \Gamma$ is not restrictive at all). Then, leakage current density σ must satisfy the Fredholm integral equation of the first kind defined on Γ:

$$1 = \frac{1}{4\pi\gamma_b} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\; d\Gamma, \quad \boldsymbol{\chi} \in \Gamma. \tag{4}$$

Finally, a weaker variational form of this equation can be written as

$$\iint_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi}) \left(\frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi}\in\Gamma} k_{bb}(\boldsymbol{\chi},\boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\ d\Gamma - 1 \right) d\Gamma \ =\ 0, \tag{5}$$

which mus hold for all members $w(\boldsymbol{\chi})$ of a suitable class of test fuctions defined on Γ (Colominas *et al.*, 1999). It is obvious that a Boundary Element approach seems to be the right choice to solve integral equation (5).

BOUNDARY ELEMENT NUMERICAL FORMULATION

2D Boundary Element Approach

For a given set of $\mathcal{N}$ trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on Γ, and for a given set of $\mathcal{M}$ 2D boundary elements $\{\Gamma^\alpha\}$, the unknown leakage current density σ and the grounding electrode surface Γ can be discretized in the form,

$$\sigma(\boldsymbol{\xi}) = \sum_{i=1}^{\mathcal{N}} \sigma_i\, N_i(\boldsymbol{\xi}), \qquad \Gamma = \bigcup_{\alpha=1}^{\mathcal{M}} \Gamma^\alpha, \tag{6}$$

and expression (3) can be approximated as

$$V_c(\boldsymbol{x}_c) = \sum_{i=1}^{\mathcal{N}} \sigma_i\, V_{c_i}(\boldsymbol{x}_c); \qquad V_{c_i}(\boldsymbol{x}_c) = \sum_{\alpha=1}^{\mathcal{M}} V_{c_i}^{\alpha}(\boldsymbol{x}_c), \quad \forall \boldsymbol{x}_c \in E_c; \quad (c = 1, C); \tag{7}$$

being $V_{c_i}^{\alpha}$:

$$V_{c_i}^{\alpha}(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \int\int_{\boldsymbol{\xi}\in\Gamma^\alpha} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\, N_i(\boldsymbol{\xi})\ d\Gamma^\alpha. \tag{8}$$

On the other hand, for a given set of $\mathcal{N}$ test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ, the variational statement (5) is reduced to the system of linear equations

$$\sum_{i=1}^{\mathcal{N}} \left(\sum_{\beta=1}^{\mathcal{M}} \sum_{\alpha=1}^{\mathcal{M}} R_{ji}^{\beta\alpha} \right) \sigma_i = \left(\sum_{\beta=1}^{\mathcal{M}} \nu_j^\beta \right) \qquad j = 1, \ldots, \mathcal{N}; \tag{9}$$

where coefficients $R_{ji}^{\beta\alpha}$ and ν_j^β are the following:

$$R_{ji}^{\beta\alpha} = \frac{1}{4\pi\gamma_b} \int\int_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi}) \int\int_{\boldsymbol{\xi}\in\Gamma^\alpha} k_{bb}(\boldsymbol{\chi},\boldsymbol{\xi})\, N_i(\boldsymbol{\xi})\, d\Gamma^\alpha d\Gamma^\beta; \tag{10}$$

$$\nu_j^\beta = \int\int_{\boldsymbol{\chi}\in\Gamma^\beta} w_j(\boldsymbol{\chi})\ d\Gamma^\beta. \tag{11}$$

In real problems, the discretization required to solve the above equations would imply a large number of degrees of freedom. On the other hand, the coefficient matrix in (9) is full and each contribution (10) requires an extremely high number of evaluations of the kernel and double integration on a 2D domain. For these reasons, some additional simplifications in the BEM approach must be introduced to reduce the computational cost (Colominas *et al.*, 1999).

