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# Evaluation of Analog Joint Source-Channel Coding Systems for Multiple Access Channels

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***Abstract*—We address the evaluation of low-complexity analog Joint Source Channel Coding (JSCC) methods for the transmission of discrete-time analog symbols over Multiple-Input Multiple-Output (MIMO) Multiple Access Channels (MAC). Analog JSCC is employed to encode the source information at each transmitter prior to be directly input to the MAC access scheme. Three channel access methods are considered to ensure the receiver is able to recover the user information: Code Division Multiple Access (CDMA), linear MMSE access codes and opportunistic access. CDMA allows the orthogonal transmission of the user data requiring only Channel State Information (CSI) at reception. On the other hand, linear MMSE access codes exploit CSI knowledge at transmission and exhibit better performance. Finally, opportunistic access also exploits CSI at transmission and allocates all MAC resources to the user with the strongest channel. This latter access scheme exhibits the best performance in terms of sum distortion although it may lead to unfair rate distributions among users.**

Index Terms: Analog Joint Source Channel Coding, Multiple Access Channels, CDMA, Linear MMSE Transceiver.

## I. Introduction

The traditional design of digital communications systems is based on the individual optimization of the source and channel encoders according to the separation principle [1]. This approach is termed Separate Source Channel Coding (SSCC) and has been shown to be optimal for both lossless compression [1] and lossy compression [2] from a theoretical perspective. Unfortunately, the complexity and delay required for practical implementations of digital SSCC systems to closely approach the optimal cost-distortion tradeoff is generally high because such systems need to employ powerful Vector Quantization (VQ) methods and long block lengths. Moreover, the optimality of the SSCC strategy does not hold in some situations like multiuser transmissions of correlated sources [3].

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An interesting alternative to drastically reduce transmission complexity and delay is to merge the operations of source and channel encoding into one single step. Over the past years, a Joint Source Channel Coding (JSCC) technique consisting in the use of a particular type of analog mappings, known as Shannon-Kotel'nikov mappings [4], [5], has been proposed for the transmission of discrete-time continuous-amplitude source symbols over AWGN channels [6]–[8]. In particular, Shannon-Kotel'nikov mappings describe the set of analog mappings based on parametrized continuous curves that fill up the entire source space efficiently. These schemes have also been shown to closely approach the optimal distortion-cost tradeoff with low complexity and delay, specially in the case of bandwidth reduction [8]–[10].

In addition to their low complexity, analog JSCC systems present a graceful performance degradation when the channel conditions for which they were designed change. Moreover, they can be continuously adapted to the channel fluctuations by simply updating the encoder parameters at the transmitter. This property constitutes a remarkable difference with respect to digital SSCC schemes, which are designed for specific channel conditions. When the Signal-to-Noise Ratio (SNR) falls below a threshold value in SSCC systems, the channel code is generally unable to correct the transmission errors and the source decoder breaks down under the presence of too many erroneous bits. Conversely, when the SNR is higher, the channel encoder actually needs to add less redundant bits to protect the information and hence more bits may be allocated to the source encoder in order to reduce the source distortion. In this case, adaptive tandem systems with separate source and channel coding can be used to cope with the fluctuations of time-varying channels. The basic premise is to continuously adapt the source distortion and the channel rate according to the instantaneous channel capacity. Notice, however, that the design of an adaptive tandem system based on the separation principle is rather difficult because it is necessary to fully redesign either the source or the channel encoder whenever either the distortion rate or the channel rate change.

These appealing properties support the suitability of analog JSCC for multiple access scenarios such as, for example, real-time transmissions in wireless communications or sensor networks. Nevertheless, most works in the literature focus on analog JSCC over Gaussian channels. Some exceptions are the evaluation of analog JSCC over Rayleigh fading channels [11], and the design of analog MIMO JSCC systems for flat-fading channels [12] and frequency-selective channels using an

OFDM modulation [13]. Although the Shannon-Kotel'nikov mappings employed for these analog JSCC systems are actually optimal for AWGN channels, the results obtained in these works also confirm their utility for fading channels and wireless communications. Nevertheless, all these schemes are designed for point-to-point communication scenarios.

In this paper, we are interested in studying the application of analog JSCC to multiuser communications. In particular, we focus on the transmission of analog data over a Multiple Access Channel (MAC). The transmission of information over a MAC is a fundamental problem in wireless communications that arises in many practical situations such as the uplink in a cellular system [14]. In a MAC scenario, multiple users simultaneously transmit their information over the same channel to a centralized receiver responsible for separating the data streams corresponding to each user. It is hence necessary to enable some mechanism to allow the users to share the MAC while ensuring an interference-free transmission of the information.

Over the past years, the design of suitable analog JSCC techniques for multiuser communications has been investigated from different points of view. Some interesting examples are [15], [16], where the design of optimal non-parametrized analog non-linear mappings for Gaussian MAC is addressed. The resulting mappings are obtained off-line for different SNR values by using an iterative procedure. In [17] a hybrid JSCC scheme is specifically proposed for the transmission of two correlated Gaussian sources over a Gaussian MAC. This scheme is referred to as Scalar Quantizer Linear Coding (SQLC) because it consists of a scalar quantizer and an optimized linear analog mapping. Analog JSCC has also been applied to relay multiple access channels in [18] where an analog network coding mapping that combines a particular type of Shannon-Kotel'nikov mapping and a sawtooth-like mapping is proposed assuming orthogonal transmission of the information. As in the case of point-to-point transmissions, most of these works focus on the case of Gaussian channels. An exception is [19], which introduces a JSCC scheme based on Channel Optimized Vector Quatization (COVQ) for the transmission of uncorrelated sources over MIMO broadcast channels. In this case, the mappings are not strictly analog because a set of discrete representation vectors are actually employed at the encoder.

In this work a different approach is presented for the practical implementation of zero-delay low-complexity analog JSCC techniques for the transmission of independent analog data over block-fading MACs. Instead of designing specific mappings for the considered MAC scenarios, we propose a distributed architecture consisting in using the parametrized analog mappings for point-to-point communications and specifically designing channel access schemes suitable for the analog JSCC transmission of discrete-time analog symbols. This approach not only simplifies the design of analog JSCC schemes for multiuser communications but also approaches the optimal distortion-cost tradeoff. Another advantage of this distributed design is its flexibility because a mapping may be easily exchanged with another at the encoding step and it is also possible to use different access schemes depending on the channel conditions.

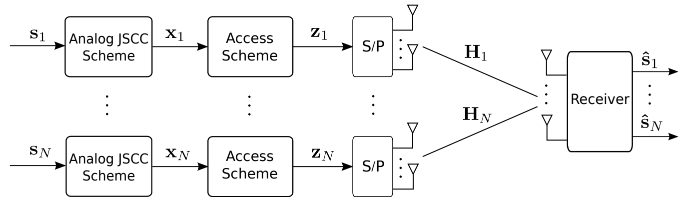


Fig. 1. Block diagram of the proposed scheme for the analog JSCC transmission over a block-fading MIMO MAC.

The rest of this paper is organized as follows. Section II introduces the system model considered along this work. Section III describes the analog JSCC transmission schemes employed to send the source information over the MIMO MAC. In Section IV we present three different channel access methods designed for analog JSCC: orthogonal Code Division Multiple Access (CDMA), linear Minimum Mean Square Error (MMSE) access codes and opportunistic access. In Section V the theoretical optimal bounds for analog JSCC transmissions over MACs are computed. Finally, Section VI presents the results of computer experiments and Section VII is devoted to the conclusions.

## II. SYSTEM MODEL

Let us consider a scenario where $N$ users transmit independent information simultaneously to a common receiver over a block-fading MAC, i.e., the channel is assumed to remain static during the transmission of a packet of symbols but independently varies from one packet to another. We address the general MIMO case where each user is equipped with $n_{T_i}$, $i = 1, \ldots, N$, transmit antennas and the centralized receiver with $n_R$ antennas. In this case, the signal at the MIMO MAC receiver is given by

$$\mathbf{y} = \sum_{i=1}^{N} \sqrt{\frac{P_i}{n_{T_i}}} \mathbf{H}_i \mathbf{z}_i + \mathbf{w}, \tag{1}$$

where $P_i$ and $\mathbf{z}_i$ are the power and the $n_{T_i} \times 1$ vector of complex-valued transmitted symbols corresponding to user $i$, respectively. $\mathbf{H}_i$ is the $n_R \times n_{T_i}$ complex-valued MIMO fading channel matrix for user $i$ and $\mathbf{w} \sim \mathcal{N}_{\mathbb{C}}(0, N_0 \mathbf{I})$ represents the Additive White Gaussian Noise (AWGN) at the receiver. Without loss of generality, we assume that the variances of the entries in $\mathbf{z}_i$ and $\mathbf{H}_i$ are equal to one. We also impose the sum power constraint $\sum_{i=1}^{N} P_i = P$.

