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Fair compensation with different social concerns for forgiveness

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Abstract Forgiveness is an ethical ideal that advocates that a fresh start should be conferred upon those individuals who have changed their preferences and regret their previous decisions. Despite the ethical debate that such an idea generates, only a few papers have dealt with this issue in depth, and they have just focused on the case of full compensation for regret. Therefore, based on efficiency, robustness, and ethical requirements, we characterise a social ordering function that formally connects the ideal of forgiveness to the problem of compensating individuals when they differ in both their preferences and their initial endowment. This social ordering allows us to rank allocations that may or may not be associated with different concerns for forgiveness. Specifically, it proposes reducing inequality between reference-comparable budget sets.

Keywords Regret · Forgiveness · Fairness · Responsibility · Social Ordering.
JEL classification: D63, D71.

1 Introduction

Whether individuals should or should not be deemed responsible for changes in their preferences has become an intense debated subject among economists and philosophers (e.g., Arneson 1989; Dworkin 2000, 2002; and Fleurbaey 1995, 2002, 2005, 2008). Regardless of what they put the focus on (resources, opportunities or capabilities), standard egalitarian theories generally consider legitimate not to compensate those individuals who change their preferences and regret their previous choices. Rewarding those who claim to regret their choices is not a trivial issue as it may generate

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incentive problems, since individuals could fake regret in order to get extra resources. Moreover, some reject this principle of forgiveness because it generates problems of unfairness. Specifically, they argue that this principle defends compensating individuals for a frugality they have never practised, and everything at the expense of those who have worked hard and have actually been frugal. This would allow the 'spendthrift' to take advantage of the situation and have the proverbial cake and eat it too (see Dworkin 2002).

Fleurbaey (2005, 2008) challenges this last ethical view. According to him, in the absence of any cost to others no-one would complain about helping individuals who regret their past choices. And the reason no-one would complain is because freedom would increase unambiguously if we give those regretful agents the possibility of choosing among an extended set of alternatives at no cost whatsoever. Therefore, Fleurbaey (2005, 2008) argues that there is no ethical, moral or fairness reason to make those individuals suffer the consequences of their wrong initial choices. He defends, then, that the problem is not really the fairness or ethical concerns, but the cost that the forgiveness ideal may have on others. However, since many redistribution and solidarity policies have been extensively justified for alternative scenarios, he argues that the fact that a forgiveness policy may entail a cost to others should not be a cause of concern either. He discards these cost issues by saying that it is only a matter of finding an adequate balance between the additional freedom obtained by those who benefit from the forgiveness ideal, and the decrease in the level of freedom experienced by those who have to fund it. Accordingly, it seems that the real problem that the principle of forgiveness raises is the fact that it may not be implemented due to incentive problems. More precisely, that the possibility that individuals have to strategically misrepresent their preferences in order to get extra resources may block the actual implementation of the principle.[1] Fleurbaey (2005, 2008) proposes dealing with this problem by means of designing an incentive-compatible forgiveness policy that would let individuals at liberty to choose different options. According to the choices that they can freely make, a social planner should control the 'excessive' level of welfare, according to a given measure, that any individual may obtain by adopting others' lifestyle. Finally, Fleurbaey (2005, 2008) presents the implementation of an incentive-compatible scheme of taxes and subsidies that grants a fresh start to those who regret their previous choices.[2] He shows that, apart from yielding higher levels of equality, the implementation of the principle of forgiveness indeed increases freedom, as individuals can overturn the consequences of their previous choices.

Together with this ideal of forgiveness, in this paper we also deal with the problem of compensating individuals who have different traits. Some of the most relevant theories of fairness and responsibility argue that inequalities in

[1] Fleurbaey (2005, 2008) suggests that those who are against the ideal of forgiveness are probably mixing this feasibility difficulty up with the former fairness concerns.

[2] A real-life example of the implementation of the forgiveness ideal is when a society tries to help those who have early dropped out of school and later want to apply to college.

agents' outcomes may contain elements for which those individuals are responsible, but also other elements for which they should not be deemed responsible. Those theories defend that individuals should only be compensated for outcome differences that are a result of the second group of elements (see Rawls 1971; Dworkin 1981a,b; Arneson 1989; Cohen 1989; and Roemer 1998).

Individual preferences are one of those elements that generate differences in the distribution of outcomes. In this paper we endorse the view, which is widely used nowadays, that considers that they are a legitimate source of inequality (e.g., Fleurbaey and Maniquet 2011). That is, we respect individuals' preferences because they reflect their opinion about what is important and what is not, and hence we hold them responsible for the way in which they decide to live their lives. It is worth stressing that there is no full unanimity about the use of this approach (see Cohen 1989; and Roemer 1998), but Fleurbaey (2008) thoroughly discusses this *responsibility cut*, and lays out sound arguments in favour of holding individuals responsible for their preferences and ambitions in life. He defends that what society should do is to let people exercise their freedom, but also to provide them with similar valuable alternatives. Apart from the ethical discussion of how to delimit the responsibility cut, endorsing this ideal of neutrality with respect to individual preferences entails interesting possibilities when analysing the principle of forgiveness. By assuming that individuals are responsible for their ambitions and life goals, which we can relate to their preferences, it is easy to design a 'forgiving' society. To do so it would only be necessary to consider that society should assess any individual's current situation in terms of preferences, or ambitions, which are different from her initial ones. For several reasons (see Fleurbaey 2005), this is harder to do in models which exclusively focus on genuine choices, such as the equal opportunity approach.[3]

After having briefly explained these ethical principles, our aim is to formally analyse how the forgiveness ideal interacts with the aforementioned compensation problem. Interestingly enough, existing models of forgiveness include neither a full axiomatic justification of the suggested solution, nor any additional source of unfairness other than regret.[4] As we have already stated, the most relevant solution to deal with the ideal of forgiveness is laid out by Fleurbaey (2005, 2008). Considering a model in which individuals are responsible for their preferences, he analyses the issue by assuming that resource egalitarianism is the final target. In such a framework, he suggests maximising, across the entire population, the minimum value of what he calls the *Equivalent Initial Share* (EIS). Such a concept is the minimum amount of resources that any individual would need to buy a bundle that

[3] See Fleurbaey (2002) and Vansteenkiste et al. (2014) for a discussion about the relevance of respecting individual preferences in relation to a model with forgiveness.

[4] An exception is Calo-Blanco (2014), who deals with the interaction between both frameworks in a specific health-related model, in which individuals differ in their health care needs.

would provide her, according to her final or *ex post* preferences, with the same level of utility as her current choice. Therefore, the key factor in his model is that any individual's current situation is compatible with her current, or *ex post*, preferences. However, besides including neither a full axiomatic justification of this equivalent measure nor any additional source of unfairness, Fleurbaey (2005, 2008) considers only full compensation for regret. That is, all the individual preferences, except for the most recent ones, are discarded in his analysis. He argues that, apart from the fact that the EIS corresponds entirely with the ideal of 'a fresh start every morning', by adopting the most generous approach towards the regretful agents it is possible to show that, even in the hardest cases, a "forgiving society is not like eating a cake and having it too". Interestingly enough, Fleurbaey (2005, 2008) acknowledges that he adopts such a view without making any normative judgment on the relative worth of old preferences versus new ones. Additionally, he also mentions the possibility of defining other measures that could balance both current and old preferences, discounting, this way, the individuals' level of regret and partially forcing them to bear the consequences of their past decisions.

