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Health and fairness with other-regarding preferences

Aitor Calo-Blanco

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Abstract This paper explores how to rank social allocations when individuals have other-regarding preferences (ORPs). Unlike the few existing studies on this issue, we focus on two different private goods, only one of which generates ORPs concerns. Specifically, individuals exhibit other-regarding views about the social health state but have standard self-centered preferences over other goods, namely consumption. Our social evaluation also incorporates a fairness view that aims to reduce inequalities that originate from factors for which individuals should not be deemed responsible. By resorting to a *non-resourcist* approach, we derive social preferences that seek to reduce individual well-being inequalities. Such differences are assessed by means of an interpersonal comparable measure that is related to an ideal situation which involves neither externalities nor unfair inequalities. We obtain that the use of the state of perfect health as the reference value leads society to give a higher priority to those who exhibit more altruistic preferences.

Keywords Health · Fairness · Other-Regarding Preferences · Social Ordering Function.
JEL classification: D62, D63, D71, I14.

1 Introduction

Behavioural economists have drawn attention to the importance of other-regarding preferences (ORPs) in the evaluation of individual well-being (see Fehr and Schmidt, 2006). Unlike standard self-centered preferences, ORPs assume that individuals care about others' situation as well as their own (e.g., Clark et al., 2008; Luttmer, 2005). As a result, the literature on

A. Calo-Blanco
Departamento de Economía, Universidade da Coruña
Campus de A Coruña, 15071 A Coruña, Spain
E-mail: aitorcalo@gmail.com

ORPs has rapidly expanded by including these individual other-regarding views about others' situation in the standard economic modelling (see Dufwenberg et al., 2011; Sobel, 2005). Nevertheless, the inclusion of such views in the branch that studies the formal analysis of redistributive results under normative principles has been barely studied. Fleurbaey (2012) exhaustively defends the view that ORPs should be taken into account, at least to some extent, in order to provide a more appropriate evaluation of social welfare. Accordingly, Decerf and Van der Linden (2016) and Treibich (2019) examine how basic fairness and efficiency principles determine the construction of social rankings when agents have heterogeneous ORPs. Following this line of research, the aim of our paper is to construct social preferences when individuals care about the overall health state in their society, and moreover they differ in both their initial resources and their health care needs.

There are, at least, two important reasons to focus on a good such as health. First, as Fleurbaey and Schokkaert (2011) emphasise, the individual trade-off between health and other goods presents a distinctive feature because one's own level of health strongly conditions one's own well-being evaluation. To deal with this trade-off, equity and fairness axioms that aim to enhance social welfare have to be anchored to a particular reference point. Fleurbaey (2005a) proposes to resort to the state of perfect health since at this reference level any agent's well-being can be measured exclusively in terms of her expenditure on other goods, avoiding, this way, comparisons between agents that are directly grounded on individual preferences. Second, there is evidence that a person's health affects the well-being of nearby individuals (e.g., Bruhin and Winkelmann, 2009; Golics et al., 2013), and hence ORPs should be relevant for the social assessment of any health care policy.

This second reason raises a crucial issue at the time of evaluating social welfare when agents have ORPs. Specifically, any resulting social ranking will depend on both the individual other-regarding views and the extent of those preferences that the planner is willing to compensate.

On the one hand, we will assume that individuals care about their own health-consumption situation as well as about the average health in society. As Fleurbaey and Schokkaert (2011) argue, there is a general consensus that health is one of the most crucial dimensions of individual well-being. Moreover, the literature on medicine has extensively defended that such a sensitive good tends to trigger strong and altruistic ORPs (e.g., Post, 2007). As suggested by Viscusi et al. (1988), these relative concerns may reach as far as the welfare of those who do not belong to the same household or to a close geographical area. Additionally, the individual willingness to pay to avoid that others are in a bad health state can be significantly high (see De Mello and Tiongson, 2009; Dickie and Messman, 2004). Therefore, we will also assume that individuals may show altruistic concerns for others' health, but not envious or negative social sentiments. Regarding the use of the average health in society, we follow Fleurbaey's (2012) idea that to care about one's relative position is a legitimate concern, and hence one can define the scope of the other-regarding views by means of such an average value. Furthermore, Frank (2005) stresses that assuming this relative position could significantly enhance overall welfare. Following this approach, Carrieri (2012) obtains that the average health state of a reference group positively affects the individual subjective well-being.

On the other hand, we will assume that the planner will take the individual other-regarding views into account at the time of designing both efficiency and fairness axioms. The case of efficiency is easy to justify as it seems only natural to promote changes that make no-one worse-off. By contrast, fairness principles that compensate individuals for their ORPs normally generate more controversy since some authors fear that they may reward extreme cases of antisocial behaviour (e.g., Harsanyi, 1982). However, in the case of health, fairness axioms are more easily justified as the effect of others' health in one's own well-being is not always fairly distributed among all agents. In such a framework social norms may promote heterogeneous other-regarding views which are worth compensating. For instance, Braakmann (2014) finds that in Germany women, but not men, suffer important well-being losses from 'spousal disability', a difference that is large and significant after controlling for household income.[1] As this author establishes, this effect could be explained by traditional gender roles as well as status concerns (see also Forssén et al., 2005; Ussher and Sandoval, 2008).[2] This result is in line with the idea that the formation of ORPs is strongly determined by socio-economic backgrounds (e.g., Bauer et al., 2014), something that justifies introducing other-regarding views when designing any public policy.

To sum up, our aim in the present paper is to construct, grounded on efficiency and fairness principles, a social ordering function that will allow us to rank all possible allocations of health and consumption in terms of both ethical and other-regarding views criteria. To implement this analysis we will adopt a normative position which assumes that ORPs over health matter for equality. Therefore, we will not follow the standard *equality of resources* models that define redistributive principles on the basis of equivalent resources dominance. On the contrary, we will resort to the so-called *non-resourcist* approach in which a particular welfare dominance criterion is used to determine the extent of the fairness principles (see Decerf and Van der Linden, 2016, for a thorough discussion of these two approaches). Similar to Treibich (2019), this alternative framework will allow us to accommodate relative views at the time of socially evaluating equality. We

[1] The literature on medicine and sociology has consistently found that female caregivers tend to suffer more negative effects on their individual subjective well-being than male caregivers (e.g., Pinquart and Sörensen, 2006; Rees et al., 2001; Yee and Schulz, 2000).

[2] Winkelmann and Winkelmann (1995) and Mendolia (2014) obtain similar subjective well-being losses for the case of partner's unemployment, and moreover that they clearly exceed the pecuniary losses. These authors also argue that such a gender asymmetry shows the traditional role distribution within the household.

will derive social preferences which establish that society should give a top priority to that agent with the lowest value of a specific measure of individual well-being. This measure will be defined as the individual relative view of an optimal hypothetical situation that entails the state of perfect health and the absence of externalities. Interestingly enough, the use of the state of perfect health as the reference value leads society to give a higher priority to those who exhibit more altruistic preferences.

The rest of the paper is organised as follows. Section 2 presents the basic components of the model while Section 3 introduces the ethical requirements that society is willing to satisfy. Section 4 characterises the social ordering function that results from these requirements. Section 5 offers the conclusions of our study. The Appendix provides the proofs.

