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Product design with attribute dependence

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Abstract

This paper studies how product design and pricing strategies are affected by the existing relationship between the characteristics that integrate the product. The analysis shows that complementarity and low substitutability encourage the provision of quality incorporated to the products and increase the quality distortion and cannibalization problems that are common in segmented markets. A two-product strategy with a common attribute is shown to be a feasible strategy for reasons other than cost savings, namely attribute dependence. In addition, menu pricing is found to be the most profitable strategy, and a commonality strategy is more profitable than a common-product strategy.

KEYWORDS

attribute dependence, market segmentation, monopoly pricing, product design, quality provision

JEL CLASSIFICATION

D42, L12, M11, M31

1 | INTRODUCTION

One of the most common assumptions in the literature on multiple characteristic products is that attributes are independent from the point of view of consumers. In reality, consumers frequently base their purchase decisions on attributes that are interrelated in the sense of having a certain degree of complementarity or substitutability, or in other words, consumer's valuations of changes in one characteristic depend on the level of other characteristics. Broadly speaking, attribute dependence implies that products are not just an addition of independent characteristics but a combination of interrelated characteristics.[1] Empirical evidence on the practical relevance of this interdependence can be found in many sectors. For example, Bajic (1993) shows that the demand for individual automobile characteristics are interdependent, Tay (2003) finds trade-offs between dimensions of spatial and quality differentiation in the hospital care market, Simon and Kadiyali (2007) study the relationship between the provision of free digital contents and the demand for print versions of magazines, Dick (2007) and Martín-Oliver and Salas-Fumás (2008) show the interdependence between advertising expenditures and density of branch network in the banking sector, and Zhang et al. (2014) consider the relationship between security and performance for software products.

The role of attribute dependence in company product placement decisions has been studied as an interaction between vertical and horizontal differentiation (Neven & Thisse, 1990; Canoy & Peitz, 1997; Degryse, 1996; Degryse & Irmen, 2001a, b; Ginsburgh & Weber, 2002; Piga & Poyago-Theotoky, 2005), and also between vertical characteristics (Baake & Boom, 2001; Novo-Peteiro, 2020; Sun et al., 2004). These papers deal with this interdependence in the context of oligopolistic markets. To our knowledge, attribute dependence has not been considered in the context of monopolistic markets.

Considering attribute dependence in a monopolistic setting allows us to analyze its role in product design and pricing strategies without the influence of strategic effects on the firms decisions that characterizes oligopolistic markets. This is relevant given that attribute dependence can exacerbate the quality differences between the varieties offered by firms; consequently it can increase their market power (for example Degryse & Irmen, 2001b or Novo-Peteiro, 2020). In this sense, the choice of a monopolistic setup is motivated by the fact that the foremost real situations of attribute dependence are industries with dominant firms that offer multi-attribute products, with these qualities being interdependent in different ways. This could be particularly important in markets where the introduction of new features and upgrades of existing ones may lead to the development of a superior product, as well as being a source of monopolization practices by companies in product line design and pricing policy. Previous studies on the role of attribute dependence analyze its impact on the intensity of product differentiation within a duopolistic setting. In that framework firms offer a single product and prices are linear. Thus, a monopolistic setting allows us to deal with two closely related issues that are not covered by the existing literature: first, the possibility of offering different varieties of a product and the implications in terms of the level of quality that is incorporated to each one of them, and second, the comparison between the different pricing strategies that are available for a firm with market power.

Anecdotal evidence about the practical relevance of our analysis can be found in digital economy; for example, in Spotify and Youtube, the major music- and video-streaming platforms, respectively, when offering a menu of services (free and premium) and letting customers choose between them (Sato, 2019), or in Google, the biggest web search engine, that provides an increasing variety of services that are interlinked with each other. Another example is Microsoft, by including its Internet browser with every copy of its operating system software, as well as software that is valuable for certain segments depending on the type of user (home users or professional users). Another example is Intel in the microprocessor industry at the beginning of this century (when its market share was about 80%), combining performance-related features of microprocessors to differentiate their lower-priced processors from higher-quality ones (Gawer & Henderson, 2007). Similar comments can be applied to the case of Adobe Systems when designing its line of products in the desktop publishing and digital-imaging markets, by offering high-end products with greater performance quality and low-end products with greater ease of use (Krishnan & Zhu, 2006).

[1]Examples and further justification of attribute dependence can be found in Degryse and Irmen (2001b) and Novo-Peteiro (2020).

There exists extensive and interrelated literature using a monopoly setup to approach industries with certain specific characteristics that are close to our analysis. Examples can be found in the literature on: (i) versioning of package software, in Bhargava and Choudhary (2008) or Mehra et al. (2014), for example, where a dimension of product quality is dependent on the length of time since the last version of the product was introduced, or Wei and Nault (2014), who study versioning of information goods by considering that consumers have individual and group tastes for quality with a multiplicative utility structure where all consumers can derive value from a shared set of product characteristics, while other characteristics provide value for specific groups; (ii) freemium, which is the analysis of business models where a core product is given away for free to a group of customers and premium variants of this product are sold to other customers (Sato, 2019; Shi et al., 2019); (iii) development-intensive products, whereby the fixed costs of development far outweigh the unit-variable costs, researched by Krishnan and Zhu (2006), for example, who look at the case of two dimensions of quality with an additive utility function that explicitly considers saturation and reservation qualities; (iv) platform products, which are made up of interdependent components that are shared across a range of the company's products, for example, Lee et al. (2020), who look at the appropriateness of different business model decisions for markets with technology-intensive supply chains by considering that the overall product quality is a multiplicative combination of technology-driven quality and system quality; (v) industries with network effects, since the quality of the product determines two dimensions of the consumer's valuation of the product that can be closely related, namely its intrinsic quality and its network value (see Jing, 2007 for a general model, and Liu et al., 2015 for an application in information goods); (vi) multi-attribute products with some kind of dependence among them, in research by Chevenaz and Jasimuddin (2017), for example, who study complementarities of advertising and quality in terms of the overall valuation of a good by consumers, Qian (2014), who analyzes a market where product or service demand depends on a set of attributes including delivery time, service level and other quality-like performances, and Orhun (2009), who studies the optimal product line design when consumers have choice set-dependent preferences, meaning that the consumer's valuation of a product's attributes depends both on the level of these attributes incorporated into that product, and their position in the available choice set of products.

The main contributions of this paper are the following. First, the paper shows the influence of the existing relationship between the characteristics that integrate a product on the provision of quality by firms. The presence of complementarity and low substitutability enhances the level of overall quality incorporated to a product under certain circumstances, and the difference between high quality and low quality products can be increased. Thus, the relevance of two well-known issues in monopoly pricing, namely, distortion of quality for the low segment and cannibalization, can be augmented in the presence of attribute dependence. In addition, the analysis shows that the provision of substitutive qualities in a product configuration (as occurs in sectors such as those previously mentioned) can be optimal. Second, the paper shows that commonality is a feasible strategy under certain conditions that have to do with the level of attribute dependence and the structure of preferences, thus complementing the usual justification for commonality strategies in the literature on product design which are based on cost savings. Third, the comparison between the optimal profits resulting from the different product configurations shows that menu pricing is the most profitable strategy and that a two-product strategy with a common attribute is more profitable than a one-product strategy with a common product for all customers. Moreover, either the selective-segment strategy or the commonality strategy could be the second best strategy, depending on the relative size of the market segment and the level of dominance in preferences.

The paper is structured as follows: Section 2 introduces assumptions and model settings that are used in the paper. Section 3 lays out the optimal product design strategies. Four strategies for product configuration are considered. In two of them, menu pricing and commonality (Subsection 3.1

and Subsection 3.2, respectively), two products are offered, one targeted at each of the segments of customers. Moreover, the firm adopts a common level for one of the qualities in both products in the commonality strategy. In the other two strategies, the selective strategy and the common-product strategy (Subsection 3.3 and Subsection 3.4, respectively), the firm offers a sole product. In the first one, only a high quality product targeted to the high quality segment of customers is offered, the low quality segment of customers not being served by the firm. In the second one, the monopolist offers a sole product for both segments. Subsection 3.5 compares optimal qualities and profits corresponding to the different strategies. The main conclusions of the paper are summarized in the last section of the paper.

2 | THE MODEL

We consider a monopolist who sells a product with two relevant characteristics, x and y. These characteristics or attributes are quality dimensions that define a product configuration. Let us assume two customer segments in the market, indexed by $i = 1, 2$ in respective proportions p and $1 - p$. The segments differ in their intensity of preference for each quality of the product. Segment i's valuation of quality j ($j = x, y$) is represented by a taste parameter denoted by θ_{ij}.

Following Novo-Peteiro (2020), we assume that each of a product's quality dimensions of a product affects the indirect utility of consumers, not only through the taste parameter for each quality, but also depending on the level of attribute dependence, that is, the existing complementarity/substitutability between both characteristics. The utility of a type-i consumer obtained from consuming one unit of the product is given by

$$U_i = \theta_{ix}x + \theta_{iy}y + \gamma xy - r \tag{1}$$

where r represents the price of the product and γ is the attribute dependence parameter. Two vertical characteristics are defined as complements (substitutes) if $\gamma > (<)0$ and independent if $\gamma = 0$.

According to the literature, two configurations of preferences can be considered (e.g. Kim & Chhajed, 2002; Vandenbosch & Weinberg, 1995). First, there is strict dominance of segment 1 when the taste parameter of both qualities is higher for that segment, that is, if $\theta_{1x} > \theta_{2x}$ and $\theta_{1y} > \theta_{2y}$. Second, there is non-strict or soft dominance of segment 1 when the taste parameter is higher in one quality, $\theta_{1x} > \theta_{2x}$, and lower in the other, $\theta_{1y} < \theta_{2y}$, with $\theta_{1x} - \theta_{2x} > \theta_{2y} - \theta_{1y}$.[2] Conversely, if $\theta_{1x} - \theta_{2x} > \theta_{2y} - \theta_{1y}$ we would have the symmetric case of soft dominance of segment 2.

Consistent with the literature, we assume that the cost of quality is convex (e.g. Moorthy, 1984; Motta, 1993; Mussa & Rosen, 1978). Specifically, we assume a quadratic cost function $(1/2)c_j j^2$ where c_j is the cost coefficient of attribute j.