All this being said, we argue in this paper that, depending on the assumptions that one could consider, there exist some scenarios in which it is worth combining the *ex ante* and the *ex post* individual preferences to define an intermediate degree of forgiveness. In other words, society should evaluate the individuals' current situation with a profile of preferences which is different from both the initial profile and the final one. For instance, the solution proposed by Fleurbaey (2005, 2008) hinges on the assumption that the new preferences are morally or cognitively superior to the old ones. Interestingly enough, he also points out that this may not be always the case, as it happens when one has gone through a process of addiction. In such scenarios he acknowledges that it is much more questionable to cater to the individual's current ambitions. Apart from this superiority of the current preferences, Fleurbaey (2005, 2008) also assumes that changes of mind cause no externalities on others, except for the taxes that they may have to pay. However, when helping those who regret their initial choices entails additional effects on others that are difficult to be compensated in terms of resources, it is more difficult to defend the ideal of 'a fresh start every morning'. Moreover, one can identify different types of regret, which can be linked to learning and informational factors, genuine changes of preferences, etc. Some of them, such as those related to the first group, generate an easy case of forgiveness. However, other sources of regret have a more difficult ethical justification, and society may, then, not be so willing to help individuals in such cases, as it happens with those sources that harm or morally damage others. Therefore, in such scenarios in which the justification of the forgiveness ideal is problematic, it is reasonable not to apply it to its full extent. A final argument in favour of considering an intermediate degree of forgiveness is that individuals may have different opinions about the kind of society they want to live in. As proposed by Fleurbaey (2005), they have to choose between "a community of egoistic self-righteous individuals who scorn those who mismanage their share of resources", and a forgiving society in which "the values of solidarity and compassion, maybe modesty as well, are cherished by individuals". The intermediate approach to forgiveness can be understood as a combination of these two views, which would represent the degree of that society's concern for forgiveness.

As a result of the previous discussion about the principle of forgiveness, our aim in the present paper is to construct, grounded on efficiency, robustness, and ethical principles, a social ordering function that allows us to rank all possible allocations on the basis of both responsibility and forgiveness criteria. Unlike previous models, this social ordering balances both *ex ante* and *ex post* individual preferences reducing, this way, the social value of their regret and partially forcing them to bear the consequences of their previous choices. Specifically, such a ranking pushes in the direction of reducing inequality between hypothetical reference budget sets that are constructed by combining initial and final preferences. This permits society to define to what extent it is willing to compensate individuals for their regretted choices. Moreover, the ranking we derive also allows for the social evaluation of alternative situations associated with different degrees of forgiveness. Therefore, this paper first introduces the axiomatic derivation of the concept of EIS, and then we extend such a measure to assess social situations characterised by different concerns for forgiveness. As it is standard in the social choice literature (e.g., Fleurbaey and Maniquet 2011), such an ordering is derived according to ethically appealing requirements that are defined by using only ordinal, and non-comparable, information about individual preferences.

The rest of the paper is organised as follows. Section 2 presents the basic components of the model, while Section 3 introduces the ethical requirements that society is willing to satisfy. Section 4 characterises the social ordering function that results from those requirements. Section 5 reviews the conclusions of this study. All proofs are relegated to the Appendix.

2 The model

Let us consider a society that consists of a finite set of individuals $N = \{1, \ldots, i, \ldots, n\}$. Each agent $i \in N$ has an initial endowment $\omega_i \in \mathbb{R}_{++}$ that she devotes to both consumption in period 1 ($x_{i1} \in \mathbb{R}_+$) and consumption in period 2 ($x_{i2} \in \mathbb{R}_+$). Let the profile of endowments in that particular society be $\omega_N = (\omega_i)_{i\in N} \in \mathbb{R}^n_{++}$,[5] and the price of general consumption in each period be a vector $q = (q_1, q_2) \in \mathbb{R}^2_{++}$, which is assumed to be fixed for all possible allocations. The individual i's *bundle* is a consumption vector $z_i = (x_{i1}, x_{i2}) \in Z$, where $Z = \mathbb{R}^2_+$ is the set of all the feasible bundles. An *allocation* defines all the individuals' bundles, that is, $z_N = (z_i)_{i\in N} \in Z^n$.

[5] A group of objects $a = (a_i)_{i\in N}$ denotes a list such as $(a_1, \ldots, a_i, \ldots, a_n)$.

Every agent $i \in N$ has well-defined preferences R_i over the space Z, which are described by a complete preorder, that is to say, a binary relation that is reflexive, transitive, and complete. The preferences, apart from being a complete preorder, must also be continuous, strictly convex, and strictly monotonic. Let $\mathcal{R}$ denote the set of such preferences, and let the expression $z_i R_i z'_i$ denote that individual i weakly prefers bundle z_i to bundle z'_i. The corresponding strict preference and indifference are denoted by P_i and I_i respectively.

Additionally, let us require individual preferences to satisfy the *single-crossing property.* That is, any two indifference curves of two different preferences cross no more than once. More precisely, for any $(x_1, x_2), (x'_1, x'_2) \in Z$, we say that individual preferences $R_j \in \mathcal{R}$ present a higher preference for future consumption than those of $R_k \in \mathcal{R}$, if they satisfy the following relations:

$$\begin{cases} x'_1 > x_1 \text{ and } (x_1, x_2) I_k (x'_1, x'_2) \Rightarrow (x_1, x_2) P_j (x'_1, x'_2) \\ x'_1 < x_1 \text{ and } (x_1, x_2) I_j (x'_1, x'_2) \Rightarrow (x_1, x_2) P_k (x'_1, x'_2). \end{cases}$$

In other words, individual j has a higher preference for future consumption than any other individual k if, for the same initial endowment, the former devotes more resources to consumption in period 2 than the latter.

A profile of preferences in this society is denoted by $R_N = (R_i)_{i \in N} \in \mathcal{R}^n$. Let us assume that agents make their choices according to some *ex ante* preferences $R^a_N = (R^a_i)_{i \in N} \in \mathcal{R}^n$, although they get their final utility from an *ex post* profile $R^t_N = (R^t_i)_{i \in N} \in \mathcal{R}^n$ that may or may not coincide with the *ex ante* preferences.