2 The framework

Our framework follows the fair social choice approach developed by Fleurbaey and Maniquet (2011). Let us consider a group of individuals $N = \{1, \ldots, n\}$ who care about two goods, consumption, $c \in \mathbb{R}_+$, and health, $h \in H = [0, 1]$. Let $h^* := 1$ denote the state of perfect health. Every individual $i \in N$ is characterised by her medical disposition $m_i \in \mathbb{R}_{++}$, which defines the amount of expenditure $m_i h \in \mathbb{R}_+$ that she needs to reach a given health state $h \in H$. Let $\mathcal{M} = [m^-, m^+]$ be the set of all the feasible medical dispositions, where m^- and m^+ determine, respectively, the best and the worst possible ones, and hence $m^- < m^+$. The population's profile of health dispositions is described by $m_N = (m_1, \ldots, m_n) = (m_i)_{i \in N} \in \mathcal{M}^n$. Agent $i \in N$ is also endowed with an amount of initial resources $\omega_i \in \mathbb{R}_{++}$ that she devotes to both consumption and medical expenditure. Let $\omega_N = (\omega_i)_{i \in N} \in \mathbb{R}^n_{++}$ be the profile of resources, and $\overline{\omega} \in \mathbb{R}_{++}$ be the average, or representative, resources in society. This value and the extreme medical dispositions in $\mathcal{M}$ are assumed to be fixed for all possible allocations. In this scenario each individual $i \in N$ has a *health-consumption bundle* $z_i = (h_i, c_i) \in Z = H \times \mathbb{R}_+$ that designates the situation in which she has a health state h_i and a level of consumption c_i. An *allocation* $z_N = (z_i)_{i \in N} \in Z^n$ is a vector that describes all the individuals' bundles. It is feasible for any (m_N, ω_N) if:

$$\textstyle\sum_{i \in N} c_i \leq \sum_{i \in N} \omega_i - \sum_{i \in N} m_i h_i.$$

Every agent $i \in N$ has well-defined preferences R_i over the space of allocations Z^n, which are described by a complete preorder, that is to say, a binary relation that is reflexive, transitive, and complete. The preferences must also be continuous, strictly convex, and strictly monotonic when the others' situation remain constant. $z_N R_i z'_N$ means that i weakly prefers z_N to allocation z'_N. Strict preference and indifference relations are denoted by P_i and I_i respectively. A profile of ORPs preferences is denoted by $R_N = (R_i)_{i \in N}$.

As we have established in the Introduction, we assume that individuals have ORPs only over health. Moreover, and to match the hypothesis that says that it is admissible to care about one's own relative position, we assume that individuals are concerned for the average health in the economy. For any allocation $z_N \in Z^n$, such a value is defined as:[3]

$$\overline{h}_z = \frac{\sum_{i \in N} h_i}{n}.$$

Consequently, let R_i^* denote agent i's preferences over the set of individual social situations $(z_i, \overline{h}_z) \in Z \times H$. Specifically:

Average Health Relative Views For all $z_N, z'_N \in Z^n$ and $i \in N$:

$$z_N R_i z'_N \Leftrightarrow (z_i, \overline{h}_z) R_i^* (z'_i, \overline{h}_{z'}).$$

For ease of exposition, we slightly abuse notation and write $(z_i, \overline{h}_z) R_i (z'_i, \overline{h}_{z'})$ to denote $(z_i, \overline{h}_z) R_i^* (z'_i, \overline{h}_{z'})$.

Since ORPs widen excessively the domain of admissible individual preferences, let us assume that they satisfy some additional properties. The first one implies that the individual ordinal evaluation of one's own choice does not depend on other agents' choices (see Dufwenberg et al., 2011).

Separability For all $z_N, z'_N \in Z^n$ and $i \in N$:

$$(z_i, z_{N \setminus \{i\}}) R_i (z'_i, z_{N \setminus \{i\}}) \Leftrightarrow (z_i, z'_{N \setminus \{i\}}) R_i (z'_i, z'_{N \setminus \{i\}}),$$

where $z_{N \setminus \{i\}} = (z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n)$.

As a result of Separability, for any $i \in N$ we can associate R_i with a unique set of *internal preferences* $\widehat{R}_i$ over the set of bundles Z, where $z_i \widehat{R}_i z'_i$ means that individual i weakly prefers bundle (h_i, c_i) to bundle (h'_i, c'_i). These preferences are continuous, convex, strictly monotonic and a complete preorder. The asymmetric and symmetric parts of $\widehat{R}_i$ are denoted by $\widehat{P}_i$ and $\widehat{I}_i$ respectively.

The second additional property that we introduce assumes that no agent prefers a situation without consumption, whatever it is that the others have. This view that life cannot be enjoyed without consumption is defended by Fleurbaey and Schokkaert (2011).

Consumption Desirability For all $z_N, z'_N \in Z^n$, $i \in N$ and $c > 0$:

$$((h_i, c), z_{N \setminus \{i\}}) P_i ((h'_i, 0), z'_{N \setminus \{i\}}).$$

Finally, we set a limit to the externalities that anyone may experience. In accordance with the arguments that we have presented in the Introduction, we exclude envious social sentiments. More precisely, no agent will strictly prefer the actual allocation to an alternative one in which she has the same bundle and the others have a better health state.

[3] A similar way of introducing relative views in a framework in which consumption is the only good in the economy is presented by Treibich (2019).

Altruism For all $z_N, z'_N \in Z^n$ and $i \in N$, if $z_i = z'_i$ and $\overline{h}_z \geq \overline{h}_{z'}$, then $z_N R_i z'_N$.

Let $\mathcal{R}$ denote the domain of profiles of ORPs that satisfy all these properties, and hence $R_N \in \mathcal{R}$.

An *economy* is then described by a list $e = (m_N, \omega_N, R_N) \in \mathcal{E}$, where $\mathcal{E}$ is the domain of all the economies that satisfy the previous assumptions. For every element in this domain social preferences allow us to compare allocations in terms of fairness and efficiency (e.g., Calo-Blanco, 2016). They are formalised as a complete ordering over all the (feasible and not feasible) allocations (see Fleurbaey and Maniquet, 2005; Maniquet and Sprumont, 2005). Specifically, a *social ordering function*, SOF hereafter, $\mathbf{R}$ maps every element in $\mathcal{E}$ to a complete *social ordering*, SO hereafter, over the set of allocations Z^n. Specifically, for any $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$, let $z_N \mathbf{R}(e) z'_N$ denote that allocation z_N is at least as good as z'_N. Strict preference is denoted by $\mathbf{P}(e)$, and the indifference relation by $\mathbf{I}(e)$.

We now introduce some basic definitions that will be useful to present our result. Let us start with the set of all the bundles that an individual can afford:

Health-Consumption Feasible Set For all $e \in \mathcal{E}$ and $i \in N$:

$$B(m_i, \omega_i) = \{(h, c) \in Z \mid m_i h + c \leq \omega_i\}.$$

Second, we introduce a measure of individual well-being for the case of self-centered preferences (see Fleurbaey, 2005a). For any $i \in N$, this measure is defined as the level of consumption $c_i^*(z_i)$ which makes that individual i remains indifferent, according to $\widehat{R}_i$, between her current bundle z_i and being perfectly healthy with income $c_i^*(z_i)$, that is,

Full-Health Equivalent Consumption (FHEC) For all $e \in \mathcal{E}$, $i \in N$ and $z_i \in Z$:

$$c_i^*(z_i) = \min\{c' \in \mathbb{R}_+ \mid (h^*, c') \widehat{R}_i (h_i, c_i)\}.$$

Let us now use these two definitions to present a particular hypothetical individual situation. Such a point will describe the FHEC that one agent would enjoy when maximising her utility, according to her internal preferences, if she had the best medical disposition m^- and the representative endowment $\overline{\omega}$. For any economy and $i \in N$, let us denote this value, which does not depend on the actual allocation, by $c_e^*(\widehat{R}_i) \in \mathbb{R}_+$. The reason why we propose this ideal point is twofold. First, given that it is defined in terms of the internal preferences this situation can be interpreted as a scenario in which the individual is free of any possible externality. Second, since this point is anchored to both the best medical disposition and the representative endowment, it will allow us to introduce a concern for inequality of opportunity in terms of health care needs and initial resources.