In this case, the communication system should be designed to recover the analog information transmitted by each user at the receiver with the lowest possible sum-distortion. Thus, the overall objective is the minimization of the sum-distortion between the recovered and source symbols of all users subject to a global power restriction given by $P$. Additional constraints on the individual rates may be necessary to satisfy certain user requirements.

Fig. 1 shows the block diagram of the multiuser communication system described by (1). As observed, the vector of discrete-time analog source symbols $\mathbf{s}_i$ corresponding to each user $i$ is first encoded using a specific analog JSCC scheme.

The resulting symbols $\mathbf{x}_i$ are then input to the access scheme and sent over the MAC using $n_{T_i}$ antennas. An adequate design of the channel access method allows the receiver to separate the information for each user from the received MAC signal. Notice that the allocation of the transmit power $P$ among the MAC users is specifically determined by the channel access scheme. Finally, an estimate of the original source symbols can be computed from the individual data streams obtained for each user. In the ensuing sections, we describe the low-complexity zero-delay analog JSCC schemes employed for the transmission of the user information and the corresponding channel access methods designed to approach the optimal performance of the MIMO MAC system under certain constraints.

## III. Analog JSCC

As previously mentioned, we consider the analog JSCC transmission of discrete-time continuous-amplitude symbols. Different from traditional digital systems, the quantization, source coding and channel coding operations are replaced by a single analog mapping. Given that analog JSCC has been shown to closely approach the optimal cost-distortion tradeoff specially for the compression case, we focus on the bandwidth reduction of the source information, although other analog JSCC mappings for bandwidth expansion can be also applied for the proposed model.

In particular, we consider that $M$ consecutive source symbols are encoded into one channel symbol at each transmitter using an analog JSCC encoder such as those described in [8]. Because these analog mappings are generally formulated for Gaussian distributions, we assume $N$ analog sources each producing i.i.d. Gaussian samples with zero mean and variance $\sigma_s^2$. In this work, we focus on two particular cases: uncoded transmission ($M = 1$) and 2:1 bandwidth reduction ($M = 2$). When $M = 1$, the source symbols $s_{ij}$, $j = 1, \ldots, k_i$, are first normalized and then input directly to the corresponding channel access scheme, i.e., $x_{ij} = s_{ij}/\sqrt{\sigma_s^2}$.

In the general case of $M > 1$, the compression operation typically involves two steps shown in Fig. 2: the mapping function $M_\delta(\cdot)$ and the stretching function $T_\alpha(\cdot)$. The mapping operation implements the $M$:1 dimension reduction by using a non-linear continuous curve that efficiently fills up the source space. Analog mappings based on the use of space-filling curves are usually referred to as Shannon-Kotel'nikov mappings [4], [5]. In the particular case of 2:1 bandwidth reduction, a spiral-like mapping was shown to be optimal for the transmission of Gaussian sources [20]. For that reason, we specifically consider a mapping based on the use of doubly interleaved Archimedean spirals [7], [8], which transform two consecutive source symbols from user $i$, i.e., $(s_{i,2j},\ s_{i,2j+1})$, into one single channel symbol $x_{ij}$. An attractive property of this mapping is that any point on the Archimedean spiral can be mathematically defined as

$$\mathbf{z}_{\delta_i}(\theta) = \left[\mathrm{sign}(\theta)\frac{\delta_i}{\pi}\theta\sin\theta, \frac{\delta_i}{\pi}\theta\cos\theta\right]^T, \qquad (2)$$

where $\delta_i$ is the distance between two neighbouring spiral arms in the curve corresponding to user $i$, while $\theta$ is the angle from

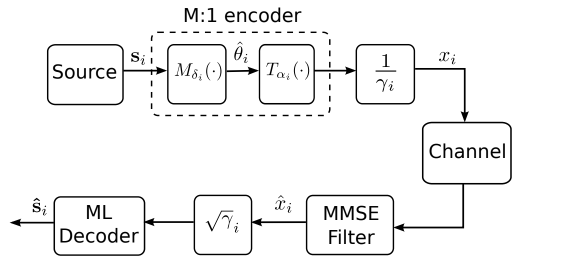


Fig. 2. Block diagram of the $M$:1 analog JSCC scheme employed for the transmission of the source symbols corresponding to user $i$.

the origin to the point $\mathbf{z} = [z_1, z_2]^T$ on the curve. Notice that the encoder parameter $\delta_i$ determines how the two-dimensional space is filled up and, therefore, how protected the source symbols are against the channel noise. Hence, it is important to use the optimum values for $\delta_i$ at the mapping operation in order to approach the optimal cost-distortion tradeoff. Since the channel conditions vary in general from one user to the other, the Archimedean spiral employed at each transmitter will be different and the parameter $\delta_i$ should be individually selected for each user.

It is possible to achieve higher compression rates ($M > 2$) by extending the Archimedean spiral for higher dimensions [10], [21]. These compression schemes are able to approach the optimal cost-distortion tradeoff although the gap between their performance and the optimal bound increases as M is larger. Similarly, Shannon-Kotel'nikov mappings can also be used for bandwidth expansion [22], although in this case the performance is further away from the optimal cost-distortion tradeoff due to the difficulty to envisage mappings that efficiently fill the entire channel space without simultaneously creating multiple neighbors that are far away in the source space.

The mapping function $M_{\delta_i}(\cdot)$ takes a source pair, $\mathbf{s}_{ij} = (s_{i,2j}, s_{i,2j+1})$, and calculates the angle from the origin to the point on the spiral that minimizes the Euclidean distance to $\mathbf{s}_{ij}$. Thus,

$$\hat{\theta}_{ij} = M_{\delta_i}(\mathbf{s}_{ij}) = \mathrm{argmin}_\theta \|\mathbf{s}_{ij} - \mathbf{z}_{\delta_i}(\theta)\|^2. \qquad (3)$$

The compressed symbols at the output of the mapping operation are then transformed by using the stretching function $T_{\alpha_i}(\hat{\theta}_{ij}) = (\hat{\theta}_{ij})^{\alpha_i}$. Notice that this transformation provides certain degrees of freedom to optimize the analog system. In [6], [8], [23], $\alpha_i = 2$ was proposed because in that case the output distribution can be approximated by a Laplacian. However, as shown in [9], the system performance can be improved if the parameter $\alpha_i$ is also optimized together with $\delta_i$. Finally, the resulting coded values are normalized by a factor $\gamma_i$ to ensure the average transmitted power is equal to one. Hence, the input symbols to the channel access scheme are given by

$$x_{ij} = \frac{T_{\alpha_i}(M_{\delta_i}(\mathbf{s}_{ij}))}{\sqrt{\gamma_i}} \qquad (4)$$

At the receiver, we determine an estimate of the transmitted source symbols using Maximum Likelihood (ML) decoding. At this point, we assume the data stream corresponding to user $i$ can be recovered from the received signal thanks to the use of an adequate channel access scheme. As shown in [24], the performance of analog JSCC systems using ML decoding closely approaches the optimal cost-distortion tradeoff with very low complexity as long as the received symbols are conveniently filtered before proceeding to the decoding. In the proposed model, the inputs at the ML decoder are obtained after applying the access scheme to recover the transmitted channel symbols and filtering the resulting symbols by using a linear MMSE detector. The utilization of this two-stage receiver structure greatly simplifies the design and the complexity of analog JSCC schemes.