In order to compare allocations we have to define, according to the above assumptions, a social ordering function $\mathbf{R}$ over all of them. To properly define such an ordering, we first need to introduce how society deals with those who change their preferences. Since our objective in this paper is to characterise a measure of social welfare that aims to help those who regret their choices, such a social ordering function cannot be determined by the profile of individual *ex ante* preferences alone. However, as we have previously discussed, we also want to take into account the possibility of not fully compensating individuals for their regretted choices. Consequently, the function $\mathbf{R}$ cannot assess social welfare based solely on the *ex post* preferences either. Therefore, let us assume that society combines the two profiles of preferences, *ex ante* and *ex post*, at the time of evaluating the individuals' final well-being. More precisely, such an evaluation is implemented by means of a reference profile of preferences R^{φ}_N selected by a function $\varphi(R^a_N, R^t_N)$, such that $\varphi : \mathcal{R}^n \times \mathcal{R}^n \to \mathcal{R}^n$. Let Φ denote the set of all the possible variations of such a function. Note that this function φ can be used to model alternative frameworks that may or may not include the principle of forgiveness. On the one hand, all the models that ascribe full responsibility for past decisions to individuals have that $\varphi(R^a_N, R^t_N) = R^a_N$. On the other hand, those approaches which assume that the individuals' final welfare should be evaluated only with their *ex post* preferences consider that

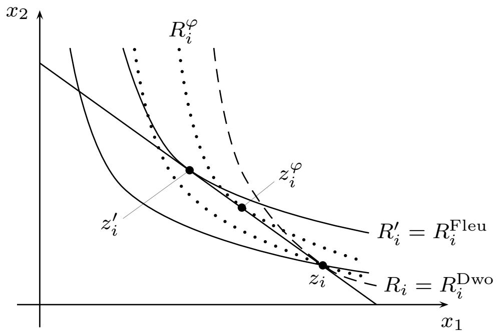


Fig. 1: Social concern for forgiveness

$\varphi(R_N^a, R_N^t) = R_N^t$. Let us take into account that when an individual does not change her preferences the *ex ante* and the *ex post* sets coincide, and hence the reference function selects that only set. The intuition for function φ is provided in Figure 1.

Let us consider an individual $i \in N$ who makes her initial choice, $z_i \in Z$, with *ex ante* preferences $R_i \in \mathcal{R}$ (dashed line), and according to a given fair share of resources. *Ex post*, she regrets her choice and wishes she had used alternative preferences $R_i' \in \mathcal{R}$ (solid curves). Dworkin's (2000) equality of resources view requires individuals to assume responsibility for their ambitions and life goals, so that he advocates giving these individuals a fair amount of resources which would help them to obtain such goals. However, as Fleurbaey (2002) points out, defining equality of resources over the whole life span at the beginning of life, as Dworkin (2000) did, can lead to similar 'unforgiving' results than those yielded by the theories that define responsibility in terms of genuine choices, such as the equality of opportunity approach. More precisely, Dworkin's (2000) view makes it possible not to help those who have mismanaged their initial resources and regret their choice, and hence it forces individual i to take full responsibility for the change in her preferences, that is $R_i^{\text{Dwo}} = R_i$. In order to avoid this particular 'unforgiving' structure, and to compensate those who have really changed their preferences, Fleurbaey (2005) proposes a modified version of the aforementioned equality of resources approach. Specifically, he requires that "people's current situations be compatible with what their current ambitions and life goals would yield from equal resources over their life span", and hence we would have that $R_i^{\text{Fleu}} = R_i'$.

As we have previously introduced, in this paper we endorse the principle of compensating those individuals who change their preferences and regret their initial choice. However, as we have previously argued too, we also want to take into account the possibility that some factors of regret may be discarded from the social compensation. Consequently, let us consider that society assesses the individual i's well-being with some intermediate preferences $R_i^{\varphi} \in \mathcal{R}$, as the ones represented in Figure 1 (dotted lines). Let us now describe such an intermediate approach. As we can observe in Figure 1, individual i would like to have her well-being evaluated with the indifference curve belonging to R'_i that passes through z_i, and hence be compensated until her *ex post* 'ideal' situation $z'_i \in Z$. However, there may exist some factors of regret for which society would not be willing to compensate this individual. As a consequence, her *ex post* preferences should be adapted or corrected in order to remove that information about the individual's regret that is not pertinent for social evaluations. Then, society would instead consider the indifference curve belonging to R_i^{φ} as the reference point for evaluation, and hence it would judge that the ideal situation for the individual i would be given by an alternative bundle $z_i^{\varphi} \in Z$, which would be closer to her initial choice. Therefore, the individual would be compensated for her regret, but not to its full extent. Hence, function φ can be seen as the degree to which the level of regret is adapted or corrected by society. It is important to stress that the exposition presented here is only valid under the single-crossing property. In this case there exists a continuum of indifference curves, belonging to different preferences, that pivoting on z_i, go from R_i to R'_i without crossing one another.

Accordingly, we write $(z_N, \varphi)\mathbf{R}(z'_N, \varphi')$ to denote that allocation z_N evaluated with social concern for forgiveness $\varphi \in \Phi$ is at least as good as z'_N under social concern for forgiveness $\varphi' \in \Phi$. $(z_N, \varphi)\mathbf{P}(z'_N, \varphi')$ means that z_N under forgiveness φ is strictly better than z'_N assessed with φ', and $(z_N, \varphi)\mathbf{I}(z'_N, \varphi')$ that they are equivalent. Let us assume that social preferences are described by a complete preorder. Note that the particular definition of $\mathbf{R}$ permits us to consider the possibility of assessing social situations with different concerns for forgiveness. The way in which this ranking is defined is similar to the notion of the *cross-profile social ordering function* introduced by Fleurbaey (2012) for the evaluation of arbitrary procedures. More precisely, none of these two social orderings resorts to standard comparisons of allocations, but to comparisons of allocations under different profiles, which in our model consist of different degrees of forgiveness. As regards the issue of helping those who regret their choices, the most natural approach is to consider that society should compare allocations that are evaluated with the same concern for forgiveness. However, the social ordering function that we have defined also includes the possibility of establishing an ordering over the set of allocations-cum-forgiveness. This would allow us, for instance, to compare societies with different degrees of forgiveness, or to evaluate who is the one who loses the most out of a change in how the concern for forgiveness is defined by one particular society. It is important to stress that we do not argue that society can choose between pairs of allocations and degrees of forgiveness such that social welfare is maximised. Our social ordering function $\mathbf{R}$ only establishes that allocations can be ranked even when they are evaluated with different given concerns for forgiveness, but not that society can actually choose between those concerns.

After having introduced the basic notation of the model, we present now a few definitions that will allow us to build our social ordering function. Let us start by defining the agent i's *consumption set* as the set of all the bundles that she can afford, given the market prices, with her initial endowment, that is:

Definition 1 For all $q \in \mathbb{R}^2_{++}, i \in N$ and $\omega_i \in \mathbb{R}_{++}$, the individual i's consumption set is:

$$B(\omega_i, q) = \{z_i \in Z : qz_i \leq \omega_i\},$$

where $qz_i = q_1 x_{i1} + q_2 x_{i2}$.

According to such a definition we introduce now a concept that will permit us to assess whether or not the individual is maximising her utility.

Definition 2 For all $\omega_N \in \mathbb{R}^n_{++}$, $q \in \mathbb{R}^2_{++}$, $i \in N$ and $R_i \in \mathcal{R}$, the individual i's set of maximisers is:

$$\overline{Z}_i(\overline{\omega}, q) = \{z_i \in Z : qz_i = \omega \leq \overline{\omega} \text{ and } z_i \in \max |_{R_i} B_i(\omega, q)\},$$

where $\overline{\omega} = \sum_{j=1}^n \omega_j$.

