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Fairness, freedom, and forgiveness in health care

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Abstract This paper focuses on the optimal allocation between health and lifestyle choices when society is concerned about forgiveness. Based on the idea of fresh starts, we construct a social ordering that permits us to make welfare assessments when it is acceptable to compensate individuals who have mismanaged their initial resources. Our social rule also allows for the inclusion of the fairness and responsibility approach in the model. Grounded on basic ethical principles, we propose the application of the minimax criterion to the existing distance between the individual's final bundle and her ideal choice.

Keywords fairness · health care · lifestyle preferences · regret · fresh start

1 Introduction

There is a clear consensus that health is one of the most crucial dimensions of individual well-being. Moreover, as Fleurbaey and Schokkaert (2009) highlight, "health care is also important because it contributes to better health, and perhaps also directly to a higher welfare level".

Assuming that individuals differ in both their preferences and their health care needs, we study how the ethical ideas of responsibility and forgiveness affect the optimal allocation between health and consumption, something that research has not yet addressed. Models of fairness and responsibility have been extensively studied (e.g., Fleurbaey 2008 and Fleurbaey and Maniquet 2011), while the ideal of forgiveness is a more recent approach that presents a not so infrequent implementation to real-life situations.[1] Therefore, it is worth modelling the two approaches at the same time.

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[1] For instance, when society helps those people who want to go back to school or when a public health service treats all individuals who are in a bad health condition, regardless of their previous lifestyle.

Regarding the first ethical ideal that inspires our analysis, theories of fairness and responsibility argue that when evaluating differences in outcome, it must be taken into account that they may contain elements of very different origin. Therefore, it is crucial to distinguish between legitimate and illegitimate sources of inequality. The former are those variables for which the individual is responsible, while the latter refer to the external circumstances that cannot be controlled by the individual. The central aim of these fairness and responsibility theories is to reduce outcome differences that are a result of illegitimate sources of inequality (e.g., Rawls 1971; Dworkin 1981a,b; Arneson 1989; Cohen 1989; Roemer 1998). Furthermore, it is argued that society should just respect outcome differences due to legitimate sources of inequality. It is well-known that both goals clash with each other (e.g., Fleurbaey 2008), we opt to adopt the first one.

Focusing on health, and following those theories, there are also legitimate and illegitimate sources of inequality. Individual preferences are one of the key factors that explain the need for health care. For instance, many of our illnesses are caused by lifestyle choices, such as too much drinking and smoking, having an unhealthy diet, or not doing enough exercise. Although there is some debate regarding purely lifestyle-based differences, we adopt the view that individual preferences are a legitimate source of inequality.[2]

An example of how to compensate individuals for illegitimate sources of inequality when they differ in their preferences about health and consumption can be found in Fleurbaey (2005a). He proposes to maximise the minimum *Healthy-Equivalent Consumption* (HEC), which is the smallest amount of consumption that any individual would accept to increase her current level of health to the state of perfect health. There are good reasons to use such a state as the reference value. In this case, all individuals can be compared directly by means of their levels of consumption, independently of their preferences. However, this would not be the case if we use a different reference point.[3] Along this line, Fleurbaey (2005a) shows that a monetary transfer principle that is not anchored at the level of perfect health may clash with the Pareto condition.

The second ethical ideal that we endorse is the notion of forgiveness. The inclusion of such a concept in the fairness literature was presented in a discussion between Dworkin (2000, 2002) and Fleurbaey (2002). Policies based on the forgiveness principle deal with individuals who experience genuine changes in their preferences and regret preceding decisions. There is still much debate as to whether such agents should be granted a fresh start, or if they should have to bear the consequences of their early choices. Some authors argue that it may be extremely unfair to deny them any kind of help. Others, such as Arneson (1989) or Dworkin (2002), assert that rewarding spendthrift individuals may generate a perverse incentive scheme.

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[2] Fleurbaey and Schokkaert (2009) list thoroughly what factors should be considered in health and health care as either legitimate or illegitimate sources of inequality.

[3] See Fleurbaey and Schokkaert (2011) for a detailed justification of the use of the level of perfect health as the reference value.

Fleurbaey (2005b) challenges the latter viewpoint. He shows that a forgiveness policy may provide more equality and more freedom, since individuals would no longer be fully forced to bear the consequences of their past decisions, and therefore they would have access to more choices.