A graphical illustration of the previous concepts is provided in Figure 1. Let us focus on individual $i \in N$ who is characterised by a low set of opportunities, due to poor values of both m_i and ω_i, and ORPs $R_i \in \mathcal{R}$ that are linked to a

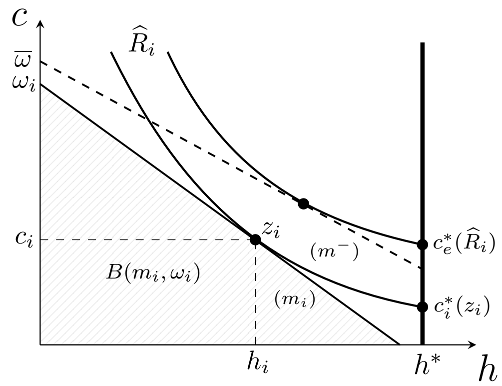


Fig. 1: The individual health-consumption space

set of internal preferences $\widehat{R}_i$. The individual's health-consumption feasible set (the lined area in the picture) is delimited by her initial resources (depicted in the vertical axis) and her medical disposition (in parenthesis below the budget line). In this set agent i chooses bundle $z_i \in Z$ according to her internal point of view (see the black solid curves). In such a scenario, $c_i^*(z_i)$ is the smallest level of consumption which i would accept in replacement of c_i in z_i, provided that she is alone in the economy and she has perfect health, that is, her FHEC (see Figure 1). The last concept, $c_e^*(\widehat{R}_i)$, is related to a situation in which the individual is endowed with both the representative resources $\overline{\omega}$ and the best medical disposition m^-. More precisely, it is defined as the FHEC associated with the bundle that maximises the agent's internal utility over this ideal budget set (delimited by the black dashed line in the picture).

Finally, we introduce our measure of individual well-being. This measure, which is based on the concept of egalitarian-equivalence (see Fleurbaey, 2005b; Pazner and Schmeidler, 1978), relates any agent's situation to a proportion of her ideal FHEC $c_e^*(\widehat{R}_i)$, specifically, to the proportion that would make her indifferent between the actual allocation and a hypothetical ideal social situation. In such a situation every individual would have her health state increased up to a perfect level. Specifically,

λ-Relative Equivalent For all $e \in \mathcal{E}$, $z_N \in Z^n$ and $i \in N$, the scalar $\lambda_i(z_N)$ satisfies:

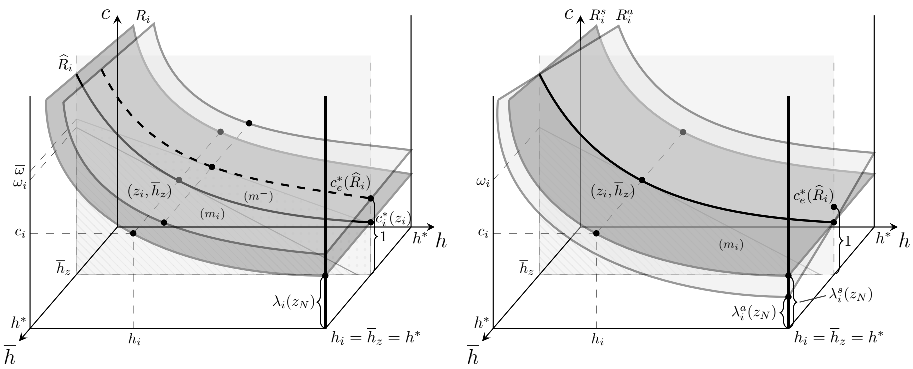
$$z_N I_i \left((h^*, \lambda_i(z_N) c_e^*(\widehat{R}_i)), (h^*, c_j)_{j \in N \setminus \{i\}} \right).$$

In settings with ORPs it is not always possible to ensure that this sort of equivalent measure exists and is unique, since its value may not be defined for those who exhibit extreme cases of other-regarding views. For instance, the well-being of one of those agents might drastically change when the others' state varies significantly, and hence her equivalent measure of well-being might go to infinity. However, as the following proposition states, in our framework $\lambda_i(z_N)$ is always well-defined for all agents:

Proposition 1 *On domain $\mathcal{E}$, for any allocation $z_N \in Z^n$ the individual i's λ-relative equivalent is finite, positive and unique.*

Let us now present a graphical illustration of this concept of λ-relative equivalent $\lambda_i(z_N)$. Figure 2a depicts the situation of agent $i \in N$, who is endowed with both a medical disposition $m_i > m^-$ and initial resources $\omega_i < \overline{\omega}$. Moreover, she belongs to a society in which the average health equals $\overline{h}_z \in H$. Since by Average Health Relative Views preferences can be defined in terms of the individual bundle and the average level of health, it is possible to represent the agent's relative views in a three-dimensional figure. According to the initial conditions of the present example we have that individual i has a smaller set of opportunities (described by the lined area) than in the reference hypothetical case, characterised by m^- and $\overline{\omega}$. Specifically, the latter set dominates the former one by the dotted area. Therefore, and according to her internal preferences $\widehat{R}_i$, agent i's actual FHEC, $c_i^*(z_i)$ (see the black solid curve), is smaller than her ideal full-health consumption $c_e^*(\widehat{R}_i)$, which is obtained with the hypothetical conditions (see the black dashed curve). Note that this assessment of individual well-being loss is exclusively related to an internal viewpoint. As regards the other-regarding views, let us assume that individual i has self-centered preferences, and hence she remains equal-off when the average health in the economy changes. This implies that any social environment that includes bundle z_i must be located in the same (three-dimensional) agent i's indifference curve, no matter the others' health (see the dark gray surface). In such a scenario i's equivalent utility would be described by a proportion $\lambda_i(z_N) \in \mathbb{R}_+$ of the ideal quantity $c_e^*(\widehat{R}_i)$. Since in this particular example the individual does not include the others' situation at the time of evaluating her own, this proportion coincides with her actual FHEC, that is, $c_i^*(z_i)$. Note that this equivalent value is also characterised by the individual's endowment since it allows her to obtain bundle z_i. Therefore, those who are endowed with a poor set of traits would be considered to have a low internal well-being. Hence, the initial profiles of resources and medical dispositions play a role in the social evaluation by introducing unfair opportunities.

Let us now analyse how the reference equivalent measure may differ according to the individual other-regarding views. Figure 2b compares the previous example with one in which the same agent exhibits altruistic preferences R_i^a instead. Hence, the individual i's alternative indifference set (represented by the light gray plane in the picture) slopes downwards as the others' health state increases, showing that she is better-off when observing such an increase. As a result of this her actual equivalent utility would not be described by $\lambda_i^s(z_N)$, which characterises the self-centered case, but by a smaller amount $\lambda_i^a(z_N)$ (see Figure 2b). Therefore, the altruistic version of agent i considers the social allocation to be equivalent to a scenario in which she is alone in the economy, she has perfect health, and she enjoys a consumption equal to $\lambda_i^a(z_N)c_e^*(\widehat{R}_i) < c_i^*(z_i)$.



(a) The λ-relative equivalent (b) Altruistic vs. self-centered preferences

Fig. 2: Individual ORPs

3 Ethical axioms

Based on the definitions that we have introduced in the previous section, let us now present the ethical principles that are desirable for our social ordering function.

The first ethical axiom is a requirement which ensures, given the individuals' ORPs, that the solution is efficient:

Strong Pareto For all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$, if $z_N R_i z'_N$ for all $i \in N$; then $z_N \mathbf{R}(e) z'_N$. If moreover, $z_N P_j z'_N$ for some $j \in N$; then $z_N \mathbf{P}(e) z'_N$.