The sequence of events is standard: for each product configuration the firm chooses the optimal level of provision of both dimensions of quality, after which the prices for the product/s are determined. Finally, consumers buy the product that gives them the highest utility.

[2]This condition is required for single-crossing property to hold: the marginal willingness to pay for quality is higher for the high-type consumers (see Anderson & Dana, 2009). In a model with two dimensions of quality, it is given by $(\partial U_1/\partial x) + (\partial U_1/\partial y) > (\partial U_2/\partial x) + (\partial U_2/\partial y)$. This property is necessary for price discrimination to be profitable.

3 | ANALYSIS

In this Section the optimal product design strategies are studied. As mentioned in the Introduction, four strategies for product configuration are considered, namely menu pricing, commonality, one product for the high segment only and one product for all customers.

3.1 | Menu pricing

The monopolist offers two products and must find the profit-maximizing pair of prices and qualities that induce type-i consumers to buy the product with quality (x_i, y_i). The monopolist maximizes its profit by choosing optimal quality levels and prices:

$$B_M = p\left(r_1 - \frac{1}{2}c_x x_1^2 - \frac{1}{2}c_y y_1^2\right) + (1-p)\left(r_2 - \frac{1}{2}c_x x_2^2 - \frac{1}{2}c_y y_2^2\right) \tag{2}$$

subject to

$$\theta_{1x}x_1 + \theta_{1y}y_1 + \gamma x_1 y_1 - r_1 \geq 0 \tag{3}$$

$$\theta_{2x}x_2 + \theta_{2y}y_2 + \gamma x_2 y_2 - r_2 \geq 0 \tag{4}$$

$$\theta_{1x}x_1 + \theta_{1y}y_1 + \gamma x_1 y_1 - r_1 \geq \theta_{1x}x_2 + \theta_{1y}y_2 + \gamma x_2 y_2 - r_2 \tag{5}$$

$$\theta_{2x}x_2 + \theta_{2y}y_2 + \gamma x_2 y_2 - r_2 \geq \theta_{2x}x_1 + \theta_{2y}y_1 + \gamma x_1 y_1 - r_1 \tag{6}$$

Constraints (3) and (4) are participation or individual rationality constraints for the first and second segments, respectively, and they ensure that each consumer gets nonnegative utility from their purchase. Constraints (5) and (6) are self-selection or incentive compatibility and they ensure that each segment prefers to buy the product intended for it rather than the product intended for the other segment. In the well-known solution of the maximization problem only (4) and (5) are binding constraints, that is:

$$r_1 = \theta_{1x}x_1 + \theta_{1y}y_1 + \gamma x_1 y_1 - (\theta_{1x} - \theta_{2x})x_2 - \left(\theta_{1y} - \theta_{2y}\right)y_2 \tag{7}$$

$$r_2 = \theta_{2x}x_2 + \theta_{2y}y_2 + \gamma x_2 y_2 \tag{8}$$

This implies that the low-type customers get zero surplus and the monopolist is not able to extract full surplus from high-type customers. After substitution of these prices into the profit function, the firm optimally chooses the level of qualities to be incorporated to the product that is offered to every segment. First-order conditions can be written as:

$$\frac{\partial B_M}{\partial j_1} = p\left(\theta_{1j} + \gamma k_1 - j_1 c_j\right) = 0 \tag{9}$$

$$\frac{\partial B_M}{\partial j_2} = \theta_{2j} + \gamma k_2 - j_2 c_j - p\left(\theta_{1j} + \gamma k_2 - j_2 c_j\right) = 0 \tag{10}$$

with $j, k = x, y$ and $j \neq k$. Second-order conditions for a local maximum require that $-pc_j < 0$, $-c_j(1-p) < 0$ and $c_x c_y - \gamma^2 > 0$.

The optimal qualities for segment 1 are

$$j_{1M}^{*}=\begin{cases}\dfrac{\gamma\theta_{1k}+c_k\theta_{1j}}{c_jc_k-\gamma^2} & \text{if } \gamma>-\dfrac{\theta_{1j}}{\theta_{1k}}c_k\\ 0 & \text{if } \gamma\le-\dfrac{\theta_{1j}}{\theta_{1k}}c_k\end{cases}\tag{11}$$

where the subscript *M* stands for "menu pricing". The optimal qualities for segment 2 are

$$j_{2M}^{*}=\begin{cases}\dfrac{\gamma(\theta_{2k}-p\theta_{1k})+c_k\left(\theta_{2j}-p\theta_{1j}\right)}{(1-p)\left(c_jc_k-\gamma^2\right)} & \text{if } \gamma(\theta_{2k}-p\theta_{1k})+c_k\left(\theta_{2j}-p\theta_{1j}\right)>0\\ 0 & \text{if } \gamma(\theta_{2k}-p\theta_{1k})+c_k\left(\theta_{2j}-p\theta_{1j}\right)\le 0\end{cases}\tag{12}$$

After substitution of optimal qualities and prices into Equation (2) we can obtain the optimal profits for the different intervals. For example, when both qualities are provided to both segments, optimal profits are

$$B_M^{*}=\frac{c_y(\theta_{2x}(p\theta_x-\theta_{2x})-p\theta_x(\theta_x-\theta_{2x}))+c_x\left(\theta_{2y}\left(p\theta_y-\theta_{2y}\right)-p\theta_y\left(\theta_y-\theta_{2y}\right)\right)+2\gamma\left(\theta_{2x}\left(p\theta_y-\theta_{2y}\right)-p\theta_x\left(\theta_y-\theta_{2y}\right)\right)}{2(1-p)\left(c_xc_y-\gamma^2\right)}$$

Next we analyze the influence of attribute dependence on the quality provision in both segments, and its implications on two typical issues in monopoly pricing when market is segmented, namely the distortion of quality for the low segment and the cannibalization problem.

High quality segment. The analysis of the relationship between the level of quality provided to the high segment and the level of attribute dependence can be summarized in the following proposition:

Proposition 1. *Consider that* $\frac{\theta_{1j}^2}{c_j}<\frac{\theta_{1k}^2}{c_k}$. *The level of quality j provided to the high quality segment is positive and is monotonically increasing in the attribute dependence for* $\gamma>-\frac{\theta_{1j}}{\theta_{1k}}c_k$ *and zero otherwise. The level of quality k provided to the high quality segment is positive for all* γ *and non-monotone in the attribute dependence with a minimum level of quality given by* $\check{k}_{1M}^{*}=\frac{\theta_{1k}^2}{2\theta_{1j}c_k}\frac{1}{1-\sqrt{\beta}}$ *with* $\beta=\frac{\theta_{1k}^2c_j}{\theta_{1j}^2c_k}$.

Proof See the Appendix.

The intuition of this result is twofold: first, the complementarity between the attributes of a product designed for the high segment enhances its overall quality, and second, this product always incorporates quality with the highest valuation adjusted to production cost. For example, if we think of an information product, we would have that it is composed of a basic or defining quality (the most valuable one), with another one incorporated depending on the level of complementarity between them. The result can be used to describe different stages in a versioning process from an initial step with the product being mainly defined by a specific component, which is complemented by others as their functionality increases.

In terms of the model, the result has to do with the existing differences between the intensity of taste parameters adjusted to their respective production cost of qualities: when the relative valuation of a quality *k* is greater than that for quality *j*, quality *k* is always provided to the high quality segment whilst quality *j* is zero below a certain level of attribute dependence. Consequently, the level of quality incorporated to the product is increasing for both qualities if the attribute dependence is sufficiently high, which implies that attribute dependence contributes to increase the aggregate quality of the

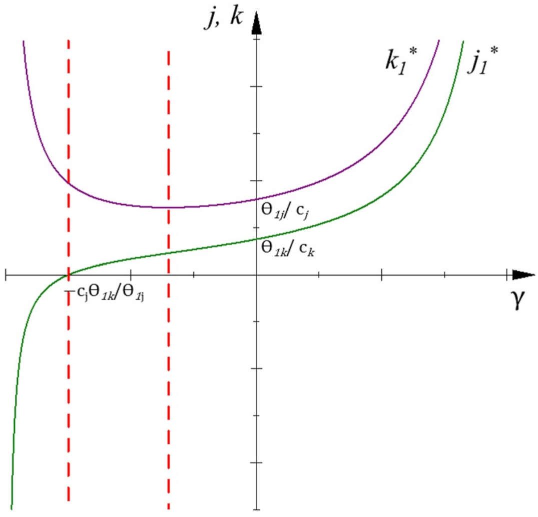

FIGURE 1 Quality and attribute dependence in the high-quality product. [Colour figure can be viewed at wileyonlinelibrary.com]

high quality product. It follows from this Proposition that the analysis of qualities is symmetric, that is, when the relationship between the level of a given quality and the level of attribute dependence is monotonically increasing, then the relationship between the level of the other quality and attribute dependence is non-monotone.

Figure 1 illustrates the Proposition and shows that we can find three different patterns for product design (delimited by the dashed lines): first, the product incorporates just one of the qualities if there is a high level of substitutability, and the level of the quality that is provided increases the higher the level of substitutability; second, for intermediate levels of substitutability both qualities are provided but the level of quality of one of them is increasing in γ whilst the other is decreasing; and third, the level provided of both qualities is increasing with respect to attribute dependence for both, complementarity and low levels of substitutability. It is important to note that not only does the firm optimally incorporates both qualities to the product that is intended for segment 1 if they are complements, but also it they have a sufficiently low degree of substitutability. The motivation for a provision of substitutive qualities is that the marginal utility of quality j, given by $\theta_{1j} + \gamma k_1$, is positive when the level of substitutability is sufficiently low. Evidently, the standard case with independent attributes is included within this third pattern.

The complete map of product design for the high quality segment is illustrated in Figure 2, where the whole set of product configurations in terms of level of attribute dependence, taste parameters and cost coefficients can be seen. For example, the area on the left within the feasible region of product configuration holds that $\theta_{1x}/\theta_{1y} < \sqrt{c_x/c_y}$ and then, according to Proposition 1, quality y is always provided and quality x is incorporated to the product if $\gamma > -c_y\theta_{1x}/\theta_{1y}$, which corresponds to the second and third previously mentioned patterns of product design in Figure 1.