To measure the regret individuals might experience, Fleurbaey (2005b) proposes a model in which all individuals have to allocate the same initial income between present and future consumption. Such a regret is measured as the difference that exists between the initial income and the *Equivalent Initial Share* (EIS). The EIS is defined as the minimum amount of monetary resources that any individual would need to buy a bundle that yields exactly the same level of utility as her current choice. The optimal redistribution policy should make the smallest EIS as large as possible. This policy entails a compromise between respecting individual preferences and giving all agents the possibility of a fresh start.

It is straightforward to show that the two measures, the EIS and the HEC, cannot be used simultaneously. Moreover, each one of them presents their own difficulties when introducing fresh starts in a health model. On the one hand, the HEC is not a good measure of regret because its value is affected by the direction in which the individual's preferences may change. On the other hand, the EIS is obtained assuming that all agents have the same trade-off between goods, which makes it possible to equalise perfectly the individuals' set of choices. However, that would not be the case if we introduce differences in health disposition, which would imply that the trade-off between goods (health and consumption) is no longer equal. In this scenario one could find two individuals with the same preferences and the same EIS, who would end up with different choices because of their different health care needs. Therefore, if we want to accommodate the ideal of forgiveness to a standard model of fairness and responsibility we should use other normative criteria than the HEC or the EIS.

An additional difficulty we should also take into account in our model is when individuals differ in their preferences. To solve this, the literature on fairness and social welfare advocates avoiding interpersonal comparisons that are based on subjective individual judgments, such as the level of satisfaction (e.g., Fleurbaey and Maniquet 2011). Therefore, any welfare evaluation must be obtained according to specific social value judgments that have to be properly defined by means of fairness conditions, which should involve individual non-comparable ordinal preferences. Consequently, the normative criterion we will derive in the present paper has to be also based on specific fairness conditions.[4]

To summarise, based on efficiency, robustness, and fairness properties, this paper singles out a specific social ordering function that ranks the preferences of a society that is concerned about individuals who regret their initial choices and/or have high health care needs. We obtain that society should give top

[4] It is worth remarking that both the HEC and the EIS follow this strategy too, albeit, as we have argued above, they are not suitable for a health model with forgiveness.

priority to that individual with the highest level of what we call *regret.* For any given individual, such a value is the (monetary) distance that exists, in healthy-equivalent consumption terms, between her current situation and the hypothetical choice she would have made with her final preferences if she had the most favourable health disposition.

The rest of the paper is organised as follows: Section 2 introduces our model. Section 3 describes the ethical requirements imposed on our social ordering rule; its derivation is presented in Section 4. Section 5 reviews the conclusions of this study. The Appendix provides the proof.

2 The model

Let us consider a model that consists of a finite set of individuals $N = \{1, \ldots, i, \ldots, n\}$. Health is a variable that ranges from 0 (full ill-health) to 1 (perfect health), that is, $h \in H = [0, 1]$, where $h^* := 1$. Consumption is interpreted as the expenditure on ordinary consumption goods (it does not include the amount of money spent on health), $c \in \mathbb{R}_+$. Every agent has a *health-consumption bundle* $z_i = (c_i, h_i) \in Z = \mathbb{R}_+ \times H$, that designates the situation in which the individual has health state h_i and consumption c_i. An *allocation* describes all individuals' bundles, that is, $z = (z_i)_{i \in N} \in Z^n$.[5]

Every individual $i \in N$ has well-defined *preferences* R_i over the space Z, which are described by a complete preorder, that is to say, a binary relation that is reflexive, transitive, and complete. The preferences, apart from being a complete preorder, must also be continuous, convex, and strictly monotonic. Let $\mathcal{R}$ denote the set of such preferences. $(c, h)R_i(c', h')$ means that individual i weakly prefers to live in a health state h with consumption c, rather than consume c' in a health state h'. Strict preference and indifference are denoted by P_i and I_i respectively.

In order to model the principle of forgiveness, let us assume that agents make their choices according to some *ex ante*, or initial, preferences, although they get their final utility from an *ex post*, or final, set that may or may not coincide with those *ex ante* preferences. Let $R^a_N = (R^a_i)_{i \in N} \in \mathcal{R}^n$ and $R^t_N = (R^t_i)_{i \in N} \in \mathcal{R}^n$ be, respectively, the *ex ante* and the *ex post* population profile of preferences.