The following axiom, Consistency, is a robustness requirement demanding that indifferent agents should not influence social preferences. The classical version of this property requires that adding or removing an agent who receives the same bundle in two different allocations does not modify the social ordering over such allocations (see D'Aspremont and Gevers, 1977; Fleurbaey and Maniquet, 2011). However, when assuming ORPs the property cannot be implemented in its original form as the situation of this indifferent agent may affect the average health level, and hence the others' relative preferences. Therefore, we propose a weaker version of this principle that additionally requires an equal average health in both allocations, and that the indifferent individual's level of health coincides with such a state. Specifically,

Consistency For all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$ such that $\overline{h}_z = \overline{h}_{z'}$, if there exists $i \in N$ with health-consumption bundles $(\overline{h}_z, c_i) = (\overline{h}_{z'}, c'_i)$, then:

$$z_N \mathbf{R}(e) z'_N \Leftrightarrow z_{N\setminus\{i\}} \mathbf{R}(e_{-i}) z'_{N\setminus\{i\}},$$

where $\mathbf{R}(e_{-i})$ is the social ordering associated with reduced population $\{1, \ldots, i-1, i+1, \ldots, n\}$.

Next, Perfect Consumption Scale Independence is a consumption invariance at the state of perfect health. Specifically, it assumes that if the consumption of all individuals in two different allocations increases in the same proportion, provided that all of them have perfect health, social preferences should not be reversed. Similar properties have been proposed by Østerdal (2005) and Hougaard et al. (2013) in contexts characterised by population profiles of health and lifetime.

Perfect Consumption Scale Independence For all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$ and $\beta \in \mathbb{R}_{++}$,

$$(h^*, c_i)_{i \in N} \mathbf{R}(e) (h^*, c'_i)_{i \in N} \Rightarrow (h^*, \beta c_i)_{i \in N} \mathbf{R}(e) (h^*, \beta c'_i)_{i \in N}.$$

These first three axioms are basic requirements derived from the social choice literature. In order to obtain our characterisation results we combine such requirements with two additional axioms that model redistribution of resources. The first one defines how to pay compensations among individuals who share their other-regarding views. Specifically, it establishes that any consumption inequality reduction between these individuals would always be desirable, provided that their health is identical. Note that this principle implies an infinite aversion to inequality since reducing differences in consumption always improves social welfare, no matter how much consumption the 'richer' agent loses. Moreover, those who are not involved in the inequality reduction have the same pre- and post-redistribution health-consumption bundles. Since the average health is the same in both allocations, these agents must remain indifferent between the two social situations. Finally, the axiom also establishes that when the two involved individuals swap their bundles social preferences do not change, that is, agents are treated anonymously. Formally,

Priority Among Equals For all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$, if there exist $j, k \in N$ with $R_j = R_k$ and $h_j = h'_j = h_k = h'_k$ such that:

$$c'_j > c_j > c_k > c'_k,$$

with $z_i = z'_i$ for all $i \neq j, k$, then $z_N \mathbf{R}(e) z'_N$; if otherwise $z_j = z'_k$ and $z'_j = z_k$, then $z_N \mathbf{I}(e) z'_N$.

Nevertheless, although Priority Among Equals addresses the differences that originate from factors which are beyond the agents' responsibility, the way in which these differences are compensated is limited by the assumption of equal preferences. To strengthen compensations for differences in initial resources and/or health care needs we introduce a second redistribution axiom that allows us to deal with those who do not share their ORPs.

As we have commented in the Introduction, in this paper we opt for a *non-resourcist* approach in which other-regarding views are assumed to be relevant in terms of equality. Consequently, our second fairness requirement aims to reduce well-being differences that take into account the agents' ORPs over the social allocation, something that is done by means of a particular well-being dominance criterion. More precisely, the axiom establishes that a transfer of consumption between someone who is better-off than in the allocation characterised by the ideal FHECs, that is $(h^*, c_e^*(\widehat{R}_i))_{i\in N}$, and another agent who is worse-off than in this hypothetical allocation cannot reduce social welfare, provided that both keep their own preference relation with respect to such a hypothetical situation. Moreover, to avoid clashes between fairness and efficiency axioms we restrict the implementation of the transfer to situations in which all individuals are perfectly healthy (see Fleurbaey, 2005a). Finally, the axiom also establishes that individuals have to be treated anonymously, that is, when the two involved agents swap similar situations social preferences do not change. Specifically:

Well-being Bound Transfer For all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$ with $h_i = h'_i = h^*$ for all $i \in N$, if there exist $j, k \in N$ and $\varepsilon \in \mathbb{R}_{++}$ such that:

$$c'_j - \varepsilon = c_j,\ c_k = c'_k + \varepsilon,$$
$$z'_N P_j z_N P_j (h^*, c_e^*(\widehat{R}_i))_{i\in N},$$
$$(h^*, c_e^*(\widehat{R}_i))_{i\in N} P_k z_N P_k z'_N,$$

with $z_i = z'_i$ for all $i \neq j, k$, then $z_N \mathbf{R}(e) z'_N$; if otherwise $c_j/c_e^*(\widehat{R}_j) = c'_k/c_e^*(\widehat{R}_k)$ and $c'_j/c_e^*(\widehat{R}_j) = c_k/c_e^*(\widehat{R}_k)$, then $z_N \mathbf{I}(e) z'_N$.

The aim of this social ordering axiom is to provide those who have an excessively low well-being with some sort of 'safety net'.[4] Note that it is defined in terms of the individual ORPs over a reference allocation in which every $i \in N$ has the ideal FHEC $c_e^*(\widehat{R}_i)$. Such a point is constructed by means of both the best possible medical disposition and the representative resources in the society. As a result of this definition, those who are endowed with a very poor set of opportunities would be assumed to have a low well-being in terms of internal preferences, and hence they should be more likely to be compensated. Therefore, Well-being Bound Transfer also shows a concern for those who have few monetary resources and/or a very low medical disposition, fully exploiting this way the agents' heterogeneity.

4 Social preferences

After having presented the ethical axioms that our society endorses, we now turn to the characterisation of the social ranking that such requirements generate. In order to execute such a task, let us first introduce the following SOF:

λ-Relative Leximin For all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$,

$$z_N \mathbf{R}^{\lambda}_{\text{lex}}(e) z'_N \Leftrightarrow (\lambda_i(z_N))_{i\in N} \geq_{\text{lex}} (\lambda_i(z'_N))_{i\in N}.$$

[4] Although different, this property bears some resemblance to the principles that impose well-being lower and upper bounds (e.g., Maniquet and Sprumont, 2005).

The $\mathbf{R}^{\lambda}_{\text{lex}}$ SOF consists of the application of the leximin criterion ($\geq_{\text{lex}}$) to the λ-relative equivalent $\lambda_i(z_N)$ (see Figure 2). Let us make use of this function to present our main result.

