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Partial privatization with endogenous choice of strategic variable

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Abstract

This paper analyzes the optimal privatization policy when firms endogenously choose their strategic variable. The level of privatization is shown to determine: (i) the choice of strategic variable, whereby an asymmetric equilibrium could emerge (either Cournot–Bertrand or Bertrand–Cournot); (ii) the stability of equilibrium when the partially privatized firm and the private firm choose quantity and price respectively as the strategic variable; and (iii) the level of welfare, whereby Cournot–Cournot and Bertrand–Cournot games could lead to a greater welfare than the Bertrand–Bertrand model.

JEL CLASSIFICATION
L00, L13, L33

1 | INTRODUCTION

Over the last decades, we have observed a worldwide wave of partial and full privatization of state-owned firms. As a consequence, competition between private firms and firms with public and private co-ownership is very common in many countries in oligopolistic markets such as airline, banking, education, healthcare and broadcasting services, among others. The way the level of privatization affects competition and welfare is an analytical issue with important practical implications as a policy instrument to improve resource allocation in imperfectly competitive markets. Specifically, this paper studies the effects of privatization policies in industries where firms endogenously choose their strategic variable. In this sense, the paper deals not only with the effects of privatization policies but also with the implications of choosing the strategic instrument when these policies are implemented. The practical relevance of the endogeneity of the choice of strategic variable has been shown by numerous studies that have identified real-world markets where firms choose to use either their price or their output level as their strategic variable (see Tremblay & Tremblay, 2019, for a survey).

An extensive literature compares the implications of price and quantity competition on company profits and welfare for environments in which the firms are private and profit maximizers and have an active role in defining the type of market competition (e.g., Häckner, 2000; Singh & Vives, 1984; Tremblay et al., 2013; Zanchettin, 2006). More recently, many papers have revisited this classic comparison in the context of mixed markets (e.g., Ghosh & Mitra, 2010; Haraguchi & Matsumura, 2016) and in the context of socially concerned firms (e.g., Kim et al., 2019; Kopel, 2015; Matsumura & Ogawa, 2014). Noticeably, some results concerning price, profit, and welfare rankings obtained for mixed markets may be different and even the opposite of those obtained in private oligopolies (Choi, 2019; Mahanta, 2019; Matsumura & Ogawa, 2012; Nakamura, 2015; Scrimitore, 2013).

Following the contribution of Matsumura (1998), some works on mixed markets endogenize the degree of privatization in order to focus on the relationship between partial privatization and different aspects both in Cournot and Bertrand settings. Examples in a Cournot setting are Matsumura and Kanda (2005) analyzing the influence of free entry, Fujiwara (2007) deals with the effects on product variety, Matsumura and Shimizu (2010) studies sequential privatization, Han and Ogawa (2012) consider the effects on advertising expenditures, and Nakamura and Takami (2015) consider that the strategies of a privatized firm are determined through bargaining between the private sector and the public sector. On the other hand, examples in a Bertrand setting are Isibashi and Kaneko (2008), who study quality competition, Xu et al. (2016), who compare emission taxes and privatization policies, and Tomaru and Wang (2018), Wang and Chiou (2018), and Choi (2019), who consider the role of lower efficiency in the public firm.

This paper contributes to this literature by analyzing the optimal privatization policy when firms endogenously choose their strategic variable (price or quantity). For this purpose, we consider a duopoly with differentiated goods where one of the firms is a private firm whilst the other is partially owned by both private owners and the public sector and maximizes the weighted average of social welfare and profits. The paper studies two different timings: In the first one, the government is a first mover, as usual in the literature on privatization in mixed oligopolies, while in the second one, firms move first. In most games about different forms of public intervention, governments are assumed to move before firms. We analyze the possibility that the intervention may have the reverse order. This could be particularly relevant when dealing with firms that have capacity to anticipate government's optimal choices. This capacity could be motivated by the size and multinational character of firms (examples can be found in the literature on strategic trade policy, such as Carmichael, 1987, or Brander, 1995), by the specificities of their production processes (e.g., technological commitments by firms, as we can see in papers on optimal taxation such as Anant et al., 1995, or Amacher & Malik, 2002), or by the commitment capacity and time-consistency problems a regulator could have when implementing long term policies (see some recent papers on privatization and liberalization policies such as Xu et al., 2017, and Lee et al., 2018). In particular, our approach is related to Xu et al. (2017), who study two cases of privatization policies, namely the government chooses the optimal level of privatization before and after firms enter the market. In our model we study optimal privatization before and after the choice of strategic variable by a privatized firm and a private firm.

Our paper shows that the degree of privatization is a determinant of:

- First, the choice of strategic variable: Besides the Cournot–Cournot equilibrium in private duopoly (Singh & Vives, 1984) and the Bertrand–Bertrand equilibrium in mixed oligopoly (Matsumura & Ogawa, 2012), our model shows that an asymmetric equilibrium can emerge (either Cournot–Bertrand or Bertrand–Cournot) for certain levels of partial privatization of the public firm.
- Second, the stability of equilibrium: Tremblay and Tremblay (2011) establish the conditions for a stable equilibrium to exist depending on the degree of differentiation when private firms play a Bertrand–Cournot game. Our paper complements that work by proving that the stability of equilibrium is determined by the degree of privatization when the partially-owned firm and the private firm choose quantity and price respectively as the strategic variable.
- Third, the welfare level. The literature states that the greatest welfare is achieved in the Bertrand–Bertrand game both for private (e.g., Singh & Vives, 1984) and mixed oligopolies (see ; Ghosh & Mitra, 2010, and Matsumura & Ogawa, 2014). Our paper shows that the Cournot–Cournot and the Bertrand–Cournot games lead to a higher welfare than the Bertrand–Bertrand model for certain degrees of privatization. As expected, when taking into account firms' endogenous choice, we find that the highest welfare level is obtained when both firms play Bertrand and the level of privatization is zero. Interestingly, this equilibrium market configuration is also obtained when firms are first-movers in the game.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 develops the subgames considered in the third stage of the game (Sections 3.1–3.4) and presents some consequences of the analysis regarding the stability (Section 3.5) and welfare (Section 3.6) in the different subgames. Section 4 presents the second stage of the game and derives the implications of endogenizing the choice of strategic variable on market configuration. Section 5 shows the first stage of the game by characterizing the government's optimal choice of the level of privatization. Section 6 explores a variation of the previous model by considering firms as the first-movers of the game. Finally, Section 7 concludes the paper.

2 | THE MODEL

Let us consider a duopoly with differentiated products. The representative consumer's utility is assumed to be similar to that of Singh and Vives (1984), that is, a quadratic function given by

$$U(q_1,q_2,m) = q_1 + q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2bq_1q_2\right) + m, \tag{1}$$

where q_i is the output of firm i ($i = 1,2$) and m is the numeraire good. The parameter $b \in [0,1]$ is an index of product differentiation with differentiation increasing as $b \to 0$. The consumer maximizes $U(q_1,q_2,m) - p_1q_1 - p_2q_2 - p_m m$ where p_i is the price of firm i and p_m is assumed to be equal to 1. In this framework, inverse and direct demands are given respectively by

$$p_i = 1 - q_i - bq_j \tag{2}$$

and

$$q_i = \left(1 - b - p_i + bp_j\right)/\left(1 - b^2\right), \tag{3}$$

with $i \neq j$. In order to simplify the analysis, we assume that firms have identical technologies with no fixed costs and constant and equal to zero marginal costs, then each firm's profit is given by its total income, that is, $\pi_i = p_iq_i$. It is assumed that Firm 2, which is a pure private firm, maximizes its profits, and Firm 1, which is a partially privatized firm, maximizes the weighted average of payoff of the government, that is, aggregated welfare, and its own profit (Bös, 1991, ch. 8; Matsumura, 1998):

$$\Phi = \alpha\pi_1 + (1-\alpha)W, \tag{4}$$

where $\alpha \in [0,1]$ represents the degree of privatization of the partially owned firm and W denotes social welfare (consumer surplus plus firms' profits), which in this framework is given by

$$W = q_1 + q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2bq_1q_2\right). \tag{5}$$

We consider a three-stage game. As mentioned in the Introduction, this paper studies two different timings that are closely related to the *ex-ante* and *ex-post* privatization policies analyzed in Xu et al. (2017). In the first timing (Section 5), the government adopts the extent of privatization that maximizes social welfare in the first stage. In the second stage, after observing the choice of the level of privatization by the government in the first stage, both firms simultaneously choose whether to adopt price or quantity as strategic variable. In the third stage, after observing the choice of the strategic contract by the rival firm in the second stage, the private firm and the mixed firm simultaneously and independently choose either their optimal price or quantity in the market. This setup is similar to the one displayed by Kopel (2015) for socially concerned firms, although the optimal values obtained for every subgame are slightly different as the objective function of a consumer-friendly firm is a combination of own profit and consumer surplus, that is, it does not take into account total welfare (i.e., rival's profit). In the second timing (Section 6), we solve the model when the firms are first-movers and the government chooses the optimal degree of privatization in the second stage after observing the choice of strategic contract by the firms in the first stage.