Note that this definition is not indexed to any specific value of ω_i. Hence, this set of maximisers consists of all the individual's optimal choices, given the fixed price vector, for any level of income that is smaller than or equal to the social endowment $\overline{\omega} \in \mathbb{R}_{++}$.

By the use of strict monotonicity of preferences, it is possible to identify a single element of the previous set that provides the individual with the same level of utility as her actual choice. Therefore, we can always relate the individual's actual level of welfare to the fraction of the overall endowment that she would need to buy a bundle that, belonging to $\overline{Z}_i(\overline{\omega}, q)$, would provide her with the same level of utility. Let us call that fraction the *proportional-income*:

Definition 3 For all $\omega_N \in \mathbb{R}^n_{++}$, $q \in \mathbb{R}^2_{++}$, $i \in N$, $R_i \in \mathcal{R}$ and $z_i \in Z$, the individual i's proportional-income is the scalar $\lambda_i(z_i, \overline{\omega}, q) \in [0, 1]$ related to a bundle $z'_i \in \overline{Z}_i(\overline{\omega}, q)$ that satisfies:

$$z'_i I_i z_i \text{ and } qz'_i = \lambda_i(z_i, \overline{\omega}, q)\overline{\omega}.$$

A graphical illustration of this concept of individual proportional-income is provided in Figure 2. Let us consider an individual $i \in N$ who has a bundle $\overline{z}_i \in Z$ which total cost, given the vector price q, amounts to the social endowment $\overline{w}$. If the individual's preferences are described by $\overline{R}_i \in \mathcal{R}$, such a bundle maximises her utility, and hence it is not possible to find another bundle that will provide her with the same level of utility at a lower cost. Consequently, with such preferences the individual's proportional-income associated with $\overline{z}_i$ is equal to 1. Let us now consider that her preferences are described by $R_i \in \mathcal{R}$ instead. For this new set there exists an alternative bundle $z_i \in Z$ that provides the individual with the same utility as $\overline{z}_i$, and which furthermore minimises her expenditure. Specifically, the cost of this bundle z_i is a fraction $\lambda_i(\overline{z}_i, \overline{\omega}, q) < 1$ of the overall endowment, a fraction that corresponds to the individual's

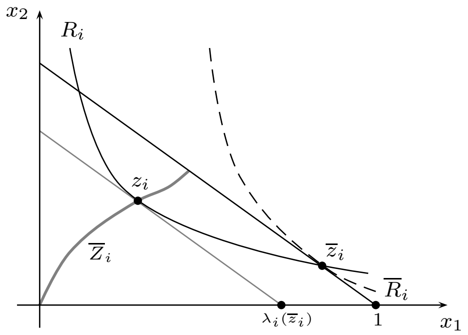


Fig. 2: The individual proportional-income

proportional-income associated with $\overline{z}_i$ and R_i. Note that, by definition, z_i must belong to $\overline{Z}_i(\overline{\omega}, q)$, which is the set that connects all the bundles that minimise the cost for any possible level of utility related to R_i (thick gray line in Figure 2). Therefore, we can identify the individual's proportional-income with the relative position of bundle z_i along such a set.

Based on the definitions that we have just introduced, in the following section we present the ethical principles that are desirable for our social ordering function.

3 Fairness requirements

When introducing the ethical principles that are desirable for our social ordering function, it is important to stress that they have to be defined in terms of the degree of forgiveness that society is willing to endorse. The first principle is a standard requirement of efficiency.

Axiom 1 *(Strong Pareto) For all $\varphi \in \Phi$, $R_N^a, R_N^t \in \mathcal{R}^n$ and $z_N, z'_N \in Z^n$, if $z_i R_i^{\varphi} z'_i$ for all $i \in N$, then $(z_N, \varphi)\mathbf{R}(z'_N, \varphi)$. If moreover, $z_j P_j^{\varphi} z'_j$ for some $j \in N$, then $(z_N, \varphi)\mathbf{P}(z'_N, \varphi)$.*

The following axiom is a robustness property demanding that indifferent agents should not influence social preferences. Specifically, it requires that adding or removing an agent who receives the same bundle in two different allocations does not modify the social ordering over such allocations (see d'Aspremont and Gevers 1977).

Axiom 2 *(Separation) For all $\varphi \in \Phi$, $R_N^a, R_N^t \in \mathcal{R}^n$ and $z_N, z'_N \in Z^n$, if there exists $i \in N$ such that $z_i = z'_i$, then:*

$$(z_N, \varphi)\mathbf{R}(z'_N, \varphi) \Leftrightarrow (z_{N\setminus\{i\}}, \varphi)\mathbf{R}_{-i}(z'_{N\setminus\{i\}}, \varphi),$$

where $z_{N\setminus\{i\}} = (z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n)$, and $\mathbf{R}_{-i}$ is the social ordering function associated with reduced population $\{1, \ldots, i-1, i+1, \ldots, n\}$.

Moreover, and similar to Arrow's condition of independence of irrelevant alternatives (see Arrow 1951), we want to limit the pieces of information about the individuals' preferences that are required to compare two different social situations. More precisely, we demand social preferences over two allocations with their own concerns for forgiveness to depend only on the indifference curves at these mentioned pairs of allocations and degrees of forgiveness (e.g., Hansson 1973; Pazner 1979; and Fleurbaey 2012).

Axiom 3 *(Independence) Let $R^a_N, R^t_N \in \mathcal{R}^n$, $z_N, z'_N \in Z^n$, and $\varphi, \varphi', \widehat{\varphi}, \widehat{\varphi}' \in \Phi$ be four degrees of forgiveness. If for all $i \in N$ and $z'' \in Z$,*

$$z_i I_i^{\varphi} z'' \Leftrightarrow z_i I_i^{\widehat{\varphi}} z''$$
$$z'_i I_i^{\varphi'} z'' \Leftrightarrow z'_i I_i^{\widehat{\varphi}'} z'',$$

then $(z_N, \varphi)\mathbf{R}(z'_N, \varphi') \Leftrightarrow (z_N, \widehat{\varphi})\mathbf{R}(z'_N, \widehat{\varphi}')$.

These first three axioms constitute a set of principles that are basic requirements derived from the social choice literature (see Fleurbaey and Maniquet 2011). In order to obtain our characterisation results we now turn to the axioms that provide the fairness properties.

The first one is the two-dimensional version of the popular Pigou-Dalton transfer axiom, which states that a mean-preserving progressive transfer increases welfare:[6]

Axiom 4 *(Transfer) For all $\varphi \in \Phi$, $R^a_N, R^t_N \in \mathcal{R}^n$ and $z_N, z'_N \in Z^n$, if there exist $j, k \in N$ and $\Delta \in \mathbb{R}^2_{++}$ such that:*

$$z'_j - \Delta = z_j \gg z_k = z'_k + \Delta,$$

with $z_i = z'_i$ for all $i \neq j, k$, then $(z_N, \varphi)\mathbf{P}(z'_N, \varphi)$.