Every individual $i \in N$ is also characterised by her health disposition. More precisely, let us assume that $\alpha_i h$, with $\alpha_i \in \mathbb{R}_{++}$, describes how much health expenditure is needed to bring this individual i to that health state $h \in H$. According to this functional form, for any pair of individuals $j, k \in N$, j is said to have a strictly better health disposition than k if and only if $\alpha_j < \alpha_k$. Let $\mathcal{A} = [\underline{\alpha}, \overline{\alpha}]$ be the set of all the feasible health dispositions, where $\underline{\alpha}$ and $\overline{\alpha}$ determine the best and the worst ones respectively. That is, $\overline{\alpha} \geq \alpha_i \geq \underline{\alpha}$ for all $i \in N$. Such limits are assumed to be fixed for all possible allocations. The population's profile of health dispositions is $A_N = (\alpha_i)_{i \in N} \in \mathcal{A}^n$. Note

[5] A group of objects $a = (a_i)_{i \in N}$ denotes a list such as $(a_1, \ldots, a_i, \ldots, a_n)$.

that the functional form of the health disposition entails two simplifications. First, the trade-off between health and consumption, although not equal, is linear for all individuals. Second, if the monetary resources are large enough, all individuals can eventually achieve the state of perfect health.

Finally, let us consider that all individuals are endowed with the same initial amount of monetary resources, $w \in \mathbb{R}_{++}$, that have to be allocated between consumption and medical expenditure. This value is also assumed to be fixed for all possible allocations.

An *economy* is denoted by $e = (R^t_N, A_N, w) \in \mathcal{E}$, where $\mathcal{E}$ is the domain of all the economies that satisfy the previous assumptions. Note that the interpretation of the concept of forgiveness we use considers that people's current situations must be evaluated taking into account only their final preferences, and hence, the viewpoint of the initial ones is totally discarded from the analysis (e.g., Fleurbaey 2005b). In order to compare allocations we have to define a social ordering $\mathbf{R}(e)$ over all of them, where $z\mathbf{R}(e)z'$ means that the allocation z is at least as good as z'. Strict preference will be denoted by $\mathbf{P}(e)$, and indifference will be denoted by $\mathbf{I}(e)$. Let us assume that social preferences are described by a complete preorder.

Let us define now the agent i's *health-consumption feasible set* as the set of bundles that she can afford, that is:

Definition 1 For all $e \in \mathcal{E}$ and $i \in N$, the individual i's health-consumption feasible set is:

$$B(w, \alpha_i) = \{(c_i, h_i) \in Z : c_i + \alpha_i h_i \leq w\}.$$

According to the individuals' preferences, we can also formally introduce the concept of *healthy-equivalent consumption*, which is the smallest level of consumption the individual would be willing to accept, according to her *ex post* preferences, to exchange her present bundle for one in which she has perfect health.

Definition 2 For all $i \in N, R^t_i \in \mathcal{R}$ and $z_i \in Z$, the individual i's healthy-equivalent consumption is the value $c^*_i(z_i)$ that satisfies:

$$c^*_i(z_i) = \min\{c' \in \mathbb{R}_+ : (c', h^*) R^t_i (c_i, h_i)\}.$$

Finally, let us describe the situation that maximises any agent's *ex post* preferences. More precisely, we define the individual i's *most preferred bundle* as the point in which she would be maximising her final preferences if she had the best health disposition. Formally:

Definition 3 For all $e \in \mathcal{E}$ and $i \in N$, the individual i's most preferred bundle is the health-consumption pair $\overline{z}_i \in Z$ that satisfies:

$$\overline{z}_i \in \max|_{R^t_i} B(w, \underline{\alpha}).$$

The reason why we propose this specific definition is twofold. First, it will allow us to describe the difference between the individual's *ex ante* choice and her ideal *ex post* bundle, introducing this way the principle of forgiveness. Second, fixing $\underline{\alpha}$ as the reference value also permits us to introduce a concern for inequality of opportunity in terms of health disposition, something that has not been considered in previous models of forgiveness.

3 The ethical principles

This section presents the ethical principles that we impose on our social ordering function. The first one is the Pareto condition, which is a minimal requirement that ensures the solution is efficient:

Axiom 1 *(Strong Pareto): For all $e \in \mathcal{E}$ and $z, z' \in Z^n$, if $z_i R_i^t z_i'$ for all $i \in N$ then $z\mathbf{R}(e)z'$. If moreover, $z_j P_j^t z_j'$ for some $j \in N$, then $z\mathbf{P}(e)z'$.*

The following principle permits us to compare two different allocations by using a subgroup of individuals, provided that the rest of the group has the same bundle in both allocations (e.g., d'Aspremont and Gevers 1977).