3 | THIRD-STAGE SUBGAMES

In this section, we discuss four possible subgames: Both firms choose quantity contracts, both firms choose price contracts, only the mixed firm chooses the quantity contract, and only the mixed firm chooses the price contract.

3.1 | Cournot game (CC)

In this case, the mixed firm and the private firm play a standard Cournot game by setting quantities. Best-reply functions are given by

$$\begin{aligned} R_1(q_2) &\equiv q_1 = \frac{1}{\alpha+1}(1-bq_2), \\ R_2(q_1) &\equiv q_2 = \frac{1}{2}(1-bq_1). \end{aligned} \tag{6}$$

These functions lead to the following expressions for the equilibrium values:

$$\begin{aligned} q_1^{CC} &= \frac{2-b}{2\alpha-b^2+2},\ q_2^{CC} = \frac{1-b+\alpha}{2\alpha-b^2+2}, \\ p_1^{CC} &= \frac{\alpha(2-b)}{2-b^2+2\alpha},\ p_2^{CC} = \frac{\alpha-b+1}{2-b^2+2\alpha}, \\ \pi_2^{CC} &= \frac{(\alpha-b+1)^2}{\left(-b^2+2\alpha+2\right)^2}, \\ W^{CC} &= \frac{2b^3-2b^2-10b\alpha-6b+3\alpha^2+14\alpha+7}{2\left(-b^2+2\alpha+2\right)^2}, \\ \Phi^{CC} &= \frac{2b^2\left[\alpha^2-(1-b)(1-\alpha)\right]+(1+\alpha)\left[2b(\alpha-3)-3\alpha^2+7\right]}{2\left(-b^2+2\alpha+2\right)^2}. \end{aligned} \tag{7}$$

Best-reply functions and the effects of variations of the level of privatization on the subgame equilibrium are illustrated in Figure A1 in Appendix A: As α increases from 0 (i.e., firm 1 would be a public firm) to 1 (i.e., firm 1 would be a fully privatized firm), the mixed firm best reply is to decrease its output for every level of its rival's output. As a consequence, the private firm output increases in equilibrium.

3.2 | Bertrand game (BB)

In this case, the mixed firm and the private firm play a standard Bertrand game by setting prices. Best-reply functions are given by

$$\begin{aligned} R_1(p_2) &\equiv p_1 = \frac{1}{\alpha+1}(\alpha-b\alpha+bp_2), \\ R_2(p_1) &\equiv p_2 = \frac{1}{2}(1-b+bp_1). \end{aligned} \tag{8}$$

These functions lead to the following expressions for the equilibrium values:

$$\begin{aligned} q_1^{BB} &= \frac{b\alpha+b^2\alpha-b^2+2}{(b+1)\left(-b^2+2\alpha+2\right)},\ q_2^{BB} = \frac{\alpha+b\alpha+1}{(b+1)\left(-b^2+2\alpha+2\right)}, \\ p_1^{BB} &= \frac{1}{2\alpha-b^2+2}\left(b+2\alpha-2b\alpha-b^2\right), \\ p_2^{BB} &= \frac{1}{2\alpha-b^2+2}\left(1-b+\alpha-b^2\alpha\right), \\ \pi_2^{BB} &= \frac{(1-b)(\alpha+b\alpha+1)^2}{(1+b)\left(-b^2+2\alpha+2\right)^2}, \\ W^{BB} &= \frac{2b^4(1-\alpha)-b^3\left(3\alpha^2+1\right)+b^2\left(-4\alpha+\alpha^2-7\right)+b\left(7\alpha^2+1\right)+3\alpha^2+14\alpha+7}{2(b+1)\left(-b^2+2\alpha+2\right)^2}, \\ \Phi^{BB} &= \frac{2b^4(1-\alpha)-b^3\left(\alpha-\alpha^2+\alpha^3+1\right)-b^2\left(\alpha-3\alpha^2+\alpha^3+7\right)+(1+\alpha)(b(1-\alpha)(3\alpha+1)-3\alpha^2+7)}{2(b+1)\left(-b^2+2\alpha+2\right)^2}. \end{aligned} \tag{9}$$

Again, best-reply functions and the effects of variations in the level of privatization on the subgame equilibrium are illustrated in

Appendix A (see Figure A2). When we move from $\alpha = 0$ to $\alpha = 1$, the equilibrium price increases for both firms because the values of p_1 and p_2 verify that $R_1(p_2;\alpha=0) = R_1(p_2;\alpha=1)$ are higher than p_1^{BB} and p_2^{BB}, respectively.

3.3 | Cournot–Bertrand game (CB)

In this case, as indicated, it is assumed that the partially owned firm and the private firm set output and price respectively as strategic variables. Best-reply functions are given by

$$\begin{aligned} R_1(p_2) &\equiv q_1 = \frac{1-b+b\alpha p_2}{\left(1-b^2\right)(1+\alpha)}, \\ R_2(q_1) &\equiv p_2 = \frac{1}{2}(1-bq_1), \end{aligned} \tag{10}$$

where we can see that the higher α, the higher firm 1's slope of the reaction function. These functions lead to the following equilibrium values:

$$\begin{aligned} q_1^{CB} &= \frac{2-2b+b\alpha}{2\alpha-b^2\alpha-2b^2+2}, q_2^{CB} = \frac{(1-b)(\alpha+b\alpha+1)}{2\alpha-b^2\alpha-2b^2+2}, \\ p_1^{CB} &= \frac{(1-b)\left(-\alpha b^2+b+2\alpha\right)}{2\alpha-b^2\alpha-2b^2+2}, p_2^{CB} = \frac{(1-b)(\alpha+b\alpha+1)}{2\alpha-b^2\alpha-2b^2+2}, \\ \pi_2^{CB} &= \frac{(1-b)^2(\alpha+b\alpha+1)^2}{\left(2\alpha-b^2\alpha-2b^2+2\right)^2}, \\ W^{CB} &= \frac{\begin{array}{c}3b^4\alpha^2+2b^3(\alpha+1)(4-\alpha)-b^2\left(6\alpha+7\alpha^2+9\right) \\ +2b\left(-7\alpha+2\alpha^2-3\right)+14\alpha+3\alpha^2+7\end{array}}{2\left(2\alpha-b^2\alpha-2b^2+2\right)^2}, \\ \Phi^{CB} &= \frac{\begin{array}{c}2b^3\left(4+\alpha-\alpha^2\right)-b^2\left(5\alpha-5\alpha^2-3\alpha^3+9\right) \\ -(\alpha+1)\left[b^4\alpha^2+2b(3-\alpha)-\left(7-3\alpha^2\right)\right]\end{array}}{2\left(2\alpha-b^2\alpha-2b^2+2\right)^2}. \end{aligned} \tag{11}$$

In this subgame, as the degree of privatization increases the mixed firm equilibrium output decreases and the private firm equilibrium price increases (see Figure A3 in Appendix A) because the values of q_1 and p_2 that verify that $R_1(p_2;\alpha=0) = R_1(p_2;\alpha=1)$ are higher than q_1^{CB} and p_2^{CB}, respectively. Similarly to Matsumura and Ogawa (2012), when $\alpha = 0$, that is, when the mixed firm becomes a public firm, it expands its output to its greatest achievable value $\left(q_1 = \frac{1}{(b+1)}\right)$, which induces the private firm price to its lowest value $\left(p_2 = \frac{1}{2(b+1)}\right)$.