Let $\hat{x}_i$ be the symbol obtained after the filtering operation. In this case, the ML estimate $\hat{\mathbf{s}}_i^{\mathrm{ML}}$ is hence calculated as the point on the Archimedean spiral that maximizes the likelihood function $p(\hat{x}_i|\mathbf{s}_i)$. In practice, ML decoding consists in first applying the inverse function $T_{\alpha_i}^{-1}(\cdot)$ to the filtered symbol $\hat{x}_i$ after de-normalization and then calculating the closest point on the spiral corresponding to the resulting value, i.e.,

$$\hat{\mathbf{s}}_i^{\mathrm{ML}} = \mathbf{z}_{\delta_i}\left(T_{\alpha_i}^{-1}\left(\sqrt{\gamma_i}\hat{x}_i\right)\right). \tag{5}$$

As mentioned in the introduction, an interesting feature of analog JSCC systems is the capacity to continuously adapt to time-varying channels by simply updating the coding parameters. In general, the performance degradation of analog JSCC is not critical when the channel information is inaccurate, although the use of the optimal values for the parameters $\delta_i$ and $\alpha_i$ at the analog encoders is required to closely approach the optimal cost-distortion tradeoff. Such optimal values actually depend on the instantaneous Channel Signal-to-Noise Ratio (CSNR) that must be estimated at the receiver and feed back to the users in order to select the proper parameters $\delta_i$ and $\alpha_i$ at the corresponding encoder. Notice that this adaptation of the coding parameters depending on the instantaneous CSNR values ensures that the optimal JSCC mapping is always employed at each channel realization. On one hand, we have empirically determined through off-line computer simulations that using $\alpha_i = 1.3$ provides a good overall performance for a wide range of CSNRs and $\delta_i$ values. On the other hand, given that the analytical optimization of $\delta_i$ is very difficult when $\alpha_i \neq 2$ [7], the optimum values for $\delta_i$ can be also determined by off-line computer simulations. The obtained values can be looked up in the tables presented in [12].

## IV. Channel Access Schemes

When sending information over a MAC, the received signal corresponding to each user is affected by the interferences from the other users. A traditional approach to deal with this situation is the utilization of a channel access method for the receiver to be able to separate the information from each user. Orthogonal access schemes have been shown to achieve the sum-capacity of a MAC when the channel is assumed to be Gaussian [14]. Nevertheless, orthogonal strategies are suboptimal in the case of fading channels. In the particular case of single-antenna fading MACs with Channel State Information (CSI) available at the transmitters, it can be shown [25] that the optimal channel access scheme allocates all available power to the user with the strongest channel. This strategy is commonly referred to as opportunistic access and is optimal in the sense that it maximizes the sum-rate in the MAC. However, opportunistic access is no longer optimal for MIMO MAC systems where the users and the common receiver employ multiple antennas. In that case, the capacity of digital SSCC communication systems over a block-fading MAC can be achieved using a scheme based on superposition coding and successive interference cancellation at the receiver [26]. In such a scheme, it is possible to decode the information from one of the users without errors as long as its rate is below the capacity limit imposed by the Signal to Interference plus Noise Ratio (SINR) for such a user. Then, the information corresponding to that user is subtracted from the incoming signal before decoding the next user. Unfortunately, this strategy is unfeasible for analog JSCC since a certain level of distortion always remains after decoding the user information in such manner that the subtracting process does not completely remove the interference of each user causing an inevitable degradation of the successive cancellation procedure.

In general, MAC access schemes can be designed according to diverse objectives such as, e.g. minimizing the transmission power while ensuring a certain distribution of the user rates or maximizing the sum-rate for a given overall transmit power constraint. In the ensuing subsections, we present three different access methods designed for the transmission of discrete-time continuous-amplitude sources over block-fading MIMO MACs using analog JSCC: orthogonal CDMA, linear MMSE access coding and opportunistic access.

When the channel is only known at the receiver, CDMA guarantees an orthogonal and simultaneous transmission of the user information to the common receiver. If CSI is assumed to be also available at the transmitters, we can exploit this knowledge to determine the optimal linear MMSE access codes to be used instead of the orthogonal CDMA codes. Finally, the opportunistic access is intended to maximize the sum-rate of the multiuser analog JSCC system regardless of per-user rate/power constraints. The advantage of the CDMA and linear MMSE schemes with respect to the opportunistic access is their ability to ensure that MAC users always transmit their data at a given individual rate, whereas the opportunistic strategy can lead to unfair distributions of the individual powers and, specially, of the user rates.

### A. Orthogonal CDMA

Let us first consider a MAC scenario where the users transmit their data with a given constant rate and the CSI is only available at the receiver. In this situation, multiple access is possible using a CDMA scheme based on orthogonal spreading codes. The advantage of CDMA with respect to other orthogonal access methods such as Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA) is that all users synchronously and simultaneously transmit over the same bandwidth while the receiver exploits the orthogonality of the spreading codes to remove the

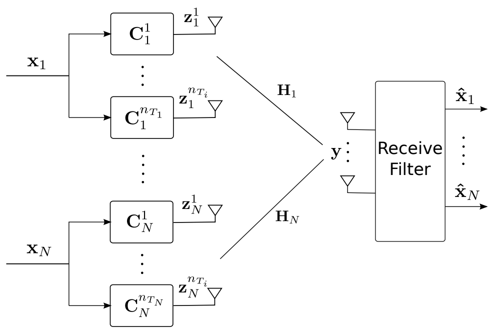


Fig. 3. Block diagram of the orthogonal CDMA access scheme proposed for analog JSCC transmission over block-fading MIMO MAC.

Multiple Access Interference (MAI). In addition, the flexible design of the proposed CDMA scheme guarantees that all users achieve the target rate.

In MIMO MAC, the spatial diversity provided by the deployment of multiple antennas at transmission and reception can be exploited to increase either the transmission rate (multiplexing gain) or the communication reliability (diversity gain). Given that analog JSCC is able to provide high transmission rates with zero-delay and minimum decoding complexity when employed for bandwidth reduction, it is more helpful to exploit the MIMO spatial diversity to improve reliability.

Let us assume user $i$ sends $k_i$ channel symbols over $K$ channel uses. Hence, $R_i = k_i/K,\ i = 1, \ldots, N$, is the $i$-th user data rate. Given that the channel responses are unknown at the transmitters, it is sensible to distribute the total available power $P$ among users according to their rate, such that the individual powers corresponding to each user are hence given by $P_i = R_i P$. Fig. 3 shows the block diagram of the proposed orthogonal CDMA scheme for the transmission of analog symbols over the block-fading MIMO MAC. As observed, the $i$-th user encodes the vector $\mathbf{x_i}$ of $k_i$ symbols at each transmit antenna using a specific spreading code. These codes can be constructed from unitary matrices such as, for example, the Hadamard matrix or the DFT matrix.

In particular, we propose to jointly design the set of $N$ user codes $\mathbf{C}_i^q,\ i = 1, \ldots, N$, corresponding to antenna $q = 1, \ldots, n_{T_i}$. We start off with a $K \times K$ unitary matrix $\mathbf{U}$ and assign $k_i$ columns to the $i$-th user so that $\sum_{i=1}^N k_i \leq K$. Thus, the access code corresponding to user $i$ at the antenna $q$, $\mathbf{C}_i^q$, will be a $K \times k_i$ matrix defined as

$$\mathbf{C}_i^q = \begin{bmatrix} \mathbf{c}_{i1}^q \\ \mathbf{c}_{i2}^q \\ \vdots \\ \mathbf{c}_{iK}^q \end{bmatrix} = \begin{bmatrix} c_{i1}^q(1) & c_{i1}^q(2) & \cdots & c_{i1}^q(k_i) \\ c_{i2}^q(1) & c_{i2}^q(2) & \cdots & c_{i2}^q(k_i) \\ \vdots & \vdots & \cdots & \vdots \\ c_{iK}^q(1) & c_{iK}^q(2) & \cdots & c_{iK}^q(k_i) \end{bmatrix} \quad (6)$$

This matrix combines the $k_i$ source symbols of user $i$, $\mathbf{x}_i = [x_{i1}, x_{i2}, \cdots, x_{ik_i}]^T$, to produce a vector of $K$ channel symbols, $\mathbf{z}_i^q = \mathbf{C}_i^q \mathbf{x}_i$. Notice that in order to ensure that $\mathbb{E}[|\mathbf{z}_i|^2] = P_i$, the columns of the matrix $\mathbf{U}$ assigned to user $i$ must be normalized by a factor $1/\sqrt{k_i}$.

If the same matrix $\mathbf{U}$ is employed to design the codes at different antennas, the orthogonality of these codes is exploited by simply sending the same channel symbols over the $n_{T_i}$ antennas. Conversely, if we consider different unitary matrices, a distinct combination of the source symbols can be sent over the MAC. A remarkable property of this design is its flexibility to achieve any distribution of the user data rates by appropriately selecting $k_i$ and the spreading factor $K$. In addition, cooperation among users is not required and, therefore, the use of these orthogonal codes enables a simple mechanism to guarantee that all users achieve a given individual rate.

