Effects of jump-diffusion models for the house price dynamics in the pricing of fixed-rate mortgages, insurance and coinsurance ☆

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Abstract

In the pricing of fixed rate mortgages with prepayment and default options, we introduce jump-diffusion models for the house price evolution. These models take into account sudden changes in the price (jumps) during bubbles and crisis situations in real estate markets. After posing the models based on partial-integro differential equations (PIDE) problems for the contract, insurance and the fraction of the total loss not covered by the insurance (coinsurance), we propose appropriate numerical methods to solve them.

Keywords: Fixed-Rate Mortgages, jump-diffusion models, option pricing, complementarity problem, numerical methods, Augmented Lagrangian Active Set formulation

1. Introduction

A mortgage is a financial contract between two parts, a borrower and a lender, in which the borrower obtains funds from the lender (a bank or a financial institution, for example) by using a risky asset as a guarantee (collateral), usually a house. We can stablish a classification in different types taking into account several characteristics of the contract such as the interest rate, the term of the loan, the amount and frequency of payments and the prepayment and default options. Moreover, the lender may have an insurance on the loan as a protection in case of default. Taking into account the interest rate, we can distinguish between fixed-rate mortgages and adjustable-rate mortgages. In the first case the interest rate and the scheduled payments are fixed during the life of the loan, whereas in the second one the contract rate is floating and it is adjusted for some period according to an index such as LIBOR or EURIBOR, for example. In this work, we focus on fixed rate mortgages with monthly payments where the interest rate is the equilibrium rate and it needs to be adjusted by using an iterative process. The loan is reimbursed through monthly payments until the cancellation of the debt at maturity date. Thus, the mortgage value is understood as the discounted value of the future monthly payments (without including a possible insurance on the loan by the lender) and the underlying stochastic factors are the interest rate and the house price. In this paper we follow the previous ones in [7, 27] where prepayment is allowed at any time during the life of the loan and default only can occur at any monthly payment date. In both previous papers a log-normal process is assumed for house price evolution so that this value evolves continuously. The consideration of geometric Brownian motion for house prices has been commonly used in the literature on mortgage insurance pricing (see [3, 19, 20, 21], among others).

However, under certain situations, such as during the relatively recent bubble or crisis phenomena in real state markets, the assumption of a geometric Brownian motion for house prices seems no longer so realistic. For example, in the recent paper [8], the consideration of U.S. national average new home price returns for single-family mortgage from January 1986 to June 2008 as shown in Figure 1, motivates the analysis of jump risk in house prices. In the time series of Figure 1, it is observed that the monthly house

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price changes more than 10 percent per month in 14 time moments. Moreover, since the subprime crisis in 2007 several significant downward jumps appear. In [8] a combination of a geometric Brownian motion and a compound Poisson process is proposed, the parameters of which are estimated by an expectation-maximum gradient algorithm from the time series of U.S. pricing data. Thus, the maximum likelihood ratio test rejects the model without jumps with a 99 percent significance level when considering the national average for new house, although does not reject the geometric Brownian motion model for the case of second hand house prices. From the jump-diffusion model, by means of a Esscher transform technique a closed formula is obtained for mortgage insurance contracts. This approach recovers the formula for mortgages in [3] for the case without jumps, which is also based on the actuarial approach in [12] assuming that the present value of the expected loss (plus a gross margin) balances the expected premium revenues and that the agents in the economy are risk neutral. Moreover, in [8] it is pointed out that models with two underlying stochastic factors (house price and interest rate) require complex numerical methods.

Thus, although geometric Brownian motion has been mainly assumed in the literature, it becomes necessary to adopt jump-diffusion models to account with the arrival of additional information in real estate markets (abnormal events, economic and financial crisis, etc) that produce sudden changes in the value of the house. This is the main innovative point of the present paper. In the quantitative finance literature, there are several examples of the valuation of financial derivatives when the underlying assets follow a jump-diffusion process (see [25], for example). Among all of them, in the present we assume that the house price dynamics is governed by Merton [24] and Kou [22] jump-diffusion models and we consider a finite number of jumps following a Poisson process. Once the particular jump-diffusion model has been chose, as in the case without jumps (see [7], the dynamic hedging methodology leads to partial integro-differential equations (PIDE) problems. In the case of fixed rate mortgages, in order to obtain the value of the mortgage, the insurance and the coinsurance a sequence of PIDE problems (one for each month) are obtained. Moreover, the prepayment option leads to free boundary problems associated to the mortgage value, so that not only the value of the mortgage has to be obtained but also the combinations of rates and house prices for which it is optimal or not to prepay (prepayment and non prepayment regions) have to be determined. The unknown boundary separating both regions is known as optimal prepayment boundary and constitutes the free boundary associated to the problem.

Concerning the numerical methods for solving PIDE problems arising in finance, in [14] the authors propose a semi-Lagrangian method for pricing American Asian options assuming jump-diffusion models for the underlying asset while in [9] an implicit finite difference method to obtain the value of options on two assets under jump-diffusion process is considered. Moreover, in order to avoid the solution of linear system with dense matrix they combine a fixed point iteration with a FFT technique. In [2], the authors obtain the value of European vanilla options under jump-diffusion models for the underlying. More precisely, they solve the PIDE for Merton and Kou models.

In some cases, the numerical schemes employed to solve the PIDE leads to a dense matrix due to the presence of jumps, so that, appropriate methods are required to solve the system as the one proposed in [26]. The complementarity problems that arise in the valuation of products of American type have been solve in the literature with different methods. For example, in [13] they propose a penalty method and in [17] they present an operator splitting method.

For solving the inequalities associated with the free boundary problem in the contract valuation problem we propose a Lagrange-Galerkin method for time and space discretization [5, 6], combined with an Augmented Lagrangian Active Set (ALAS) algorithm jointly with the explicit treatment of the integral term [11], thus maintaining the same matrix than in the absence of jumps for the house value. For solving the PIDE problems associated to insurance and coinsurance pricing we consider the same Lagrange-Galerkin method with the same explicit treatment of the integral term. The equilibrium interest rate is obtained by the solution of a nonlinear equation with a variable secant method.

This paper is organized as follows. In Section 2 we briefly describe the pricing model under consideration as well as the mortgage contract related aspects. In Section 3 we describe the different numerical solution techniques. Finally, in Section 4 we present some numerical results allowing to compare the two different jump-diffusion models.