3.4 | Bertrand–Cournot game (BC)

Now, it is assumed that the partially owned firm sets its price and the private firm sets its output level. Best-reply functions are given by

$$\begin{aligned} R_1(q_2) &\equiv p_1 = \frac{\alpha}{1+\alpha}(1-bq_2), \\ R_2(p_1) &\equiv q_2 = \frac{1}{2\left(1-b^2\right)}(1-b+bp_1), \end{aligned} \tag{12}$$

where we can see that the higher α, the higher the absolute value of firm 1's slope of the reaction function. These functions lead to the following expressions for the equilibrium values:

$$\begin{aligned} q_1^{BC} &= \frac{2-b^2-b}{2\alpha-b^2\alpha-2b^2+2}, q_2^{BC} = \frac{1-b+\alpha}{2\alpha-b^2\alpha-2b^2+2}, \\ p_1^{BC} &= \frac{2\alpha-b\alpha-b^2\alpha}{2\alpha-b^2\alpha-2b^2+2}, p_2^{BC} = \frac{\left(1-b^2\right)(\alpha-b+1)}{2\alpha-b^2\alpha-2b^2+2}, \\ \pi_2^{BC} &= \frac{\left(1-b^2\right)(\alpha-b+1)^2}{\left(2\alpha-b^2\alpha-2b^2+2\right)^2}, \\ W^{BC} &= \frac{\begin{array}{c}b^4(2\alpha+1)+6b^3(\alpha+1)-2b^2\left(6\alpha+\alpha^2+4\right) \\ -2b(5\alpha+3)+3\alpha^2+14\alpha+7\end{array}}{2\left(2\alpha-b^2\alpha-2b^2+2\right)^2}, \\ \Phi^{BC} &= \frac{\begin{array}{c}(\alpha+1)\left(b^4+2b(\alpha-3)-\left(3\alpha^2-7\right)\right) \\ -2b^2\left(b\left(\alpha^2-3\right)-(\alpha+2)\left(\alpha^2-2\right)\right)\end{array}}{2\left(2\alpha-b^2\alpha-2b^2+2\right)^2}. \end{aligned} \tag{13}$$

Best-reply functions and the effects of variations of the level of privatization (from $\alpha = 0$ to $\alpha = 1$) on the subgame equilibrium are illustrated in Figure A4 in Appendix A. As the level of privatization increases, both the mixed firm equilibrium price and the private firm equilibrium output increase. Noticeably, for $\alpha = 0$, the mixed firm (in this case, the public firm) best-reply function matches the vertical axis for $q_2 \in [0, 1/b]$, then this firm produces the perfectly competitive level of output (i.e., its equilibrium price equals marginal cost, that is zero in our model).

3.5 | Stability of subgame equilibrium

In this subsection, we present a result on the relationship between the level of privatization and the stability of the equilibrium obtained for the previous subgames. Following Dixit (1986) and Tremblay and Tremblay (2011), a stable equilibrium requires that $\left|\partial^2\pi_i/\partial s_i^2\right| > \left|\partial^2\pi_i/\partial s_i \partial s_j\right|$ with $i,j = 1,2$ and $i \neq j$, being $s_i = p_i, q_i$. This condition holds for any b in both Cournot–Cournot and Bertrand–Bertrand games. Moreover, stability in the Bertrand–Cournot game requires a low-enough level of product differentiation, as stated in Tremblay and Tremblay (2011) for two private firms. In other words, we find that stability in the Cournot–Cournot, Bertrand–Bertrand, and Bertrand–Cournot games does not depend on the degree of privatization. However, we find that the degree of privatization plays an important role in the stability of the equilibrium in the Cournot–Bertrand game. Specifically, we find that privatization does not impact the stability of the subgame if product differentiation is sufficiently high; however, if the product differentiation is low enough, a low degree of privatization is required for stability. This result can be expressed by the following proposition:

Proposition 1. Stability of equilibrium. When the privatized firm chooses quantity and the private firm

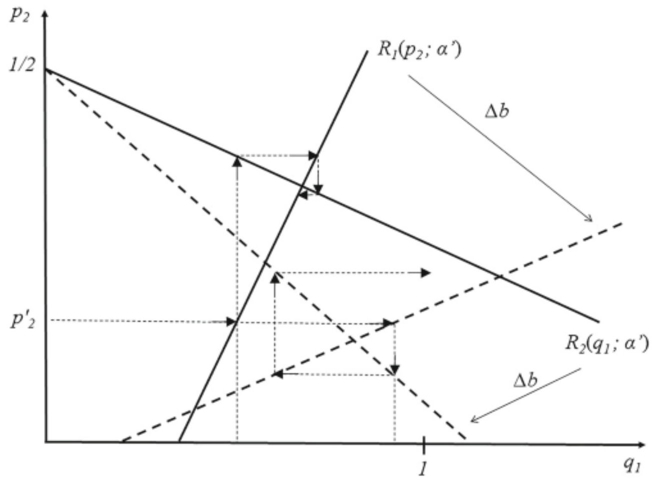

FIGURE 1 Stability of equilibrium in the CB subgame

chooses price, the equilibrium is stable for any α if $b<\frac{\sqrt{17}-1}{4}$ and for $\alpha<\frac{1-b^2}{b^2+b-1}$ if $b>\frac{\sqrt{17}-1}{4}$.

Proof. See Appendix A. ■

This result complements Proposition 1 in Tremblay and Tremblay (2011), which establishes the sufficient product differentiation to support equilibrium stability in a private duopoly with a Cournot-type firm and a Bertrand-type firm. The situations described in Proposition 1 are represented in Figure 1.

The figure is drawn for a high degree of privatization (i.e., α' is close to 1). We have two pairs of reactions functions, depending on the level of product differentiation. The reaction functions with continuous lines correspond to a market with a high product differentiation (i.e., a low b) whilst discontinuous lines correspond to low product differentiation (i.e., b is close to 1). As we can see, if we start at a point such as p_2', the equilibrium is unstable when product differentiation is low because the adjustment process does not converge to the equilibrium, and conversely, it is stable when product differentiation is high.

3.6 | Welfare effects

This subsection presents some consequences of the previous analysis on the resulting level of welfare in every subgame. The literature states that in both private and mixed oligopoly the highest welfare is achieved when both firms play Bertrand (Ghosh & Mitra, 2010; Matsumura & Ogawa, 2012; Singh & Vives, 1984, among others). However, this result does not always hold when we consider a partially-owned firm. Specifically, we find that for certain degrees of privatization, both the Cournot–Cournot and Bertrand–Cournot strategies lead to a greater welfare level than the Bertrand–Bertrand strategy. This result is expressed in the following proposition:

Proposition 2. Privatization and welfare. It is verified that

a) $\exists \alpha \in (0,1)$ such that $W^{BB} < W^{BC}$.
b) $\exists \alpha \in (0,1)$ such that $W^{BB} < W^{CC}$.

Proof. See Appendix A. ■

In contrast to conventional wisdom, this proposition states that the highest level of welfare does not always correspond to Bertrand competition, but it can be obtained for intermediate levels of privatization when both firms are quantity setters (similar to Proposition 2 in Kopel 2015), and also when the privatized firm chooses the price contract and the private firm chooses the quantity contract. To explain these differences in the level of welfare, notice that (5) can be rewritten as $W=(q_1+q_2)-(1+b)(q_1+q_2)^2/4-(1-b)(q_1-q_2)^2/4$. First, it is easy to see that, both in CC and in BC, the overall optimal quantity in the market increases compared to BB for certain values of α and b. Second, it holds that $q_1^{CC}-q_2^{CC}>q_1^{BB}-q_2^{BB}$, and moreover that $q_1^{BC}-q_2^{BC}>q_1^{BB}-q_2^{BB}$ under the same condition required for $q_1^{BC}+q_2^{BC}>q_1^{BB}+q_2^{BB}$. Consequently, the third term in W determines the subgame with the highest welfare: for CC subgame, a low b is required for $W^{CC}-W^{BB}>0$, and for BC subgame, we have that $W^{BC}-W^{BB}>0$ for a combination of intermediate values of α and b. In addition, it must be noticed that our analysis is consistent with the available literature when the extreme values of the level of privatization $\alpha=0$ and $\alpha=1$ are considered: We obtain that the highest level of welfare corresponds to Bertrand competition both in a private duopoly (Singh & Vives, 1984) and in mixed oligopoly (Ghosh & Mitra, 2010; Matsumura & Ogawa, 2012). Finally, we can easily see that $W^{CC}<W^{BC}$ for any value of α and b in the relevant interval.

4 | SECOND-STAGE GAME

This section deals with each firm's optimal choice of strategic variable. This choice can be displayed in terms of the level of privatization that the government chooses at the first stage of the game. For that purpose, let us define $\alpha_{1M}(b)$ as any α such that $\Phi^{BB}-\Phi^{CB}=0, \alpha_{2M}(b)$ as any α such that $\Phi^{CC}-\Phi^{BC}=0, \alpha_{1P}(b)$ as any α such that $\pi_2^{CB}-\pi_2^{CC}=0$, and $\alpha_{2P}(b)$ as any α such that $\pi_2^{BB}-\pi_2^{BC}=0$. The full expression of these critical values can be found in Appendix A where $\alpha_{2P}(b)<\alpha_{2M}(b)<\alpha_{1M}(b)<\alpha_{1P}(b)$ along the interval $\alpha, b \in [0,1]$ is also shown to hold. We can write the following proposition.

Proposition 3. Endogenous choice of strategic variable.

- If $\alpha<\alpha_{2P}(b)$, then BB is dominant strategy Nash equilibrium.
- If $\alpha_{2P}(b)<\alpha<\alpha_{2M}(b)$, then BC is the equilibrium of the game.