Approximated 1D Boundary Element Approach

The specific geometry of the grounding systems in practice (the kind, size and disposition of the electrodes) allows to assume that the leakage current density is constant around the cross section of the cylindrical electrode (ANSI/IEEE, 1986; Navarrina *et al.*, 1992). With this hypothesis of circumferential uniformity, it is possible to obtain approximated expressions of potential (3). Thus, being L the whole set of axial lines of the electrodes, $\widehat{\boldsymbol{\xi}}$ the orthogonal projection over the axis of a given generic point $\boldsymbol{\xi} \in \Gamma$, $\phi(\widehat{\boldsymbol{\xi}})$ the conductor diameter, $P(\widehat{\boldsymbol{\xi}})$ the perimeter of the cross section at $\boldsymbol{\xi}$, and $\widehat{\sigma}(\widehat{\boldsymbol{\xi}})$ the approximated leakage current density at this point (assumed uniform around the cross section), expression (3) results in

$$\widehat{V}_c(\boldsymbol{x}_c) = \frac{1}{4\gamma_b} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\; dL, \quad \forall \boldsymbol{x}_c \in E_c; \quad (c = 1, C); \qquad (12)$$

being $\bar{k}_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}})$ the average of integral kernel $k_{bc}(\boldsymbol{x}_c, \widehat{\boldsymbol{\xi}})$ around the cross section at $\widehat{\boldsymbol{\xi}}$ (Aneiros, 1996)

Now, since the leakage current is not exactly uniform around the cross section, boundary condition $V_c(\boldsymbol{\chi}) = 1$, $\boldsymbol{\chi} \in \Gamma$ will not be strictly satisfied at every point $\boldsymbol{\chi} \in \Gamma$, and variational form (5) will not verify anymore. However, if we restrict the class of trial functions to those with circumferential uniformity (i.e. $w(\boldsymbol{\chi}) = \widehat{w}(\widehat{\boldsymbol{\chi}})\ \forall \boldsymbol{\chi} \in P(\widehat{\boldsymbol{\chi}})$), for all members $\widehat{w}(\widehat{\boldsymbol{\chi}})$ of a suitable class of test functions defined on L, it must hold the weaker variational form

$$\frac{1}{4\gamma_b} \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}}) \left[\int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL \right] dL = \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}})\, dL, \qquad (13)$$

where $\bar{\bar{k}}_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ is the average of kernel $k_{bb}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ around cross sections at points $\widehat{\boldsymbol{\chi}}$ and $\widehat{\boldsymbol{\xi}}$ (Colominas *et al.*, 1999; Colominas *et al.*, 1999-b).

The resolution of the above variational statement requires the discretization of the domain, in this case the axial lines of the electrodes. Consequently, the whole set of the axial lines and the unknown approximated leakage current density $\widehat{\sigma}$ can be discretized for given sets of 1D boundary elements and trial functions defined on L, and we can obtain a discretized version for approximated potential expression (12). Finally, for a suitable selection of test functions defined on L, statement (13) is reduced to a system of linear equations similar to (9), but its coefficients imply integration on a 1D domain (Colominas *et al.*, 1999). Therefore, computing effort of this approximated 1D approach is drastically reduced in comparison with the 2D one, since the 1D discretization will be much more simple. Furthermore, averaged kernels $\bar{k}_{bc}(\cdot,\cdot)$, $\bar{k}_{bb}(\cdot,\cdot)$ and $\bar{\bar{k}}_{bb}(\cdot,\cdot)$, can be evaluated by using suitable unexpensive approximations developed by Colominas, 1999, for the computation of average kernels involved in the grounding analysis in uniform soil models.

Finally, it is important to remark that computation of remaining line integrals is not obvious and standard quadratures cannot be used due to the ill-conditioning of integrands. However, it is possible to perform suitable arrangements in the final expressions of the matrix coefficients (Aneiros, 1996), so that we can use the highly efficient analytical integration techniques derived by Navarrina *et al.*, 1992 and Colominas *et al.*, 1999 for grounding systems in uniform soils.

This numerical formulation has been implemented in the CAD system for earthing analysis "TOTBEM" developed by (Casteleiro *et al.*, 1994) in recent years, being possible to compute accurately grounding grids in uniform and layered soil models of electrical substations of medium/big sizes, with acceptable computing requirements in memory storage and CPU time (Colominas *et al.*, 1999). In the case of grounding systems embedded in two layered soils, the computing effort required can be very high, specially when conductivities of soil layers are very different. In cases like these, the rate of convergence in the computation of average kernels $\bar{k}_{bc}$, $\bar{k}_{bb}$ and $\bar{\bar{k}}_{bb}$ is very low, and it is necessary to evaluate a large number of terms of the series in order to obtain accurate results (Aneiros, 1999).