2. Mathematical modelling under jump-diffusion processes

2.1. Models for the underlying stochastic factors

In order to model the evolution of the logarithmic house price at time t, $X_t = \ln(H_t)$, where H_t denotes the house value, we consider the following stochastic differential equation (SDE):

$$dX_t \;=\; (\mu - \frac{\sigma_H^2}{2} - \delta)\, dt \;+\; \sigma_H dZ_t^H \;+\; d\left(\sum_{i=1}^{N_t} V_i\right), \tag{1}$$

where μ, δ and σ_H denote the house-price appreciation average rate, the dividend yield provided by the house (by hiring or using it, for example) and the house price volatility respectively, while Z_t^H is the Wiener process. Moreover, in the jump part of the model $(N_t)_{t\geq 0}$ denotes a Poisson process with parameter $\tilde{\lambda}$ and (V_i) is a sequence of square integrable, independent and identically distributed random variables, so that Z_t^H, N_t and (V_i) are independent. We notice that according to [14], [23] and [24], for example, the stochastic differential equation can be written in the original variable H_t in the form

$$dH_t \;=\; (\mu - \delta) H_t\, dt \;+\; \sigma_H H_t dZ_t^H \;+\; H_t\, d\left(\sum_{i=1}^{N_t} (Y_i - 1)\right), \tag{2}$$

where $Y_i = \exp(V_i)$.

Under a risk neutral probability measure, we can obtain the equivalent SDEs in the logarithmic house price and in the house price respectively:

$$dX_t \;=\; (r_t - \frac{\sigma_H^2}{2} - \delta - \tilde{\lambda}\tilde{\kappa})\, dt \;+\; \sigma_H dZ_t^H \;+\; d\left(\sum_{i=1}^{N_t} V_i\right), \tag{3}$$

$$dH_t \;=\; (r_t - \delta - \tilde{\lambda}\tilde{\kappa}) H_t\, dt \;+\; \sigma_H H_t\, dZ_t^H \;+\; H_t\, d\left(\sum_{i=1}^{N_t} (Y_i - 1)\right), \tag{4}$$

where $\tilde{\kappa} = \mathbb{E}[\exp(V_i)] - 1$.

Additionally, we assume that the interest rate follows the CIR following process [10]:

$$dr_t \;=\; \kappa(\theta - r_t)\, dt \;+\; \sigma_r \sqrt{r_t}\, dZ_t^r, \tag{5}$$

where κ denotes the speed of mean reversion to the long term rate θ and σ_r is the interest rate volatility. Wiener processes Z_t^H and Z_t^r could be correlated with correlation coefficient ρ (i.e. $dZ_t^H dZ_t^r = \rho dt$) to incorporate possible correlation between interest rate and house price. Time dependent correlation can be also considered in the same way, while stochastic correlation would imply an additional spatial-like variable in the forthcoming PIDE formulation.

2.1.1. Partial integral differential equation (PIDE) formulation

In the case without jumps, by using a dynamic hedging technique in [7], a partial differential equation (PDE) model for pricing any asset depending on house price and interest rate is posed. In the here treated jump-diffusion models for house prices, if we denote the value of any asset depending on house price and interest rate by $F_t = F(t, H_t, r_t)$ then standard techniques based on Ito formulas for jump-diffusion process prove that the function F satisfies the following PIDE (see Cont and Tankov [11], for example):

$$\begin{aligned}\partial_t F + \frac{1}{2}\sigma_H^2 H^2 \partial_{HH} F + \rho\sigma_H\sigma_r H\sqrt{r}\partial_{Hr} F + \frac{1}{2}\sigma_r^2 r \partial_{rr} F + (r-\delta) H \partial_H F + \kappa(\theta - r)\partial_r F - rF \\ + \int_{-\infty}^{\infty} \tilde{\lambda}\left[F(t, H\exp(y), r) - F(t,H,r) - H(\exp(y) - 1)\partial_H F(t,H,r)\right]\nu(y)dy \;=\; 0,\end{aligned} \tag{6}$$

where subindexes in symbol ∂ indicate partial derivatives.

Moreover, in order to completely define the model, we must also specify the distribution of jump sizes. For this purpose, we will consider either Merton model [24] or Kou model [22]. More precisely, under Merton model (V_i) are taken from the normal distribution $(N(\mu_j, \gamma_j^2))$, with the density

$$\nu(y) = \nu_m(y) = \frac{1}{\gamma_j\sqrt{2\pi}} \exp\left(-\frac{(y-\mu_j)^2}{2\gamma_j^2}\right), \tag{7}$$

where μ_j is the mean jump size and γ_j is the standard deviation of the jump size, whereas under Kou model the set (V_i) corresponds to a distribution with double-exponential density

$$\nu(y) = \nu_k(y) = \begin{cases} q\alpha_2 \exp(\alpha_2 y), & y < 0 \\ p\alpha_1 \exp(-\alpha_1 y), & y \geq 0, \end{cases} \tag{8}$$

where p, q, α_1 and α_2 are positive constants such that $p+q=1$ and $\alpha_1 > 1$. Note that, p and q represent the probabilities of upward and downward jumps, respectively.

Since $\nu(y)$ is the probability density function of the jump amplitude V_i, then

$$\int_{-\infty}^{\infty} \nu(y)dy = 1.$$

Moreover, we can compute the expectations for Merton and Kou models

$$\mathbb{E}_m[\exp(V_i)] = \int_{-\infty}^{\infty} \exp(y)\nu_m(y)dy = e^{\mu_j+\gamma_j^2/2},$$

$$\mathbb{E}_k[\exp(V_i)] = \int_{-\infty}^{\infty} \exp(y)\nu_k(y)dy = \frac{p\alpha_1}{\alpha_1-1} + \frac{q\alpha_2}{\alpha_2+1}.$$

Therefore, the PIDE (6) can be written in the form

$$\begin{aligned} &\partial_t F + \frac{1}{2}\sigma_H^2 H^2 \partial_{HH} F + \rho\sigma_H\sigma_r H\sqrt{r}\partial_{Hr}F + \frac{1}{2}\sigma_r^2 r\partial_{rr}F \\ &+(r-\delta-\tilde{\lambda}\tilde{\kappa})H\partial_H F + \kappa(\theta-r)\partial_r F - (r+\tilde{\lambda})F + \tilde{\lambda}\int_{-\infty}^{\infty} F(t, H\exp(y), r)\nu(y)dy = 0, \end{aligned} \tag{9}$$

where $\tilde{\kappa} = e^{\mu_j+\gamma_j^2/2} - 1$ or $\tilde{\kappa} = \frac{p\alpha_1}{\alpha_1-1} + \frac{q\alpha_2}{\alpha_2+1} - 1$ for Merton or Kou models, respectively.

Note that with respect to the PDE model in [7], there is an integral term in the equation due to the presence of jumps. This term makes the PIDE more difficult to solve than the corresponding PDE. In a forthcoming section we show how to discretize this integral in order to find a numerical solution of the PIDE problem.