Note that when the taste parameter adjusted to production cost is similar for both qualities (i.e. $\theta_{1x}/\theta_{1y}. = \sqrt{c_x/c_y}$), the firm optimally provides both qualities for any feasible γ (that is, even for the highest level of substitutability that is compatible with SOC).

Low quality segment. The results of the analysis concerning the relationship between the level of attribute dependence and the level of quality provided to the low segment can be summarized in the following proposition:

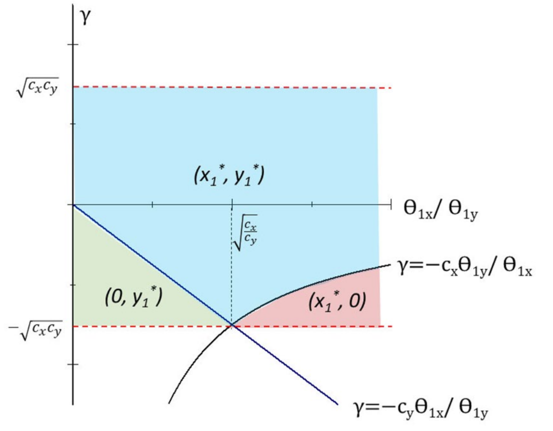

FIGURE 2 Product configuration for the high-quality segment. [Colour figure can be viewed at wileyonlinelibrary.com]

Proposition 2. *Consider that* $\Delta^2 < \frac{c_j}{c_k}$ *with* $\Delta = \frac{\theta_{2j} - p\theta_{1j}}{\theta_{2k} - p\theta_{1k}}$*. Then, if* $p < \theta_{2k}/\theta_{1k}$*, the level of quality j provided to the low quality segment is monotonically increasing in* γ *and quality k is always provided and is non-monotone and convex in* γ*. If* $p > \theta_{2k}/\theta_{1k}$*, quality j is monotonically decreasing in* γ *and quality k is not provided to the low quality segment.*

Proof See the Appendix.

The intuition of this proposition is that the provision of quality in the product designed for the low quality segment is focused on the quality with higher attractiveness whilst other qualities are provided depending on the relative characteristics of the customer segments. Similar to the intuition given for the high-quality product when mentioning information products like software, we have that low quality product is mainly defined by a basic component whilst other components are included or not in the package depending on the relative size and taste preference of the customer segments (see Krishnan & Zhu, 2006 for the example of Adobe Systems).

In order to understand the two parts of this Proposition, one must bear in mind that the ratio $\left(\theta_{2j} - p\theta_{1j}\right)^2/c_j$ can be interpreted as the relative attractiveness of providing quality j to the low quality segment when the relative size of the high quality segment is sufficiently low (i.e. $p < \theta_{2j}/\theta_{1j}$): the ratio increases (decreases) as the cost coefficient of that quality decreases (increases) and the difference between the relative taste parameter (θ_{1j}/θ_{2j}, that is, the level of dominance) and the proportion of high quality customers (p) increases (decreases). However as the relative size of the low quality segment decreases (i.e. $p > \theta_{2j}/\theta_{1j}$), a higher value of the ratio has the opposite meaning, that is, it represents lower relative attractiveness. Figures 3 and 4 illustrate this Proposition. Similarly to the high segment, we have that the situation for quality x is symmetric to that for quality y.

Figure 3 represents the first part of Proposition 2 and corresponds to the case that the relative size of the high quality segment is sufficiently small (i.e. $p < \theta_{2k}/\theta_{1k}$). In that context, the condition given by $\frac{(\theta_{2j} - p\theta_{1j})^2}{c_j} < \frac{(\theta_{2k} - p\theta_{1k})^2}{c_k}$ means that producing k is more attractive to the firm than producing j, and then the product for the low quality customers always incorporates quality k, whilst quality j is

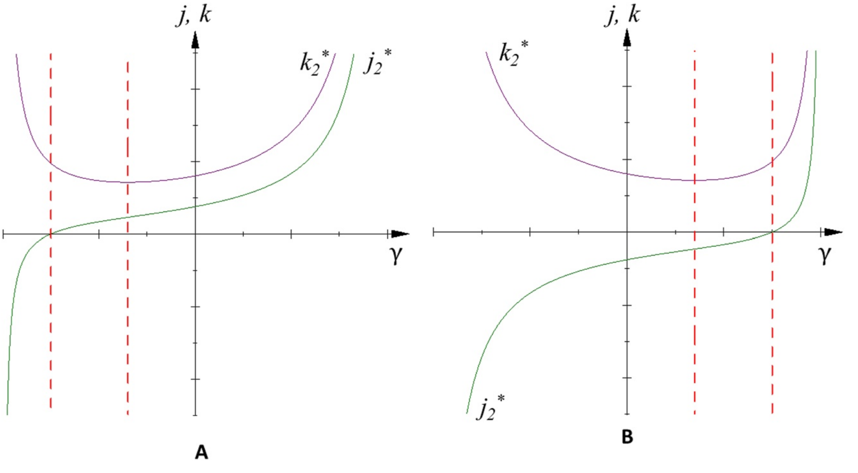

FIGURE 3 Quality and attribute dependence in the low quality segment when $p < \theta_{2k}/\theta_{1k}$. [Colour figure can be viewed at wileyonlinelibrary.com]

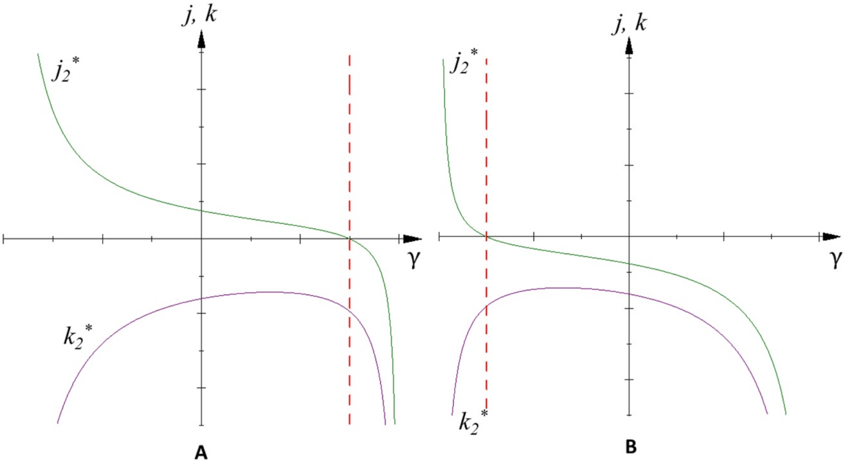

FIGURE 4 Quality and attribute dependence in the low quality segment when $p > \theta_{2k}/\theta_{1k}$. [Colour figure can be viewed at wileyonlinelibrary.com]

only incorporated if the attribute dependence is big enough. Two situations can be distinguished in this Figure:

- The threshold for γ is negative if the proportion of high quality customers is small enough, specifically if $p < \min\left\{\frac{\theta_{2k}}{\theta_{1k}}, \frac{\theta_{2j}}{\theta_{1j}}\right\}$ (see Panel A in Figure 3). Thus, a positive provision of both qualities is also a feasible outcome for segment 2 (and not only for segment 1, as shown above) both for complementary and substitutive qualities when the proportion of high quality customers is sufficiently small.
- The threshold for γ is positive for intermediate values of p, specifically if $\frac{\theta_{2j}}{\theta_{1j}} < p < \frac{\theta_{2k}}{\theta_{1k}}$ (see Panel B in Figure 3). Then, a sufficiently high level of complementarity is required for the low quality product to incorporate both qualities. Note that in this case not only is k more efficient for the firm, but the dominance of quality j in segment 1 needs to be higher than that for quality k (or alternatively, the valuation of k in segment 2 needs to be higher in relative terms than that of j).

Thus, the first part of the Proposition is similar to that found for the high quality segment: a positive amount of the quality that is more efficient in a production cost perspective relative to the preference parameters of every segment and to the proportion of high quality customers is always provided, and moreover, there is a minimum level of this quality provided by the firm. The other quality is incorporated to the product or not, depending on the level of attribute dependence. Thus, we have that it is also possible to find an optimal solution in the low quality segment, characterized by a positive provision of both qualities, both when qualities are complements and also if they have a certain degree of substitutability.

Figure 4 corresponds to the case in which the relative size of the high quality segment is sufficiently great (i.e. $p > \theta_{2k}/\theta_{1k}$). In that context, the condition $\frac{(\theta_{2j} - p\theta_{1j})^2}{c_j} < \frac{(\theta_{2k} - p\theta_{1k})^2}{c_k}$ means that k is less attractive than j to the firm. Consequently, quality k is not incorporated to the product and quality j is incorporated to a limited extent, depending on the level of attribute dependence:

- If complementarity is sufficiently big when $\frac{\theta_{2k}}{\theta_{1k}} < p < \frac{\theta_{2j}}{\theta 1j}$(see Panel A in Figure 4), and
- If substitutability is sufficiently high when $p > \max\left\{\frac{\theta_{2k}}{\theta_{1k}}, \frac{\theta_{2j}}{\theta_{1j}}\right\}$(see Panel B in Figure 4).

Thus, under the conditions described for the second part of this Proposition, we have that the provision of quality to the low segment follows two patterns of product configuration: first, only one of the qualities is included in the product, and second, the low quality segment is not served by the monopolist.

The feasible set of product configurations for the low quality segment is illustrated in Figure 5, where the horizontal axis represents the ratio $\Delta = \frac{\theta_{2x} - p\theta_{1x}}{\theta_{2y} - p\theta_{1y}}$. The value of this ratio depends on the proportion of high quality customers and the relative dominance of segment 1 in every quality. By using this ratio, the analysis of the resulting product configurations can be summarized in two cases: first, the dominance of segment 1 in quality x is higher than the dominance in quality y (i.e. $\theta_{1x}/\theta_{2x} > \theta_{1y}/\theta_{2y}$), which is represented in panel A of this Figure, and second, the dominance of segment in quality y is higher than dominance in quality x (i.e. $\theta_{1x}/\theta_{2x} < \theta_{1y}/\theta_{2y}$), represented in panel B.