Theorem 1 *On domain $\mathcal{E}$, a SOF satisfies Strong Pareto, Consistency, Perfect Consumption Scale Independence, Priority Among Equals and Well-being Bound Transfer if and only if it coincides with $\mathbf{R}^{\lambda}_{\text{lex}}$.*

This outcome provides a characterisation of the social preferences. More precisely, Theorem 1 shows that a society which endorses all the ethical axioms that we have previously introduced should rank any two allocations according to $\mathbf{R}^{\lambda}_{\text{lex}}$. Such an ordering shows a compensation flavour for those who exhibit more altruistic preferences, in other words, under similar conditions social preferences tend to give more weight to those who are more concerned for the others' situation. Additionally, if we also assumed agents with envious preferences,[5] the reference measure of well-being $\lambda_i(z_N)$ would dictate that such agents should be given, in general, a low priority. It is important to stress that the λ-relative equivalent utility crucially depends on the use of h^* as the reference health state. Fleurbaey (2005a) and Fleurbaey and Schokkaert (2011) thoroughly justify that there are good reasons to use such a level as the reference one, especially at the time of establishing individual well-being comparisons. Interestingly enough, Theorem 1 provides a new argument to focus on the state of perfect health. If agents actually suffer unfair negative externalities when other individuals are in a bad health condition, the choice of h^* tends to induce larger compensations to those who suffer the most from such an externality. If, on the contrary, society focused on a different reference point, social preferences would turn its priority to those who do not suffer when the others' health worsens.

5 Discussion

The aim of the present paper is to develop a framework that allows us to make fair welfare evaluations when agents have ORPs and health is one of the dimensions of individual well-being. To do so we have assumed that agents can only show altruistic preferences over the average health in society. Grounded on this framework, we have proposed a comparable measure of individual well-being which is defined as the relative view of an optimal hypothetical personal situation that entails the state of perfect health and the absence of externalities.

Next, by endorsing some robustness and fairness principles we have characterised a social ordering function that gives absolute priority to that agent with the lowest value for our comparable well-being measure. These social preferences exhibit two features that are worth mentioning. First, they allow us to compare individuals with different ORPs and different personal

[5] The λ-relative equivalent for such agents would be a positive value that may go to $+\infty$.

traits introducing, this way, the equality of opportunity principle in the model (see Bossert et al., 1999). Second, by using the state of perfect health as the reference value, something that is usual in the literature, the resulting social ranking establishes that those who have more altruistic preferences should receive larger compensations. Interestingly enough, this result does not support the general idea that basing normative criteria on ORPs may unfairly benefit agents with envious or antisocial preferences, which happens to be one of the main arguments against compensating relative views.

Appendix

Proof of Proposition 1

For any $e \in \mathcal{E}$, let us assume an allocation $z_N \in Z^n$. By *Consumption Desirability*, each individual $i \in N$ prefers a strictly positive level of consumption than no consumption at all, whatever the others' situation is. Therefore, $((h_i, c), \overline{h}_z) P_i ((h^*, 0), h^*)$ for any $c > 0$. Hence, due to monotonicity in one's own consumption $\lambda_i(z_N)$ cannot be negative.

As regards the existence of $\lambda_i(z_N)$, by continuity, monotonicity and the previous result there exists $\widehat{\lambda} \in \mathbb{R}_+$ such that for all $\lambda \leq \widehat{\lambda}$ we have that $z_N R_i \left((h^*, \lambda c_e^*(\widehat{R}_i)), h^*\right)$. Moreover, by *Altruism*, monotonicity and the choice of h^* as the reference value it is always possible to find $\widetilde{\lambda} \in \mathbb{R}_{++}$ such that $\left((h^*, \lambda c_e^*(\widehat{R}_i)), h^*\right) R_i z_N$ for all $\lambda \geq \widetilde{\lambda}$. Since both $\widehat{\lambda}$ and $\widetilde{\lambda}$ exist for any $i \in N$, and given that $\mathbb{R}_+$ is a connected space, by continuity there exits $\lambda_i(z_N) \in \mathbb{R}_+$ such that $z_N I_i \left((h^*, \lambda_i(z_N) c_e^*(\widehat{R}_i)), h^*\right)$.

Finally, let us assume that for some individual $i \in N$ there exist $\lambda_i(z_N), \lambda_i'(z_N) \in \mathbb{R}_+$ such that $\lambda_i(z_N) > \lambda_i'(z_N)$. By monotonicity in one's own consumption $\left((h^*, \lambda_i(z_N) c_e^*(\widehat{R}_i)), h^*\right) P_i \left((h^*, \lambda_i'(z_N) c_e^*(\widehat{R}_i)), h^*\right)$. Therefore, the individual is not indifferent between these two allocations, and hence $\lambda_i(z_N)$ and $\lambda_i'(z_N)$ cannot both at the same time represent her preferences over the allocation $z_N \in Z^n$.

Consequently, the λ-relative equivalent $\lambda_i(z_N)$ is well-defined for all agents, that is, this value exists and it is positive and unique.

Proof of Theorem 1

The proof of Theorem 1 is based on the characterisation of the leximin criterion proposed by Hammond (1976), and it is split in four steps. The version of the proof that we present here relies on the assumption that the population can vary.

-Step 1: Indifference of $\mathbf{R}^{\lambda}_{\text{lex}}$

For any economy $e \in \mathcal{E}$, let us assume two allocations $z_N, z'_N \in Z^n$ and two individuals $j, k \in N$ such that, without loss of generality, $\lambda_k(z'_N) = \lambda_j(z_N) < \lambda_k(z_N) = \lambda_j(z'_N) < 1$, and $\lambda_i(z_N) = \lambda_i(z'_N)$ for all $i \neq j, k$. When the values of the λ-relative equivalents are different to the ones proposed here the same proof can be induced by means of *Perfect Consumption Scale Independence*. We will show that under such conditions $z_N \mathbf{I}(e) z'_N$. Opposite to the desired result, let us assume that $z_N \mathbf{P}(e) z'_N$.

Due to the properties of the ORPs it is possible to define two perfect health allocations $\widehat{z}_N, \widehat{z}'_N \in Z^n(h^*)$[6] such that for all $i \in N$:

$$\lambda_i(\widehat{z}_N) = \lambda_i(z_N) \text{ and } \lambda_i(\widehat{z}'_N) = \lambda_i(z'_N).$$

Note that since $\lambda_i(z_N)$ is a valid measure of the individual well-being, one has that $\widehat{z}_i = \widehat{z}'_i$ for all $i \neq j, k$. Hence, if we apply *Strong Pareto* (which implies *Pareto Indifference*) we have that $\widehat{z}_N \mathbf{P}(e) \widehat{z}'_N$.

Let us now introduce two additional individuals a and b such that $R_a = R_j \in \mathcal{R}$ and $R_b = R_k \in \mathcal{R}$. Let us also assume, with $\beta > 1$, the following levels of consumption (see Figure 3a):

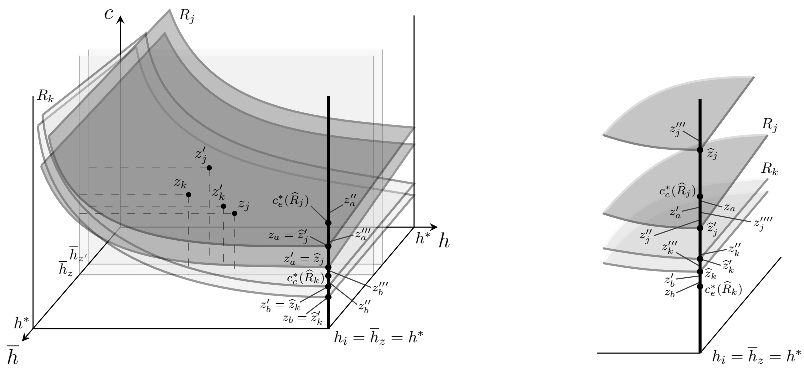
$$\begin{aligned}
&c_a = \widehat{c}'_j > c'_a = \widehat{c}_j,\ c'_b = \widehat{c}_k > c_b = \widehat{c}'_k,\\
&c''_a = \beta c_a,\ c'''_a = \beta c'_a,\ c''_b = \beta c_b,\ c'''_b = \beta c'_b,\\
&c''_a / c^*_e(\widehat{R}_j) = c'''_b / c^*_e(\widehat{R}_k) > 1,\ c'''_a / c^*_e(\widehat{R}_j) = c''_b / c^*_e(\widehat{R}_k) < 1.
\end{aligned}$$