2.2. Mortgage contract

Following the same notation as in [7], the equal monthly payment dates are denoted by T_m, $m = 1, ..., M$, where M is the number of months. Assuming that $T_0 = 0$, let $\Delta T_m = T_m - T_{m-1}$ the duration of month m, c is the fixed contract rate and $P(0)$ is the initial loan (i.e. the principal at $t = T_0 = 0$), the fixed mortgage payment (MP) is given by:

$$MP = \frac{(c/12)(1+c/12)^M P(0)}{(1+c/12)^M - 1}, \tag{10}$$

For $m = 1, ..., M$, the unpaid loan just after the (m-1)th payment date is

$$P(m-1) = \frac{((1+c/12)^M - (1+c/12)^{m-1})P(0)}{(1+c/12)^M - 1}. \tag{11}$$

If $t_m = t - T_{m-1}$ denotes the time elapsed at month m (which starts at $t = T_{m-1}$), let $\tau_m = \Delta T_m - t_m$ be the time until T_m. This change of variable transforms equation (9) into another one associated with an initial value problem. More precisely, the mortgage value to the lender during month m, $V(\tau_m, H, r)$, without including the insurance the lender has on the loan, satisfies the PIDE

$$\begin{aligned} & -\partial_{\tau_m} F + \frac{1}{2}\sigma_H^2 H^2 \partial_{HH} F + \rho\sigma_H\sigma_r H\sqrt{r}\partial_{Hr} F + \frac{1}{2}\sigma_r^2 r\partial_{rr} F \\ +(r-\delta-\tilde{\lambda}\tilde{\kappa})H\partial_H F + \kappa(\theta-r)\partial_r F - (r+\tilde{\lambda})F + \tilde{\lambda}\int_{-\infty}^{\infty} F(\tau_m, H\exp(y), r)\nu(y)dy &= 0 \end{aligned} \tag{12}$$

for $0 \le \tau_m \le \Delta T_m$, $0 \le H < \infty$, $0 \le r < \infty$. We clarify a certain abuse of notation: if $\overline{F}$ denotes the solution of (9) and F the solution of (12) then $F(\tau_m, H, r) = \overline{F}(T_m - \tau_m, H, r)$.

Next, we will take into account the prepayment and default options. The option to default only happens at payment dates when the borrower does not pay the monthly amount MP. The option to prepay can be exercise at any time during the life of the loan. If the borrower fully amortizes the mortgage at time τ_m by paying the amount (which includes the total remaining debt plus an early termination penalty):

$$TD(\tau_m) = (1+\Psi)(1+c(\Delta T_m - \tau_m))P(m-1), \tag{13}$$

where Ψ is the prepayment penalty factor.

Hereafter we denote by V, I and CI the functions defining the values of the mortgage, the insurance and the coinsurance, respectively. Clearly, they can be considered as particular cases of assets depending on interest rate and house price. The option of early prepayment implies that V satisfies a complementarity problem associated to the PIDE, while C and CI satisfy a PIDE with the appropriate conditions at monthly payment dates.

The mortgage pricing problem starts from the value of the mortgage at maturity ($t = T_M$), just before the last payment, given by

$$V(\tau_M = 0, H, r) = \min(MP, H) \tag{14}$$

while at the other payment dates, it is given by

$$V(\tau_m = 0, H, r) = \min(V(\tau_{m+1} = \Delta T_{m+1}, H, r) + MP, H), \tag{15}$$

where $1 \le m \le M-1$.

If the borrower defaults, which occurs when the mortgage value is equal to the house value, the lender will lose the promised future payments. Then, the lender might have taken an insurance against default which would cover a fraction of the loss associated with default. This asset has no value for the borrower. Actually, it is part of the lender's portfolio, as indicated in [27] this asset adds to the lender's position in the contract. In order to obtain the value of the insurance to the lender, denoted by $I(\tau_m, H, r)$, we must solve equation (12) with suitable payment date conditions. In order to pose them, we assume that in case of default the insurer accepts to pay a fraction γ of the currently unpaid balance up to a maximum indemnity, Γ. Therefore, depending on if default occurs or not, the insurance value at the maturity of the loan is

$$I(\tau_M = 0, H, r) = \begin{cases} \min(\gamma(MP - H), \Gamma) & \text{(Default)} \\ 0 & \text{(No default)} \end{cases} \tag{16}$$

At earlier payment dates, the value of the insurance is

$$I(\tau_m = 0, H, r) = \begin{cases} \min(\gamma[TD(\tau_m = 0) - H], \Gamma) & \text{(Default)} \\ I(\tau_{m+1} = \Delta T_{m+1}, H, r) & \text{(No default)} \end{cases} \tag{17}$$

where $1 \le m \le M-1$.

The fraction of the potential loss not covered by the insurance is referred as the coinsurance. At each payment date, the coinsurance is the difference between the values of the potential loss and the insurance coverage. In this case, in order to price the coinsurance, $CI(\tau_m, H, r)$, equation (12) must be solved again with suitable conditions. At maturity, the value of the coinsurance is

$$CI(\tau_M = 0, H, r) = \begin{cases} \max((1-\gamma)(MP - H), (MP - H) - \Gamma) & \text{(Default)} \\ 0 & \text{(No default)} \end{cases} \tag{18}$$

At earlier payment dates, the value of the coinsurance is

$$CI(\tau_m = 0, H, r) = \begin{cases} \max((1-\gamma)[TD(\tau_m = 0) - H], [TD(\tau_m = 0) - H] - \Gamma) & \text{(Default)} \\ CI(\tau_{m+1} = \Delta T_{m+1}, H, r) & \text{(No default)} \end{cases} \tag{19}$$

where $1 \le m \le M - 1$.

At origination, the equilibrium condition explained in [7] needs to be satisfied in order to avoid arbitrage. Formally, this condition states that

$$V(\tau_1 = \Delta T_1, H_{initial}, r_{initial}; \Psi, c) + I(\tau_1 = \Delta T_1, H_{initial}, r_{initial}; \Psi, c) = (1 - \xi)P(0) \tag{20}$$

where Ψ is the prepayment penalty and ξ is the arrangement fee. Thus, the determination of the unknown fixed rate c of the contract solves the nonlinear equation (20) and is obtained by using a secant variable iterative algorithm, which requires the solution of the problems for V and I from maturity up to origination date at each iteration. In practice, we observed the convergence of the method for a not too restrictive choice of the two interest rates for the initialization.

In Section 2.3 we write the problem satisfied by the value of the mortgage, V, while at the beginning of Section 3 we write the problems satisfied by insurance, I, and coinsurance, CI (after a localization procedure).