- If $\alpha_{2M}(b) < \alpha < \alpha_{1M}(b)$, then no equilibrium exists.
- If $\alpha_{1M}(b) < \alpha < \alpha_{1P}(b)$, then CB is the equilibrium of the game.
- If $\alpha_{1P}(b) < \alpha$, then CC is dominant strategy Nash equilibrium.

Proof. See Appendix A. ■

This proposition states that the degree of privatization of the partly owned firm plays a crucial role in the determination of the equilibrium of the game. The result generalizes both Proposition 1 in Matsumura and Ogawa (2012), which establishes that choosing the price contract is the dominant strategy when public and private firms compete, and also Singh and Vives (1984), who prove that Cournot competition is a dominant strategy for the two firms in a private duopoly with substitutive goods.

The intuition of this proposition is based on the varying of the partially privatized firm's objective function under different degrees of privatization and the changes in the demand elasticity for a firm caused by changes in the choice of strategic variable by its rival. For example, for $\alpha = 1$, we have the standard symmetric private duopoly, where quantity is dominant strategy for both firms, that is, *CC* is the optimal market configuration (Singh & Vives, 1984). As firm 1 becomes more public (i.e., α decreases), welfare is increasingly important in its objective function and market outcomes are characterized by higher quantities and lower prices. Then, below a certain value of α, the private firm has incentives to switch to a price contract because the resulting reduction of prices are lower (i.e., the privatized firm faces a more elastic demand). This argument can be reversed for $\alpha = 0$, the standard mixed oligopoly, where the price contract is dominant strategy for both firms, that is, *BB* is the optimal market configuration (Matsumura & Ogawa, 2012): As the level of privatization increases, the subsequent reduction of quantity gives incentives to the private firm to switch to the quantity contract because in this manner the increasingly privatized firm faces a less elastic demand and the resulting prices are higher. Finally, there is a region between the asymmetric market configurations (*BC* and *CB*) where no equilibrium exists because, for example, from *BC*, as α increases, the privatized firm finds optimal to switch to the quantity contract (because profits are increasingly important in its objective function), but the private firm would change to price contract in that case, and so on.

5 | FIRST-STAGE GAME

Finally, a question arises: Which is the optimal degree of privatization from a welfare point of view? In order to answer this question, it must be taken into account that, as we have shown in the second stage of the game, the level of privatization chosen by the government affects firms' optimal strategy, and consequently welfare. Then, the first-stage decision must consider that fact that by choosing a degree of privatization, the government is inducing a type of competition which generates a level of welfare.

The following proposition relates the level of privatization with both the endogenous choice of the strategic variable and the aggregated welfare level.

Proposition 4. Privatization, endogenous choice of strategic variable and welfare. The market configuration BB is the equilibrium of the game and leads to the highest welfare level if $\alpha^{BB} = 0$.

Proof. See Appendix A. ■

The intuition of this proposition rests on the fact that there is a conflict between the level of privatization that maximizes welfare and the market configuration that firms would choose when that level of privatization is set by the government: If the government were to choose the level of α such that $W^{BC} > W^{BB}$ (see Proposition 2), the resulting market configuration would not be *BC* but *BB*, as shown in Proposition (3), and consequently the resulting welfare would be suboptimal. In fact, the critical value of α below which the level of privatization that gives a dominant strategy Nash equilibrium for *BB* (i.e. $\alpha_{2P}(b)$) is the same as the level of privatization below which the level of welfare for *BC* is higher than the level of welfare for *BB*. In other words, Propositions 3 and 2 show that if $\alpha < \alpha_{2P}(b)$, *BB* is the equilibrium market configuration, and it holds that $W^{BB}(\alpha) > W^{i}(\alpha)$ (with $i = CC, BC, CB$), respectively. More specifically, the optimal choice for the government in this first stage is $\alpha^{BB} = 0$, which implies that both firms play Bertrand and the highest level of welfare is achieved when the mixed firm becomes a fully state-owned firm.

6 | FIRMS MOVE FIRST

As explained in the Introduction, there are many real-world situations where government decisions can be made both before and after business decisions. In this section, we assume a different timing for the game: firms move first. Again, we have a three-stage model. In the first stage, the firms choose simultaneously their strategic variable. In the second stage, after observing the choice of strategic contract by the firms in the first stage, the government chooses the optimal degree of privatization for each subgame. In the third stage, the private firm and the mixed firm simultaneously and independently choose either their optimal price or quantity in the market, on the basis of their decisions in the first stage. Obviously this third stage is identical to the third stage displayed in Section 3.

The optimal privatization policy differs depending on the equilibrium market configuration, that is, on the firms' choice of strategic variable. In other words, we can find an optimal level of privatization for each one of the subgames considered in the third stage of the game. Then the second stage is a standard maximization of the third-stage equilibrium welfare that gives the following optimal values of α in each subgame (See Appendix A):

$$\alpha^{CC} = \frac{b(1-b)}{4-3b},$$
$$\alpha^{BB} = 0,$$
$$\alpha^{CB} = \begin{cases} \alpha_1^{CB} = 0 & \text{if } b < \left(\sqrt{17}-1\right)/4 \\ \alpha_2^{CB} = (1-b^2)/(b^2+b-1) & \text{if } b > \left(\sqrt{17}-1\right)/4, \end{cases} \quad (14)$$
$$\alpha^{BC} = \frac{b(1-b^2)}{4+b-3b^2-b^3}.$$

The optimal level α^{CC} is in accordance with Matsumura (1998), who proved that a partial privatization of the public firm is optimal from the aggregated welfare point of view. More specifically, Matsumura (1998) considers a Cournot-type model with homogeneous product and decreasing returns. When these elements are considered in the *CC*-subgame ($b=1$ and $c_i(q_i)=cq_i^2$), we also obtain that a partial privatization is optimal; in particular, we obtain that partial privatization is optimal for $\alpha = 2c/(1+6c+4c^2)$. Our analysis shows that partial privatization is also optimal in the asymmetric subgames: First, it is optimal for all b in the *BC* subgame, and second, it is optimal for low product differentiation in the *CB* subgame. As explained in Appendix A, the optimal level of privatization for *CB* is a consequence of Proposition 1, which establishes an upper limit for α for stability to exist in that subgame. The optimal level of privatization for *BB* is similar to Wang and Chiou (2018), who consider differences in efficiency between a mixed firm and a private firm in a Bertrand setting and obtain that the optimal privatization policy is complete nationalization when the public firm cost is close to the private firm cost. In our analysis, a complete nationalization of the mixed firm is also optimal in the *CB* subgame if product differentiation is not sufficiently low.

The relationship between both the optimal levels of privatization and the associated welfare is summarized in the following proposition:

> **Proposition 5. Optimal privatization and welfare ranking.** The optimal level of privatization for every subgame follows this ranking: $\alpha_2^{CB} > \alpha^{BC} > \alpha^{CC} > \alpha^{BB} = \alpha_1^{CB} = 0$. These optimal levels imply that $W^{BC}(\alpha^{BC}) > W^{BB}(\alpha^{BB}) > W^{CC}(\alpha^{CC}) > W^{CB}(\alpha_i^{CB})$.
>
> *Proof.* See Appendix A. ■

Firms's decision at the first stage of the game is strategic because of two reasons: First, each firm's decision on its strategic variable must take into account its rival's options, and second, the outcome of this interaction involves a government decision on the level of privatization in the following stage of the game. Thus, this first stage can be represented by the following payoff matrix:

$\downarrow\Phi, \pi_2 \rightarrow$	C	B
C	$\Phi^{CC}(\alpha^{CC}), \pi_2^{CC}(\alpha^{CC})$	$\Phi^{CB}(\alpha_i^{CB}), \pi_2^{CB}(\alpha_i^{CB})$
B	$\Phi^{BC}(\alpha^{BC}), \pi_2^{BC}(\alpha^{BC})$	$\Phi^{BB}(\alpha^{BB}), \pi_2^{BB}(\alpha^{BB})$

After comparing optimal profits resulting from the second stage of the game, we find that choosing price as strategic variable is dominant strategy for the private firm, whilst choosing price is dominant strategy for the mixed firm only if the level of product differentiation is not sufficiently low, specifically, if $b < 0.85984$. Otherwise, the mixed firm does not have a dominant strategy and chooses quantity (price) when the private firm chooses quantity (price). Consequently, we obtain the following result:

> **Proposition 6.** Bertrand–Bertrand is the equilibrium market configuration when the private and the mixed firm are first-movers.
>
> *Proof.* See Appendix A. ■

A Bertrand–Bertrand market configuration is an usual outcome in the literature on privatization policies as the policy maker is primarily concerned with welfare maximization (Matsumura & Ogawa, 2012). However, this proposition provides the same result in a different setting, when the firms make their decisions on their strategic variable in the first stage of the game. In this case, they anticipate the government decision (given by $\alpha^{BB}=0$) and *BB* is the equilibrium of the game and consequently the timing of the game is neutral.