The area where both qualities are provided in Panel A of Figure 5 corresponds to the second and third patterns of product design in Panel A of Figure 3, and the area where both qualities are provided in Panel B of Figure 5 corresponds to the second and third patterns of product design in Panel B of Figure 3. Similarly, the area where no quality is provided to this segment in Panel A of Figure 5 corresponds to the second pattern of product design in Panel A of Figure 4 and the area where both qualities are provided in Panel B of Figure 5 corresponds to the second pattern in Panel B of Figure 4. For product configurations that incorporate just one dimension of quality, the correspondence between figures is similar, and results from switching j and k in Figures 3 and 4 by x and y. For example, for $j = x$ the combination $(x_2, 0)$ in Panel A and Panel B in Figure 5 corresponds to the first pattern in Panel B and Panel A of Figure 4, respectively.

This Figure can be also interpreted in terms of the proportion of high quality customers. This proportion is decreasing in Panel A as we move to the right. We can see that as p and the degree of substitutability is increasing, the provision of quality decreases, and either only one quality is provided, or none. Contrary to that panel, the proportion of high quality customers is increasing in panel B as we move to the right in this figure. The provision of quality is lower than in the previous case, as the combination of qualities implies zero provision in one quality or even both, except for a high enough level of complementarity and a low enough proportion of high quality customers.

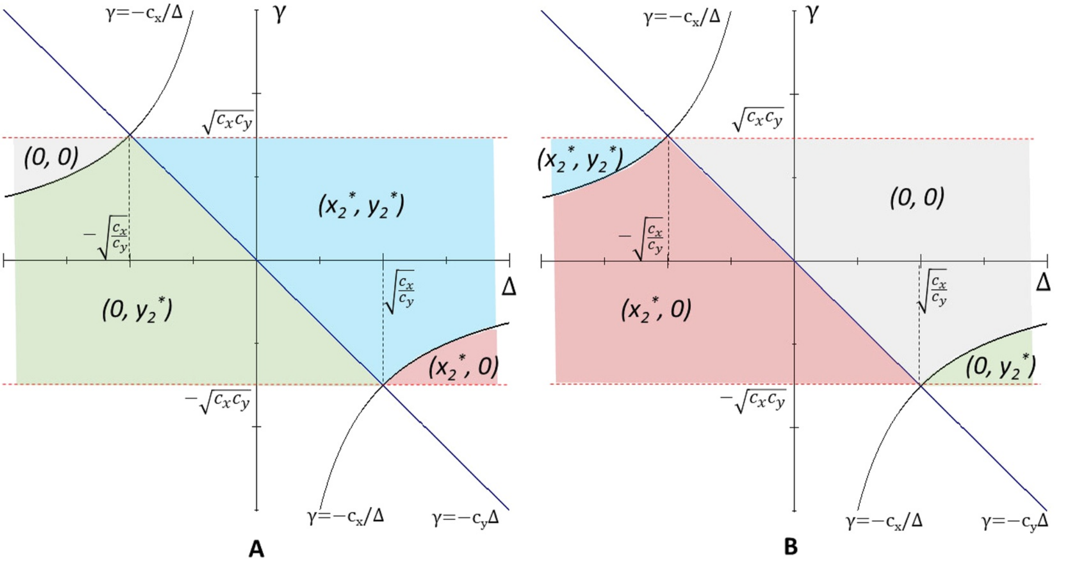

FIGURE 5 Product configuration for the low quality segment. [Colour figure can be viewed at wileyonlinelibrary.com]

Distortion of quality. As is usual in these models (e.g. Moorthy, 1984; Mussa & Rosen, 1978), we obtain the "efficient" solution for the high quality segment (see Equation (9)), while the first-order condition for the low quality segment includes a negative term in the right-hand side (see Equation (10)), which implies that the monopolist distorts low quality downwards compared with the efficient solution.

Simple calculus shows that the difference between the efficient quality for the low quality segment[3] (denoted by j_O^*) and j_{2M} can be written as

$$j_O^* - j_{2M}^* = p\frac{\gamma(\theta_{1k} - \theta_{2k}) + c_k\left(\theta_{1j} - \theta_{2j}\right)}{(1-p)\left(c_j c_k - \gamma^2\right)} = \frac{p}{1-p}\left(j_{1M}^* - j_O^*\right),$$

that is, the quality distortion is positive[4] and proportional to the level of attribute dependence and, according to the literature (e.g. Desai et al., 2001), it also originates in the relative size of every segment of customers and by costs and taste parameters.

Note that this expression is similar to that corresponding to j_{2M}^* (just removing p from the numerator of Equation (12)). Thus, the previous analysis on the relationship between optimal quality for low segment and attribute dependence applies here, that is, first, we again have two cases for that relationship, a monotone increasing relationship and a non-monotone and convex relationship, and second, the analysis for quality y is symmetric to that for quality x. In other words, when the distortion in quality x is monotonically increasing in γ, then the distortion in quality y is non-monotone and convex in γ, and vice versa. Consequently, the analysis of quality distortion requires observation of both qualities at the same time. For that purpose, we calculate the overall quality distortion:

$$x_O - x_{2M} + y_O - y_{2M} = \frac{p}{1-p}\frac{\gamma\left(\theta_{1x} - \theta_{2x} + \theta_{1y} - \theta_{2y}\right) + c_y(\theta_{1x} - \theta_{2x}) + c_x\left(\theta_{1y} - \theta_{2y}\right)}{c_x c_y - \gamma^2}$$

[3]This level can be obtained by analogy from Equation (11) and it is calculated in Section 3.4.

[4]As we shall see, the commonality strategy studied in Section 3.2 corresponds to a zero distortion in one of the dimensions of quality, the common one.

which is positive for $\gamma > -\Theta$ with $\Theta = c_x c_y \frac{\frac{\theta_{1x}-\theta_{2x}}{c_x} + \frac{\theta_{1y}-\theta_{2y}}{c_y}}{\theta_{1x}-\theta_{2x}+\theta_{1y}-\theta_{2y}}$. The meaning of the ratio Θ is analogous to that of the divergence index in Kim and Chhajed (2002), that is, a measure of the degree of dominance in every quality adjusted to production costs: the ratio is increasing in every production cost, and moreover it is increasing (decreasing) in the level of dominance of a given quality if its production cost is lower (more expensive) than the production cost of the other quality. Thus, the ratio is an integrated measure of the level of divergence of segments and qualities, as it incorporates the level of dominance in preferences and the production costs of qualities: for example, higher dominance of the cheapest quality means higher divergence, and higher dominance of the most expensive quality means lower divergence. Because the ratio can be either positive or negative, a low (high) level of divergence is associated to low (high) values of Θ^2. Note that a positive Θ can be associated to both strict and sufficiently low non-strict dominance, and a negative Θ requires non-strict dominance. Thus, both low and high divergence are compatible with both strict and non-strict dominance.

Our analysis of the overall quality distortion gives the following result:

Proposition 3. *The distortion of a low quality product is proportional to the level of attribute dependence and to the degree of divergence between segments: if the level of divergence is sufficiently low, that is, $\Theta^2 < c_x c_y$, then the distortion of quality is increasing in γ, and if the level of divergence is sufficiently high, that is, $\Theta^2 > c_x c_y$, we have two subcases: (i) if $-\Theta < -\sqrt{c_x c_y}$ then the quality distortion is increasing in γ for any $\gamma > -\Theta\left(1 + \sqrt{1 - \frac{c_x c_y}{\Theta^2}}\right)$ and is decreasing otherwise; and (ii) if $-\Theta > \sqrt{c_x c_y}$ then the quality distortion is increasing for $\gamma < -\Theta\left(1 - \sqrt{1 - \frac{c_x c_y}{\Theta^2}}\right)$ and is decreasing otherwise.*

Proof See the Appendix.

This proposition complements Propositions 1 and 2 as it refers to the differential effect of attribute dependence in high and low quality products depending on the similarity between segments and the relative production costs of qualities: when the distortion of quality increases, we have that the influence of attribute dependence is higher for the high quality segment, and vice versa. The proposition comprises two different situations. First, when taste parameters of the segments and production costs of qualities are sufficiently similar, the aggregate quality distortion is always increasing in the level of attribute dependence. Second, when taste parameters of the segments and production costs of qualities diverge, the aggregate quality distortion can either increase or decrease depending on the level of attribute dependence. For example, positive and high values of Θ are associated to a positive impact of attribute dependence on aggregate quality distortion if qualities are complements, or if they have a low substitutability. Conversely, for high (in absolute value) and negative values of Θ, aggregate quality only increases in γ for sufficiently high levels of substitutability (recall that in this case the product incorporates just one dimension of quality).

Then, according to the literature, the distortion of quality depends on the level of preference for every quality of every segment and on the production cost of every quality. In addition, our analysis shows that the difference of aggregate quality between high quality and low quality products can be exacerbated by the existing relationship between the qualities that are incorporated to the product. As we will see these results have important implications for the problem of cannibalization that is studied next.

Cannibalization. The literature on product design has analyzed the cannibalization problem for a monopolist who faces consumers who differ in their quality valuations: higher-valuation consumers may find it beneficial to buy lower-quality products rather than the higher quality products targeted to them. The notion of cannibalization can be captured through the amount of surplus that must be offered to the switching segment (Desai, 2001; Kim et al., 2013; Moorthy & Png, 1992). This amount is denoted by C_M and corresponds to the negative terms of Equation (7), that is, $C_M = (\theta_{1x} - \theta_{2x})x^*_{2M} + \left(\theta_{1y} - \theta_{2y}\right)y^*_{2M}$: high-type consumers buy the high quality but at a reduced price.

Based on the previous analysis on the relationship between optimal quality and attribute dependence, we can write the following Proposition:

Proposition 4. *Consider that* $\Delta^2 < \frac{c_j}{c_k}$ *with* $\Delta = \frac{\theta_{2j} - p\theta_{1j}}{\theta_{2k} - p\theta_{1k}}$. *If* $p < \theta_{2k}/\theta_{1k}$, *then attribute dependence increases the level of cannibalization if* $\gamma > -\frac{c_j}{\Delta}\left(1 \pm \sqrt{1 - \frac{c_k}{c_j}\Delta^2}\right)$. *If* $p > \theta_{2k}/\theta_{1k}$, *then attribute dependence decreases the level of cannibalization if* $\gamma < - c_k\Delta$.