At the output of the CDMA scheme we hence obtain a vector of $K$ coded symbols, $\mathbf{z}_i^q = \mathbf{C}_i^q \mathbf{x_i} = [z_{ik}^q, \ldots, z_{iK}^q]^T,\ q = 1, \ldots, n_{T_i}$, for each antenna at the transmitters. These symbols are then transmitted sequentially over the block-fading MIMO MAC using the channel $K$ times. Finally, the receiver utilizes the sequences of $K$ symbols received at the $n_R$ antennas to recover the information corresponding to each user by using a proper filter.

An alternative way to describe the proposed CDMA scheme is to construct an overall MAC access code for each user by stacking the set of $n_{T_i}$ orthogonal codes employed at each antenna. Hence, we introduce $\tilde{\mathbf{C}}_i = [\mathbf{C_i^1};\ \ldots; \mathbf{C_i^{n_{T_i}}}]$ as the $n_{T_i} K \times k_i$ matrix that represents the MAC access code for the $i$-th user. If $\mathbf{x}_i = [x_{i1}, x_{i2}, \cdots, x_{ik_i}]^T$ is the vector of source symbols corresponding to user $i$, the vector of $n_{T_i} K$ coded symbols can be obtained as $\mathbf{z}_i = \tilde{\mathbf{C}}_i \mathbf{x}_i$. Notice that this vector can be actually expressed as the concatenation of the channel symbols corresponding to each antenna, thus $\mathbf{z}_i = [\mathbf{z}_i^1, \mathbf{z}_i^2, \ldots, \mathbf{z}_i^{n_{T_i}}]^T$. These sequences of coded symbols are first spatially multiplexed over the $n_{T_i}$ transmit antennas and then sent over the block-fading MIMO MAC along $K$ time instants. Therefore, the received signal vector at time $k$ is given by

$$\mathbf{y}_k = \sum_{i=1}^N \sqrt{\frac{P_i}{n_{T_i}}} \mathbf{H}_i \mathbf{z}_{ik} + \mathbf{w}_k \quad 1 \leq k \leq K, \quad (7)$$

where $\mathbf{z}_{ik} = [z_{ik}^1, z_{ik}^2, \ldots, z_{ik}^{n_{T_i}}]^T$ and $\mathbf{w}_k = [w_{1k}, w_{2k}, \ldots, w_{n_R k}^{n_{T_i}}]^T$ are the $n_{T_i}$ transmitted symbol vector of user $i$ and the noise at time $k$, respectively. Since the channel is unknown at the transmitters, the user power $P_i$ is uniformly distributed among the $n_{T_i}$ antennas.

The MIMO MAC system model given by (7) can be reformulated as an equivalent model. Indeed, let us define

$$\mathbf{A}_{pq} = [\sqrt{\frac{P_1}{n_{T_1}}} (\mathbf{H}_1)_{p,q} \mathbf{C}_1^q, \cdots, \sqrt{\frac{P_N}{n_{T_N}}} (\mathbf{H}_N)_{p,q} \mathbf{C}_N^q],$$

where $(\mathbf{H}_i)_{p,q}$ is the $(p, q)$ entry of $\mathbf{H}_i$ that represents the channel gain between the antennas $q$ and $p$ of user $i$ with $p = 1, \ldots, n_R$ and $q = 1, \ldots, n_{T_i}$. We now build the matrix $\tilde{\mathbf{H}}$ stacking the matrices $\mathbf{A}_{pq}$ as follows

$$\tilde{\mathbf{H}} = \left[ \sum_{i=1}^{n_T} \mathbf{A}_{1i}\ ;\ \sum_{i=1}^{n_T} \mathbf{A}_{2i}\ ;\ \ldots\ ;\ \sum_{i=1}^{n_T} \mathbf{A}_{n_R i} \right].$$

If we now stack the source symbols from the $N$ users into one single vector $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T, \cdots, \mathbf{x}_N^T]^T$, (7) can be rewritten as

$$\mathbf{y} = \tilde{\mathbf{H}} \mathbf{x} + \mathbf{w}, \quad (8)$$

where the $\mathbf{y}$ vector comprises the symbols received at the $n_R$ antennas along $K$ intervals, $\tilde{\mathbf{H}}$ is the equivalent channel matrix obtained from the orthogonal codes and the channel paths, and $\mathbf{w} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_K]^T$ is the AWGN vector.

We now assume that a linear filter $\mathbf{G}$ is employed at the MAC receiver to produce estimates of the transmitted symbols from the vector of received symbols $\mathbf{y}$, i.e., $\hat{\mathbf{x}} = \mathbf{G}\mathbf{y}$. Given that the source symbols are transmitted using analog JSCC and the objective is the sum-distortion minimization, we consider the linear MMSE filter given by

$$\mathbf{G} = \left(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + N_0 \mathbf{I}_K\right)^{-1} \tilde{\mathbf{H}}^H, \tag{9}$$

where the superscript $^H$ denotes the Hermitian operator. The linear filter $\mathbf{G}$ is able to separate the information corresponding to each user because of the orthogonal properties of the spreading codes. As explained in the previous section, ML decoding can be then applied to the resulting symbols at the filter output to obtain the corresponding estimate of the analog source symbols transmitted by each user.

Notice that the linear MMSE filter is optimum for Gaussian symbols but, in this case, the distribution of the analog coded symbols is not necessarily Gaussian because the analog JSCC mapping based on the Archimedean spiral implies non-linear transformations of the source symbols. As we will show in the results Section, linear MMSE filtering provides good performance with an affordable complexity and delay for analog communications. The design of the non-linear optimum MMSE filter would require knowledge of the channel symbols probability as well as the calculation of a complex integral by numerical methods. Given that the MMSE filter is applied to obtain an estimate of each received symbols, the overall complexity of the analog system would significantly increase with respect to the linear case.

## B. Linear MMSE Access Codes

We now assume that the block-fading channel response for each user, $\mathbf{H}_i$, is perfectly known at the corresponding transmitter. Given that the objective is the sum-distortion minimization, CSI can be exploited to obtain the linear access codes that minimize the sum-MSE between the input and output of the channel. This is a reasonable approach if we have in mind that linear MMSE filtering together with ML decoding performs extremely well in point-to-point communications [12], [24].

The CDMA scheme presented in Fig. 3 is also valid when considering linear MMSE access codes, with the only difference that the orthogonal codes employed at the transmitters are now substituted by the linear MMSE access codes. We continue to assume that the $i$-th user transmits at the fixed rate $R_i = k_i/K, i = 1, \ldots, N$. We now express the received MAC signal as

$$\mathbf{y} = \sum_{i=1}^{N} \frac{1}{\sqrt{n_{T_i}}} \tilde{\mathbf{H}}_i \mathbf{F}_i \mathbf{x}_i + \mathbf{w}, \tag{10}$$

where $\mathbf{F}_i$ is an $(n_T K \times k_i)$ matrix that represents the non-orthogonal access code for user $i$ and $\tilde{\mathbf{H}}_i$ is a $(n_R K \times n_T K)$ block-diagonal matrix where the $i$-th $n_R \times n_T$ element of the diagonal is the MIMO channel corresponding to user $i$, $\mathbf{H}_i$. Since the transmitted symbols are assumed to be normalized, the access codes $\mathbf{F}_i$ should satisfy the transmit sum-power constraint, i.e., $\sum_{i=1}^{N} ||\mathbf{F}_i||_F^2 = P$, where $||\cdot||_F$ represents the Frobenius norm of a matrix.