It is a well-known fact that in this sort of settings the Transfer principle clashes with Pareto efficiency (e.g., Fleurbaey and Trannoy 2003). In order to accommodate equality of resources to efficiency principles we have opted to introduce weaker versions of the Transfer axiom (e.g., Fleurbaey and Maniquet 2011). The easiest way of executing such a task is to restrict the axiom only to individuals who have the same reference preferences:

Axiom 5 *(Equal Preferences Transfer) For all $\varphi \in \Phi$, $R^a_N, R^t_N \in \mathcal{R}^n$ and $z_N, z'_N \in Z^n$, if there exist $j, k \in N$ and $\Delta \in \mathbb{R}^2_{++}$ such that $R^{\varphi}_j = R^{\varphi}_k$ and:*

$$z'_j - \Delta = z_j P^{\varphi}_j z_k = z'_k + \Delta,$$

with $z_i = z'_i$ for all $i \neq j, k$, then $(z_N, \varphi)\mathbf{P}(z'_N, \varphi)$.

The problem with this last principle is that it does not permit us to define transfers that improve welfare when individuals differ in their reference preferences. Therefore, in order to make comparisons between any two individuals, regardless of their preferences, we have to introduce an additional axiom.

[6] Vector inequalities are denoted $\geq, >, \gg$.

This additional ethical requirement is an equality of resources principle that establishes that a mean-preserving progressive transfer that reduces the inequality of budgets will increase welfare. However, as we have learnt from the discussion between Dworkin (2000) and Fleurbaey (2005), one can follow different interpretations at the time of comparing resources when the principle of forgiveness is involved. Therefore, in order to avoid potential problems with alternative interpretations of forgiveness, we have focused on a less demanding idea of resource egalitarianism which considers transfers only among those agents who are not mismanaging their initial endowment according to the profile R_N^{φ}. Note that, as we have previously commented, for those who do not change their preferences there is no possible ambiguity with the profile R_N^{φ} as their *ex ante* and *ex post* preferences are identical. Hence, we can relate this additional axiom to those who do not regret their initial choice. Let us then define $\overline{Z}_i^{\varphi}(\overline{\omega}, q)$ as the individual i's reference set of maximisers, the one associated with the preferences that determine the social concern for forgiveness. Then, we propose a fairness requirement that aims to reduce inequality between reference budgets:

Axiom 6 *(Between-Maximisers Transfer) For all $\omega_N \in \mathbb{R}^n_{++}$, $\varphi \in \Phi$, $R_N^a, R_N^t \in \mathcal{R}^n$, $q \in \mathbb{R}^2_{++}$ and $z_N, z'_N \in Z^n$, if there exist $j, k \in N$ with $z_j, z'_j \in \overline{Z}_j^{\varphi}(\overline{\omega}, q)$ and $z_k, z'_k \in \overline{Z}_k^{\varphi}(\overline{\omega}, q)$, and for some $\delta \in \mathbb{R}_{++}$ such that:*

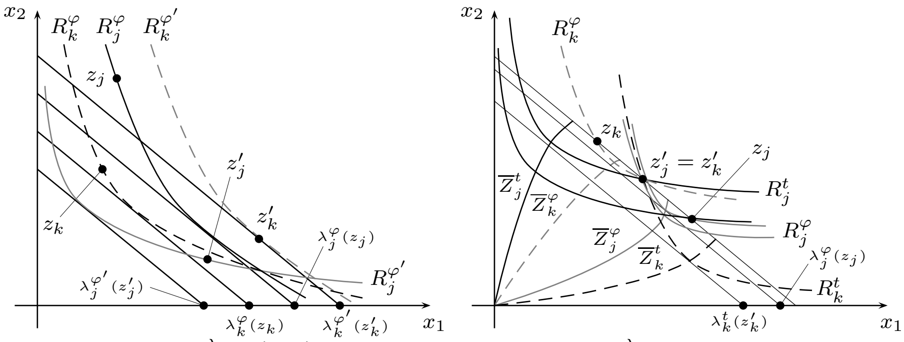
$$qz'_j - \delta = qz_j \geq qz_k = qz'_k + \delta,$$

with $z_i = z'_i$ for all $i \neq j, k$, then $(z_N, \varphi)\mathbf{P}(z'_N, \varphi)$.

This axiom reads as follows. By focusing on this particular group of individuals, their consumption set can be used to identify, in an unambiguous way, pairs of agents such that one is relatively 'richer' than the other. Then, the Between-Maximisers Transfer axiom establishes that a budget transfer from the former to the latter will increase welfare. That is, this particular group of agents should ideally receive the same value of resources, and then be left free to choose their preferred bundle.

The EIS proposed by Fleurbaey (2005) defines the unambiguous ranking in terms of the set $\overline{Z}_i^{t}(\overline{\omega}, q)$ as a function of the profile R_N^t, whereas the Dworkin's (2000) solution is presented in terms of $\overline{Z}_i^{a}(\overline{\omega}, q)$ defined by R_N^a. As we will characterise in the next section, the combination of all the axioms that we have previously introduced will allow us to relate any individual i's well-being to her specific set $\overline{Z}_i^{\varphi}(\overline{\omega}, q)$, and hence to the unambiguous rank between 'rich' and 'poor' people, whether she regrets her choice or not. Therefore, resource equalisation is characterised in our model by considering an intermediate approach to forgiveness which ranges between the two benchmark sets of maximisers proposed by Dworkin (2000) and Fleurbaey (2005).

Finally, note that the way in which the fairness axioms are introduced allows us to take into account both endowment differences and the level of regret. Therefore, such ethical principles aim to compensate individuals for the two aspects.



(a) $(z_N, \varphi)\mathbf{P}^{\lambda}_{\min}(z'_N, \varphi')$ (b) $\mathbf{P}^{\lambda}_{\min}$ and EIS

Fig. 3: The λ-Endowment Maximin function

4 Social preferences

In this section we proceed to characterise the social ranking that satisfies the axioms that we have previously described. Let us introduce, then, the following social ordering function:

Social Ordering Funcion 1 (λ-Endowment Maximin) *For all* $\omega_N \in \mathbb{R}^n_{++}$, $\varphi, \varphi' \in \Phi$, $R^a_N, R^t_N \in \mathcal{R}^n$, $q \in \mathbb{R}^2_{++}$ *and* $z_N, z'_N \in Z^n$,

$$\min_{i\in N} \lambda_i^{\varphi}(z_i, \overline{\omega}, q) > \min_{i\in N} \lambda_i^{\varphi'}(z'_i, \overline{\omega}, q) \Rightarrow (z_N, \varphi)\mathbf{P}^{\lambda}_{\min}(z'_N, \varphi').$$

This function identifies social welfare with the smallest value of a given reference measure. It is important to stress that such a measure is explicitly defined for each allocation on the basis of its own specific degree of forgiveness. This social ordering function considers the minimum percentage of the overall endowment that an individual would need to buy another bundle that would provide her, according to the social concern for forgiveness φ, with the same level of utility as her actual choice. More precisely, this measure is the individual's proportional-income defined in terms of her reference preferences, that is, $\lambda_i^{\varphi}(z_i, \overline{\omega}, q)$.

We present now the scenario in which the λ-Endowment Maximin social ordering function is obtained to evaluate social welfare.