Axiom 2 *(Separation): For all $e \in \mathcal{E}$ and $z, z' \in Z^n$, if there exists $i \in N$ such that $z_i = z_i'$, then:*

$$z\mathbf{R}(e)z' \Leftrightarrow z_{-i}\mathbf{R}(e_{-i})z'_{-i},$$

where $z_{-i} = (z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n)$, and e_{-i} is the economy with reduced population $\{1, \ldots, i-1, i+1, \ldots, n\}$.

These two axioms are basic requirements derived from the social choice literature. Characterisation results would only be obtained when these basic requirements are combined with axioms modeling transfers. For instance, the result obtained by Fleurbaey (2005a) is given by a restricted version of the Pigou-Dalton axiom. More precisely, a transfer from a rich individual to a poor one would always be desirable, provided that either they are in perfect health or they have the same preferences. Fleurbaey (2005a) shows that a reasonable social objective is to apply the maximin criterion to healthy-equivalent consumptions.

In a more general framework, Valletta (2009) obtains a social ordering that applies the maximin criterion to the level of resources needed for agents to accept the highest level of personal endowment or talent (e.g., health condition) instead of their current situation.[6] His result is based, partly, on the assumption that transfers do not need to add up to zero; that is, provided that the individuals have the same preferences, it does not matter how much wealth the rich lose, as long as the poor are better-off and still poorer than the rich. Specifically:

[6] If such a particular amount of resources does not exist, he proposes to use the lowest level of talent that would make an agent accept a null transfer instead of her current situation.

Axiom 3 *(Equal Preferences Priority): For all $e \in \mathcal{E}$ and $z, z' \in Z^n$, if there exist $j, k \in N$ with $R_j^t = R_k^t$ such that:*

$$z'_j P_j^t z_j P_j^t z_k P_k^t z'_k,$$

with $z_i = z'_i$ for all $i \neq j, k$; then $z\mathbf{P}(e)z'$.

This principle implies an infinite aversion to inequality because monetary transfers are always desirable so long as the 'rich' remain 'richer' than the 'poor'. Therefore, ideally, two equally responsible individuals should end up with an equal level of utility.

Finally, in order to shape the idea of fresh starts into a fairness and responsibility framework we introduce two additional axioms.

The first of these two axioms requires that for any two allocations in which all agents have perfect health, if the consumption of all individuals increases in the same amount, the social preferences over those two allocations does not change.[7] Therefore:

Axiom 4 *(Linearity): For all $e \in \mathcal{E}$, $z, z' \in Z^n$ and $\beta > 0$,*

$$[(h^*, c_i)_{i \in N}]\mathbf{R}(e)[(h^*, c'_i)_{i \in N}] \Rightarrow [(h^*, c_i + \beta)_{i \in N}]\mathbf{R}(e)[(h^*, c'_i + \beta)_{i \in N}].$$

The second axiom we introduce at this point is a minimal requirement of solidarity from the agents who have an 'excessive' welfare level. It requires that a monetary transfer from one agent A who is better-off than in her most preferred bundle to another agent B who is worse-off than in her own ideal situation, will not reduce social welfare, provided that the individuals involved in the transfer have perfect health and that both keep the preference relation with respect to their own most preferred bundle. Similar principles have been laid out in order to impose welfare lower and upper bounds (e.g., Moulin 1987 and Maniquet and Sprumont 2005). Likewise, the axiom we propose entails a sort of 'safety net' for those who are worst-off than in their most preferred bundle. Then:

Axiom 5 *(Minimal Solidarity) For all $e \in \mathcal{E}$ and $z, z' \in Z^n$, if there exist $j, k \in N$ and $\varepsilon > 0$ such that $h_j = h_k = h'_j = h'_k = h^*$, with $z'_j P_j^t z_j P_j^t \overline{z}_j$ and $\overline{z}_k P_k^t z_k P_k^t z'_k$, where,*

$$\begin{cases} c_j = c'_j - \varepsilon \\ c_k = c'_k + \varepsilon, \end{cases}$$

with $z_i = z'_i$ for all $i \neq j, k$; then $z\mathbf{R}(e)z'$.