Similarly to the previous section, we further elaborate the signal model of the proposed MIMO MAC system. If the source symbols of all users are compacted into a single vector $\mathbf{x} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_N^T]^T$ and by defining the matrices $\tilde{\mathbf{H}} = [\frac{1}{\sqrt{n_{T_1}}}\tilde{\mathbf{H}}_1, \ldots, \frac{1}{\sqrt{n_{T_N}}}\tilde{\mathbf{H}}_N]$ and $\mathbf{F} = \text{blockdiag}^1\{\mathbf{F}_i\}_{i=1}^N$, (10) can be rewritten as

$$\mathbf{y} = \tilde{\mathbf{H}}\mathbf{F}\mathbf{x} + \mathbf{w}. \tag{11}$$

At the receiver, a linear filter $\mathbf{G}$ is employed to obtain an estimate of the transmitted symbols from vector $\mathbf{y}$, i.e., $\hat{\mathbf{x}} = \mathbf{G}\mathbf{y}$. According to this model, the optimal linear codes $\mathbf{F}_i$ and the receiver filter $\mathbf{G}$ that minimize the sum-MSE between the output of the analog encoder and the input of the analog decoder can be determined by solving the following constrained minimization problem

$$\underset{\mathbf{F},\mathbf{G}}{\text{argmin}}\, \mathbb{E}\left[\text{tr}(\mathbf{e}\mathbf{e}^H)\right] \qquad \text{s.t.} \quad ||\mathbf{F}||_F^2 = P, \tag{12}$$

where $\mathbb{E}[\cdot]$ and $\text{tr}(\cdot)$ denote the expectation and trace operators, respectively, and $\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$ represents the error vector. Given that the estimated symbol vector $\hat{\mathbf{x}}$ is calculated as

$$\hat{\mathbf{x}} = \mathbf{G}\mathbf{y} = \mathbf{G}\tilde{\mathbf{H}}\mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{w},$$

the error vector is hence given by

$$\mathbf{e} = \mathbf{x} - [\mathbf{G}\tilde{\mathbf{H}}\mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{w}]. \tag{13}$$

In the literature, the sum-MSE minimization for a MAC system using linear filters has been specifically addressed in several works [27]–[29]. As an example, [29] uses the idea of alterning optimization to pose an iterative algorithm where the linear MMSE filters are updated in an alternating fashion. Rather than this option, we consider the approach described in [27] where a projected gradient algorithm is employed to update the transmit filters of all users and implicitly the receive filter simultaneously at the each iteration. The computational cost of this algorithm is lower than that of other alternating algorithms while the convergence speed is also improved at high SNRs.

The expected sum-MSE at the output of the receiver filter can be directly computed from (13) as

$$\begin{aligned}\epsilon = \text{tr}(\mathbf{e}\mathbf{e}^H) &= \sum_{i=1}^{N} k_i - \text{tr}\left[\mathbf{G}\tilde{\mathbf{H}}\mathbf{F} + \mathbf{F}^H\tilde{\mathbf{H}}^H\mathbf{G}^H\right] \\ &+ \text{tr}\left[\mathbf{G}\tilde{\mathbf{H}}\mathbf{F}\mathbf{F}^H\tilde{\mathbf{H}}^H\mathbf{G}^H + N_0\mathbf{G}\mathbf{G}^H\right]. \end{aligned} \tag{14}$$

Notice that the linear receive filter $\mathbf{G}$ can be computed from the overall code matrix $\mathbf{F}$ and the equivalent channel matrix $\tilde{\mathbf{H}}$ as

$$\mathbf{G} = \mathbf{F}^H\tilde{\mathbf{H}}^H(N_0\mathbf{I}_{n_R K} + \tilde{\mathbf{H}}\mathbf{F}\mathbf{F}^H\tilde{\mathbf{H}}^H)^{-1}.$$

[1] The operator blockdiag$\{\mathbf{M}_i\}$ constructs a diagonal supermatrix in which the diagonal elements are given by the matrices $\mathbf{M}_i$ and the off-diagonal elements are zero matrices.

If we define the auxiliary matrix $\mathbf{X} = N_0\mathbf{I}_{n_R K} + \tilde{\mathbf{H}}\mathbf{F}\mathbf{F}^H\tilde{\mathbf{H}}^H$, the expression for the receive filter can be rewritten as $\mathbf{G} = \mathbf{F}^H\tilde{\mathbf{H}}^H\mathbf{X}^{-1}$, and the calculation of the sum-MSE in (14) simplifies to

$$\epsilon = \sum_{i=1}^{N} k_i - N + N_0 \operatorname{tr}\left[\mathbf{X}^{-1}\right]. \qquad (15)$$

We now use an iterative gradient algorithm to update the linear codes (and implicitly the receive filter) with the following iteration step

$$\mathbf{F}^{(l+1)} = \left[\mathbf{F}^{(l)} - \frac{\lambda^{(l)}}{s^{(l)}}\nabla^*\epsilon(\mathbf{F}^{(l)})\right], \qquad (16)$$

where $s^{(l)}$ is the step size and $\nabla^*(\,\cdot\,)$ is the conjugate nabla operator that generates all conjugate Jacobi matrices. Such operator is computed as

$$\nabla^*\epsilon(\mathbf{F}^{(l)}) = \frac{\partial\epsilon(\mathbf{F}^{(l)})}{\partial\mathbf{F}^H} = -N_0\tilde{\mathbf{H}}^H\mathbf{X}^{-2}\tilde{\mathbf{H}}\mathbf{F}^{(l)}.$$

The factor $\lambda^{(l)}$ in (16) is introduced to reduce the number of iterations for the algorithm to converge in the high SNR region where the sum-MSE cost function is practically flat and, as a consequence, the norm of the Jacobi matrices are small [27]. At each iteration $l$, a normalization of $\mathbf{F}^{(l)}$ is also required to satisfy the imposed sum-power constraint. Such a normalization actually consists of the orthogonal projection of the linear codes obtained at iteration $l$ into the subspace defined by the set of feasible solutions that satisfy the sum-power constraint. The fraction of transmit power allocated to each user is specifically determined by its corresponding linear MMSE access code, such that $\sum_{i=1}^{N} P_i = P$, because this access scheme is conveniently designed to satisfy the global power constraint.

Notice that the access codes $\mathbf{F}_i$ can be actually interpreted as a type of spatio-temporal precoders where the data symbols are combined in the spatial and temporal domain to produce the whole set of coded symbols. The implementation of this MAC access method also requires CSI available at both sides. Thus, we assume that the receiver is able to correctly estimate the channels of all users and then feed back the information of each transmitter to the corresponding user. As shown in [30], [31], the latency in the transmission of the channel information over the feedback channel can be significantly reduced by using different analog techniques.

### C. Opportunistic Access

We finally address a more generic scenario where perfect CSI is available at both sides and, additionally, the constraints previously imposed on the individual user rates are eliminated, i.e., MAC users are allowed to transmit the information at any rate not set a priori. In the single-antenna case, it has been shown [25] that the sum-capacity of a block-fading SISO MAC is maximized when the total transmit power $P$ is dynamically allocated to the user with the highest channel gain at each time block. Such a scheme is usually referred to as opportunistic. Hence the opportunistic access actually transforms the MAC problem into a point-to-point transmission where only the user with the best channel transmits at each time instant and, therefore, the received signal is not affected by MAI.

In the case of MIMO MAC, the allocation of the available power to the user with the best channel is no longer the optimal strategy when the objective is to maximize the sum-capacity [32], [33]. In that case, the spatial diversity provided by the use of multiple antennas can be employed to allow multiple users to transmit simultaneously over the block-fading MIMO MAC. As explained in previous sections, it is however preferable to exploit the spatial diversity to improve the reliability of analog JSCC transmissions rather than increasing the system sum-rate. For that reason, the opportunistic access is also a suitable scheme for the transmission of continuous samples over the MAC using analog JSCC and multiple antennas at transmission. Thus, in the opportunistic access to the MIMO MAC only the user $i$ with the best channel at each time block transmits its information over the $n_{T_i}$ antennas using the total power $P$.

As in the single-antenna case, the considered MIMO MAC model with opportunistic access is transformed into a point-to-point MIMO communication and, therefore, the received symbols are given by

$$\mathbf{y} = \sqrt{\frac{P}{n_{T_i}}}\bar{\mathbf{H}}\mathbf{z}_i + \mathbf{w}, \qquad (17)$$

where $\bar{\mathbf{H}}$ corresponds to the response of the MIMO channel with largest capacity at each block time. Such channel response is determined as follows

$$\bar{\mathbf{H}} = \{\mathbf{H}_i \mid \max_i\left[C(\mathbf{H}_i)\right], \quad i = 1, \ldots, N\}. \qquad (18)$$

where $C(\mathbf{H}_i) = \log\det\left(\mathbf{I}_{n_R} + \frac{P}{n_{T_i}N_0}\mathbf{H}_i\mathbf{H}_i^H\right)$ represents the capacity of the MIMO channel for the $i$-th user without considering the interferences caused by the remaining users.