2.3. The free boundary problem under jump-diffusion models

In the presence of jumps for the house value, let us consider the following linear operator:

$$\begin{aligned} \mathcal{L}V &= \partial_{\tau_m} V - \frac{1}{2}\sigma_H^2 H^2 \partial_{HH} V - \rho\sigma_H\sigma_r H\sqrt{r}\partial_{Hr} V - \frac{1}{2}\sigma_r^2 r \partial_{rr} V \\ &\quad -(r - \delta - \tilde{\lambda}\tilde{\kappa})H\partial_H V - \kappa(\theta - r)\partial_r V + (r + \tilde{\lambda})V - \tilde{\lambda}\int_{-\infty}^{\infty} V(\tau_m, H\exp(y), r)\nu(y)dy. \end{aligned} \tag{21}$$

So, the free boundary problem associated with the valuation of the mortgage contract can be written in terms of the linear complementarity problem:

$$\mathcal{L}V \le 0, \quad (TD(\tau_m) - V(\tau_m, H, r)) \ge 0, \quad (\mathcal{L}V)(TD(\tau_m) - V(\tau_m, H, r)) = 0. \tag{22}$$

The option to prepay can be exercised at any time during the lifetime of the mortgage contract (which exhibits an American or early exercise feature). If $V = TD$ then it is optimal for the borrower to prepay (prepayment region), otherwise $\mathcal{L}V = 0$ and we are inside the non prepayment region. Note that in this way we implicitly assume that the holder acts maximizing the mortgage valued to the borrower, so that the mortgage rate corresponds to the worst case cost of hedging the risk. In practice, banks have their own pre-payment models, however these models seemed not enough suitable during the recent crisis. From the point of view of risk management the pre-payment model here considered results the most useful. However, more recently in [28], the authors develop a new prepayment model which includes a decision time in the valuation of a fixed rate mortgage: instead of prepaying when $V = TD$, the borrower waits until $V \ge TD$ for the decision time and then prepays. In this case an additional variable for decision time arises, thus increasing in one time dimension the complexity in the numerical solution. This approach can be understood as replacing the American call option exercise by a variant of Parisian call option exercise.

3. Numerical solution

The PIDE is initially posed on the unbounded domain of positive interest rates and house prices. So, as in the case without jumps, we approximate it by a bounded domain formulation and we impose boundary conditions in the new boundaries. Note that the domain of integration in the integral term also needs to be localized. For this purpose, we introduce the following changes of variables and notation:

$$x_1 = \frac{H}{H_\infty}, \quad x_2 = \frac{r}{r_\infty}, \quad \bar{x} = \ln(x_1), \tag{23}$$

where both H_∞ and r_∞ are sufficiently large suitably chosen real numbers. Let $\Omega = (0, x_1^\infty) \times (0, x_2^\infty)$, with $x_1^\infty = x_2^\infty = 1$. Then, let us denote the Lipschitz boundary by $\Gamma = \partial\Omega$ such that $\Gamma = \bigcup_{i=1}^{2}(\Gamma_i^- \cup \Gamma_i^+)$, where:

$$\Gamma_i^- = \{(x_1, x_2) \in \Gamma \mid x_i = 0\}, \ \Gamma_i^+ = \{(x_1, x_2) \in \Gamma \mid x_i = x_i^\infty\}, \ i = 1, 2.$$

Next, taking into account the new variables we write the equation (12) in divergence form in the bounded domain. As in [27], we consider the case $\rho = 0$. Thus, the initial-boundary value problem for the insurance and coinsurance can be written in the form:

Find $J : [0, \Delta T_m] \times \Omega \to \mathbb{R}$ such that

$$\partial_{\tau_m} J + \vec{v} \cdot \nabla J - Div(A \nabla J) + lJ - \tilde{\lambda} \int_{y_{min}}^{y_{max}} \bar{J}(\tau_m, \bar{x}_1 + y, x_2)\nu(y)dy = f \qquad \text{in } (0, \Delta T_m) \times \Omega \,, \tag{24}$$

$$\frac{\partial J}{\partial x_1} = g_1 \qquad \text{on } (0, \Delta T_m) \times \Gamma_1^+ \,, \tag{25}$$

$$\frac{\partial J}{\partial x_2} = g_2 \qquad \text{on } (0, \Delta T_m) \times \Gamma_2^+ \,, \tag{26}$$

where $J = I, CI$, $\bar{J}(\tau_m, \bar{x}_1 + y, x_2) = J(\tau_m, \exp(\bar{x}_1 + y), x_2)$ and the appropriate initial condition for each month is given by the equations (16) and (17) when we are pricing the insurance and by the equations (18) and (19) in the case of valuing the coinsurance.

Furthermore, for the complementarity problem associated with the mortgage value during month m, we can pose the following mixed formulation:

Find $V : [0, \Delta T_m] \times \Omega \to \mathbb{R}$ and $Q : [0, \Delta T_m] \times \Omega \to \mathbb{R}$ satisfying the PIDE

$$\partial_{\tau_m} V + \vec{v} \cdot \nabla V - Div(A \nabla V) + lV - \tilde{\lambda} \int_{y_{min}}^{y_{max}} \bar{V}(\tau_m, \bar{x}_1 + y, x_2)\nu(y)dy + Q = f \quad \text{in } (0, \Delta T_m) \times \Omega \,, \tag{27}$$

the complementarity conditions

$$V \leq TD, \quad Q \geq 0, \quad Q(TD - V) = 0 \ \text{ in } (0, \Delta T_m) \times \Omega, \tag{28}$$

where Q is the Lagrange multiplier (slack variable) associated to the inequality constraints in (22). Thus, when $Q > 0$ we are in the prepayment region. Moreover, we consider the boundary conditions

$$\frac{\partial V}{\partial x_1} = g_1 \qquad \text{on } (0, \Delta T_m) \times \Gamma_1^+ \,, \tag{29}$$

$$\frac{\partial V}{\partial x_2} = g_2 \qquad \text{on } (0, \Delta T_m) \times \Gamma_2^+, \tag{30}$$

where $\bar{V}(\tau_m, \bar{x}_1 + y, x_2) = V(\tau_m, \exp(\bar{x}_1 + y), x_2)$ and the initial condition for each month is given by the equations (14) or (15).

For both problems, the involved data is defined as follows

$$A = \begin{pmatrix} \frac{1}{2}\sigma_H^2 x_1^2 & 0 \\ 0 & \frac{1}{2}\sigma_r^2 \frac{x_2}{r_\infty} \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} (\sigma_H^2 - x_2 r_\infty + \delta + \tilde{\lambda}\tilde{\kappa})x_1 \\ (\frac{1}{2}\sigma_r^2 - \kappa(\theta - x_2 r_\infty))/r_\infty \end{pmatrix} \tag{31}$$

$$l = x_2 r_\infty + \tilde{\lambda}, \quad f = 0,\ g_1 = 0,\ g_2 = 0. \tag{32}$$

Remark 3.1. *Note that the differential part of the PIDE is defined in the domain $[0, x_1^\infty] \times [0, x_2^\infty]$, using the discrete grid $0 = x_{1_0}, x_{1_1}, \cdots, x_{1_q} = x_1^\infty$. Since $\ln(x_{1_0}) = -\infty$, we choose $y_{min} = \ln(x_{1_1})$ and $y_{max} = \ln(x_{1_q})$ as it is proposed in [15].*

3.1. Time discretization

For the time discretization we consider the method of characteristics. A first order version of this method has been introduced in [16] and [4], for example. This version has been used in [29] for vanilla options and in [14] for American Asian options with jumps.