According to the assumption $z_N \mathbf{P}(e) z'_N$, by *Strong Pareto* and *Consistency* $(\widehat{z}_N, (h^*, c_a), (h^*, c_b)) \mathbf{P}(e) (\widehat{z}'_N, (h^*, c_a), (h^*, c_b))$. By applying *Priority Among Equals* twice $(\widehat{z}'_N, (h^*, c'_a), (h^*, c'_b)) \mathbf{I}(e) (\widehat{z}_N, (h^*, c_a), (h^*, c_b))$. By *Transitivity* and *Consistency* $((h^*, c'_a), (h^*, c'_b)) \mathbf{P}(e) ((h^*, c_a), (h^*, c_b))$. Finally, by *Perfect Consumption Scale Independence* we conclude that $((h^*, c'''_a), (h^*, c'''_b)) \mathbf{P}(e) ((h^*, c''_a), (h^*, c''_b))$. However, using *Well-being Bound Transfer* we obtain that $((h^*, c''_a), (h^*, c''_b)) \mathbf{I}(e) ((h^*, c'''_a), (h^*, c'''_b))$, which yields the desired contradiction.

-Step 2: Strict preference of $\mathbf{R}^{\lambda}_{\text{lex}}$

For any $e \in \mathcal{E}$, let us assume two allocations $z_N, z'_N \in Z^n$ and two individuals $j, k \in N$ such that, without loss of generality,

[6] Where $Z^n(h^*)$ denotes the set of allocations in which $h_i = h^*$ for all $i \in N$. These allocations can always be constructed since any individual's level of health, that is h^*, cannot be larger than the aggregate state in society.



(a) Step 1

(b) Step 2

Fig. 3: Proof of Theorem 1

$\lambda_j(z'_N) < \lambda_k(z_N) < \lambda_j(z_N) < \lambda_k(z'_N)$, and $\lambda_i(z_N) = \lambda_i(z'_N)$ for all $i \neq j, k$. First, we need to prove that it must be the case that $z_N \mathbf{P}(e) z'_N$. Opposite to the desired result, let us assume that $z'_N \mathbf{R}(e) z_N$.

Due to the basic axioms of the ORPs it is possible to define two perfect health allocations $\widehat{z}_N, \widehat{z}'_N \in Z^n(h^*)$ such that for all $i \in N$:

$$\lambda_i(\widehat{z}_N) = \lambda_i(z_N) \text{ and } \lambda_i(\widehat{z}'_N) = \lambda_i(z'_N).$$

Once again, $\widehat{z}_i = \widehat{z}'_i$ for all $i \neq j, k$, and hence $\widehat{z}'_N \mathbf{R}(e) \widehat{z}_N$ by *Strong Pareto*.

To obtain our result we distinguish between two possible scenarios according to the value of $\lambda_j(z'_N)$. We first assume $\lambda_j(z'_N) < 1$. Let us introduce two individuals a and b with ORPs $R_a = R_j$ and $R_b = R_k$. Let us also assume, with $\beta, \varepsilon \in \mathbb{R}_{++}$, the following levels of consumption:

$$\begin{aligned}
&c''_j = \beta \widehat{c}'_j,\ c'''_j = \beta \widehat{c}_j,\ c''_k = \beta \widehat{c}'_k,\ c'''_k = \beta \widehat{c}_k,\\
&c_a = c'_a + \varepsilon,\ c_b = c'_b - \varepsilon,\\
&c'''_j > \widehat{c}_j > c^*_e(\widehat{R}_j) > c_a > c'_a > c''''_j > c''_j > \widehat{c}'_j,\\
&c''_k > \widehat{c}'_k > c'''_k > \widehat{c}_k > c'_b > c_b > c^*_e(\widehat{R}_k).
\end{aligned}$$

By continuity and *Perfect Consumption Scale Independence*, such bundles can always be found (see Figure 3b). Given the initial assumption $z_N \mathbf{R}(e) z'_N$, by *Strong Pareto* $\widehat{z}'_N \mathbf{R}(e) \widehat{z}_N$. By *Perfect Consumption Scale Independence* $((h^*, \beta\widehat{c}'_i)_{i \neq j,k}, (h^*, c''_j), (h^*, c''_k)) \mathbf{R}(e) ((h^*, \beta\widehat{c}_i)_{i \in N\setminus\{j,k\}}, (h^*, c'''_j), (h^*, c'''_k))$. By *Consistency* it is straightforward to check that $((h^*, c''_j), (h^*, c''_k), (h^*, c_a), (h^*, c_b)) \mathbf{R}(e) ((h^*, c'''_j), (h^*, c'''_k), (h^*, c_a), (h^*, c_b))$. If we apply *Priority Among Equals* twice

$((h^*,c_j''''),(h^*,c_k'''),(h^*,c_a'),(h^*,c_b'))\mathbf{R}(e)((h^*,c_j''),(h^*,c_k''),(h^*,c_a),(h^*,c_b))$. By *Strong Pareto* $((h^*,c_j'''),(h^*,c_k'''),(h^*,c_a'),(h^*,c_b'))\mathbf{P}(e)((h^*,c_j''''),(h^*,c_k'''),(h^*,c_a'),(h^*,c_b'))$. Finally, by *Transitivity* $((h^*,c_j'''),(h^*,c_k'''),(h^*,c_a'),(h^*,c_b'))\mathbf{P}(e)((h^*,c_j'''),(h^*,c_k'''),(h^*,c_a),(h^*,c_b))$. However, by *Well-being Bound Transfer* axiom we obtain that $((h^*,c_j'''),(h^*,c_k'''),(h^*,c_a),(h^*,c_b)\mathbf{R}(e)((h^*,c_j'''),(h^*,c_k'''),(h^*,c_a'),(h^*,c_b'))$, which yields the desired contradiction.

Let us now derive a similar contradiction for $\lambda_j(z'_N)\geq 1$. To do so we assume, once again, two individuals a and b with ORPs $R_a=R_j$ and $R_b=R_k$. Let us also assume, with $\beta,\varepsilon\in\mathbb{R}_{++}$, the following levels of consumption:

$$\begin{aligned}
&c_a''=\beta c_a,\ c_a'''=\beta c_a',\ c_b''=\beta c_b,\ c_k'''=\beta c_k',\\
&c_a=c_a'+\varepsilon,\ c_b=c_b'-\varepsilon,\\
&\widehat{c}_j>c_a''>c_a'''>c_j''>\widehat{c}_j'\geq c_e^*(\widehat{R}_j)>c_a>c_a',\\
&\widehat{c}_k'>\widehat{c}_k>c_b'''>c_b''>c_b'>c_b>c_e^*(\widehat{R}_k).
\end{aligned}$$