At the receiver, we can directly calculate an estimate of the source symbols transmitted by the best user since the received signal is not affected by interferences. As mentioned in Section III, low-complexity ML decoding provides an excellent performance for analog JSCC transmissions when the received symbols are previously filtered to reduce the impact of the channel noise [24]. A linear filter is hence used at the receiver to first obtain an estimate of the transmitted symbols for the best user $\mathbf{z}_i$ from the vector of $n_R$ received symbols as $\hat{\mathbf{z}}_i = \mathbf{G}\mathbf{y}$. We consider a conventional linear MMSE filter for MIMO systems, i.e.,

$$\mathbf{G} = \left(\bar{\mathbf{H}}^H\bar{\mathbf{H}} + N_0\mathbf{I}_{n_{T_i}}\right)^{-1}\bar{\mathbf{H}}^H.$$

A remarkable disadvantage of the opportunistic strategy with respect to the previous proposed access schemes is that it can lead to unfair situations in terms of rate distribution and power consumption, because a specific user may experience a better channel during a large number of channel realizations. Such a user consequently achieves a higher individual rate, whereas the remaining users must wait to transmit their information and can undergo large delays on the communication. There are many practical situations where this unfair behaviour of opportunistic access is unacceptable.

## V. OPTA for Analog JSCC Systems over MAC

When transmitting analog information, the optimal distortion-cost tradeoff is referred to as the Optimum Performance Theoretically Attainable (OPTA) and is calculated by equating the rate distortion function of the source and the capacity of the channel [34]. In the MAC, we can consider either the maximum data rate at which each individual user can transmit or the total sum-rate for the set of all users. In the former approach, we should calculate a different upper bound (OPTA) for each user in terms of the rate distortion function of the corresponding source and the maximum individual rate such a user can achieve. Conversely, in the second case we just calculate a single OPTA curve that describes the optimal performance of the overall MAC system.

Rather than minimizing the individual signal distortion corresponding to each user, we are hence interested in the overall distortion of the set of received signals. In such a case, the sum-distortion can be defined as

$$D_{\text{sum}} = \sum_{i=1}^{N} D_i \tag{19}$$

as long as we consider a linear distortion metric such as the MSE. The index $N$ represents the number of active users transmitting over the MAC and $D_i$ is the distortion observed in the recovered signal for each user $i$. The performance of analog JSCC systems in MAC communications can be measured in terms of the average Signal-to-Distortion Ratio (SDR) with respect to the CSNR. The average SDR is given by

$$\overline{\text{SDR}} = \frac{\sigma_s^2}{(\sum_{i=1}^{N} D_i)/N} = \frac{\sigma_s^2}{D_{\text{sum}}/N},$$

where $D_{\text{sum}}/N$ is the average distortion and the $N$ sources are assumed to have the same variance $\sigma_s^2$ for simplicity. The overall CSNR is directly computed as the quotient between the transmit power and the noise variance, thus

$$\text{CSNR} = \frac{P}{N_0}.$$

If the sources are also independent, the OPTA bound can be calculated by equating the rate distortion function of the sources and the sum-capacity of the MAC [3], i.e.,

$$M \sum_{i=1}^{N} R(D_i) = K R_{\text{sum}}, \tag{20}$$

where $R(D_i)$ is the rate distortion function of user $i$ given a distortion target $D_i$, and $R_{\text{sum}}$ is the sum-capacity of the MAC, i.e., the maximum sum transmission rate that the set of all users can achieve. The terms $M$ and $K$ represent the bandwidth of the sources and the channel, respectively, and therefore such values determine the expansion/compression factor applied when the source symbols are mapped into the channel symbols at the transmitters.

Finally, if we assume that the $N$ sources are also identically distributed, the average contribution of each individual user to the sum-distortion is the same, thus $D_i = (D_{\text{sum}}/N) \;\; \forall i$. In such a case, the general expression for the OPTA given by (20) can be simplified as follows

$$MNR(D) = KR_{\text{sum}}, \tag{21}$$

where $D = D_{\text{sum}}/N$ is the average distortion for all users. Notice that the product $MNR(D)$ specifies the total number of source symbols required to be transmitted by the set of $N$ users along $K$ channel uses to obtain the sum-distortion $D_{\text{sum}}$ after decoding the received MAC signal.

In this work, we focus on the transmission of i.i.d. complex-valued Gaussian symbols with zero mean and variance $\sigma_s^2$. If we consider the MSE as the distortion metric, the rate distortion function is given by [35]

$$R(D) = \max\left[0, \log\left(\frac{\sigma_s^2}{D}\right)\right] = \max\left[0, \log(\overline{\text{SDR}})\right]. \tag{22}$$

As mentioned in the description of the system model, we assume block-fading channels that remain static during the transmission of a packet of symbols but independently vary from one packet to another. The different channel realizations are also assumed to be generated from a stationary and ergodic process. In such a case, the individual rates users may achieve for a given realization of the block-fading MIMO MAC, $R_i$, $i = 1, \ldots, N$, are restricted by the following set of $N$ inequalities

$$R_i \leq \log\det\left(\mathbf{I}_{n_R} + \frac{1}{N_0}\mathbf{H}_i\mathbf{K}_i\mathbf{H}_i^H\right), \tag{23}$$

where the term $\mathbf{K}_i$ represents the transmit covariance matrix of the $i$-th user for the specific channel realization $\mathbf{H}_i$. As shown in [14], the maximum sum-rate to be achieved by the set of all users is determined as

$$R_{\text{sum}} = \sum_{i=1}^{N} R_i \leq \log\det\left(\mathbf{I}_{n_R} + \frac{1}{N_0}\sum_{i=1}^{N}\mathbf{H}_i\mathbf{K}_i\mathbf{H}_i^H\right). \tag{24}$$

In the case of multiple transmit antennas, it is not feasible to find $N$ covariance matrices $\mathbf{K}_i$ that simultaneously maximize the individual rates given by the inequalities in (23) and the sum-rate in (24) [14]. In general, the achievable sum-rate is therefore defined by the particular allocation strategy employed by the users to send the information over the MIMO MAC. For example, it can be shown that the achievable capacity region for digital multiuser systems is obtained by using the dirty paper coding signaling technique [36]. Nevertheless, as already mentioned in the introduction, decoding procedures based on the successive cancellation of the user interferences cannot be successfully applied to analog JSCC transmissions.

For this reason, we specifically consider the achievable sum-rates corresponding to the linear MMSE access coding and the opportunistic access in order to calculate two practical upper bounds for the performance of the proposed transmission model. For ergodic channels the sum-rate of the analog JSCC system with linear MMSE access can be directly obtained by replacing the covariance matrices with the corresponding optimal codes and averaging over all channel realizations, i.e.,

$$R_{\text{sum}} \leq \mathbb{E}_{\mathbf{H}}\left[\log\det\left(\mathbf{I}_{n_R} + \frac{1}{N_0}\sum_{i=1}^{N}\mathbf{H}_i\mathbf{F}_i\mathbf{H}_i^H\right)\right]. \tag{25}$$

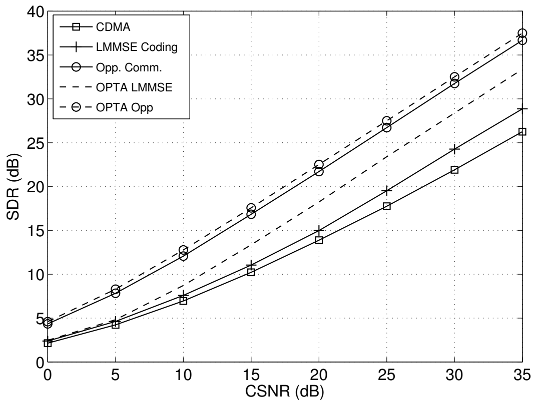


Fig. 4. Performance of the proposed access schemes for SISO MAC and 4 users with uncoded analog transmission ($M = 1$).

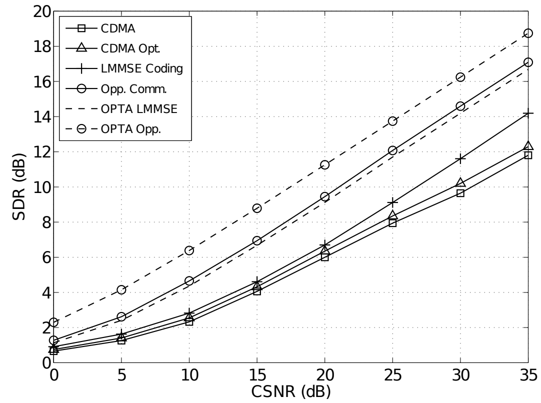


Fig. 5. Performance of the proposed access schemes for SISO MAC and 4 users with 2:1 bandwidth reduction using the Archimedean spiral ($M = 2$).