Proof See the Appendix.

This proposition complements Proposition 2 as cannibalization is defined in terms of the quality provided to the low quality segment, thus the intuition is similar. The interpretation of this Proposition can be made by means of Figure 3. Cannibalization is increasing in attribute dependence for low quality products which incorporate both dimensions of quality, because in that case the provision of both qualities is increasing in attribute dependence. This situation can be associated to complementarity and low substitutability if the proportion of high quality customers is sufficiently low (i.e. $p < \min\left\{\frac{\theta_{2k}}{\theta_{1k}}, \frac{\theta_{2j}}{\theta_{1j}}\right\}$), and is associated to sufficiently high complementarity for intermediate values of that proportion of customers (i.e. $\frac{\theta_{2k}}{\theta_{1k}} < p < \frac{\theta_{2j}}{\theta 1j}$). Conversely, a decreasing cannibalization is associated to a situation where only one of the qualities is incorporated to the product and its optimal level is decreasing in attribute dependence (see Figure 4). There is an exception for the second pattern of product configuration included in Panel B of Figure 3: cannibalization is increasing in the interval of values of γ where $j^*_2 = 0$ and k^*_2 is increasing.

3.2 | Commonality

Existing research on market segmentation and product line design emphasises commonality strategy, that is, when one attribute or quality is offered at the same level in several products whilst the others are provided at different levels in each product (e.g. Desai et al., 2001; Kim et al., 2013). Broadly speaking, this strategy is a way of meeting diverse customer needs with less cost due to economies of scale in procurement, production and distribution (Kim & Chhajed, 2000). Examples of commonality are abundant in many industries such as automotive, computers and electronics (Wong et al., 2021). In this Section, we analyze the relationship between commonality strategy and attribute dependence independently of cost motivations. We study commonality as an ex-ante decision on product design configuration when the presence of a common attribute is induced by supply-side factors such as the existence of common components in the provision of a given service, technological standards, or by other factors such as regulatory constraints, etc.[5]

Let us assume that $x_1 = x_2 = x_C$ being x_C the level of quality x that is provided at the same level to each segment.

The monopolist profits are

$$B_C = p\left(r_1 - \frac{1}{2}c_y y_1^2\right) + (1-p)\left(r_2 - \frac{1}{2}c_y y_2^2\right) - \frac{1}{2}c_x x_C^2 \tag{13}$$

where prices for this case result from substitution of $x_1 = x_2 = x_C$ in Equations (7) and (8), and then r_1 can be rewritten as $r_1 = (\theta_{1y} + \gamma x_C)(y_1 - y_2) + r_2$.

The resulting optimal qualities are given by

$$x_C^* = \begin{cases} \dfrac{\gamma\theta_{2y} + c_y\theta_{2x}}{c_x c_y - \gamma^2} & \text{if } \gamma > -\dfrac{\theta_{2x}}{\theta_{2y}}c_y \\ 0 & \text{if } \gamma \le -\dfrac{\theta_{2x}}{\theta_{2y}}c_y \end{cases} \tag{14}$$

$$y_{1C}^* = \frac{1}{c_y}\left(\theta_{1y} + \gamma x_C^*\right) \tag{15}$$

$$y_{2C}^* = \frac{\gamma\theta_{2x} + c_x\theta_{2y}}{(1-p)\left(c_x c_y - \gamma^2\right)} - \frac{p}{1-p}y_{1C}^* \tag{16}$$

$$B_C^* = \frac{\left(p\theta_y^2(1-p) + \left(\theta_{2y} - p\theta_y\right)^2\right)\left(c_x c_y - \gamma^2\right) + \left(\gamma\theta_{2y} + c_y\theta_{2x}\right)^2(1-p)}{2c_y\left(c_x c_y - \gamma^2\right)(1-p)}.$$

These optimal values allow us to obtain the following proposition:

Proposition 5. *The common quality j is set at the efficient level of the low quality segment and the non-common quality is given by* $k_{iC}^* = k_{iM}^*(\gamma = 0) + \frac{\gamma}{c_k} j_C^*$.

Proof See the Appendix.

According to Desai et al. (2001), we obtain that the common quality x_C^* is set at the efficient level of quality for the low segment. However, our results concerning the non-common quality differ when attribute dependence is considered. Specifically, those authors find that the two quality levels of the non-common attribute, y_{1C}^* and y_{2C}^*, are identical to those offered by the firm in a selective strategy and in a common product strategy, respectively (see below Subsections 3.3 and 3.4). We obtain that, first, the two optimal levels for the non-common quality are identical to those obtained in a menu pricing strategy when attribute dependence is not considered in the analysis (i.e. qualities are independent), and second, complementarity (substitutability) increases (decreases) the level of the quality

[5]In our model, a commonality strategy can also be an endogenous outcome within a menu pricing strategy as it is a feasible optimal outcome under certain conditions related to the preference structure. Specifically, we have that $j_{1M}^* = j_{2M}^*$ if $\gamma = -c_k\left(\theta_{1j} - \theta_{2j}\right)/(\theta_{1k} - \theta_{2k})$, that is, if there is either a certain level of substitutability between qualities with strict dominance, or a certain level of complementarity with soft dominance.

that is not made common in the same amount for both segments. Of course, the analysis for quality y as the common quality is symmetric to the previous one.

As a consequence, commonality intensifies cannibalization with both strict and non-strict dominance, that is, regardless of the preference structure. We can see this simply by comparing the intensity of cannibalization under menu pricing (C_M) and under product commonality (C_C)[6]:

$$C_M - C_C = -p\frac{\left(c_y(\theta_{1x} - \theta_{2x}) + \gamma\left(\theta_{1y} - \theta_{2y}\right)\right)^2}{c_y\left(c_x c_y - \gamma^2\right)(1-p)} < 0$$

In other words, according to Kim and Chhajed (2013), a commonality strategy does not relieve cannibalization in absence of cost savings.

Concerning the conditions under which it is more profitable for the firm to make a given quality common, we find that making quality x common is more profitable than making quality y common when it holds that $\frac{c_x}{c_y} > \frac{(\theta_{1x}-\theta_{2x})^2}{(\theta_{1y}-\theta_{2y})^2}$ and vice versa. This result is in line with Proposition 6 in Desai et al. (2001).

The distortion of quality is obviously zero for the common quality in this strategy and the non-common quality diverges from the efficient level. Specifically we have that $y_O - y_{2C} = p\frac{\theta_y - \theta_{2y}}{c_y(1-p)}$. We can find moderate levels of non strict dominance which hold that $-\sqrt{c_x/c_y} < (\theta_x - \theta_{2x})/\left(\theta_y - \theta_{2y}\right) < \sqrt{c_x/c_y}$ (so making quality x common is more profitable than making quality y) that hold that $y_O - y_{2C} < 0$, that is a negative distortion of quality. This situation is more feasible the higher c_x/c_y.

3.3 | Single product for high-segment only

In this section, we consider the selective-segment strategy, that is, the monopolist produces only the high quality product (x_1, y_1) and only consumers from segment 1 buy the product. Then, only high segment is binding, that is, constraint 3 holds at equality. This means that the price charged by the firm is given by:

$$r_1 = \theta_{1x}x_1 + \theta_{1y}y_1 + \gamma x_1 y_1, \tag{17}$$

and the profit function is

$$B_H = p\left(r_1 - \frac{1}{2}c_x x_1^2 - \frac{1}{2}c_y y_1^2\right). \tag{18}$$

The resulting optimal qualities are identical to those obtained in menu pricing for the high segment:

$$j_H^* = \begin{cases} \dfrac{\gamma\theta_{1k} + c_k\theta_{1j}}{c_j c_k - \gamma^2} & \text{if } \gamma > -\dfrac{\theta_{1j}}{\theta_{1k}}c_k \\ 0 & \text{if } \gamma \le -\dfrac{\theta_{1j}}{\theta_{1k}}c_k \end{cases} \tag{19}$$

with $j, k = x, y$ and $j \neq k$. We can see that $j_H^* = j_{1M}^*$, and subsequently, the analysis on the influence of attribute dependence is identical to that displayed in Subsection 3.1 for the high quality product.

[6]Similarly to C_M, C_C is defined as $C_C = (\theta_{1x} - \theta_{2x})x_C^* + \left(\theta_{1y} - \theta_{2y}\right)y_{2C}^*$.

Optimal profits are then given by

$$B_H^* = \begin{cases} \dfrac{p\left(\theta_{1j}^2 c_k + \theta_{1k}^2 c_j + 2\gamma\theta_{1j}\theta_{1k}\right)}{2\left(c_j c_k - \gamma^2\right)} & \text{if } \gamma > \max\left\{-\dfrac{\theta_{1j}}{\theta_{1k}}c_k, -\dfrac{\theta_{1k}}{\theta_{1j}}c_j\right\} \\ p\dfrac{\gamma\theta_{1k} + c_k\theta_{1j}}{c_j c_k - \gamma^2}\left(\theta_{1j} - \dfrac{1}{2}c_j\dfrac{\gamma\theta_{1k} + c_k\theta_{1j}}{c_j c_k - \gamma^2}\right) & \text{if } -\dfrac{\theta_{1k}}{\theta_{1j}}c_j > \gamma > -\dfrac{\theta_{1j}}{\theta_{1k}}c_k \\ 0 & \text{if } \gamma \le \min\left\{-\dfrac{\theta_{1j}}{\theta_{1k}}c_k, -\dfrac{\theta_{1k}}{\theta_{1j}}c_j\right\} \end{cases}$$

where the intermediate case represents the situation when only quality j is provided.

3.4 | Single product for both segments

In this section, we consider a common-product strategy, that is, the monopolist produces a single product (x, y) designed and priced for both types of customers. Then, only low segment is binding, otherwise the low segment would not buy the good because these customers would get negative surplus, that is, constraint Equation (4) holds at equality. The maximum price the firm can charge is then given by Equation (8), and the profit function is

$$B_O = r_2 - \frac{1}{2}c_x x_2^2 - \frac{1}{2}c_y y_2^2 \tag{20}$$

The resulting optimal qualities are given by

$$j_O^* = \begin{cases} \dfrac{\gamma\theta_{2k} + c_k\theta_{2j}}{c_j c_k - \gamma^2} & \text{if } \gamma > -\dfrac{\theta_{2j}}{\theta_{2k}}c_k \\ 0 & \text{if } \gamma \le -\dfrac{\theta_{2j}}{\theta_{2k}}c_k \end{cases} \tag{21}$$

with $j, k = x, y$ and $j \neq k$. This is the efficient level of quality j for the low quality segment, and then coincides with the optimal common quality in the commonality strategy studied in Section 3.2.