One again, by *Strong Pareto* $\widehat{z}'_N\mathbf{R}(e)\widehat{z}_N$. By *Consistency* $((h^*,\widehat{c}_j'),(h^*,\widehat{c}_k'),(h^*,c_a''),(h^*,c_b''))\mathbf{R}(e)((h^*,\widehat{c}_j),(h^*,\widehat{c}_k),(h^*,c_a''),(h^*,c_b''))$. By *Priority Among Equals* $((h^*,c_j''),(h^*,\widehat{c}_k'),(h^*,c_a'''),(h^*,c_b''))\mathbf{R}(e)((h^*,\widehat{c}_j'),(h^*,\widehat{c}_k'),(h^*,c_a''),(h^*,c_b''))$, and moreover by *Strong Pareto* we also have $((h^*,\widehat{c}_j),(h^*,\widehat{c}_k'),(h^*,c_a'''),(h^*,c_b''))\mathbf{P}(e)((h^*,c_j''),(h^*,\widehat{c}_k'),(h^*,c_a'''),(h^*,c_b''))$. Applying *Priority Among Equals* $((h^*,\widehat{c}_j),(h^*,\widehat{c}_k),(h^*,c_a'''),(h^*,c_b'''))\mathbf{R}(e)((h^*,\widehat{c}_j),(h^*,\widehat{c}_k'),(h^*,c_a'''),(h^*,c_b''))$. Finally, by Transitivity $((h^*,\widehat{c}_j),(h^*,\widehat{c}_k),(h^*,c_a'''),(h^*,c_b'''))\mathbf{P}(e)((h^*,\widehat{c}_j),(h^*,\widehat{c}_k),(h^*,c_a''),(h^*,c_b''))$. However, by *Well-being Bound Transfer* axiom $((h^*,c_a),(h^*,c_b))\mathbf{R}(e)((h^*,c_a'),(h^*,c_b'))$, which by *Perfect Consumption Scale Independence* leads to $((h^*,c_a''),(h^*,c_b''))\mathbf{R}(e)((h^*,c_a'''),(h^*,c_b'''))$. By *Consistency* we get the desired contradiction $((h^*,\widehat{c}_j),(h^*,\widehat{c}_k),(h^*,c_a''),(h^*,c_b''))\mathbf{R}(e)((h^*,\widehat{c}_j),(h^*,\widehat{c}_k),(h^*,c_a'''),(h^*,c_b'''))$.

In the final part of this second step of the proof we show that for any pair of allocations $z_N,z'_N\in Z^n$ such that $\min_{i\in N}\lambda_i(z_N)>\min_{i\in N}\lambda_i(z'_N)$, social preferences over these two allocations are characterised by $z_N\mathbf{P}(e)z'_N$. Let us then assume $z_N,z'_N\in Z^n$ with $\min_{i\in N}\lambda_i(z_N)>\min_{i\in N}\lambda_i(z'_N)$. Due to the properties of the ORPs it is possible to define two perfect health allocations $x_N,x'_N\in Z^n(h^*)$ such that for all $i\in N$ we have $\lambda_i(z_N)>\lambda_i(x_N)$ and $\lambda_i(x'_N)>\lambda_i(z'_N)$. Additionally, let $i_0\in N$ be such that

$\lambda_{i_0}(z'_N) = \min_{i \in N} \lambda_i(z'_N)$, where

$$\lambda_i(x'_N) > \lambda_i(x_N) > \lambda_{i_0}(x_N) > \lambda_{i_0}(x'_N).$$

Let $Q = N \setminus \{i_0\}$, and let us define a sequence of allocations $(x^q_N)_{1 \leq q \leq |Q|+1}$ such that for all $i \neq i_0$

$$c_i^{x^i} = \cdots = c_i^{x^1} = c_i^{x'},$$
$$c_i^{x} = c_i^{x^{|Q|+1}} = \cdots = c_i^{x^{i+1}},$$

and

$$c_i^{x'} = c_{i_0}^{x^1} < \cdots < c_{i_0}^{x^i} < \cdots < c_i^{x^{|Q|+1}} = c_i^{x}.$$

This sequence implies that $\lambda_q(x^q_N) > \lambda_q(x^{q+1}_N) > \lambda_{i_0}(x^{q+1}_N) > \lambda_{i_0}(x^q_N)$, with $\lambda_i(x^q_N) = \lambda_i(x^{q+1}_N)$ for all $i \neq q, i_0$. Consequently, and as we have previously proved, it must be the case that $x^{q+1}_N \mathbf{P}(e) x^q_N$ for all $q \in Q$. Moreover, by construction of x and x'_N we have both $z_N \mathbf{P}(e) x^{|Q|+1}_N$ and $x^1_N \mathbf{P}(e) z'_N$, and hence by *Transitivity* we finally get that $z_N \mathbf{P}(e) z'_N$.

-Step 3: Lexicographic order of $\mathbf{R}^{\lambda}_{\text{lex}}$

We extend the previous result in order to meet the lexicographic criterion, that is, we show that whenever $(\lambda_i(z_N))_{i \in N} \geq_{\text{lex}} (\lambda_i(z_N))_{i \in N}$, with at least one strict inequality, then $z_N \mathbf{P}(e) z'_N$. Without loss of generality, let us assume that there exist $j, k \in N$ such that $\min_{i \in N} \lambda_i(z_N) = \lambda_j(z_N) = \lambda_k(z'_N) = \min_{i \in N} \lambda_i(z'_N) < 1$. If the value of this minimum equivalent is different to the one proposed here, we can construct the allocations by means of *Strong Pareto* and *Perfect Consumption Scale Independence.* Additionally, let us assume that $\min_{i \in N \setminus \{j\}} \lambda_i(z_N) = \lambda_k(z_N) > \lambda_j(z'_N) = \min_{i \in N \setminus \{k\}} \lambda_i(z'_N)$. The fact that these values are given by agents j and k can always be generated by Step 1 and *Strong Pareto.* To obtain the desired result we need to distinguish between four different cases.

Case 1: $\lambda_j(z'_N) < \lambda_k(z_N) < 1$. By *Strong Pareto* $z^*_N \mathbf{I}(e) z_N$, where $z^*_N \in Z^n(h^*)$ is the allocation in which for every individual i we have both $h_i = h^*$ and $z^*_N I_i z_N$. Likewise, we obtain $z^{*'}_N \mathbf{I}(e) z'_N$. Let us now include one additional agent a who shares ORPs with individual j, that is $R_a = R_j \in \mathcal{R}$, and who enjoys a health-consumption bundle $z^*_a = (h^*, c^*_a) \in Z$ such that $c^*_a > c^*_e(\widehat{R}_j)$. According to *Consistency* and *Well-being Bound Transfer* $(z^{*'}_{N \setminus \{k\}}, z^{*''}_k, z^{*'}_a) \mathbf{I}(e) (z^{*'}_N, z^*_a)$, where $z^{*''}_k, z^{*'}_a \in Z$ are the (perfect health) bundles that satisfy, respectively, $c^{*''}_k / c^*_e(\widehat{R}_k) = c^*_a / c^*_e(\widehat{R}_j) > 1$ and $c^{*'}_k / c^*_e(\widehat{R}_k) = c^{*'}_a / c^*_e(\widehat{R}_j) < 1$. By *Priority Among Equals*

$(z^{*'}_{N\setminus\{j,k\}}, z^*_j, z^{*''}_k, z^{*''}_a)\mathbf{I}(e)(z^{*'}_{N\setminus\{k\}}, z^{*''}_k, z^{*'}_a)$, where $z^{*''}_a = z^{*'}_j \in Z$. If we apply *Well-being Bound Transfer* $(z^{*'}_{N\setminus\{j,k\}}, z^*_j, z^{*'''}_k, z^*_a)\mathbf{I}(e)(z^{*'}_{N\setminus\{j,k\}}, z^*_j, z^{*''}_k, z^{*''}_a)$, where $z^{*'''}_k \in Z$ is the bundle that satisfies $c^{*'''}_k / c^*_e(\widehat{R}_k) = c^{*''}_a / c^*_e(\widehat{R}_j) < 1$. According to the values of the λ-relative equivalents and *Strong Pareto*, we know from Step 2 that $z^*_{N\setminus\{j\}}\mathbf{P}(e)(z^{*'}_{N\setminus\{j,k\}}, z^{*'''}_k)$. If we apply *Consistency* and *Transitivity* $(z^*_N, z^*_a)\mathbf{P}(e)(z^{*'}_{N\setminus\{j,k\}}, z^*_j, z^{*'''}_k, z^*_a)\mathbf{I}(e)(z^{*'}_N, z^*_a)$. By means of *Consistency* and *Transitivity* we reach the desired result $z_N\mathbf{P}(e)z'_N$.