Theorem 1 *A social ordering function satisfying Strong Pareto, Separation, Independence, Equal Preferences Transfer and Between-Maximisers Transfer is a* λ*-Endowment Maximin function for a* $q \in \mathbb{R}^2_{++}$.

Theorem 1 characterises the individual proportional-income as the proper measure to make welfare assessments when different degrees of forgiveness are considered. Note that when such degrees are anchored to the *ex post* preferences, the theorem provides an axiomatic derivation of the concept of EIS. Figure 3a illustrates this social ordering function.

Let us consider two allocations, $z_N, z'_N \in Z^n$, each one of them consisting of the bundles of two different individuals j and k. Let us also assume that the first allocation is evaluated with a social degree of forgiveness $\varphi \in \Phi$, while the second one is assessed with a different concern for forgiveness $\varphi' \in \Phi$. As we can observe in Figure 3a, the minimum proportional-income in the first scenario, $\lambda_k^{\varphi}(z_k, \overline{\omega}, q)$, is higher than that smallest one in the second situation, $\lambda_j^{\varphi'}(z'_j, \overline{\omega}, q)$. In other words, the minimum level of the specific reference individual well-being in z_N is larger than the respectively minimum specific one in allocation z'_N. Then, Theorem 1 states that allocation z_N evaluated with the degree of forgiveness φ is socially preferred to an alternative situation defined by z'_N and φ', that is, $(z_N, \varphi)\mathbf{P}^{\lambda}_{\min}(z'_N, \varphi')$.

After having presented the general way in which the λ-Endowment Maximin ranks pairs of allocation-cum-forgiveness, let us now draw an explicit comparison between this ranking and the social ordering function originally proposed by Fleurbaey (2005). In order to perform such a comparison in a clear fashion, let us focus, once again, on a society in which there are only two individuals, j and k (see Figure 3b). Let us also assume that the individual j's *ex post* preferences are given by the set $R_j^t \in \mathcal{R}$, whereas the individual k's ones are characterised by $R_k^t \in \mathcal{R}$. Let us consider that this society's aim is to rank, according to a given concern for forgiveness, an allocation $z_N \in Z^n$, which can be thought as the individuals' initial choice out of an equal amount of resources, with respect to an alternative allocation $z'_N \in Z^n$. According to Fleurbaey's (2005) view of forgiveness, society should resort to the *ex post* profile of preferences to define the reference measure of individual well-being. Then, such a view would consist of assessing the individuals' situation by means of their relative position on their own set of maximisers, being those two sets characterised by the individuals' *ex post* preferences, that is, $\overline{Z}_j^t(\overline{\omega}, q)$ and $\overline{Z}_k^t(\overline{\omega}, q)$ (in solid and dashed black, respectively, in Figure 3b). Actually, this relative position corresponds to the concept of EIS defined by Fleurbaey (2005). As we can observe in the picture, this solution ranks allocation z'_N above z_N, that is $(z'_N, \varphi^t)\mathbf{P}^{\lambda}_{\min}(z_N, \varphi^t)$, where φ^t denotes the Fleurbaey's (2005) *ex post* degree of forgiveness. The graphic intuition for this result is that each individual is undoubtedly closer to her respectively sole reference set of maximisers with the bundle associated with allocation z'_N, than with that one related to z_N. However, the result may differ as one moves away from Fleurbaey's (2005) view of forgiveness. Let us now consider that the individuals' factors of regret are adapted by society in such a way that their final well-being is evaluated, instead, with a reference profile $R_N^{\varphi} \in \mathcal{R}^n$, which is different from the *ex post* one. As a consequence, the individuals' relative situation should now be assessed by means of $\overline{Z}_j^{\varphi}(\overline{\omega}, q)$ and $\overline{Z}_k^{\varphi}(\overline{\omega}, q)$ (respectively, solid and dashed gray sets in Figure 3b). As we can observe in the picture, with this different approach we obtain that $(z_N, \varphi)\mathbf{P}^{\lambda}_{\min}(z'_N, \varphi)$. That is, opposite to Fleurbaey's (2005) approach, the conclusion is now that allocation z_N should be ranked above z'_N. The reason for this opposite result lies in the particular shape of the sets of maximisers in each specific view of forgiveness. In the view with preferences R_N^{φ} the reference set of maximisers is, for both agents, in between the two possible bundles and hence the result is not so clear as in the previous case. However, if we compare the individuals' proportional-income in the two possible allocations, we can check that the minimum value in z_N, which is given by $\lambda_j^{\varphi}(z_j, \overline{\omega}, q)$, is larger than the minimum one associated with allocation z'_N, which would be given by individual j and bundle z'_j. Therefore, as society is now more meticulous when defining the factors that trigger the compensation for regret, it ends up sticking to the *ex ante* choices rather than to an alternative allocation that is preferred by the two individuals, and that would have been ranked above by the approach proposed by Fleurbaey (2005). Then, we have that $\overline{Z}_j^{\varphi}(\overline{\omega}, q)$ and $\overline{Z}_k^{\varphi}(\overline{\omega}, q)$ are relatively closer to the sets of maximisers that would characterise the *ex ante* view of forgiveness rather than to those which would define the EIS measure of individual well-being.

It is important to stress that the result obtained in Theorem 1, as so does the EIS, hinges on the fact that prices are constant for all possible allocations, otherwise the social ordering would not yield a robust ranking. Additionally, as Fleurbaey and Maniquet (2011) have pointed out, to allow prices to vary will lead to a conflict between Pareto and Separation axioms. Moreover, it is only logical to assume that prices are fixed since they are related to the trade-off between present and future consumption (e.g., Fleurbaey 2005).

Finally, it is worth remarking the fact that the λ-Endowment Maximin social ordering function does meet the *laissez-faire* criterion. More precisely, in the particular case of uniform initial endowment and no regret, the social ranking would propose the absence of redistribution. The reason for this result is that in such a particular scenario, and according to the ethical requirements that society endorses, individuals would be considered to have an equivalent well-being in terms of these two only aspects that call for social compensation, namely endowment and regret. In other words, the individuals would be identified with the same relative position along their own reference set of maximisers. On the contrary, alternative rankings that are robust to changes in the trade-off between present and future consumption, such as those grounded on the Pazner and Schmeidler's (1978) concept of egalitarian-equivalence, fail to satisfy this *laissez-faire* ideal.

5 Conclusion

Forgiveness is an ethical ideal that defends giving a fresh start to those who have mismanaged their initial resources. The aim of this paper is to shape such an ideal into a standard model of fairness and responsibility. From minimally egalitarian principles we have constructed a social ordering function that allows us to compare allocations associated with different degrees of forgiveness. We have obtained that the function gives absolute priority to what it considers, according to a specific criterion, to be the worst-off individual. Such a criterion is an intermediate approach to forgiveness that is constructed by using both *ex ante* and *ex post* individual preferences. The specific way in which these preferences are combined defines the extent to which society is concerned for forgiveness.