Optimal profits are then given by

$$B_O^* = \begin{cases} \dfrac{c_k\theta_{2j}^2 + c_j\theta_{2k}^2 + 2\gamma\theta_{2j}\theta_{2k}}{2\left(c_j c_k - \gamma^2\right)} & \text{if } \gamma > \max\left\{-\dfrac{\theta_{2j}}{\theta_{2k}}c_k, -\dfrac{\theta_{2k}}{\theta_{2j}}c_j\right\} \\ \left(\dfrac{\gamma\theta_{2k} + c_k\theta_{2j}}{c_j c_k - \gamma^2}\right)\left(\theta_{2j} - \dfrac{1}{2}c_j\dfrac{\gamma\theta_{2k} + c_k\theta_{2j}}{c_j c_k - \gamma^2}\right) & \text{if } -\dfrac{\theta_{2k}}{\theta_{2j}}c_j > \gamma > -\dfrac{\theta_{2j}}{\theta_{2k}}c_k \\ 0 & \text{if } \gamma \le \min\left\{-\dfrac{\theta_{2j}}{\theta_{2k}}c_k, -\dfrac{\theta_{2k}}{\theta_{2j}}c_j\right\} \end{cases}$$

where the intermediate case represents the situation when only quality j is provided.

3.5 | Comparative analysis

In this subsection, we identify the relationships between optimal qualities and profits obtained under the different strategies. Concerning optimal qualities, the analysis shows the following identities:

(i) $j_H^* = j_{1M}^*$ which correspond to the efficient level of quality j for the high quality segment.
(ii) $j_O^* = pj_{1M}^* + (1-p)j_{2M}^*$, that is, the optimal level of quality j provided in the "only one product for both segments" strategy, j_O^*, is a weighted linear combination of the optimal level of quality y provided in a menu pricing strategy, where the relative size of every segment of customers is the weighting factor. Note that this expression can be rewritten as $j_O^* - j_{2M}^* = p\left(j_{1M}^* - j_{2M}^*\right)$; consequently the analysis on quality distortion displayed in Section 3.1 can also be applied to the comparison between high quality and low quality product in a menu pricing strategy, which implies that the difference in quality can be exacerbated by attribute dependence.
(iii) $j_O^* = j_C^*$ being j the common quality in a commonality strategy. This level of quality corresponds to the efficient level of quality j for the low quality segment.
(iv) $k_O^* = pk_{1C}^* + (1-p)k_{2C}^*$ being k the non-common quality in a commonality strategy. That is, the optimal level of the quality k provided in the "only one product for both segments" strategy, k_O^*, is a linear combination of the optimal qualities on the non-common quality in a commonality strategy, where the relative size of every segment of customers is the weighting factor.

Concerning the optimal profits resulting from every strategy, we can write the following result:

Proposition 6. *$B_O^* < B_C^* < B_M^*$ and $B_H^* < B_M^*$ with $B_H^* \lesseqgtr B_O^*$ and $B_H^* \lesseqgtr B_C^*$.*

Proof See the Appendix.

The Proposition states that menu pricing is always the most profitable strategy when viable, and when it is not, the optimal pricing strategy is either the selective-segment strategy or the commonality strategy, depending on the relative attractiveness of the high quality segment and its relative size. The intuition of this result is that, if there is attribute dependence and average production costs are constant for both qualities, then flexibility in the qualities allows the monopolist to choose both quality levels in a more efficient and profitable way (i.e. according to the preference configuration). This is particularly relevant for commonality as it could be an optimal strategy for reasons other than cost-savings, contrary to what has been reported in the extant literature.

The difference in profitability between selling high quality to the high segment only and selling low quality to all customers can be written as $B_H - B_O = \frac{A+\gamma B}{2(c_x c_y - \gamma^2)}$ where $A = c_x c_y \left[p\left(\frac{\theta_x^2}{c_x} + \frac{\theta_y^2}{c_y}\right) - \left(\frac{\theta_{2x}^2}{c_x} + \frac{\theta_{2y}^2}{c_y}\right)\right]$ and $B = 2\left(p\theta_x\theta_y - \theta_{2x}\theta_{2y}\right)$. The term A is the difference in attractiveness between the selective strategy and the common product strategy adjusted to production costs, and B is the level of aggregate dominance of the high quality segment adjusted to its size. We can see that the threshold for this difference is given by $\gamma_0 = -A/B$, and then, we have two situations: first, if γ_0 does not belong to the feasible interval for γ (i.e. $A^2/B^2 > c_x c_y$) then $B_H > B_O$ when A is positive and vice versa; and second, if γ_0 belongs to the feasible interval for γ (i.e. $A^2/B^2 < c_x c_y$) then we have that if $p > (\theta_{2x}\theta_{2y})/(\theta_x\theta_y)$ then $B_H - B_O > 0$ for any $\gamma > \gamma_0$, and negative otherwise, and if $p < (\theta_{2x}\theta_{2y})/(\theta_x\theta_y)$ then $B_H - B_O > 0$ for any $\gamma < \gamma_0$, and negative otherwise (see the Proof of Proposition 6 for a detailed explanation). We can see that the level of attribute dependence reinforces the influence of the aggregate dominance of the high quality segment on the difference of profitability between H and O. For example, we can see that this influence is positive when qualities are complements if the aggregate dominance of the high quality segment is positive and vice versa.

The comparison of profitability between the selective strategy and the commonality strategy can be written as $B_H - B_C = \frac{A+\gamma B}{2(c_x c_y - \gamma^2)} - \frac{p(\theta_y - \theta_{2y})^2}{2c_y(1-p)}$ where the first term (which corresponds to $B_H - B_O$) can be either positive or negative as we have just seen, and the second (which corresponds to $B_O - B_C$) is always negative. Then, we have that, for example, $B_H > B_C$ requires that $B_H > B_O$.

4 | CONCLUSIONS

The analysis presented in the paper shows that the introduction of attribute dependence in consumer preferences over multi-attribute products yields new qualitative results in the context of product design and pricing in a monopolistic market. The existence of interdependence between characteristics is found to be a determining factor of quality provision. Specifically, complementarity and low substitutability between qualities is an incentive for firms to improve the level of quality incorporated to the products, and can contribute to exacerbate the differences between high quality product and low quality product in monopolistic markets. Consequently, it enhances the distortion of quality and cannibalization problems that are inherent to monopoly pricing when market demand is segmented. Interestingly, this increased difference between high quality and low quality products runs parallel to the *maxmax* solutions obtained in product differentiation models in oligopolistic markets when attribute dependence is considered (maximum differentiation in both qualities, in contrast to *maxmin* solutions that are usual in the literature, i.e. maximum differentiation in one quality and minimum in the other). In addition, the analysis contributes to improve our understanding about why some firms incorporate attributes that are substitutes in a given product (in sectors such as those mentioned in the Introduction).

The paper shows that menu pricing is the most profitable strategy and that a two-product strategy with a common attribute is more profitable than a common-product strategy for all customers. In this sense, another relevant conclusion of the paper is that the feasibility of a commonality strategy may be based on reasons other that cost motivations, in particular on the existence of attribute dependence under certain conditions related to the structure of customer preferences.

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APPENDIX

Proof of Proposition 1

The relationship between the optimal level of quality x provided to the high quality segment and attribute dependence for the high quality segment is given by

$$\frac{\partial x_{1M}^*}{\partial \gamma} = \frac{\gamma^2\theta_{1y} + 2\gamma\theta_{1x}c_y + \theta_{1y}c_xc_y}{\left(c_xc_y - \gamma^2\right)^2}$$

that equals to zero for $\gamma = c_y\frac{\theta_{1x}}{\theta_{1y}}\left(-1 \pm \sqrt{1-\beta}\right)$ with $\beta = \frac{\theta_{1y}^2 c_x}{\theta_{1x}^2 c_y}$. Then we can consider two cases:

- Case 1: $\beta > 1$, that is, if $\frac{\theta_{1x}^2}{c_x} < \frac{\theta_{1y}^2}{c_y}$, which is equivalent to $-\sqrt{c_xc_y} < -\frac{\theta_{1x}}{\theta_{1y}}c_y$. This means that the threshold $-\frac{\theta_{1x}}{\theta_{1y}}c_y$ given by Equation (11) is a feasible value for γ. Then, when $\gamma \leq -\frac{\theta_{1x}}{\theta_{1y}}c_y$ the optimal level of quality x is zero, and when $\gamma > -\frac{\theta_{1x}}{\theta_{1y}}c_y$ the optimal level of quality x is positive and it is positively related to attribute dependence.

- Case 2: $\beta < 1$, that is, if $\frac{\theta_{1x}^2}{c_x} > \frac{\theta_{1y}^2}{c_y}$, which is equivalent to $-\frac{\theta_{1x}}{\theta_{1y}}c_y < -\sqrt{c_x c_y}$.This means that the threshold $-\frac{\theta_{1x}}{\theta_{1y}}c_y$ is not a feasible value for γ, and therefore quality x is always provided by the monopolist to the high segment for any γ. After analyzing the roots of $\frac{\partial x_{1M}^*}{\partial \gamma}$, which are given by $\gamma = c_y \frac{\theta_{1x}}{\theta_{1y}}\left(-1 \pm \sqrt{1-\beta}\right)$, simple calculus shows that only $\gamma = c_y \frac{\theta_x}{\theta_y}\left(-1 + \sqrt{1-\beta}\right)$ is a feasible value, and moreover, that root is a minimum. After substitution in Equation (11), we obtain the associated level of quality, given by $\check{x}_{1M}^* = \frac{\theta_{1y}^2}{2\theta_{1x}c_y}\frac{1}{\sqrt{1-\beta}-1}$.