APPLICATION EXAMPLE

The example that we present is the analysis of the Santiago II grounding system (close to the city of Santiago de Compostela in Spain), by using a uniform soil model and a two layer one. The earthing system is shown in figure 2 and its characteristics is summarized in table I. The numerical model used in this analysis has been a Galerkin formulation, and the grid has been discretized in 582 linear elements.

Table I.—Santiago II Substation: Characteristics and Numerical Model

Data		**Numerical Model**	
Number of Electrodes :	534	Type of Approach :	Galerkin
Number of Vertical Rods :	24	Type of 1D Element :	Linear
Electrode Diameter :	11.28 mm	Number of Elements :	582
Vertical Rod Diameter :	15.00 mm	Degrees of Freedom :	386
Installation Depth :	0.75 m		
Vertical Rod Length :	4 m		
Max. Grid Dimensions :	230 m×195 m		
GPR :	10 kV		

Results, such as the equivalent resistance and total surge current obtained with the BEM approach by using uniform and two layer soil models can be found in table II, and potential distributions on earth surface in both cases in figure 3. As it is shown, results noticeably vary when different soil models are used, so grounding parameters (such as equivalent resistance, the touch, step and mesh voltages, etc.) may significantly change. For this reason, in order to assure the safety of the installation, it will be essential to perform the grounding analysis with this BEM approaches although the computing cost increases, in cases where conductivity changes markedly with depth.

Finally, it is important to remark that in the case of Santiago II grounding system, its analysis by using a two layer soil model is very complicated because part of the grid is buried in the upper layer and other in the lower. Therefore, the final implementation of the numerical formulation must be performed with care, in order to take into account the different arrangements of electrodes in the soil.

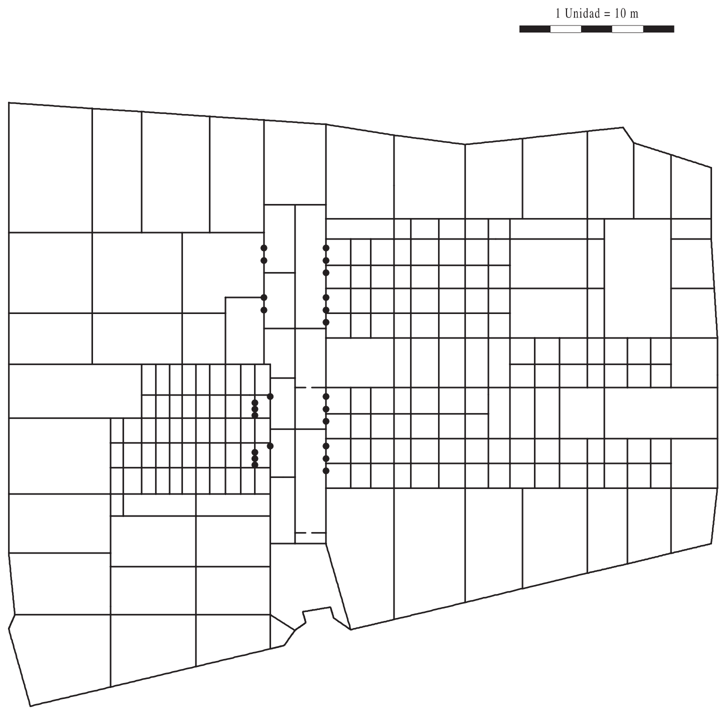

Fig. 2.—Santiago II Substation: Plan of the Grounding Grid (Vertical rods in black points).

Table II.—Santiago II Substation: Results by using different soil models

Two Layer Soil Model		**Uniform Soil Model**	
Upper Layer Resistivity :	200 Ω m	—	
Lower Layer Resistivity :	60 Ω m	—	
Height of Upper Layer :	1.2 m	Earth Resistivity :	60 Ω m
Fault Current :	5.61 kA	Fault Current :	6.73 kA
Equivalent Resistance :	0.1782 Ω	Equivalent Resistance :	0.1486 Ω
CPU Time (AXP 4000):	13.35 min.	CPU Time (AXP 4000):	7.7 s.

CONCLUSIONS

In this paper we have presented a numerical formulation based on the BEM for the analysis of earthing systems embedded in layered soils. Taking into account the real geometry of these systems, the general 2D approach can be rewritten in terms of an approximated 1D version. Moreover, since suitable arrangements can be done in the discretized expressions, it is possible to use the same analytical integration techniques developed by the authors for grounding analysis in uniform soils. Finally, the BEM approach proposed has been applied to the practical case of a grounded system in an equivalent two-layer soil model. The feasibility of this methodology has been demonstrated with its application to a practical example, obtaining highly accurate results in the earthing analysis of electrical substations of medium/big sizes by using layered soil models.