In the case of opportunistic access, the sum-rate actually corresponds to the ergodic capacity of the MIMO system where the channel response is always given by the strongest channel $\bar{\mathbf{H}}$ at each time block and the power used to transmit is the total available power $P$. Thus,

$$R_{\text{sum}} \leq \mathbb{E}_{\mathbf{H}} \left[ \log\det \left( \mathbf{I}_{n_R} + \frac{P}{n_{Ti} N_0} \bar{\mathbf{H}} \bar{\mathbf{H}}^H \right) \right]. \tag{26}$$

## VI. Simulation results

In this section, the results of several computer experiments are presented to illustrate the performance of the three different access schemes considered along the paper: orthogonal CDMA, linear MMSE access coding and opportunistic access.

We start considering a SISO scenario where all transmitters and the receiver are equipped with a single antenna. In particular, we consider $N = 4$ distributed users sending analog Gaussian symbols to a common receiver over a block-fading MAC. We assume the channel responses are modeled as complex-valued zero-mean cicularly-symmetric Gaussian random variables, i.e. Rayleigh fading. Channel realizations are statistically independent from one user to another and from one block to another. At each transmitter, the analog source symbols are first mapped into the channel symbols using an analog JSCC scheme and then input to the corresponding MAC access scheme. We consider two particular analog JSCC strategies: uncoded transmission, i.e., $(M = 1)$, and a 2:1 compression scheme based on the Archimedean spiral, i.e., $(M = 2)$. In the latter case, the encoder parameters $\delta_i$ and $\alpha_i$ are always optimized for linear MMSE and oportunistic access, whereas we consider two different possibilities for CDMA: optimum values for these parameters depending on the instantaneous CSNRs, or the use of those values corresponding to the average expected CSNR. We also assume users transmit with constant rates $R_1 = 2/16$, $R_2 = 4/16$, $R_3 = 4/16$ and $R_4 = 6/16$ in CDMA and linear MMSE coding, whereas these rates are dynamically adapted in opportunistic access.

Figs. 4 and 5 show the system performance in terms of average SDR with respect to CSNR for uncoded transmission and 2:1 bandwidth reduction of the source symbols, respectively. The OPTA bounds corresponding to both the case of linear MMSE coding and opportunistic access are also plotted. Both curves are calculated from (20) using the proper expression for the achievable sum-capacity given by (25) and (26), respectively. Notice that the OPTA bound for linear MMSE access codes is lower than that with opportunistic access. This is because the linear codes have been actually designed to minimize the sum-MSE between the output of the analog encoder and the input of the analog decoder. For this reason, they do not necessarily maximize the MAC sum-capacity, while opportunistic access is the optimal strategy for the single antenna case in terms of sum-rate maximization.

As observed, performance substantially improves when the CSI at the transmitters is exploited to design the optimal linear access codes. For uncoded transmission (Fig. 4), this improvement is strictly due to the utilization of the linear MMSE codes and is about 2 dB for high and medium CSNRs. Hence the behaviour of the CDMA scheme is quite satisfactory considering that it does not utilize CSI at the transmitters and the lower complexity of the orthogonal codes. For 2:1 compression (Fig. 5), the curve corresponding to CDMA without parameter optimization also remains about 2 dB below the curve of the linear MMSE access, whereas the performance gain due to optimization of the encoder parameters is about $0.4$ dB. The good performance provided by the CDMA system without parameter optimization is motivated by the graceful performance degradation of the analog JSCC schemes.

As also shown in Figs. 4 and 5, opportunistic access closely approaches the corresponding OPTA curve and provides the best results for the whole range of CSNRs. System performance improves specially in the high CSNR region and for uncoded transmissions. However, it is important to remember that such a strategy does not allow us to guarantee the target data rates to be achieved over the MAC users during the transmissions. These rates are optimally allocated according to the channel conditions of the users, which may lead to an unfair distribution of both the user rates and the power consumption.

We next consider a MIMO MAC scenario where $N = 4$ users with $n_{T_i} = 2$ $\forall i$ antennas send their data to the centralized receiver also equipped with $n_R = 2$ antennas. As in the single-antenna scenario, the MAC is assumed to

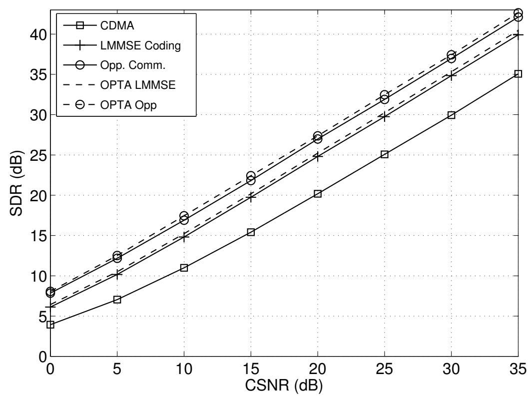


Fig. 6. Performance of the proposed access schemes for 4 users with uncoded transmission over 2x2 MIMO MAC.

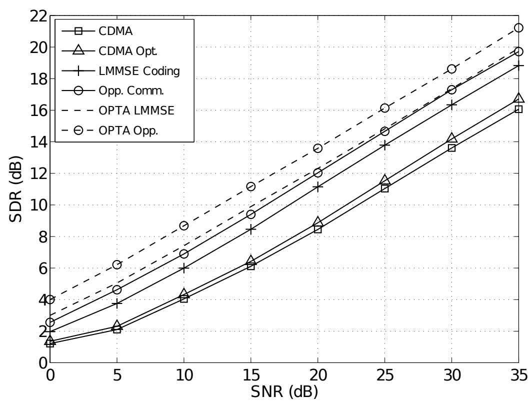


Fig. 7. Performance of the proposed access schemes for 4 users with 2:1 bandwidth reduction over 2x2 MIMO MAC.

be block-fading and Rayleigh distributed. In addition, the users transmit at the same rate as in the SISO case, i.e., $R_1 = 2/16$, $R_2 = 4/16$, $R_3 = 4/16$ and $R_4 = 6/16$ for CDMA and linear MMSE coding whereas such rates and the available power are dynamically allocated in opportunistic access.

Figs. 6 and 7 show the performance of the analog JSCC system for the three considered MAC access methods when the source symbols are transmitted either uncoded or compressed with rate 2:1. The OPTA bounds corresponding to the linear MMSE and opportunistic access schemes are again included in the figures. In the MIMO case, opportunistic access is not the optimal strategy that maximizes the sum-rate although the associated OPTA represents a suitable upper bound for the performance of the proposed analog JSCC systems. As observed, simulation results resemble those obtained in the SISO scenario. On one hand, the use of optimal linear MMSE access codes rather than CDMA substantially improves the overall performance in the high and medium CSNR region, about 5 dB for uncoded transmission and over 2 dB for 2:1 bandwidth reduction. Notice the performance gain with respect to the SISO scenario that is motivated because the channel diversity of the MIMO MAC is significantly larger than in the single-antenna case and, therefore, this diversity can be properly exploited to design the MMSE codes. The gap from

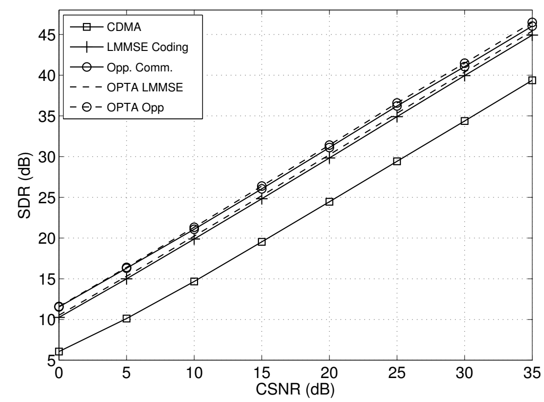


Fig. 8. Performance of the proposed access schemes for 4 users with uncoded transmission over 4x4 MIMO MAC.

the linear MMSE performance curve to the corresponding OPTA curve is drastically reduced down to less than 1 dB for 1:1 and about 1 dB for 2:1. We have also observed that this gap is even smaller as the number of MAC users becomes larger. On the other hand, the gain due to the parameter optimization in CDMA is similar to that of the SISO scenario (less than 1 dB), which reaffirms the robustness of the CDMA scheme even when the transmitters do not have any information about the channel. In addition, the performance difference between the linear MMSE coding and the opportunistic access is reduced with respect to the SISO case and both performance curves are quite close to each other. Nevertheless, the opportunistic method continues to provide the best average SDR for the whole CSNR range and approaches more closely the performance upper bound.