Now, if we repeat the same analysis for the level of quality y provided to the high quality segment, we can see that it is symmetric to the previous one. Specifically

$$\frac{\partial y_{1M}^*}{\partial \gamma} = \frac{\gamma^2\theta_{1x} + 2\gamma\theta_{1y}c_x + \theta_{1x}c_x c_y}{\left(\gamma^2 - c_x c_y\right)^2}$$

that equals zero for $\gamma = -c_x \frac{\theta_{1y}}{\theta_{1x}}\left(1 \pm \sqrt{1-\frac{1}{\beta}}\right)$. Again, we have two cases depending on the sign of $1-\frac{1}{\beta}$. It is easy to see that case 1 for quality x corresponds to case 2 for quality y, and vice versa.

Proof of Proposition 2

The relationship between the optimal level of quality x and attribute dependence for the low quality segment is given by

$$\frac{\partial x_2^*}{\partial \gamma} = \frac{\left(\gamma^2 + c_x c_y\right)\left(\theta_{2y} - p\theta_{1y}\right) + 2\gamma c_y(\theta_{2x} - p\theta_{1x})}{\left(c_x c_y - \gamma^2\right)^2(1-p)}$$

that equals zero for $\gamma = -c_y\Delta\left(1 \pm \sqrt{1-\frac{c_x}{c_y\Delta^2}}\right)$ with $\Delta = \frac{\theta_{2x} - p\theta_{1x}}{\theta_{2y} - p\theta_{1y}}$. We have two cases:

- Case 1: if $1-\frac{c_x}{c_y\Delta^2} < 0$, we have that $\frac{\left(\theta_{2x}-p\theta_{1x}\right)^2}{c_x} < \frac{\left(\theta_{2y}-p\theta_{1y}\right)^2}{cy}$, then the relationship between the optimal level of x_2 and γ is monotonic. There are two possibilities: that relationship is monotonically increasing if $p < \theta_{2y}/\theta_y$, and it monotonically decreasing if $p > \theta_{2y}/\theta_y$. Note that the threshold for x_{2M} to be positive is $\gamma > -\frac{c_y(\theta_{2x}-p\theta_{1x})}{\theta_{2y}-p\theta_{1y}}$ if $\theta_{2y} - p\theta_{1y} > 0$, and $\gamma < -\frac{c_y(\theta_{2x}-p\theta_{1x})}{\theta_{2y}-p\theta_{1y}}$ if $\theta_{2y} - p\theta_{1y} < 0$. Simple calculus show that these thresholds belong to the feasible set of γ (according to SOC) when $\frac{\left(\theta_{2x}-p\theta_{1x}\right)^2}{c_x} < \frac{\left(\theta_{2y}-p\theta_{1y}\right)^2}{cy}$. Consequently, case 1 comprises two situations: first, when $\theta_{2y} - p\theta_{1y} > 0$ we have that x_{2M} is positive and increasing in γ if $\gamma > -\frac{c_y(\theta_{2x}-p\theta_{1x})}{\theta_{2y}-p\theta_{1y}}$ and x_{2M} is not provided otherwise; and second, when $\theta_{2y} - p\theta_{1y} < 0$ we have that x_{2M} is positive and decreasing in γ if $\gamma < -\frac{c_y(\theta_{2x}-p\theta_{1x})}{\theta_{2y}-p\theta_{1y}}$ and x_{2M} is not provided otherwise.
- Case 2: if $1-\frac{c_x}{c_y\Delta^2} > 0$, we have that $\frac{\left(\theta_{2x}-p\theta_{1x}\right)^2}{c_x} > \frac{\left(\theta_{2y}-p\theta_{1y}\right)^2}{c_y}$. Then the threshold for x_{2M} to be positive does not belong to the feasible set of γ, and consequently x_{2M} is always positive. Moreover, the

relationship between the optimal level of x_2 and γ is non monotonic and $\gamma_0 = -c_y\Delta\left(1 - \sqrt{1 - \frac{c_x}{c_y\Delta^2}}\right)$ is a minimum (the other root of $\partial x_2^*/\partial\gamma$ does not belong to the feasible set of values of γ). This implies that there is a minimum level of provision of quality x given by $\check{x}_2 = \frac{\gamma_0(\theta_{2y} - p\theta_{1y}) + c_y(\theta_{2x} - p\theta_{1x})}{(1-p)\left(c_x c_y - \gamma_0^2\right)}$.

By repeating the same analysis for the optimal level of quality y provided to the high quality segment, we can see that it is symmetric to the previous one. Specifically

$$\frac{\partial y_1^*}{\partial\gamma} = \frac{\left(\gamma^2 + c_x c_y\right)(\theta_{2x} - p\theta_{1x}) + 2\gamma c_x\left(\theta_{2y} - p\theta_{1y}\right)}{\left(c_x c_y - \gamma^2\right)^2(1-p)}$$

that equals to zero for $\gamma = \frac{-c_x}{\Delta}\left(1 \pm \sqrt{1 - \frac{c_y\Delta^2}{c_x}}\right)$. Again, we have two cases depending on the sign of $1 - \frac{c_y\Delta^2}{c_x}$. It is easy to see that case 1 for quality x corresponds to case 2 for quality y, and vice versa.

Proof of Proposition 3

The relationship between the overall quality of the product and the level of attribute dependence is given by

$$\frac{\partial(x_1 - x_2)}{\partial\gamma} + \frac{\partial(y_1 - y_2)}{\partial\gamma} = \frac{p}{1-p}\frac{\left(\gamma^2 + c_x c_y\right)\left(\theta_{1x} - \theta_{2x} + \theta_{1y} - \theta_{2y}\right) + 2\gamma\left(c_x\left(\theta_{1y} - \theta_{2y}\right) + c_y(\theta_{1x} - \theta_{2x})\right)}{\left(c_x c_y - \gamma^2\right)^2}$$

which equals zero for $\gamma = -\Theta\left(1 \pm \sqrt{1 - \frac{c_x c_y}{\Theta^2}}\right)$. The numerator of the expression is a convex function with a minimum for $\gamma = -\Theta$. Then, we have two situations:

- $\Theta^2 < c_x c_y$, that is, the level of divergence is sufficiently low, which implies that the numerator is always positive, that is, the distortion of the overall quality of the product is always increasing in the attribute dependence;
- $\Theta^2 > c_x c_y$, that is, the level of divergence is sufficiently high, which implies that there is a minimum at $\gamma = -\Theta$ (recall that $\theta_x - \theta_{2x} + \theta_y - \theta_{2y} > 0$), which does not belong to the feasible values for γ. Because Θ can be either positive or negative, there are two cases:

(i) $-\Theta < -\sqrt{c_x c_y}$ which implicitly requires that $\frac{\theta_{1x} - \theta_{2x}}{c_x} + \frac{\theta_{1y} - \theta_{2y}}{c_y} > 0$. Then, the distortion of the overall quality of the product is always increasing in the attribute dependence for any $\gamma > -\Theta\left(1 + \sqrt{1 - \frac{c_x c_y}{\Theta^2}}\right)$ and decreasing otherwise, and

(ii) $\sqrt{c_x c_y} < -\Theta$ which implicitly requires that $\frac{\theta_{1x} - \theta_{2x}}{c_x} + \frac{\theta_{1y} - \theta_{2y}}{c_y} < 0$. Then, the distortion of the overall quality of the product is always increasing in the attribute dependence for any $\gamma < -\Theta\left(1 - \sqrt{1 - \frac{c_x c_y}{\Theta^2}}\right)$ and decreasing otherwise.

Proof of Proposition 4

The relationship between C and γ can be written as:

$$\frac{\partial C_M}{\partial \gamma} = (\theta_{1x} - \theta_{2x})\frac{\partial x_2^*}{\partial \gamma} + \left(\theta_{1y} - \theta_{2y}\right)\frac{\partial y_2^*}{\partial \gamma}$$

The two parts of the Proposition are a consequence of combining the effects of γ on the optimal level of every quality incorporated to the low quality product. As stated in Proposition 2, there are two critical values for γ in the first part of this Proposition, one for a low proportion of high quality consumers, given by $p < \min\left\{\frac{\theta_{2k}}{\theta_{1k}}, \frac{\theta_{2j}}{\theta_{1j}}\right\}$ which corresponds to substitute qualities, and other for an intermediate proportion of high quality consumers, given by $\frac{\theta_{2k}}{\theta_{1k}} < p < \frac{\theta_{2j}}{\theta 1j}$ which corresponds to complementary qualities. Moreover, note that in the second part of the Proposition the low quality product is not offered when γ is higher than that critical value.

Proof of Proposition 5

From Equations (11) and (12) it is easy to see that $y_{1M}^*(\gamma = 0) = \frac{\theta_{1y}}{c_y}$ and $y_{2M}^*(\gamma = 0) = \frac{\theta_{2y} - p\theta_{1y}}{(1-p)c_y}$.Then, after simple substitutions we can rewrite y_{1C}^* and y_{2C}^* as $y_{1C}^* = y_{1M}^*(\gamma = 0) + \frac{\gamma}{c_y}x_C^*$ and $y_{2C}^* = y_{2M}^*(\gamma = 0) + \frac{\gamma}{c_y}x_C^*$.

Proof of Proposition 6

- "Single product for high-segment only" versus "menu pricing":

$$B_H - B_M = -\frac{1}{2}\frac{c_y(p\theta_{1x} - \theta_{2x})^2 + c_x\left(p\theta_{1y} - \theta_{2y}\right)^2 + 2\gamma\left(p\theta_{1y} - \theta_{2y}\right)(p\theta_{1x} - \theta_{2x})}{(1-p)\left(c_x c_y - \gamma^2\right)}$$

where the numerator is negative if

(i) $\gamma < -\frac{c_y\left(p\theta_{1x} - \theta_{2x}\right)^2 + c_x\left(p\theta_{1y} - \theta_{2y}\right)^2}{2\left(\theta_{2y} - p\theta_{1y}\right)\left(\theta_{2x} - p\theta_{1x}\right)}$when $\left(\theta_{2y} - p\theta_{1y}\right)(\theta_{2x} - p\theta_{1x}) > 0$, or if

(ii) $\gamma > -\frac{c_y\left(p\theta_{1x} - \theta_{2x}\right)^2 + c_x\left(p\theta_{1y} - \theta_{2y}\right)^2}{2\left(\theta_{2y} - p\theta_{1y}\right)\left(\theta_{2x} - p\theta_{1x}\right)}$ when $\left(\theta_{2y} - p\theta_{1y}\right)(\theta_{2x} - p\theta_{1x}) < 0$.