7 | CONCLUSIONS

Is it socially advantageous and desirable to privatize public firms? This paper analyzes the extent to which the degree of privatization of public firms can be used as a policy instrument to affect the type of competition and to improve welfare in an imperfectly competitive market. The model studies optimal privatization policy when firms make an endogenous choice of strategic variable (either price or quantity) in a duopoly with differentiated products, made up of a pure private firm and a partially privatized firm, which is jointly owned by both public and private sectors. The main contribution of our paper is providing different results by determining the influence of the level of privatization on the stability of equilibrium, the existence of asymmetric equilibrium outcomes and the rankings of both welfare levels and degrees of privatization that result in the different subgames.

The article has important implications for public policy, since it shows that privatization is not neutral in terms of welfare: Changes in the level of privatization involve changes in the way firms compete; as a consequence, the resulting level of welfare is affected. In particular, when the endogenous choice of strategic variable is taken into account, we show that the highest welfare level is obtained when both firms play Bertrand and the level of privatization is zero. Remarkably, this result is obtained when the government is the first-mover in the game, and also when the firms are able to anticipate the government's decision on the level of privatization. The results underline the importance of public firms to improve welfare when they are used as a price competition enhancing tool in those sectors where firms can choose their strategic variable. Thus, the implications of our analysis are concerned with two policy instruments, namely, the level of privatization of the industry and the strategic variable chosen by the mixed firm.

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

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APPENDIX A

Reaction functions in the different subgames

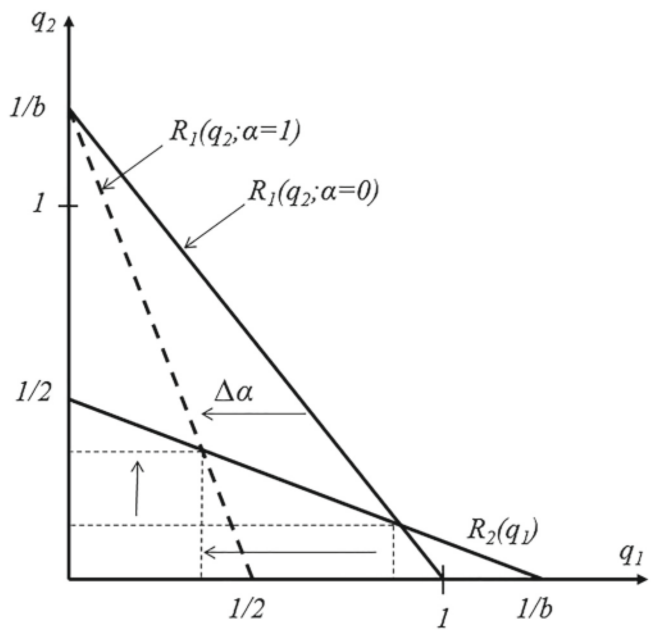

FIGURE A1 Cournot–Cournot subgame

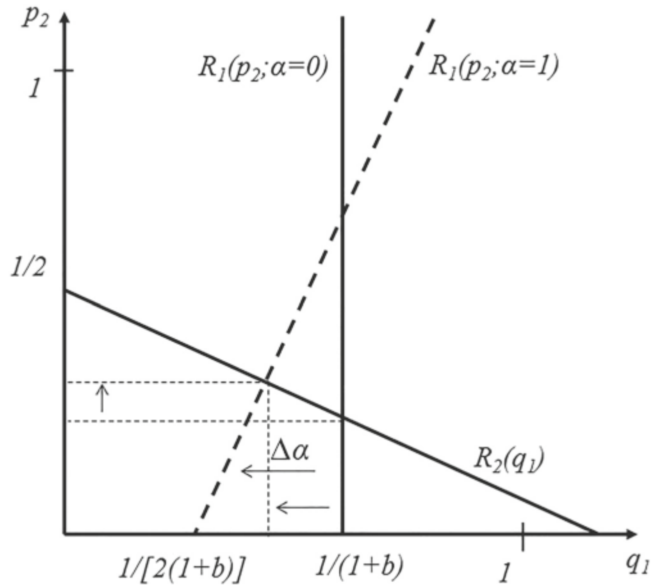

FIGURE A3 Cournot–Bertrand subgame

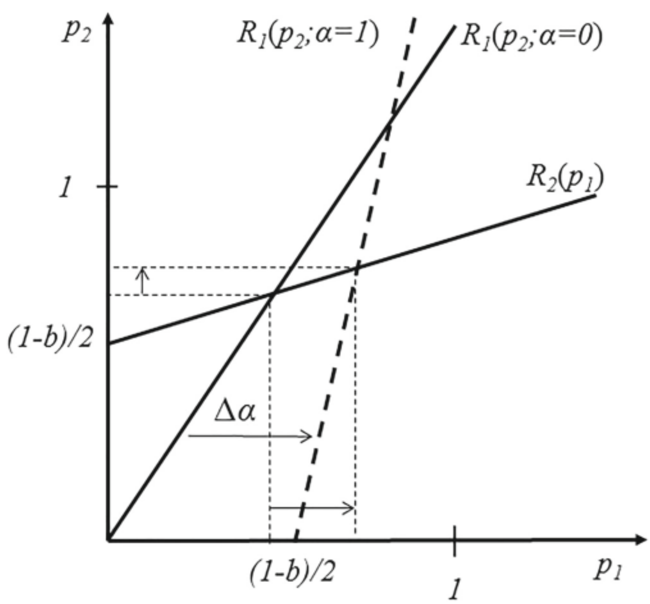

FIGURE A2 Bertrand–Bertand subgame

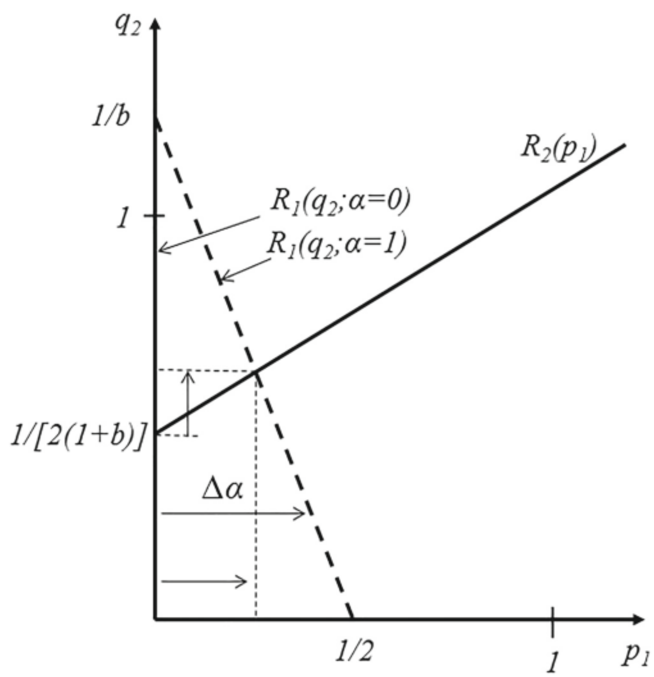

FIGURE A4 Bertrand–Cournot subgame

Proof of Proposition 1. As stated by Dixit (1986) and Tremblay and Tremblay (2011), a stable equilibrium in a Cournot–Bertrand model requires that $|\pi_{ii}| > |\pi_{ij}|$. In our case, when the privatized firm chooses quantity and the private firm chooses price, the equilibrium is stable if $(1+\alpha)(1-b^2) > b\alpha$. This condition becomes $\alpha > -(1-b^2)/(1-b^2-b)$ if $(1-b^2-b) > 0$ and $\alpha < -(1-b^2)/(1-b^2-b)$ if $(1-b^2-b) < 0$. In the first case, the condition holds for any α because the right-hand side of the inequality is negative. In the second case, we have that the condition always holds if $\frac{-(1-b^2)}{1-b^2-b} > 1$, that is, if $b < \frac{\sqrt{17}-1}{4}$. Therefore, in the Cournot–Bertrand subgame the equilibrium is stable for any α if $b \in \left(0, \frac{\sqrt{17}-1}{4}\right)$, and for $\alpha < \frac{1-b^2}{b^2+b-1}$ if $b \in \left(\frac{\sqrt{17}-1}{4}, 1\right)$.