A Proof of Theorem 1

In order to prove this theorem we first need to present the following two lemmas:

Lemma 1 *If a social ordering function satisfies Strong Pareto, Independence and Equal Preferences Transfer, then for all* $\varphi \in \Phi$, $R_N^a, R_N^t \in \mathcal{R}^n$ *and* $z_N, z'_N \in Z^n$, *if there exist* $j, k \in N$ *with* $R_j^\varphi = R_k^\varphi$ *such that:*

$$z'_j P_j^\varphi z_j P_j^\varphi z_k P_k^\varphi z'_k,$$

with $z_i = z'_i$ *for all* $i \neq j, k$, *then* $(z_N, \varphi)\mathbf{P}(z'_N, \varphi)$.

For proof of Lemma 1 see Fleurbaey and Maniquet (2006).

Lemma 2 *If a social ordering function satisfies Between Maximisers Transfer, then for all* $\omega_N \in \mathbb{R}_{++}^n$, $\varphi \in \Phi$, $R_N^a, R_N^t \in \mathcal{R}^n$, $q \in \mathbb{R}_{++}^2$ *and* $z_N, z'_N \in Z^n$, *such that for all* $i \in N$ $z_i, z'_i \in \overline{Z}_i^\varphi(\overline{\omega}, q)$ *and moreover,*

$$qz_i = \frac{1}{n}\sum_{j=1}^{n} qz'_j,$$

then $(z_N, \varphi)\mathbf{R}(z'_N, \varphi)$.

Let us assume that in allocation z'_N the value of the social endowment is not equally distributed, and hence there exist $j, k \in N$ such that $qz'_j > qz_j$ and $qz_k > qz'_k$. By *Between Maximisers Transfer* we can induce a third allocation $z''_N \in Z^n$ that presents a higher level of social welfare than z'_N, that is $(z''_N, \varphi)\mathbf{P}(z'_N, \varphi)$. More precisely, let us take a transfer $\delta \in \mathbb{R}_{++}$ such that $qz'_j - \delta = qz''_j \geq qz''_k = qz'_k + \delta$, with $z''_j \in \overline{Z}_j^\varphi(\overline{\omega}, q)$, $z''_k \in \overline{Z}_k^\varphi(\overline{\omega}, q)$ and $z''_i = z'_i$ for all $i \neq j, k$. We can then design the transfer δ such that either $z''_j = z_j$, or $z''_k = z_k$, or both $z''_j = z_j$ and $z''_k = z_k$. In the first case we have both $z''_j = z_j$ and $qz_k > qz''_k$, hence it must exist at least one individual $a \in N$ such that $qz''_a > qz_a$, and so the line of argument presented above can be repeated. If on the contrary we have $z''_k = z_k$ and $qz''_j > qz_j$, then there exists at least one individual $b \in N$ such that $qz_b > qz''_b$, and the same line of argument can be applied once more. Finally, if by any chance we get both $z''_j = z_j$ and $z''_k = z_k$, then either $z''_i = z_i$ for all $i \neq j, k$, or there exist $a, b \in N$ such that $qz''_a > qz_a$ and $qz_b > qz''_b$.

We can turn now to prove Theorem 1. The proof is split in two steps.

-***Step 1*:** For any $\omega_N \in \mathbb{R}_{++}^n$, $\varphi \in \Phi$, $R_N^a, R_N^t \in \mathcal{R}^n$ and a price vector $q \in \mathbb{R}_{++}^2$, let us consider, without loss of generality, two allocations $z_N, z'_N \in Z^n$ and two individuals $j, k \in N$ such that $\lambda_j^\varphi(z'_j) < \lambda_k^\varphi(z_k) < \lambda_j^\varphi(z_j) < \lambda_k^\varphi(z'_k)$,[7] and $z_i = z'_i$ for all $i \neq j, k$. We need to prove that it must be the case that $(z_N, \varphi)\mathbf{P}(z'_N, \varphi)$. Opposite to the desired result, let us assume that $(z'_N, \varphi)\mathbf{R}(z_N, \varphi)$.

Let us now introduce two additional individuals b, c such that $R_b^a = R_j^a$, $R_b^t = R_j^t$, $R_c^a = R_k^a$ and $R_c^t = R_k^t$. Let us assume too, that there exist $z''_j, z''_k \in Z$, $z_b, z'_b, z''_b \in \overline{Z}_b^\varphi(\overline{\omega}, q)$, and $z_c, z'_c, z''_c \in \overline{Z}_c^\varphi(\overline{\omega}, q)$ such that:

$$z_j P_j^\varphi z_b P_b^\varphi z'_b P_b^\varphi z''_b P_b^\varphi z''_j P_j^\varphi z'_j,$$
$$z'_k P_k^\varphi z_k P_k^\varphi z''_k P_k^\varphi z'_c P_c^\varphi z''_c P_c^\varphi z_c.$$

[7] For ease of notation, throughout the whole proof we will write $\lambda_i^\varphi(z_i)$ instead of $\lambda_i^\varphi(z_i, \overline{\omega}, q)$.

Moreover, there exists $\delta \in \mathbb{R}_{++}$ such that:

$$qz'_c - \delta = qz_c > qz_b = qz'_b + \delta \Rightarrow \lambda_c^\varphi(z'_c) > \lambda_c^\varphi(z_c) > \lambda_b^\varphi(z_b) > \lambda_b^\varphi(z'_b),$$

and $z_i = z''_i$ for all $i \neq j, k, b, c$.

According to the initial assumptions and the *Separation* axiom we get that $((z'_N, z_b, z_c), \varphi)\mathbf{R}((z_N, z_b, z_c), \varphi)$. Applying *Lemma 1* twice we obtain that $((z'_{N\setminus\{j,k\}}, z''_j, z''_k, z''_b, z''_c), \varphi)\mathbf{P}((z'_N, z_b, z_c), \varphi)$. *Strong Pareto* implies that $((z'_{N\setminus\{j,k\}}, z_j, z_k, z''_b, z''_c), \varphi)\mathbf{P}((z'_{N\setminus\{j,k\}}, z''_j, z''_k, z''_b, z''_c), \varphi)$. Finally, by *Transitivity* and *Separation* we have that $((z''_b, z''_c), \varphi)\mathbf{P}((z_b, z_c), \varphi)$. However, by *Strong Pareto* we have that $((z'_b, z'_c), \varphi)\mathbf{P}((z''_b, z''_c), \varphi)$, while according to *Between-Maximisers Transfer* we get that $((z_b, z_c), \varphi)\mathbf{P}((z'_b, z'_c), \varphi)$. *Transitivity* induces then that $((z_b, z_c), \varphi)\mathbf{P}((z''_b, z''_c), \varphi)$, which yields the desired contradiction.

Finally, we can design a series of allocations that would allow us to prove that whenever there exist $z_N, z'_N \in Z^n$ such that $\min_{i\in N} \lambda_i^\varphi(z_i) > \min_{i\in N} \lambda_i^\varphi(z'_i) \Rightarrow (z_N, \varphi)\mathbf{P}(z'_N, \varphi)$. Let us take then two allocations $z_N, z'_N \in Z^n$ such that $\min_{i\in N} \lambda_i^\varphi(z_i) > \min_{i\in N} \lambda_i^\varphi(z'_i)$.