[7] Østerdal (2005) and Hougaard et al. (2013) obtain some of their results using the idea of equal social gains. However, the axiom they use is different from the one we propose here. More precisely, they consider that "for any health distribution with a pair of individuals (j and k) at a given equal health state, a gain in life years for individual j is, in social terms, as equally good as a gain in life years for individual k for the same number of years".

4 Social ordering and forgiveness

Having defined the basic elements of our model, we now proceed to describe the social ordering that, without violating the ideal of responsibility, aims to give those who regret their previous choices a fresh start. To carry out this analysis we will first introduce a function that provides us with a specific (monetary) measure of the individual's utility loss as a result of not being in her most preferred bundle.

Definition 4 For all $e \in \mathcal{E}, i \in N$ and $z_i \in Z$, the individual i's regret function is:

$$\rho_i(z_i) = c_i^*(\overline{z}_i) - c_i^*(z_i).$$

Note that such a gap can be caused by changes in the individual's preferences and/or by a poor health disposition. This function permits us to present the main result of this paper.

Proposition 1 *If social preferences satisfy Strong Pareto, Separation, Equal Preferences Priority, Linearity and Minimal Solidarity then, for any economy* $e \in \mathcal{E}$ *and allocations* $z, z' \in Z^n$ *we have that:*

$$\max_i \rho_i(z_i) < \max_i \rho_i(z_i') \Rightarrow z\mathbf{P}(e)z'.$$

Specifically, this social ordering minimises the maximum value of the regret function across the population. A graphical representation of this measure is given in Figure 1.

Let us consider two individuals, P and S, who are endowed with the best health disposition possible but have different preferences. In their initial choices, P_0 and S_0 respectively, individual P expends more on health than agent S. Let us also include an individual H who shares preferences with agent P, but has the worst health care needs. Her initial choice would be H_0, and her utility loss due to a poorer health disposition could be measured by means of her regret function ρ_{H_0}. Finally, we will also include a fourth individual, G, who is endowed with the best health disposition and shares her initial choice with agent S, although she gets utility according to individual P's preferences. The cost of her mistake can also be measured by her regret function ρ_{G_0}. Taking $z_0 = (P_0, H_0, S_0, G_0)$ as a starting point, Proposition 1 states that to move to an alternative allocation like $z_1 = (P_1, H_1, S_1, G_1)$ would improve social welfare, since the largest value of the regret functions would be smaller than that of the initial allocation (see Figure 1).

A remarkable feature of our social ordering is that the final result is driven by the hypothetical most preferred bundle, and hence is blind about the individuals' actual health disposition. The equal opportunity principle in health is, therefore, also included in our model since the most preferred bundle is exactly the same for all individuals with the same *ex post* preferences, regardless of their health dispositions.

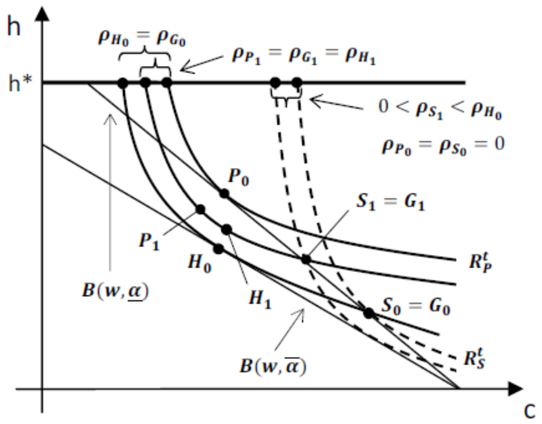

Fig. 1: Minimisation of the regret function

5 Concluding remarks

Forgiveness is a largely unstudied approach that advocates compensating those individuals who regret their past choices. After initially discussing the difficulties of implementing such a principle together with the fairness and responsibility ideal into a health-related scenario, we have made use of some ethical principles to reach a compromise between forgiveness and responsibility. From this we have derived a social ordering function that gives absolute priority to that individual with the highest level of regret. Such a regret, or utility loss, is defined as the monetary difference, in full health equivalent terms, between the individual's actual bundle and the choice she would make with her true preferences, if she had the best health disposition possible. This has allowed us to compare individuals with different preferences and different personal traits, namely health care needs, introducing, in this way, the opportunity principle in the model.