Proof of Proposition 2.

a) Simple calculations show that $sign(W^{BB} - W^{BC}) = signAB$ with $A = \alpha^2\left(-16b^2 - b^3 + 3b^4 + 16\right) + \alpha\left(-24b^2 - 2b^3 + 9b^4 + 2b^5 + 16\right) - b^3\left(1 - b^2\right)$ and $B = b\alpha + \alpha^2 + b\alpha^2 + b^2 - 1$. It holds that:

(i) $A = 0$ for $\alpha_1^{BC}(b) = \frac{-(-24b^2-2b^3+9b^4+2b^5+16) \pm \sqrt{(-24b^2-2b^3+9b^4+2b^5+16)^2 + 4(-16b^2-b^3+3b^4+16)b^3(1-b^2)}}{2(-16b^2-b^3+3b^4+16)}$, where we have two roots, one with +, denoted by $\alpha_{1+}^{BC}(b)$, and one with −, denoted by $\alpha_{1-}^{BC}(b)$. Note that A is convex for $b \in [0,1]$ because $-16b^2 - b^3 + 3b^4 + 16 = (1-b)(-3b^3 - 2b^2 + 14b + 14) + 2 > 0$. Moreover $-24b^2 - 2b^3 + 9b^4 + 2b^5 + 16 = (1-b)\left(-2b^4 - 11b^3 - 9b^2 + 15b + 15\right) + 1 > 0$. It follows that $\alpha_{1-}^{BC} < 0$ and $\alpha_{1+}^{BC} > 0$, and then, only α_{1+}^{BC} is relevant as the level of privatization is defined for $\alpha \in [0,1]$. Consequently, $A < 0$ for $\alpha_{1-}^{BC} < \alpha < \alpha_{1+}^{BC}$ and $A > 0$ otherwise. Interestingly, negative roots should be taken into account when studying socially concerned firms: if the firm maximizes $V_i = \pi_i + \alpha_i CS$ (e.g., in Kopel, 2015), the value of α_i obtained for $A = 0$ (and for $B = 0$) could be either positive or negative, reflecting the level of "consumer friendliness" that characterizes the firm objective function.

(ii) $B = 0$ for $\alpha_2^{BC}(b) = \frac{-b \pm \sqrt{b^2 - 4(1+b)(b^2-1)}}{2(1+b)}$. Again we have two roots. We can see that the root with +, $\alpha_{2+}^{BC}(b)$, is positive and the root with −, $\alpha_{2-}^{BC}(b)$, is negative, and moreover B is a convex function. Consequently $B < 0$ if $\alpha_{2-}^{BC}(b) < \alpha < \alpha_{2+}^{BC}(b)$ and $B > 0$ otherwise.

(iii) $\alpha_{1+}^{BC}(b) < \alpha_{2+}^{BC}(b)$ in the relevant interval of values for α and b, as illustrated in Figure A5, where these critical values for α are represented in the $\alpha - b$ plane. Then A and B are both negative for $\alpha < \alpha_{1+}^{BC}(b)$, and both positive for $\alpha > \alpha_{2+}^{BC}(b)$. Moreover, if $\alpha_{1+}^{BC}(b) < \alpha < \alpha_{2+}^{BC}(b) A$ is positive and B is negative, and then $W^{BC} > W^{BB}$. Conversely, if $\alpha_{1+}^{BC}(b) > \alpha$ or if $\alpha > \alpha_{2+}^{BC}(b)$, it holds that $W^{BC} < W^{BB}$.

b) Simple calculations show that $W^{BB} - W^{CC} = b\frac{\alpha^2(b+1)(4-3b) - 2\alpha(b+2)(b-1)^2 + b(1-b)}{2(b+1)(2\alpha - b^2 + 2)^2}$

The denominator is strictly positive in the relevant interval of α and b. Then, $sign(W^{BB} - W^{CC})$ equals to the sign of the numerator. Moreover, because $(1+b)(4-3b) > 0$, the numerator is a convex function with two roots given by $\alpha_1^{CC}(b) = \frac{2(b+2)(1-b)^2 \pm \sqrt{(-2(b+2)(1-b)^2)^2 - 4(1+b)(4-3b)b(1-b)}}{2(1+b)(4-3b)}$, which are illustrated in the α-b plane in Figure A6. Specifically, the root with −, $\alpha_{1-}^{CC}(b)$,

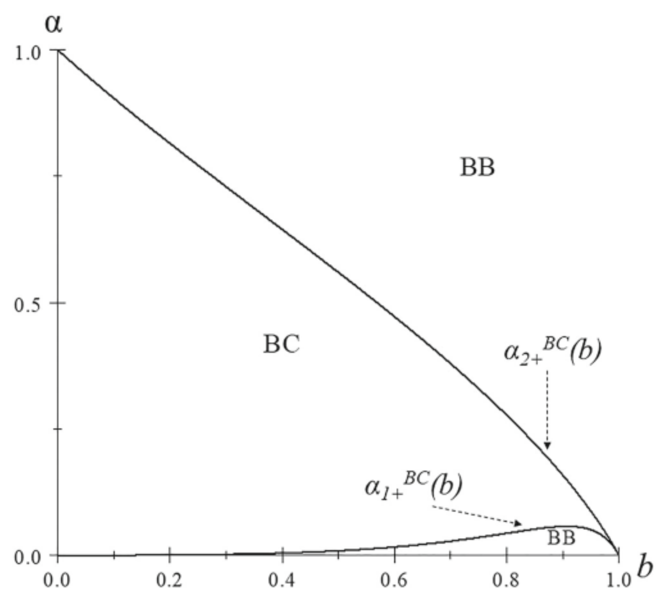

FIGURE A5 Critical levels of α in $W^{BB} - W^{BC}$

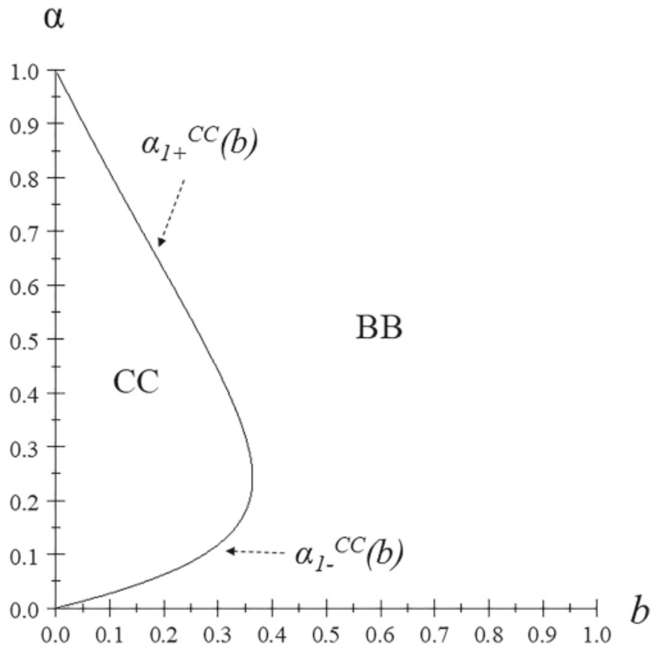

FIGURE A6 Critical levels of α in $W^{BB}-W^{CC}$

corresponds to the increasing part of the curve, and the root with $+$, α_{1+}^{CC}, is the decreasing part. Consequently, we have that if $\alpha_{1-}^{CC}(b) < \alpha < \alpha_{1+}^{CC}(b)$ it holds that $W^{BB}-W^{CC} < 0$.

Proof of Proposition 3. $\pi_2^{CC}-\pi_2^{CB}=\frac{b^2\left[\alpha^2-(1-b\alpha)(1-b)\right]\left[\alpha^2\left(4-3b^2\right)+\alpha(b-1)(b-2)(b+2)^2+(1-b)\left(4-3b^2\right)\right]}{\left(-2\alpha+b^2-2\right)^2\left(-2\alpha+b^2\alpha+2b^2-2\right)^2}$

The first bracket in the numerator has two roots given by $\frac{1}{2}\left(b(b-1)\pm\sqrt{(1-b)(2-b)\left(b^2+b+2\right)}\right)$. The root with $+$ belongs to the relevant interval of α, and the root with $-$ does not (it is negative). Moreover, the first bracket is a convex function in α, and then, it is positive if $\alpha > \alpha_{1P}(b)=\frac{1}{2}\left(b(b-1)+\sqrt{(1-b)(2-b)\left(b^2+b+2\right)}\right)$. The second bracket in the numerator also has two roots, given by

$\frac{1}{8-6b^2}\left((b-1)(2-b)(b+2)^2\pm b\sqrt{(b-1)\left(4b+b^2-4\right)\left(-b^2+b^3+4\right)}\right)$.

Both roots are negative. Moreover, because the second bracket is also a convex function (i.e., $4-3b^2>0$), it follows that it is positive in the relevant interval of α. Then, we can conclude that $\pi_2^{CC}-\pi_2^{CB}>0$ if $\alpha>\alpha_{1P}(b)$ and vice versa.