Because of the strict monotonicity of the preferences, one can find two allocations $x_N, x'_N \in Z^n$ such that $\lambda_i^\varphi(z_i) > \lambda_i^\varphi(x_i)$ and $\lambda_i^\varphi(x'_i) > \lambda_i^\varphi(z'_i)$ for all $i \in N$. Moreover, there exists i_0 such that for all $i \neq i_0$:

$$\lambda_i^\varphi(x'_i) > \lambda_i^\varphi(x_i) > \lambda_{i_0}^\varphi(x_{i_0}) > \lambda_{i_0}^\varphi(x'_{i_0}).$$

Let $Q = N \setminus \{i_0\}$ and let us assume a sequence of allocations $(x_N^q)_{1\leq q\leq|Q|+1}$ such that:

$$\begin{array}{ll} \lambda_i^\varphi(x_i^q) = \lambda_i^\varphi(x'_i), & \forall i \in Q: \ i \geq q, \\ \lambda_i^\varphi(x_i^q) = \lambda_i^\varphi(x_i), & \forall i \in Q: \ i < q, \end{array}$$

while

$$\lambda_{i_0}^\varphi(x_{i_0}) = \lambda_{i_0}^\varphi(x_{i_0}^{|Q+1|}) > \lambda_{i_0}^\varphi(x_{i_0}^{|Q|}) > \ldots > \lambda_{i_0}^\varphi(x_{i_0}^1) = \lambda_{i_0}^\varphi(x'_{i_0}).$$

This implies that $\lambda_{i_0}^\varphi(x_{i_0}^q) < \lambda_{i_0}^\varphi(x_{i_0}^{q+1}) < \lambda_q^\varphi(x_q^{q+1}) < \lambda_q^\varphi(x_q^q)$, while for all $j \neq q, i_0$, we have that $\lambda_j^\varphi(x_j^q) = \lambda_j^\varphi(x_j^{q+1})$. As we have previously proved, it must be the case that $(x_N^{q+1}, \varphi)\mathbf{P}(x_N^q, \varphi)$, for all $q \in Q$. According to the initial assumptions, by *Strong Pareto* we have that $(z_N, \varphi)\mathbf{P}(x_N^{|Q|+1}, \varphi)$ and $(x_N^1, \varphi)\mathbf{P}(z'_N, \varphi)$. Finally, by *Transitivity* we have that $(z_N, \varphi)\mathbf{P}(z'_N, \varphi)$.

-***Step 2*:** Let us show now that whenever there exist $\omega_N \in \mathbb{R}^n_{++}$, $\varphi, \varphi' \in \Phi$, $R_N^a, R_N^t \in \mathcal{R}^n$, $q \in \mathbb{R}^2_{++}$ and $z_N, z'_N \in Z^n$ such that $\min_{i\in N} \lambda_i^\varphi(z_i) > \min_{i\in N} \lambda_i^{\varphi'}(z'_i) \Rightarrow (z_N, \varphi)\mathbf{P}(z'_N, \varphi')$. Let $a, b, c, d, e, f, g \in \mathbb{R}_{++}$ be such that:

$$\min_{i\in N} \lambda_i^\varphi(z_i) > g > e > d > b > a > \min_{i\in N} \lambda_i^{\varphi'}(z'_i),$$

and:

$$c = \frac{b + (n-1)d}{n}, \quad f = \frac{(n-1)e + g}{n}.$$

Let $x_N, y_N, x'_N, y'_N, y''_N \in Z^n$ be defined as:

$$\begin{array}{c} qx_1 = g\overline{\omega}, \ \text{ and for all } i \neq 1, \ qx_i = e\overline{\omega}; \\ \text{for all } i, \ qx'_i = f\overline{\omega}; \\ qy_1 = b\overline{\omega}, \ \text{ and for all } i \neq 1, \ qy_i = d\overline{\omega}; \\ \text{for all } i, \ qy'_i = c\overline{\omega}; \\ \text{for all } i, \ qy''_i = a\overline{\omega}. \end{array}$$

Let $\varphi^1, \varphi^2, \varphi^3, \varphi^4 \in \Phi$ be such that for all $i \in N$ and $z'' \in Z$,

$$\begin{array}{c} y''_i I_i^{\varphi^1} z'' \Leftrightarrow y''_i I_i^{\varphi'} z''; \ \ y_i I_i^{\varphi^2} z'' \Leftrightarrow y_i I_i^{\varphi^1} z''; \ \ y'_i I_i^{\varphi^3} z'' \Leftrightarrow y'_i I_i^{\varphi^2} z''; \\ x_i I_i^{\varphi^4} z'' \Leftrightarrow x_i I_i^{\varphi^3} z''; \ \ x'_i I_i^{\varphi^4} z'' \Leftrightarrow x'_i I_i^{\varphi} z''. \\ y''_i \in \overline{Z}_i^{\varphi'}(\overline{\omega}, q); \ \ y_i \in \overline{Z}_i^{\varphi^1}(\overline{\omega}, q); \ \ y'_i \in \overline{Z}_i^{\varphi^2}(\overline{\omega}, q); \\ x_i \in \overline{Z}_i^{\varphi^3}(\overline{\omega}, q); \ \ x'_i \in \overline{Z}_i^{\varphi^4}(\overline{\omega}, q). \end{array}$$

Since for all i, $y''_i \in \overline{Z}_i^{\varphi'}(\varpi, q)$ and since $\min_{i\in N} \lambda_i^{\varphi'}(y''_i) < a$, we obtain, following step 1, that $(y''_N, \varphi')\mathbf{P}(z'_N, \varphi')$. By *Independence* $(y''_N, \varphi^1)\mathbf{P}(z'_N, \varphi')$, and since $y_i \in \overline{Z}_i^{\varphi^1}(\varpi, q)$ and $b > a$, by *Strong Pareto* $(y_N, \varphi^1)\mathbf{P}(y''_N, \varphi^1)$. By *Independence* $(y_N, \varphi^2)\mathbf{P}(y''_N, \varphi^1)$. Since $y_i I_i^{\varphi^1} q \Leftrightarrow y_i I_i^{\varphi^2} q$ for all i, then $y_i \in \overline{Z}_i^{\varphi^2}(\varpi, q)$ for all i, and hence by *Lemma 2* $(y'_N, \varphi^2)\mathbf{P}(y_N, \varphi^2)$. By *Independence* $(y'_N, \varphi^3)\mathbf{P}(y_N, \varphi^2)$, and by *Strong Pareto* $(x_N, \varphi^3)\mathbf{P}(y'_N, \varphi^3)$. By *Independence* $(x_N, \varphi^4)\mathbf{P}(y'_N, \varphi^3)$, and by *Independence* and *Lemma 2* one has that $(x'_N, \varphi^4)\mathbf{P}(x_N, \varphi^4)$. By *Independence* $(x'_N, \varphi)\mathbf{P}(x_N, \varphi^4)$, whereas by *Independence* and *Strong Pareto* one obtains that $(z_N, \varphi)\mathbf{P}(x'_N, \varphi)$. Finally, by *Transitivity* $(z_N, \varphi)\mathbf{P}(z'_N, \varphi')$.

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