First, we define the characteristics curve through $\mathbf{x} = (x_1, x_2)$ at time $\bar{\tau}_m$, $X(\mathbf{x}, \bar{\tau}_m; s)$, which satisfies:

$$\frac{\partial}{\partial s} X(\mathbf{x}, \bar{\tau}_m; s) = \vec{v}(X(\mathbf{x}, \bar{\tau}_m; s)),\ X(\mathbf{x}, \bar{\tau}_m; \bar{\tau}_m) = \mathbf{x}. \tag{33}$$

For $N > 1$ let us consider the time step $\Delta\tau_m = \Delta T_m / N$ and the time mesh points $\tau_m^n = n\Delta\tau_m$, $n = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots, N$. The material derivative approximation by characteristics method is given by:

$$\frac{DF}{D\tau_m} = \frac{F^{n+1} - F^n \circ X^n}{\Delta\tau_m},$$

where $F = CI,\ I\,, V$ and $X^n(\mathbf{x}) := X(\mathbf{x}, \tau_m^{n+1}; \tau_m^n)$. In view of the expression of the velocity field the components of $X^n(\mathbf{x})$ can be analytically computed:

$$\begin{aligned} X_1^n(\mathbf{x}) &= x_1 \exp\left(-\left(\sigma_H^2 + \delta + \frac{\sigma_r^2}{2\kappa} - \theta + \tilde{\lambda}\tilde{\kappa}\right)\Delta\tau_m\right) \times \\ &\quad \exp\left(\left(\frac{-x_2 r_\infty}{\kappa} - \frac{\sigma_r^2}{2\kappa^2} + \frac{\theta}{\kappa}\right)(\exp(-\kappa\Delta\tau_m) - 1)\right) \\ X_2^n(\mathbf{x}) &= \left(-\frac{\sigma_r^2}{2\kappa r_\infty} + \frac{\theta}{r_\infty}\right)(1 - \exp(-\kappa\Delta\tau_m)) + x_2\exp(-\kappa\Delta\tau_m) \end{aligned}$$

In the present work we use a more recent version of the method of characteristics combined with a Crank-Nicolson technique for time discretization. The analysis of the method for a more general equation has been addressed in [5, 6]. More precisely, we consider a Crank-Nicolson scheme around $\big(X(\mathbf{x}, \tau_m^{n+1}; \tau_m), \tau_m\big)$ for $\tau_m = \tau_m^{n+\frac{1}{2}}$. So, for $n = 0, \ldots, N-1$, the time discretized equation for $F = I$, CI, V and $P = 0$ can be written as follows:

Find F^{n+1} such that:

$$\begin{aligned} &\frac{F^{n+1}(\mathbf{x}) - F^n(X^n(\mathbf{x}))}{\Delta\tau_m} - \frac{1}{2}Div(A\nabla F^{n+1})(\mathbf{x}) - \frac{1}{2}Div(A\nabla F^n)(X^n(\mathbf{x})) \\ &+\frac{1}{2}(l\,F^{n+1})(\mathbf{x}) + \frac{1}{2}(l\,F^n)(X^n(\mathbf{x})) - \tilde{\lambda}\int_{y_{min}}^{y_{max}} \bar{F}^n(\bar{x}_1 + y, x_2)\nu(y)dy \;=\; 0, \end{aligned} \tag{34}$$

where $\bar{F}^n(\bar{x}_1+y, x_2) = F^n(e^{\bar{x}_1+y}, x_2)$. Note that the integral term is evaluated at the previous time step, thus avoiding the presence of a full matrix in the linear systems associated with the fully discretized problems [11]. After some computations, we can write a variational formulation for the semi-discretized problem as follows:

Find $F^{n+1} \in H^1(\Omega)$ such that, for all $\Psi \in H^1(\Omega)$:

$$
\begin{aligned}
&\int_\Omega F^{n+1}(\mathbf{x})\Psi(\mathbf{x})d\mathbf{x} + \frac{\Delta\tau_m}{2}\int_\Omega (A\nabla F^{n+1})(\mathbf{x})\nabla\Psi(\mathbf{x})d\mathbf{x} + \frac{\Delta\tau_m}{2}\int_\Omega lF^{n+1}(\mathbf{x})\Psi(\mathbf{x})d\mathbf{x} = \\
&\int_\Omega F^n(X^n(\mathbf{x}))\Psi(\mathbf{x})d\mathbf{x} - \frac{\Delta\tau_m}{2}\int_\Omega (\nabla X^n)^{-1}(\mathbf{x})(A\nabla F^n)(X^n(\mathbf{x}))\nabla\Psi(\mathbf{x})d\mathbf{x} \\
&-\frac{\Delta\tau_m}{2}\int_\Omega lF^n(X^n(\mathbf{x}))\Psi(\mathbf{x})d\mathbf{x} + \frac{\Delta\tau_m}{2}\int_\Gamma \tilde{g}^n(\mathbf{x})\Psi(\mathbf{x})dA_{\mathbf{x}} \\
&+\frac{\Delta\tau_m}{2}\int_{\Gamma_{1+}} \bar{g}_1^{n+1}(\mathbf{x})\Psi(\mathbf{x})dA_{\mathbf{x}} + \frac{\Delta\tau_m}{2}\int_{\Gamma_{2+}} \bar{g}_2^{n+1}(\mathbf{x})\Psi(\mathbf{x})dA_{\mathbf{x}} \\
&-\frac{\Delta\tau_m}{2}\int_\Omega Div((\nabla X^n)^{-T}(\mathbf{x}))(A\nabla F^n)(X^n(\mathbf{x}))\Psi(\mathbf{x})d\mathbf{x} \\
&+\Delta\tau_m\tilde{\lambda}\int_\Omega\int_{y_{min}}^{y_{max}} \bar{F}^n(\bar{x}_1+y, x_2)\nu(y)dy\Psi(\mathbf{x})d\mathbf{x}
\end{aligned}
\tag{35}
$$

where ∇X^n can be analytically computed, $\bar{g}_1^{n+1}(\mathbf{x}) = g_1^{n+1}(\mathbf{x})a_{11}(\mathbf{x}) = 0$, $\bar{g}_2^{n+1}(\mathbf{x}) = g_2^{n+1}(\mathbf{x})a_{22}(\mathbf{x}) = 0$,