$\pi_2^{BC}-\pi_2^{BB}=\frac{b^2(b+2)(1-b)\left[\alpha^2(1+b)(2-b)+\alpha\left(4-3b^2\right)+(2-b)\left(1-b^2\right)\right]\left[\alpha^2(1+b)+b\alpha+b^2-1\right]}{(b+1)\left(-2\alpha+b^2-2\right)^2\left(-2\alpha+b^2\alpha+2b^2-2\right)^2}$

The first bracket in the numerator has two roots given by $\frac{-4+3b^2\pm b\sqrt{4-3b^2-4b+4b^3}}{2(b+1)(2-b)}$ and both are negative. Then, because the expression into the first brackets is a convex function (i.e., $(1+b)(2-b)>0$), it holds that this expression is positive for any $\alpha>0$. The second bracket in the numerator also has two roots, given by $\frac{-b\pm\sqrt{4b-3b^2-4b^3+4}}{2(1+b)}$. The root with $+$ belongs to the relevant interval of α, and the one with $-$ does not (because it is negative). Because the expression inside the second brackets is a convex function we have that that expression is positive for any $\alpha>\alpha_{2P}(b)=\frac{-b+\sqrt{4b-3b^2-4b^3+4}}{2(1+b)}$. Then, we can conclude that $\pi_2^{BC}-\pi_2^{BB}>0$ if $\alpha>\alpha_{2P}(b)$ and vice versa. Note that $\alpha_{2P}(b)=\alpha_{2+}^{BC}(b)$.

$$\Phi^{CC}-\Phi^{BC}=\frac{b^2(1+\alpha)(1+\alpha-b)C}{2\left(-2\alpha+b^2-2\right)^2\left(-2\alpha+b^2\alpha+2b^2-2\right)^2}$$

with $C=\left(4-b^2\right)(1-b)(\alpha^2(2b+1)-\left(1-b^2\right))+\alpha\left(4-3b^2\right)(\alpha^2-(1-b)^2)$. Then $sign(\Phi^{CC}-\Phi^{BC})=signC$, being C cubic in α with a real root denoted by $\alpha_{1M}(b)$.

$\Phi^{CB}-\Phi^{BB}=\frac{b^2(1-b)(\alpha+1)(\alpha+b\alpha+1)D}{2(b+1)\left(-2\alpha+b^2-2\right)^2\left(-2\alpha+b^2\alpha+2b^2-2\right)^2}$ with $D=\left(4-b^2\right)(1+b)(\alpha^3-\alpha(b-1)^2)+\left(4-3b^2\right)(\alpha^2(2b+1)-\left(1-b^2\right))$. Then $sign(\Phi^{CB}-\Phi^{BB})=signD$, being D cubic in α with a real root denoted by $\alpha_{2M}(b)$.

Figure A7 plots these critical values for α and b in the relevant interval $[0,1]$ and shows that it holds that $\alpha_{2P}(b)<\alpha_{2M}(b)<\alpha_{1M}(b)<\alpha_{1P}(b)$. After comparing these critical values we can write that:

- For any $\alpha<\alpha_{2P}(b)$ it is verified that $\pi_2^{BB}>\pi_2^{BC}$ and $\Phi^{BB}>\Phi^{CB}$, $\forall b\in[0,1]$, that is, playing Bertrand is dominant strategy for the two firms and therefore BB is the equilibrium of the game.
- If $\alpha_{2P}(b)<\alpha<\alpha_{2M}(b)$, then $\pi_2^{BC}>\pi_2^{BB}$ and $\Phi^{BC}>\Phi^{CC}$, $\forall b\in[0,1]$, that is, no firm has incentives for changing unilaterally its strategy from BC and, therefore BC is the equilibrium strategy.
- If $\alpha_{2M}(b)<\alpha<\alpha_{1M}(b)$, it can be argued as follows: CC is not equilibrium because $\pi_2^{CC}<\pi_2^{CB}$; BB is not equilibrium because $\pi_2^{BB}<\pi_2^{BC}$; BC is not equilibrium because $\Phi^{BC}<\Phi^{CC}$; and CB is not equilibrium because $\Phi^{CB}<\Phi^{BB}$. Therefore at least one firm has incentives to deviate from any combination of strategies and there is no equilibrium for the game.
- If $\alpha_{1M}(b)<\alpha<\alpha_{1P}(b)$, then $\pi_2^{CB}>\pi_2^{CC}$ and $\Phi^{CB}>\Phi^{BB}$, $\forall b\in[0,1]$, and consequently CB is the equilibrium of the game in this case.
- Finally, for any $\alpha_{1P}(b)<\alpha$ it is verified that $\pi_2^{CC}>\pi_2^{CB}$ and $\Phi^{CC}>\Phi^{BC}$, $\forall b\in[0,1]$, that is, playing Cournot is dominant strategy for the two firms and CC is the equilibrium of the game.

Proof of Proposition 4. First, from Proposition 2 we know that $W^{BB}<W^{CC}$ if $\alpha_{1-}^{CC}(b)<\alpha<\alpha_{1+}^{CC}(b)$. Simple calculus show that $\alpha_{1+}^{CC}(b)<\alpha_{1P}(b)$. Then, according to Proposition 3, if the government chooses a level of privatization $\alpha<\alpha_{1+}^{CC}(b)$ the equilibrium market configuration is *BB*. In other words, there is no intersection between the interval of α that holds that $W^{BB}<W^{CC}$ and the interval where *CC* is the equilibrium market configuration.

Second, from Proposition 2 we know that $W^{BB}<W^{BC}$ if $\alpha_{1+}^{BC}(b)<\alpha<\alpha_{2+}^{BC}(b)$ and from Proposition 3 we know that $\alpha_{2+}^{BC}(b)=\alpha_{2P}(b)$. Then, according to Proposition 3, if the government chooses a level of privatization $\alpha<\alpha_{2+}^{BC}(b)$ the equilibrium market configuration is *BB*. This means that there is no intersection between the interval of α that holds that $W^{BB}<W^{BC}$ and the interval of α where *BC* is the equilibrium market configuration.

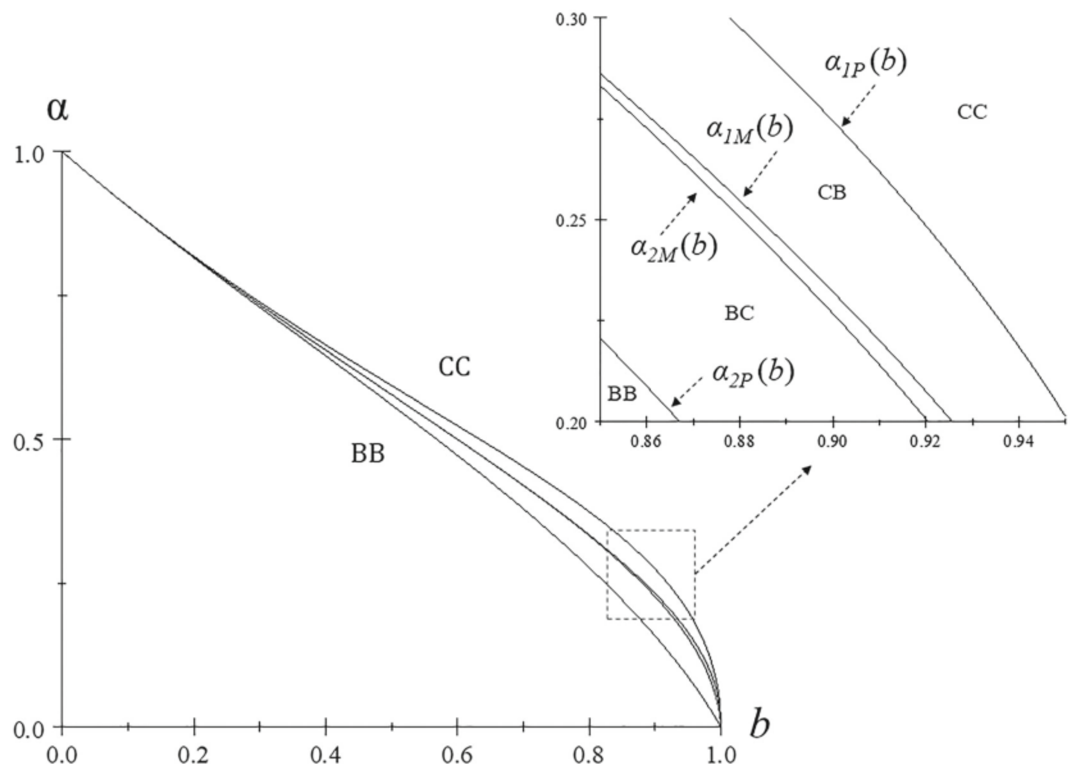

FIGURE A7 Optimal market configuration and level of privatization

Third, it is can be easily proved that $W^{BB} > W^{CB}$ for all $\alpha \in [0,1]$.