Case 2: $\lambda_k(z_N) = 1 > \lambda_j(z'_N)$ or $\lambda_k(z_N) > 1 > \lambda_j(z'_N)$. The final result can be easily obtained by using the same line of reasoning that we have applied in the previous case.

Case 3: $\lambda_k(z_N) > \lambda_j(z'_N) > 1$. By *Strong Pareto* $z^*_N\mathbf{I}(e)z_N$ and $z^{*'}_N\mathbf{I}(e)z'_N$, where z^*_N and $z^{*'}_N$ are constructed exactly as before. According to the values of the λ-relative equivalents and *Strong Pareto* $z^*_{N\setminus\{j\}}\mathbf{P}(e)(z^{*'}_{N\setminus\{j,k\}}, z^{*'''}_k)$, where $z^{*'''}_k \in Z$ is the bundle that satisfies $c^{*'''}_k / c^*_e(\widehat{R}_k) = c^{*'}_j / c^*_e(\widehat{R}_j) > 1$, and by *Consistency* $z^*_N\mathbf{P}(e)(z^{*'}_{N\setminus\{j,k\}}, z^*_j, z^{*'''}_k)$. Given that $\lambda_j(z^*_N) = \lambda_k(z^{*'}_N) < 1$ and $\lambda_j(z^{*'}_N) = \lambda_k(z^{*'''}_N) > 1$, by *Well-being Bound Transfer* $(z^{*'}_{N\setminus\{j,k\}}, z^*_j, z^{*'''}_k)\mathbf{I}(e)z^{*'}_N$. Again, by means of *Strong Pareto* and *Transitivity* we reach the desired result $z_N\mathbf{P}(e)z'_N$.

Case 4: $\lambda_k(z_N) > \lambda_j(z'_N) = 1$. Once more, we construct $z^*_N, z^{*'}_N \in Z^n$. Next, let us assume alternative allocations $z^{*''}_N, z^{*'''}_N \in Z^n$ such that for all $i \in N$ we have that $h_i = h^*$, and moreover $c^{*''}_i = \beta c^*_i$ and $c^{*'''}_i = \beta c^{*'}_i$, with $\beta \in (0,1)$. By continuity it is possible to find such allocations with $\lambda_k(z^{*''}_N) > 1$ and $\lambda_j(z^{*'''}_N) < 1$. From previous results we know that $z^{*''}_N\mathbf{P}(e)z^{*'''}_N$. By *Perfect Consumption Scale Independence* $z^*_N\mathbf{P}(e)z^{*'}_N$, and then by *Strong Pareto* and *Transitivity* we reach again the desired result $z_N\mathbf{P}(e)z'_N$.

-Step 4: Independence of the SO axioms

To conclude the proof we additionally show that the five normative requirements are independent, and hence if one of them is dropped it is possible to find a SOF which does not satisfy Theorem 1.

1) Drop *Strong Pareto*: Take $\mathbf{R}$ defined as the application of the lexicographic minimax criterion over $(\lambda_i(z_N))_{i\in N}$. This rule establishes that a group of objects $a_N \in \mathbb{R}^n$ dominates any other group $b_N \in \mathbb{R}^n$ if the highest value in a_N is lower than the highest value in b_N. If they are identical, then society eliminates these maximal elements and compares the highest values in the reduced allocations, and so on. The fact that $\mathbf{R}$ satisfies *Consistency* and *Perfect Consumption Scale Independence* is straightforward. According to *Well-being Bound Transfer*, social welfare does not decrease when we move

from allocation z'_N to z_N such that $\lambda_j(z'_N) > \lambda_j(z_N) > 1 > \lambda_k(z_N) > \lambda_k(z'_N)$. Therefore, this axiom is also satisfied by $\mathbf{R}$. The case of *Priority Among Equals* is shown in a similar way.

2) Drop *Consistency*: Consider $\mathbf{R}^{\lambda^+}_{\text{lex}}$ defined exactly as $\mathbf{R}^{\lambda}_{\text{lex}}$ but assuming, for all $i \in N$, that $c^*_e(\widehat{R}_i)$ is constructed using m^+ instead of m^-. Let $\mathbf{R}$ now coincide with $\mathbf{R}^{\lambda^+}_{\text{lex}}$ if there exists $i \in N$ such that $m_i = m^+$, and with $\mathbf{R}^{\lambda}_{\text{lex}}$ otherwise. With the exception of *Consistency*, all axioms are satisfied by $\mathbf{R}$ since it is constructed exactly as $\mathbf{R}^{\lambda}_{\text{lex}}$.
3) Drop *Perfect Consumption Scale Independence*: Take $\mathbf{R}$ defined as the application of the lexicographic minimax criterion over $(\rho_i(z_N))_{i\in N}$, where $\rho_i(z_N) = c^*_e(\widehat{R}_i)(1-\lambda_i(z_N))$, for all $i \in N$. The fact that $\mathbf{R}$ satisfies *Strong Pareto* and *Consistency* is straightforward. According to *Well-being Bound Transfer*, social welfare does not decrease when we move from allocation z'_N to z_N such that $c^*_e(\widehat{R}_j)(1-\lambda_j(z'_N)) > c^*_e(\widehat{R}_j)(1-\lambda_j(z_N)) > 0 > c^*_e(\widehat{R}_k)(1-\lambda_k(z_N)) > c^*_e(\widehat{R}_k)(1-\lambda_k(z'_N))$. Therefore, this axiom is also satisfied by $\mathbf{R}$. The case of *Priority Among Equals* is shown in a similar way.
4) Drop *Priority Among Equals*: Let $\mathbf{R}$ be defined such that for all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$, $z_N\mathbf{R}(e)z'_N$ if $\sum_{i\in N}\mu_i(z_N) \geq \sum_{i\in N}\mu_i(z'_N)$, where $\mu_i(z_N) = \lambda_i(z_N)c^*_e(\widehat{R}_i)$. The fact that $\mathbf{R}$ satisfies *Strong Pareto*, *Consistency* and *Perfect Consumption Scale Independence* is straightforward. Note that $\mu_i(z_N)$ is the individual i's level of consumption when she has perfect health. Therefore, since *Well-being Bound Transfer* is defined by assuming balanced transfers this axiom is also satisfied by $\mathbf{R}$. In other words, no transfer reverses the social preferences.
5) Drop *Well-being Bound Transfer*: Take $\mathbf{R}$ defined such that for all $e \in \mathcal{E}$ and $z_N, z'_N \in Z^n$, $z_N\mathbf{R}(e)z'_N$ if $(\mu_i(z_N))_{i\in N} \geq_{\text{lex}} (\mu_i(z'_N))_{i\in N}$, where $\mu_i(z_N)$ is defined as in the previous case. The fact that $\mathbf{R}$ satisfies all the four remaining axioms is straightforward. Note that *Perfect Consumption Scale Independence* changes $\mu_i(z_N)$, that is, the individual i's level of consumption when she has perfect health, in the same proportion for all individuals, and hence the lexicographic ranking does not change.

Consequently, any SOF that satisfies *Strong Pareto*, *Consistency*, *Perfect Consumption Scale Independence*, *Priority Among Equals* and *Well-being Bound Transfer* is grounded on the application of the leximin criterion over the λ-relative equivalents. Other social rankings may also satisfy all these requirements, although the combination of the five axioms single out $\mathbf{R}^{\lambda}_{\text{lex}}$.

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