$$
\tilde{g}^n(\mathbf{x}) := \begin{cases}
-\left[(\nabla X^n)^{-T}\right]_{21}(\mathbf{x})a_{22}(X^n(\mathbf{x}))\frac{\partial F}{\partial x_2}(X^n(\mathbf{x})) & on \quad \Gamma_1^- \\
0 & on \quad \Gamma_2^- \\
\left[(\nabla X^n)^{-T}\right]_{22}(\mathbf{x})a_{22}(X^n(\mathbf{x}))g_2^n(X^n(\mathbf{x})) & on \quad \Gamma_2^+ \\
\left[(\nabla X^n)^{-T}\right]_{11}(\mathbf{x})a_{11}(X^n(\mathbf{x}))g_1^n(X^n(\mathbf{x})) & \\
+\left[(\nabla X^n)^{-T}\right]_{21}(\mathbf{x})a_{22}(X^n(\mathbf{x}))\frac{\partial F}{\partial x_2}(X^n(\mathbf{x})) & on \quad \Gamma_1^+
\end{cases}
\tag{36}
$$

and

$$
Div((\nabla X^n)^{-T}(\mathbf{x})) = \begin{pmatrix} 0 \\ \dfrac{r_\infty}{\kappa}(1-\exp(\kappa\Delta\tau_m)) \end{pmatrix}. \tag{37}
$$

3.2. Spatial discretization and nonlinear terms

For spatial discretization we consider piecewise quadratic Lagrange finite elements. For the numerical integration of the terms appearing in this finite elements discretization, we use a 9-points quadrature formula which implies a lumped mass matrix computation when dealing with this term.

In order to deal with the nonlinearities in the free boundary problem associated with prepayment option, we apply to the mixed formulation (27)-(30) the Augmented Lagrangian Active Set (ALAS) algorithm proposed in [18] and explained in detail in [7] for the case without jumps in the house price. In summary, the ALAS iterative method computes sequences that converge to the mortgage value V, the Lagrange multiplier Q and the prepayment and non-prepayment regions, thus allowing also to identify an approximation to the optimal prepayment boundary.

3.3. Approximation of the integral term

In order to approximate the integral term that appears in the PIDE due to the presence of jumps, we use a suitable numerical integration procedure. More precisely, we use the classical composite trapezoidal rule with $m+1$ points in the following way:

$$\int_{y_{min}}^{y_{max}} \bar{F}^n(\bar{x}_1+y,x_2)\nu(y)dy \approx$$
$$\frac{h}{2}\left[\bar{F}^n(\bar{x}_1+y_{min},x_2)\nu(y_{min})+\bar{F}^n(\bar{x}_1+ymax,x_2)\nu(y_{max})+2\sum_{j=1}^{m-1}\bar{F}^n(\bar{x}_1+k_j,x_2)\nu(k_j)\right],$$

where $k_j=y_{min}+jh$ for $j=1,...,m-1$ and $h=\dfrac{y_{max}-y_{min}}{m}$.

4. Numerical results

In order to solve the fixed rate mortgage valuation problem, we need to specify a set of parameters related to the stochastic models, contract characteristics and insurance. All of them are based on the existent literature (see [1] and [27], for example) and are shown in Table 1. Moreover, concerning the numerical methods employed to solve the problem, we consider the parameters collected in Table 2. In order to compare the results obtained with Merton and Kou models we need a certain matching between the density functions of the normal distribution (Merton) and of the double-exponential distribution (Kou). For this purpose, we consider the parameters involved in the jump-diffusion models which are proposed in [15]. For a more realistic application, these parameter could be estimated by analogous techniques to those ones proposed in [8] form suitable market data.

In Tables 3, 4 and 5 we show the values of the mortgage, the insurance and the coinsurance taking into account jumps for the house value and when the loan term is 15 years. The values were computed for different initial interest rates (spot interest rates) and arrangement fees. In Table 3 we consider Merton jump-diffusion model for house price dynamics, in Table 4 we use Kou jump-diffusion process and in Table 5 we assume a geometric Brownian motion (which does not allow jumps in the prices). As expected, in the presence of jumps the value of the contract is lower than without jumps whereas the value of the insurance and the coinsurance are higher. Note that the presence of jumps increases uncertainty in the house price, thus depreciating the mortgage price.

Figures 2, 3 and 4 illustrate the free boundary at origination with Merton and Kou jump-diffusion models and without jumps in the house price, respectively. We take into account the fixed parameters of the model shown in Table 1. In this case, the interest rate volatility is 10%, the house price volatility is 5%, the maturity of the loan is 25 years and the spot rate is 8%. Under Merton model, the obtained adjusted fixed rate is 14.5892% whereas with Kou model this rate is 14.3926%. When using a geometric Brownian motion for house price dynamics the adjusted rate falls to the significantly lower value of 9.3969%. In all figures the prepayment (coincidence) region appears in red and the non prepayment (non coincidence) region is shown in blue. Note that prepayment region is located in the part of the domain with lower rates and higher house prices, which results reasonable from the financial point of view: in this part of the computational domain it is better to fully prepay the loan and refinance at lower market interest rates if necessary. We note that the prepayment region is nearly rectangular in the case without jumps while it is nearly elliptic in both jump-diffusion models.

Concerning the numerical convergence of the Crank-Nicolson characteristics discretization method, first note that the numerical analysis for the initial boundary value problem under rather general conditions on a PDE operator has been developed in [5, 6], where second order convergence in space and time is theoretically proved. The order of the method for a complementarity problem and/or PIDE being an open problem. However, in the present work we need to combine the method with additional numerical techniques: ALAS algorithm for the complementarity condition and composite trapezoidal rule for the nonlocal term, so that the second order convergence is lost as illustrated by the forthcoming tables. However, we prefer to maintain Crank-Nicolson characteristics which a bit better accurate than a possible alternative fully implicit method.

In order to illustrate the convergence properties, we evolve in time from the mortgage maturity ($T=15$) until the first day of the last moth (i.e. $t_0=14+11/12(\approx 14.92)$) so that we just test the behaviour of the method for the PIDE complementarity problem. More precisely, in the forthcoming tables we consider the values at the point $(t_0, H, r) = (t_0, 83333, 0.13)$ which is located outside the prepayment region. In Table 6 we show the data of the quadrangular finite element meshes.

Following the ideas in [14], we compute the following indicator of order convergence

$$R = \frac{V(h/2) - V(h)}{V(h/4) - V(h/2)},$$

the parameter h just indicating that we start with a level of refinement in time and space and divide by two both the time and finite element mesh steps to get the results for $h/2$. Thus, $R = 2$ corresponds to linear convergence while $R = 4$ corresponds to quadratic convergence. Table 7 shows the obtained results for Crank-Nicolson while Table 8 exhibits the ones for the fully implicit method, thus illustrating that only first order is achieved in both cases with a bit better results in the first case.