Consequently, Bertrand–Bertrand is the only optimal subgame that provides the highest level of welfare. Moreover, we have that $\partial W^{BB}/\partial \alpha < 0$, and then the highest level of welfare is achieved for $\alpha = 0$.

Optimal levels of privatization

Standard maximization of equilibrium welfare gives the following optimal values of α for each subgame:

- *CC* subgame: $\frac{\partial W^{CC}}{\partial \alpha} = (b-2)\frac{-b+4\alpha-3b\alpha+b^2}{\left(2\alpha-b^2+2\right)^3} = 0$ implies that $\alpha^{CC} = \frac{b(1-b)}{4-3b}$, and $\frac{\partial^2 W^{CC}(\alpha^{CC})}{\partial \alpha^2} = \frac{1}{(b-2)^2}\frac{(3b-4)^4}{\left(3b^2-4\right)^3} < 0$. $W^{CC}(\alpha^{CC}) = \frac{7-6b}{2(4-3b^2)}$.
- *BB* subgame: $\frac{\partial W^{BB}}{\partial \alpha} = (b-1)^3(b+2)\frac{b+4\alpha+3b\alpha+b^2}{(b+1)\left(2\alpha-b^2+2\right)^3} < 0$ for all $\alpha \in [0,1]$. Then $\alpha^{BB} = 0$. $W^{BB}(\alpha^{BB}) = \frac{b-7b^2-b^3+2b^4+7}{2\left(b^2-2\right)^2(b+1)}$.
- *CB* subgame: $\frac{\partial W^{CB}}{\partial \alpha} = (b-1)^2\left(b+2b^2-2\right)\frac{-b-4\alpha+b\alpha+3b^2\alpha}{\left(-2\alpha+b^2\alpha+2b^2-2\right)^3} = 0$ for $\hat{\alpha} = \frac{b}{b+3b^2-4} < 0$, with $\frac{\partial W^{CB}(\hat{\alpha})}{\partial \alpha} = \frac{(b-1)^6}{\left(b+2b^2-2\right)^2}\frac{(3b+4)^4}{\left(3b^2-4\right)^3} < 0$. Then $\hat{\alpha}$ is a maximum but it is not a feasible level of privatization because it is negative. Note that $\operatorname{sign}\frac{\partial W^{CB}}{\partial \alpha} = \operatorname{sign}\left(b+2b^2-2\right)$ in the relevant interval of values for α and b, then if $b > (<)\frac{\sqrt{17}-1}{4}$ it holds that $\frac{\partial W^{CB}}{\partial \alpha} > (<)0$. Consequently if $b < \frac{\sqrt{17}-1}{4}$ then $\alpha^{BC} = 0$, and if $b > \frac{\sqrt{17}-1}{4}$, then $\alpha^{BC} = \frac{1-b^2}{b^2+b-1}$, which is the threshold for α for equilibrium stability to exist in the *CB* subgame (see Proposition 1). $W^{CB}(\alpha_1^{CB}) = \frac{8b+7}{8(b+1)^2}$, $W^{CB}(\alpha_2^{CB}) = \frac{4b+15b^2+8b^3+3b^4+6b^5+3b^6-4}{2b^2(b+2)^2(b+1)^2}$.
- *BC* subgame: $\frac{\partial W^{BC}}{\partial \alpha} = (b-1)^2(b+2)\frac{-b+4\alpha+b\alpha-3b^2\alpha-b^3\alpha+b^3}{\left(-2\alpha+b^2\alpha+2b^2-2\right)^3} = 0$ implies that $\alpha^{BC} = \frac{b(1-b^2)}{4+b-3b^2-b^3}$, and $\frac{\partial^2 W^{BC}(\alpha^{BC})}{\partial \alpha^2} = -\frac{1}{(b-1)(b+2)^2}\frac{\left(-b+3b^2+b^3-4\right)^4}{(b+1)^3\left(3b^2-4\right)^3} < 0$. $W^{BC}(\alpha^{BC}) = \frac{-b+5b^2+b^3-7}{2(b+1)\left(3b^2-4\right)}$.

Proof of Proposition 5. Proof is straightforward from comparisons between optimal values of α given by (14) and between the resulting welfare for each subgame:

- Comparison of the optimal level of privatization: $\alpha_2^{CB} - \alpha^{CC} = \frac{(1-b)\left(-2b+4b^2+b^3-4\right)}{(3b-4)\left(b+b^2-1\right)} > 0$ (note that $b+b^2-1 > 0$ for the values of b that define α_2^{CB}), $\alpha_2^{CB} - \alpha^{BC} = -\frac{2(b+2)(b-1)^2(b+1)^2}{\left(b+b^2-1\right)\left(-b+3b^2+b^3-4\right)} > 0$, $\alpha^{CC} - \alpha^{BC} = -\frac{b^4(1-b)}{(3b-4)\left(-b+3b^2+b^3-4\right)} < 0$. It follows that $\alpha_2^{CB} > \alpha^{BC} > \alpha^{CC} > \alpha^{BB} = \alpha_1^{CB} = 0$.
- Comparison of the optimal levels of welfare: $W^{BB}(\alpha^{BB}) - W^{BC}(\alpha^{BC}) = -\frac{1}{2}b^2(b+1)\frac{(b-1)^3}{\left(3b^2-4\right)\left(b^2-2\right)^2} < 0$, $W^{CC}(\alpha^{CC}) - W^{BB}(\alpha^{BB}) = \frac{1}{2}b^2(b-1)\frac{2b^2-3}{(b+1)\left(3b^2-4\right)\left(b^2-2\right)^2} < 0$, $W^{CB}(\alpha_1^{CB}) - W^{CC}(\alpha^{CC}) = \frac{1}{8}\frac{b^2}{\left(3b^2-4\right)(b+1)^2} < 0$, $W^{CB}(\alpha_2^{CB}) - W^{CC}(\alpha^{CC}) = \frac{1}{2}\frac{\left(-2b-6b^2+2b^3+3b^4+4\right)^2}{b^2\left(3b^2-4\right)(b+2)^2(b+1)^2} < 0$. It follows that $W_{BC}(\alpha^{BC}) > W_{BB}(\alpha^{BB}) > W_{CC}(\alpha^{CC}) > W_{CBi}(\alpha_i^{CB})$.

Proof of Proposition 6. $\Phi^{CC}(\alpha^{CC}) - \Phi^{BC}(\alpha^{BC}) = \frac{1}{2}b^2(1-b)\frac{-12b-19b^2+8b^3+6b^4+16}{(3b-4)(b+1)\left(3b^2-4\right)\left(-b+3b^2+b^3-4\right)} < (>)0$ if $b < (>)0.85984$,

$$\Phi^{CB}(\alpha_1^{CB}) - \Phi^{BB}(\alpha^{BB}) = \frac{1}{8}b^2\frac{3b^2-4}{(b+1)^2\left(b^2-2\right)^2} < 0,$$

$\Phi^{CB}(\alpha_2^{CB}) - \Phi^{BB}(\alpha^{BB}) = \frac{1}{2}\frac{-20b-108b^2+7b^3+169b^4+43b^5-97b^6-41b^7+18b^8+11b^9+b^{10}+16}{b\left(b+b^2-1\right)(b+2)^2(b+1)^2\left(b^2-2\right)^2} < 0$ for $b > 0.32881$ (recall that α_2^{CB} stands for $b > 0.78078$).

Then, choosing price is dominant strategy for the private firm if $b < 0.85984$.

$$\pi_2^{CC}(\alpha^{CC}) - \pi_2^{CB}(\alpha_1^{CB}) = \frac{b^2}{4}\frac{7b^2-8}{(b+1)^2\left(3b^2-4\right)^2} < 0,$$

$$\pi_2^{CC}(\alpha^{CC}) - \pi_2^{CB}(\alpha_2^{CB}) = \left(-2b+14b^2+2b^3-3b^4-12\right)\frac{-2b-6b^2+2b^3+3b^4+4}{(b+2)^2(b+1)^2\left(3b^2-4\right)^2} < 0 \text{ for } \alpha, b \in [0,1],$$

$$\pi_2^{BC}(\alpha^{BC}) - \pi_2^{BB}(\alpha^{BB}) = -\frac{b^2\left(-7b^2+b^4+8\right)(b-1)^2}{\left(b^2-2\right)^2\left(3b^2-4\right)^2} < 0.$$

Then, choosing price is dominant strategy for the private firm.