A Appendix: Proofs

A.1 Proof of Proposition 1

Proof This proof is based on the result obtained by Valletta (2009). For any economy $e \in \mathcal{E}$, let us consider two allocations $z, z' \in Z^n$ and two different individuals $j, k \in N$ with $R_j^t \neq R_k^t$ such that, without loss of generality, $\rho_j(z'_j) > \rho_j(z_k) > \rho_k(z_j) > \rho_k(z'_k)$, and $z_i = z'_i$ for all $i \neq j, k$. When the relations are different to the one proposed here, the proof is either analogous or immediate.

The proof splits in two steps.

First we need to prove that it must be the case that $z\mathbf{P}(e)z'$. Contrary to the desired result, let us assume that $z'\mathbf{R}(e)z$.

• Case i: Let us consider that $\rho_j(z'_j) > 0$. Let us introduce two individuals $b, c \in N$ with *ex post* preferences $R_b^t = R_j^t$ and $R_c^t = R_k^t$. Let us also assume that there exist

$z_j'', z_j''', z_k'', z_k''', z_b, z_b', z_c, z_c' \in Z$ with $h_j'' = h_j''' = h_j'''' = h_k'' = h_k''' = h_b = h_b' = h_c = h_c' = h^*$ and $\beta, \varepsilon > 0$ such that:

$$\begin{gathered} c_j'' = c_j^*(z_j') + \beta;\ c_j''' = c_j^*(z_j) + \beta \\ c_k'' = c_k^*(z_k') + \beta;\ c_k''' = c_k^*(z_k) + \beta \\ c_b = c_b' + \varepsilon;\ c_c = c_c' - \varepsilon \\ z_j''' P_j^t \overline{z}_j P_j^t z_b P_j^t z_b' P_j^t z_j'''' P_j^t z_j'' P_j^t z_j P_j^t z_j' \\ z_k'' P_k^t z_k''' P_k^t z_c' P_k^t z_c P_k^t \overline{z}_k P_k^t z_k' P_k^t z_k. \end{gathered}$$

Since $\rho_j(z_j') > \rho_j(z_k)$, and because of continuity, such bundles can always be found. According to the initial assumptions, if we apply *Strong Pareto*, *Separation* and *Linearity* it is straightforward to check that $(z_{-\{j,k\}}, z_j'', z_k'')\mathbf{R}(e)(z_{-\{j,k\}}, z_j''', z_k''')$. Because of *Separation*, we can also add identical agents in both allocations without altering the social ordering, that is, $(z_{-\{j,k\}}, z_j'', z_k'', z_b, z_c)\mathbf{R}(e)(z_{-\{j,k\}}, z_j''', z_k''', z_b, z_c)$. If we apply *Equal Preferences Priority* we get that $(z_{-\{j,k\}}, z_j'''', z_k'', z_b', z_c)\mathbf{P}(e)(z_{-\{j,k\}}, z_j'', z_k'', z_b, z_c)$, and moreover $(z_{-\{j,k\}}, z_j'''', z_k''', z_b', z_c')\mathbf{P}(e)(z_{-\{j,k\}}, z_j'''', z_k'', z_b', z_c)$. Using *Strong Pareto* we have that $(z_{-\{j,k\}}, z_j''', z_k''', z_b', z_c')\mathbf{P}(e)(z_{-\{j,k\}}, z_j'''', z_k''', z_b', z_c')$. Finally, by *Transitivity* we obtain that $(z_{-\{j,k\}}, z_j''', z_k''', z_b', z_c')\mathbf{P}(e)(z_{-\{j,k\}}, z_j''', z_k''', z_b, z_c)$. However, if we apply the *Minimum Solidarity* axiom it is straightforward to obtain that $(z_{-\{j,k\}}, z_j''', z_k''', z_b, z_c)\mathbf{R}(e)(z_{-\{j,k\}}, z_j''', z_k''', z_b', z_c')$, which yields the desired contradiction.

• Case *ii*: Let us consider now that $\rho_j(z_j') \leq 0$. Let us again introduce two individuals $b, c \in N$ with *ex post* preferences $R_b^t = R_j^t$ and $R_c^t = R_k^t$. Let us also assume that there exist $z_j'', z_b, z_b', z_b'', z_b''', z_c, z_c', z_c'', z_c''' \in Z$ with $h_j'' = h_b = h_b' = h_b'' = h_b''' = h_c = h_c' = h_c'' = h_c''' = h^*$ and $\beta > 0$ such that:

$$\begin{gathered} c_b = c_b' + \varepsilon;\ c_c = c_c' - \varepsilon \\ c_b'' = c_b + \beta;\ c_b''' = c_b' + \beta \\ c_c'' = c_c + \beta;\ c_b''' = c_c' + \beta \\ z_j P_j^t z_b'' P_j^t z_b''' P_j^t z_j'' P_j^t z_j' P_j^t \overline{z}_j P_j^t z_b P_j^t z_b' \\ z_k' P_k^t z_k P_k^t z_c''' P_k^t z_c'' P_k^t z_c' P_k^t z_c P_k^t \overline{z}_k. \end{gathered}$$