5. Conclusions

In this paper we consider the valuation of a fixed rate mortgage with prepayment and default options and where the house value is supposed to be driven by a jump-diffusion process. The assumption of jump-diffusion models instead of pure diffusion ones (GBM process) seems more reasonable under certain situations in real estate markets and has not been previously addressed in the literature for fixed rate mortgage pricing model with two underlying stochastic factors. More precisely, we assume that the jumps follow Merton and Kou models, although it can be extended to other Levy processes. In both cases, a set of PIDE problems arises and we pose the models to price the mortgage value and other components, such as the insurance and the coinsurance. Next, we propose appropriate numerical methods based on Lagrange-Galerkin formulations to solve the problems combined with the ALAS algorithm to deal with the non-linearities associated with the free boundary problem that appears in the contract pricing due to the prepayment option. Moreover, the integral term that arises due to the presence of jumps is explicitly treated. Furthermore, we adjust the fixed rate of the mortgage by using an iterative process.

For both jump-diffusion models in the house price, we show some numerical examples to illustrate the behaviour of the methods and the quantitative and qualitative properties of the solutions, as well as the difference with respect to the pure diffusion model. In particular, if we assume jump-diffusion dynamics for the house value then the contract price decreases and the insurance and coinsurance prices increase, as expected. Actually, the same qualitative behaviour is observed in [8]. Finally, we include figures which represent the optimal prepayment boundary separating the region where it is optimal to prepay the loan from the one where it is not.

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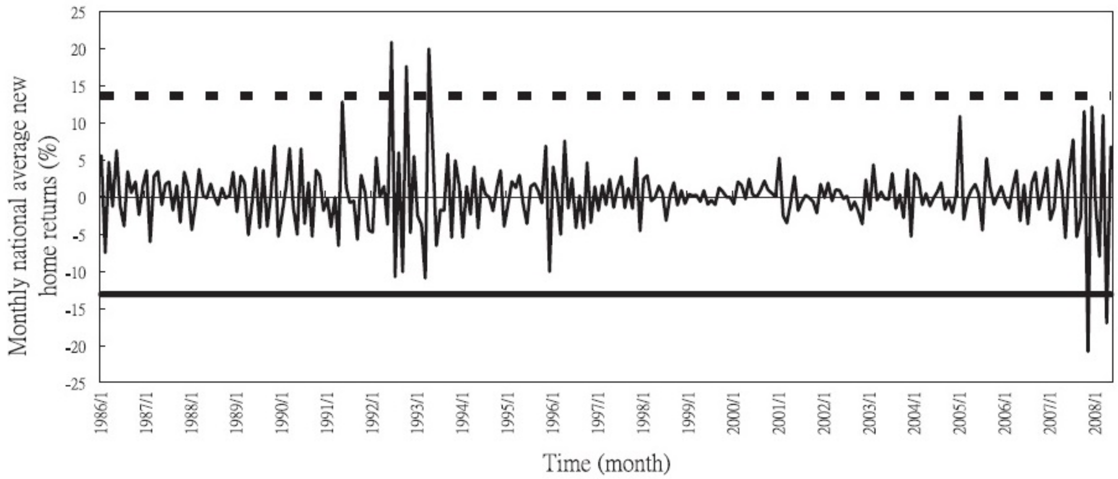

Figure 1: U.S. Time Series of new home price returns for single family mortgage 1986-2008

House price and interest rate models data	
Steady state spot rate, θ	10 %
Speed of reversion, κ	25 %
House service flow, δ	7.5%
Correlation coefficient, ρ	0
Parameter of Poisson process, $\tilde{\lambda}$	0.1
Mean of jump size (Merton), μ_j	-0.1
Standard deviations of jump size (Merton), γ_j	0.45
Probability of upward jump (Kou), p	0.3445
Parameter (Kou), α_1	3.0465
Parameter (Kou), α_2	3.0775
Contract specifications	
Initial value of the house, $H_{initial}$	100000€
Ratio of the loan to value	95%
Initial estimate for contract rate, c_0	10%
Prepayment penalty, Ψ	5%
Insurance	
Guaranteed fraction of total loss, γ	80%
Cap, Γ	$20\% H_{initial}$

Table 1: Fixed parameters in the mortgage valuation model

Computational domain	
H_∞	200000€
r_∞	40 %
Finite elements mesh data	
Number of elements	576
Number of nodes	2401
Time discretization	
Time steps per month	30
ALAS algorithm	
Parameter β	10000

Table 2: Numerical resolution parameters

Loan (years)	spot rate r(0)	ξ	Contract rate c	Contract value V	Insurance I	Coinsurance CI
15	8%	0%	14.4301%	91730	3270	2402
		0.5%	14.3121%	91282	3243	2402
		1%	14.1966%	90840	3210	2402
		1.5%	14.0815%	90396	3179	2402
	10%	0%	15.4554%	92050	2950	2190
		0.5%	15.3245%	91588	2937	2190
		1%	15.1965%	91132	2918	2190
		1.5%	15.0698%	90674	2901	2188
	12%	0%	16.5677%	92360	2640	1992
		0.5%	16.4168%	91892	2633	1998
		1%	16.2706%	91428	2622	1998
		1.5%	16.1271%	90960	2615	1998

Table 3: Contract rate, mortgage contract, insurance and coinsurance values for $\sigma_r = 5\%, \quad \sigma_H = 5\%$ different contract specifications under Merton jump-diffusion model for the house value

Loan (years)	spot rate r(0)	ξ	Contract rate c	Contract value V	Insurance I	Coinsurance CI
15	8%	0%	14.2355%	92090	2910	2092
		0.5%	14.1191%	91647	2878	2090
		1%	14.0045%	91202	2848	2090
		1.5%	13.8920%	90759	2816	2090
	10%	0%	15.2618%	92404	2596	1909
		0.5%	15.1339%	91949	2576	1910
		1%	15.0078%	91492	2558	1910
		1.5%	14.8838%	91036	2539	1908
	12%	0%	16.3824%	92714	2286	1744
		0.5%	16.2317%	92244	2281	1744
		1%	16.0861%	91778	2272	1748
		1.5%	15.9438%	91314	2261	1746

Table 4: Contract rate, mortgage contract, insurance and coinsurance values for $\sigma_r = 5\%, \quad \sigma_H = 5\%$ and different contract specifications under Kou jump-diffusion model for the house value

Loan (years)	spot rate r(0)	ξ	Contract rate c	Contract value V	Insurance I	Coinsurance CI
15	8%	0%	9.0839%	94549	449	112
		0.5%	8.9911%	94116	410	103
		1%	8.8992%	93663	386	96
		1.5%	8.8119%	93230	345	86
	10%	0%	10.0782%	94656	343	84
		0.5%	9.9696%	94208	317	79
		1%	9.8634%	93764	288	72
		1.5%	9.7579%	93316	260	66
	12%	0%	11.1662%	94691	309	76
		0.5%	11.0389%	94274	249	62
		1%	10.9203%	93870	181	45
		1.5%	10.8006%	93422	154	38