Moreover, SOC requires that $-\sqrt{c_x c_y} < \gamma < \sqrt{c_x c_y}$. Then, it is required that

(i) $-\sqrt{c_x c_y} < -\frac{c_y\left(p\theta_{1x} - \theta_{2x}\right)^2 + c_x\left(p\theta_{1y} - \theta_{2y}\right)^2}{2\left(\theta_{2y} - p\theta_{1y}\right)\left(\theta_{2x} - p\theta_{1x}\right)}$, and

(ii) $\sqrt{c_x c_y} > -\frac{c_y\left(p\theta_{1x} - \theta_{2x}\right)^2 + c_x\left(p\theta_{1y} - \theta_{2y}\right)^2}{2\left(\theta_{2y} - p\theta_{1y}\right)\left(\theta_{2x} - p\theta_{1x}\right)}$.

Simple calculus show that both inequalities imply that

$$0 > \left(c_y(p\theta_{1x} - \theta_{2x})^2 - c_x\left(p\theta_{1y} - \theta_{2y}\right)^2\right)^2$$

which is false, that is, the numerator is always positive in the feasible range of values for γ. Consequently, menu pricing is more profitable than "single product for high-segment only".

- "Single product for both segments" versus "menu pricing":

$$B_O - B_M = -\frac{1}{2}p\frac{c_y(\theta_{1x}-\theta_{2x})^2 + c_x\left(\theta_{1y}-\theta_{2y}\right)^2 + 2\gamma(\theta_{1x}-\theta_{2x})\left(\theta_{1y}-\theta_{2y}\right)}{(1-p)\left(c_xc_y-\gamma^2\right)}$$

where the numerator is negative if

(i) $\gamma < -\frac{c_y(\theta_{1x}-\theta_{2x})^2 + c_x(\theta_{1y}-\theta_{2y})^2}{2(\theta_{2y}-\theta_{1y})(\theta_{2x}-\theta_{1x})}$ when $\left(\theta_{2y}-\theta_{1y}\right)(\theta_{2x}-\theta_{1x}) > 0$, or if

(ii) $\gamma > -\frac{c_y(\theta_{1x}-\theta_{2x})^2 + c_x(\theta_{1y}-\theta_{2y})^2}{2(\theta_{2y}-\theta_{1y})(\theta_{2x}-\theta_{1x})}$ when $\left(\theta_{2y}-\theta_{1y}\right)(\theta_{2x}-\theta_{1x}) < 0$.

Moreover, SOC requires that $-\sqrt{c_xc_y} < \gamma < \sqrt{c_xc_y}$. Then, it is required that

(i) $-\sqrt{c_xc_y} < -\frac{c_y(\theta_{1x}-\theta_{2x})^2 + c_x(\theta_{1y}-\theta_{2y})^2}{2(\theta_{2y}-\theta_{1y})(\theta_{2x}-\theta_{1x})}$ when $\left(\theta_{2y}-\theta_{1y}\right)(\theta_{2x}-\theta_{1x}) > 0$ and

(ii) $\sqrt{c_xc_y} > -\frac{c_y(\theta_{1x}-\theta_{2x})^2 + c_x(\theta_{1y}-\theta_{2y})^2}{2(\theta_{2y}-\theta_{1y})(\theta_{2x}-\theta_{1x})}$ when $\left(\theta_{2y}-\theta_{1y}\right)(\theta_{2x}-\theta_{1x}) < 0$.

Simple calculus show that both inequalities imply that

$$0 > c_y^2(\theta_{1x}-\theta_{2x})^4 + c_x^2\left(\theta_{1y}-\theta_{2y}\right)^4 + (\theta_{1x}-\theta_{2x})^2\left(\theta_{1y}-\theta_{2y}\right)^2 c_xc_y$$

which is false, that is, the numerator is always positive in the feasible range of values for γ. Consequently, menu pricing is more profitable than "single product for both segments".

- "Commonality" versus "menu pricing":

$$B_C - B_M = -\frac{p\left(-\gamma\theta_{2y} - c_y\theta_{2x} + \gamma\theta_{1y} + \theta_{1x}c_y\right)^2}{2c_y\left(c_xc_y-\gamma^2\right)(1-p)} < 0.$$

- "Commonality" versus "single product for both segments":

$$B_C - B_O = \frac{p\left(\theta_{1y}-\theta_{2y}\right)^2}{2c_y(1-p)} > 0.$$

- "Single product for high-quality segment only" versus "single product for both segments":

$B_H - B_O = \frac{p\left(\theta_x^2c_y + \theta_y^2c_x + 2\gamma\theta_x\theta_y\right) - \left(c_y\theta_{2x}^2 + c_x\theta_{2y}^2 + 2\gamma\theta_{2x}\theta_{2y}\right)}{2\left(c_xc_y-\gamma^2\right)}$, which equals to zero for $\gamma_0 = -\frac{A}{B}$ with $A = c_y\left(p\theta_x^2 - \theta_{2x}^2\right) + c_x\left(p\theta_y^2 - \theta_{2y}^2\right)$ and $B = 2\left(p\theta_x\theta_y - \theta_{2x}\theta_{2y}\right)$. A is the difference in attractiveness between the selective strategy and the common product strategy adjusted to production costs (selling high quality to the high segment only vs. selling low quality to all customers) and B is the level of aggregate dominance of the high quality segment adjusted to its size (note that $-B$ decreases as the size of the segment and/or its aggregate dominance increases and vice versa). We can distinguish two cases:

- Case 1: γ_0 is not a feasible value of γ (i.e. $c_xc_y - \gamma_0^2 < 0$, that is, $\frac{A^2}{B^2} > c_xc_y$) then $B_H - B_O$ is either positive or negative. Two subcases: first, $B_H - B_O > 0$ if $A > 0$, because we have that

$B_H(\gamma=0) - B_O(\gamma=0) > 0$ when $A > 0$ (then it follows that the difference is positive for all γ); and second, $B_H - B_O < 0$ if $A < 0$ because we have that $B_H(\gamma=0) - B_O(\gamma=0) < 0$ when $A < 0$.

- Case 2: γ_0 is a feasible value of γ (i.e. $c_x c_y - \gamma_0^2 > 0$, that is, $\frac{A^2}{B^2} < c_x c_y$), then $B_H - B_O$ can be positive or negative in the relevant interval for γ. After calculating $\frac{\partial(B_H - B_O)}{\partial\gamma} = \frac{1}{2}\frac{2A\gamma + B\gamma^2 + Bk}{(k-\gamma^2)^2} = 0$ we find that the candidates to extreme values are $\gamma = \gamma_0\left(1 \pm \sqrt{1 - \frac{c_x c_y}{\gamma_0^2}}\right)$ and they do not belong to the feasible set of values for γ (these extreme values are not real values for this case in the relevant interval), that is, $B_H - B_O$ is a monotone function in the relevant interval for γ. Again we have two subcases: first, if $2A\gamma + B\gamma^2 + Bc_x c_y > 0$, then $B_H - B_O$ is an increasing function and it is positive for any $\gamma > \gamma_0$ and negative otherwise (the derivative is positive for $\gamma = 0$ if $B > 0$, that is, if $p > \frac{\theta_{2x}\theta_{2y}}{\theta_x\theta_y}$); and second, if $2A\gamma + B\gamma^2 + Bc_x c_y < 0$, then $B_H - B_O$ is a decreasing function, and it is positive for any $\gamma < \gamma_0$ and negative otherwise (the derivative is negative for $\gamma = 0$ if $B < 0$, that is, if $p < \frac{\theta_{2x}\theta_{2y}}{\theta_x\theta_y}$).
- "Single product for high-quality segment only" versus "commonality":

$$B_H - B_C$$
$$= \frac{p}{2}\frac{\theta_{1x}^2 c_y + \theta_{1y}^2 c_x + 2\gamma\theta_{1x}\theta_{1y}}{c_x c_y - \gamma^2} - \frac{\left(p\theta_{1y}^2(1-p) + \left(\theta_{2y} - p\theta_{1y}\right)^2\right)\left(c_x c_y - \gamma^2\right) + \left(\gamma\theta_{2y} + c_y\theta_{2x}\right)^2(1-p)}{2c_y\left(c_x c_y - \gamma^2\right)(1-p)}$$
$$= \frac{p\gamma^2\left(\theta_{1y} - \theta_{2y}\right)^2 + 2\gamma c_y(1-p)\left(-\theta_{2x}\theta_{2y} + p\theta_{1x}\theta_{1y}\right) + c_y^2(1-p)\left(p\theta_{1x}^2 - \theta_{2x}^2\right) - c_x c_y\left(p\theta_{1y} - \theta_{2y}\right)^2}{2c_y\left(c_x c_y - \gamma^2\right)(1-p)}$$

where we have that $B_H - B_C = 0$ for

$$\gamma_0 = -\frac{2c_y(1-p)\left(-\theta_{2x}\theta_{2y} + p\theta_{1x}\theta_{1y}\right)}{2p\left(\theta_{1y} - \theta_{2y}\right)^2}\left(1 \pm \sqrt{1 - \frac{4p\left(\theta_{1y} - \theta_{2y}\right)^2\left(c_y^2(1-p)\left(p\theta_{1x}^2 - \theta_{2x}^2\right) - c_x c_y\left(p\theta_{1y} - \theta_{2y}\right)^2\right)}{\left(2c_y(1-p)\left(-\theta_{2x}\theta_{2y} + p\theta_{1x}\theta_{1y}\right)\right)^2}}\right)$$

Then there are two situations:
- $B_H - B_C > 0$ if

$$\frac{c_y\left((1-p)\left(p\theta_{1x}\theta_{1y} - \theta_{2x}\theta_{2y}\right)\right)^2}{p\left(\theta_{1y} - \theta_{2y}\right)^2} < c_y(1-p)\left(p\theta_{1x}^2 - \theta_{2x}^2\right) - c_x\left(p\theta_{1y} - \theta_{2y}\right)^2$$

- $B_H - B_C < 0$ if $\gamma \in (\gamma_{0+}, \gamma_{0-})$ and $B_H - B_C > 0$ otherwise.