Again, since $\rho_j(z_j') > \rho_j(z_k)$, and because of continuity, such bundles can always be found. According to the initial assumptions, if we apply *Separation* we have that $(z_{-\{j,k\}}, z_j', z_k', z_b'', z_c'')\mathbf{R}(e)(z_{-\{j,k\}}, z_j, z_k, z_b'', z_c'')$. Because of *Equal Preferences Priority* it is straightforward to see that, $(z_{-\{j,k\}}, z_j'', z_k', z_b''', z_c'')\mathbf{P}(e)(z_{-\{j,k\}}, z_j', z_k', z_b'', z_c'')$. Using *Strong Pareto* we have that $(z_{-\{j,k\}}, z_j, z_k', z_b''', z_c'')\mathbf{P}(e)(z_{-\{j,k\}}, z_j'', z_k', z_b''', z_c'')$. Applying *Equal Preferences Priority* once more we get that $(z_{-\{j,k\}}, z_j, z_k, z_b''', z_c''')\mathbf{P}(e)(z_{-\{j,k\}}, z_j, z_k', z_b''', z_c'')$. Finally, by *Transitivity* we obtain that $(z, z_b''', z_c''')\mathbf{P}(e)(z, z_b'', z_c'')$. However, if we apply the *Minimum Solidarity* axiom we have that $(z_b, z_c)\mathbf{P}(e)(z_b', z_c')$, and because of *Linearity* we obtain that $(z_b'', z_c'')\mathbf{P}(e)(z_b''', z_c''')$. Using *Separation* we get the desired contradiction $(z, z_b'', z_c'')\mathbf{P}(e)(z, z_b''', z_c''')$.

In the second step of the proof we need to show that whenever there exist $z, z' \in Z^n$ such that $\max_i \rho_i(z_i) < \max_i \rho_i(z_i')$, this implies that $z\mathbf{P}(e)z'$. Let us take now two allocations $z, z' \in Z^n$ such that $\max_i \rho_i(z_i) < \max_i \rho_i(z_i')$. By *monotonicity* of preferences, we can find two allocations $x, x' \in Z^n$ in which for all $i \in N, h_i = h^*, z_i P_i^t x_i$ and $x_i' P_i^t z_i'$. Moreover, there exists i_0 such that for all $i \neq i_0$

$$\rho_i(x_i') < \rho_i(x_i) < \rho_{i_0}(x_{i_0}) < \rho_{i_0}(x_{i_0}').$$

Let $Q = N \setminus \{i_0\}$ and let us assume a sequence of allocations $(x^q)_{1 \leq q \leq |Q|+1}$ such that

$$\begin{aligned} c_i^*(x_i^q) &= c_i^*(x_i'), \quad \forall i \in Q:\ i \geq q \\ c_i^*(x_i^q) &= c_i^*(x_i), \quad \forall i \in Q:\ i < q, \end{aligned}$$

and for i_0 let us have

$$c_{i_0}^*(x_{i_0}') = c_{i_0}^*(x_{i_0}^1) < c_{i_0}^*(x_{i_0}^2) < \ldots < c_{i_0}^*(x_{i_0}^{|Q|}) < c_{i_0}^*(x_{i_0}^{|Q|+1}) = c_{i_0}^*(x_{i_0}).$$

This implies that $\rho_{i_0}(x^q_{i_0}) > \rho_{i_0}(x^{q+1}_{i_0}) > \rho_q(x^{q+1}_q) > \rho_q(x^q_q)$, while for all $j \neq q, i_0$ we have that $\rho_j(x^q_j) = \rho_j(x^{q+1}_j)$. As we have previously proved, for all $q \in Q$ it must be the case that $x^{q+1}\mathbf{P}(e)x^q$. According to the initial assumptions, $z\mathbf{P}(e)x^{|Q|+1}$ and $x^1\mathbf{P}(e)z'$. Finally, by transitivity we have that $z\mathbf{P}(e)z'$.

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