Table 5: Contract rate, mortgage contract, insurance and coinsurance values for $\sigma_r = 5\%, \quad \sigma_H = 5\%$ different contract specifications with geometric Brownian motion for the house value (without jumps)

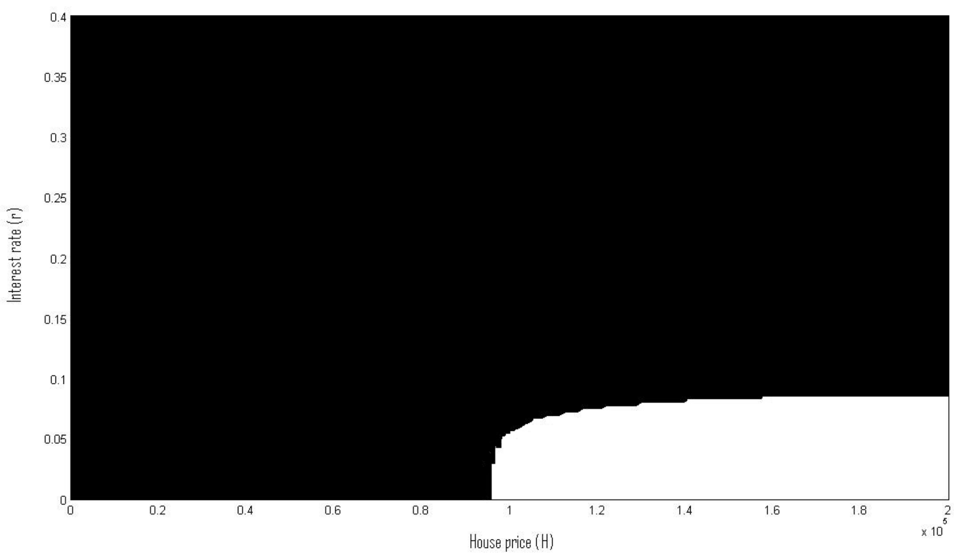

Figure 2: Free boundary at origination when Merton jump-diffusion model for the house value is considered

	N. Elem	**N. Nodes**
Mesh 8	64	289
Mesh 12	144	625
Mesh 24	576	2401
Mesh 48	2304	9409
Mesh 96	9216	37249

Table 6: FEM meshes data

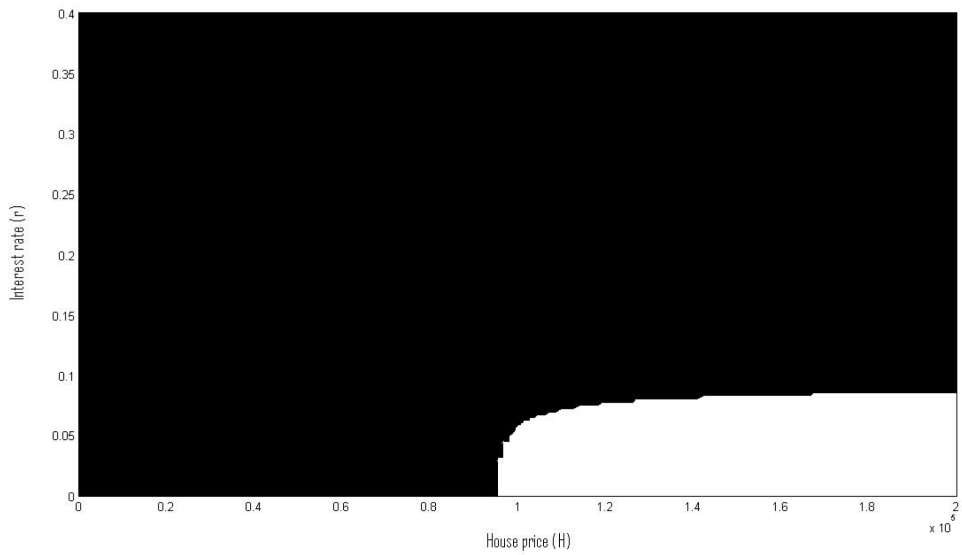

Figure 3: Free boundary at origination when Kou jump-diffusion model for the house value is considered

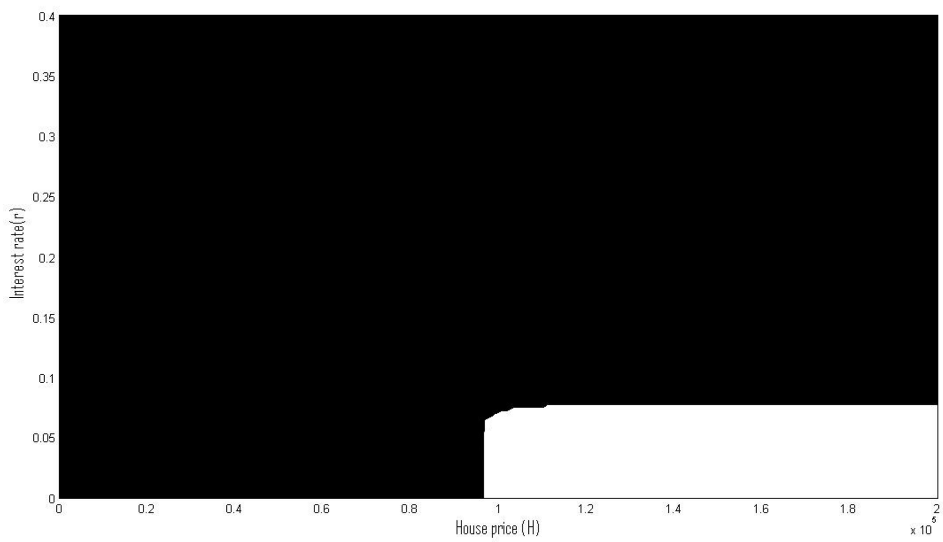

Figure 4: Free boundary at origination when a geometric Brownian motion for the house value is considered (model without jumps)

Time steps	**FE Mesh**	**Value**	**R**
15	**8**	1330.361966	-
30	**12**	1330.355001	-
60	**24**	1330.353948	6.619062
120	**48**	1330.353596	2.982661
240	**96**	1330.353423	2.039254

Table 7: Illustration of order of convergence at the point $(t_0, H, r) = (t_0, 83333, 0.13)$ for the Crank-Nicolson characteristics method

Time steps	FE Mesh	Value	R
15	**8**	1330.371808	-
30	**12**	1330.359937	-
60	**24**	1330.356419	3.386653
120	**48**	1330.354837	2.218456
240	**96**	1330.354045	1.997664

Table 8: Illustration of order of convergence at the point $(t_0, H, r) = (t_0, 83333, 0.13)$ for the fully implicit characteristics method