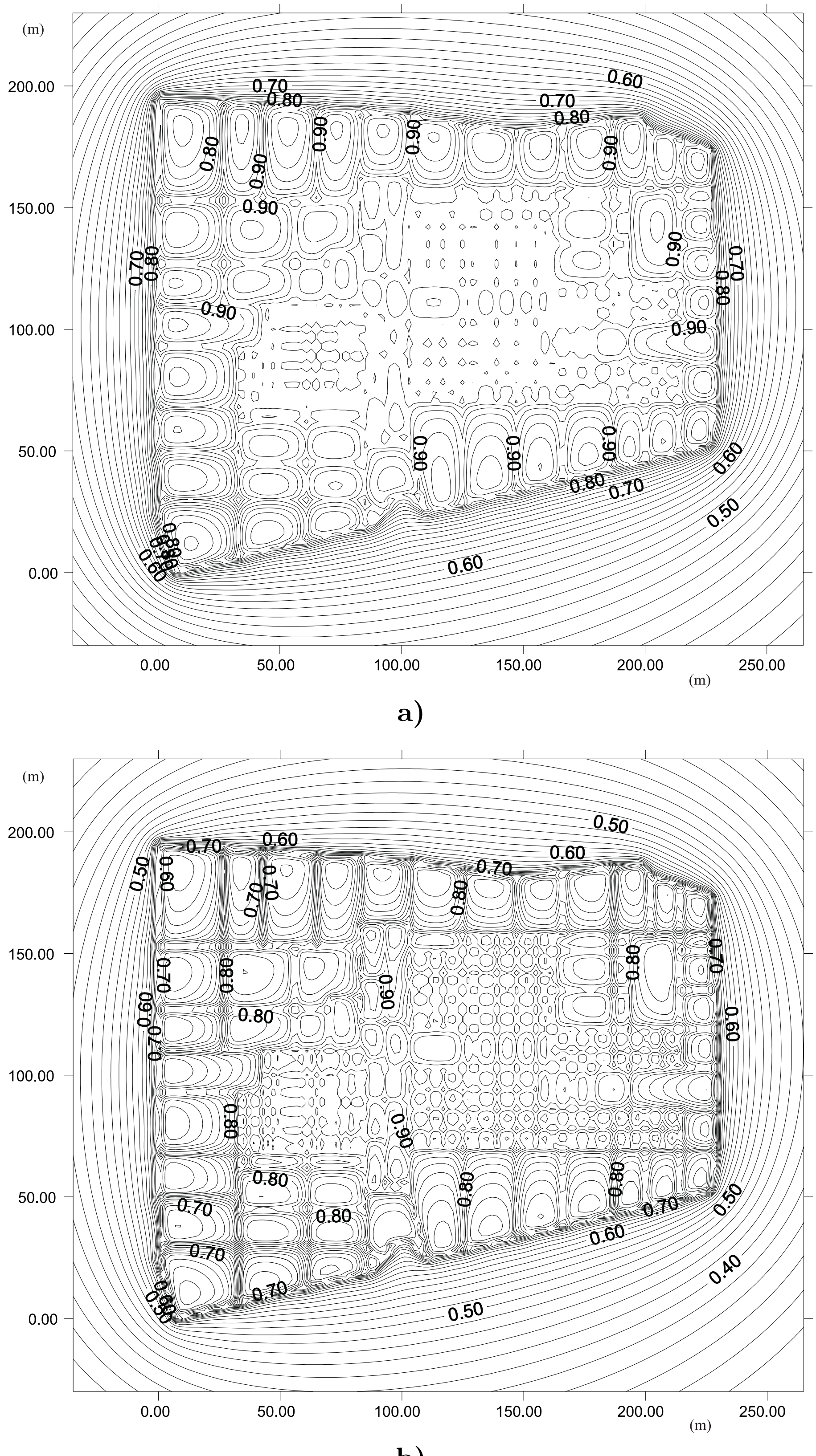

Fig. 3.—Santiago II Substation: Potential distribution (×10 kV) on earth surface obtained by using: **a)** an uniform soil model, and **b)** a two-layer soil model.

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