In order to evaluate the impact of increasing the number of antennas, we address the case where $N = 4$ users transmit their analog information using $n_{T_i} = 4\ \forall i$ antennas and the receiver also utilizes $n_R = 4$ antennas to acquire the user data from the block-fading Rayleigh MAC. As in previous experiments, users transmit with fixed data rates $R_1 = 2/16$, $R_2 = 4/16$, $R_3 = 4/16$ and $R_4 = 6/16$ for CDMA and linear MMSE access whereas those rates are dynamically adapted in opportunistic access. As already mentioned, it is preferable to exploit the additional antennas to improve the transmission reliability rather than increasing the data throughput.

Figs. 8 and 9 show the performance curves of the three evaluated access schemes for uncoded transmission and 2:1 compression of Gaussian samples, respectively. As expected, the use of two additional antennas at both the transmitters and the receiver leads to a significant increase of the system performance for the three access methods. This improvement is specially remarkable for linear MMSE access, where in the case of 4 antennas the performance gap between linear MMSE access and CDMA without parameter optimization (about 6 dB) is larger than for the case of 2 antennas (4 dB). In addition, the SDR curves of linear MMSE and opportunistic access are very close to each other again and to the corresponding performance upper bounds.

As observed, opportunist access provides the best results in

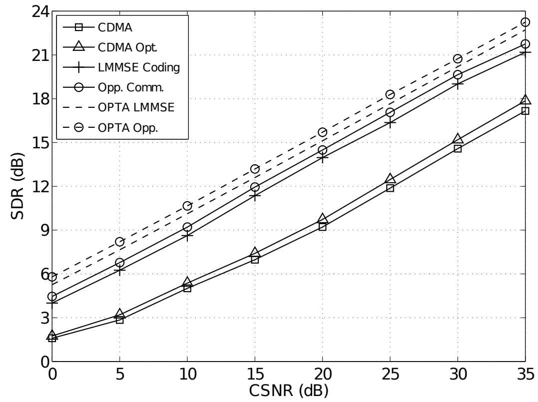


Fig. 9. Performance of the proposed access schemes for 4 users with 2:1 bandwidth reduction over 4x4 MIMO MAC.

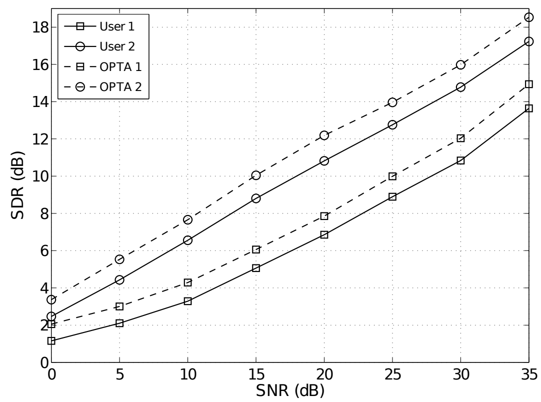


Fig. 10. Performance of the opportunistic access schemes for 2 users with 2:1 bandwidth reduction over 2x2 asymmetrical MIMO MACs.

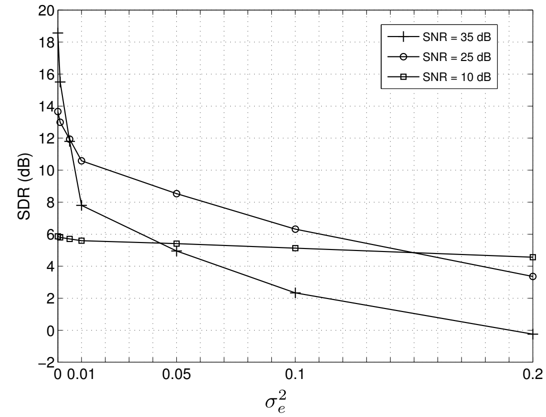


Fig. 11. Peformance degradation for the 2:1 analog JSCC system with linear MMSE access over $2 \times 2$ MIMO MACs.

the three considered scenarios. In those cases, user channels are generated according to Rayleigh distributions with the same statistics and, therefore, the average allocation of the power consumption and data rates among the users tends to be symmetrical. However, this theoretical symmetry is not very realistic in practical situations where different users usually transmit over fading channels with different characteristics. This situation motivates an asymmetrical distribution of powers and rates depending on the variable conditions of the user channels. For this reason, we proceed to assess the opportunistic scheme when MAC users experience fading channels with different statistical properties. In particular, we focus on the case of two users with opportunistic access over $2 \times 2$ MIMO MACs. The channel realizations for the two users are synthetically generated to guarantee that, on average, the eigenvalues of one MIMO channel are considerably more dispersive than those corresponding to the other channel. Thus, both channels are generated to preserve the same average power but the distribution of such a power among the eigenmodes of each channel is radically different.

Fig. 10 presents the individual performance of the two users when the source data is 2:1 compressed and transmitted over the $2 \times 2$ MIMO MAC with the opportunistic scheme. We considered an asymmetric MIMO block-fading MAC where the eigenvalue spread of the MIMO channel realizations for user 2 is assumed to be on average 10 times larger than that of user 1. The channel response eigenvalues of both users are normalized so that both transmit with the same power. The individual OPTAs corresponding to each user are calculated according to (26) and plotted in Fig. 10. As shown in the same figure, both users still approach their corresponding OPTA curves although user 2 exhibits a superior performance for the whole CSNR region. In fact, its SDR curve is more than 3 dB above that of user 1 for high and medium CSNRs. Computer simulations reveal that the average percentages of power assigned to users 1 and 2 are around 70% and 30%, respectively. This is because on average the channel capacity of user 1 is larger than that of user 2. Situations in which the capacity experienced by user 2 is larger than that of 1 are less likely to happen but when they occur the capacity of user 2 is rather large. This explains why at the end the OPTA corresponding to user 2 is significantly larger than that of user 1.

So far perfect CSI is always assumed to calculate the linear filters for the considered MAC model. An interesting issue is to determine the impact of using inaccurate channel information to design the linear MMSE access scheme. We now consider a simplified model where the estimates of the channel matrices are given by $\hat{\mathbf{H}}_i = \mathbf{H}_i + \mathbf{E}$. The term $\mathbf{E}$ is the $n_R \times n_T$ error matrix whose entries $e_{ij} \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_e^2)$ follow an i.i.d. Gaussian distribution with zero mean and variance $\sigma_e^2$. The estimated matrices $\hat{\mathbf{H}}_i$ are now utilized to jointly design the linear MMSE filters instead of the real channel matrices $\mathbf{H}_i$. Notice that imperfect CSI is considered at both the transmitters and the receiver of the MAC system. Perfect CSI can be assumed in the design of the linear detector but, in that case, the algorithm proposed in [27] would no longer be applicable.

Fig. 11 illustrates the degradation of the system performance with respect to the error variance $\sigma_e^2$ for three different CSNRs: 35 dB, 25 dB and 10 dB. We specifically consider 4 users transmitting their data over a $2 \times 2$ MIMO MAC using a 2:1 analog JSCC scheme. The performance curves show the SDR provided by the analog system as $\sigma_e^2$ increases. As observed, the analog JSCC system is rather sensitive to the use of inaccurate channel information for high CSNRs. From

Fig. 11, it is also possible to appreciate that the performance degradation lowers with the CSNR and it is minimum for medium and low CSNRs.

## VII. Conclusion

In this work, we have studied the application of analog JSCC techniques for the transmission of discrete-time continuous-amplitude symbols over block fading MIMO MAC. For this communication problem, we have proposed a distributed architecture that basically consists in the separate optimization of the mapping operation and of the MAC access scheme. Given that the joint design of both operations is very complex and probably impractical, this approach makes the implementation of practical analog JSCC systems feasible for multiuser communications over fading channels. Well-known single-user analog mappings have been considered for the encoding step, while three different methods have been proposed for the access scheme: orthogonal CDMA, linear MMSE codes and opportunistic access. Linear MMSE and opportunistic access exploit the CSI at the transmitters to reduce the sum-distortion of the received symbols, whereas CDMA can be applied even when the channel is unknown at the transmitters because it makes use of orthogonal spreading codes to ensure the separation of the users information at the receiver. CDMA and the linear MMSE access scheme are specially suitable for MAC systems with per-user rate constraints as well as they provide a flexible framework to obtain any distribution of the individual user rates. The evaluation of the proposed MAC transmission methods by means of computer simulations show that the performance can be significantly improved if the optimal linear MMSE access codes are employed instead of the orthogonal ones, although CDMA also obtains satisfactory results even when the encoder parameters are not optimized. The opportunistic access provides the best results in terms of sum-distortion, but it may lead to unfair distributions of the user rates and power consumption.

## Disclaimer

The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government.

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