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Transmission of Analog Information Over the Multiple Access Relay Channel Using Zero-Delay Non-Linear Mappings

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ABSTRACT We consider the zero-delay encoding of discrete-time analog information over the Multiple Access Relay Channel (MARC) using non-linear mapping functions. On the one hand, zero-delay non-linear mappings are capable to deal with the multiple access interference (MAI) caused by the simultaneous transmission of the information. On the other, the relaying operation is a Decode-and-Forward (DF) strategy where the decoded messages are merged into a single message using a specific continuous mapping depending on the correlation level of the source information. At the receiver, an approximated Minimum Mean Squared Error (MMSE) decoder is developed to obtain an estimate of the transmitted source symbols which exploits the information received from the relay node in combination with the messages received from the transmitters through the direct links. The resulting system provides better performance than the other alternative encoding strategies for the MARC with similar complexity and delay and also approaches the performance of theoretical strategies which require a significantly higher delay and computational cost.

INDEX TERMS Combined source-channel coding, relay networks, multiuser channels, network coding.

I. INTRODUCTION

The transmission of information from a source node to a destination point is limited by the fact that the physical signals undergo a power attenuation when they are transmitted over a specific propagation medium. This limitation may be alleviated introducing intermediate nodes capable of performing different operations to improve the quality of the received signals at the destination point. In this sense, relaying provides an efficient mechanism to increase system capacity and/or coverage by enabling communication over larger distances or around obstacles. The most general relay scenario consists of a communication network where a set of source nodes exchange data with another set of nodes through several layers of relay nodes [1]. In this paper, we focus on a particular scenario known as the MARC where several users aim at communicating their information to a common receiver with the help of an intermediate relay [2].

The first attempt to model the relaying problem is [3], where the communication system comprises three main nodes, namely, a source, a destination and a relay. This scenario is known as the relay channel and it has been widely studied in the literature. Cover and El Gamal [4] determined bounds on the capacity of the degraded relay channel considering different relaying strategies. This seminal paper was the starting point for extensive research on the design of suitable transmission schemes for the relay channel. As a result, several relaying strategies have been proposed for the relay channel such as Amplify-and-Forward (AF) [5], [6], Compression-and-Forward (CF) [4], DF [4], or Quantize-Map-and Forward (QMC) [7]–[9]. In the case of AF, the relay simply sends a scaled version of the received information, whereas the rest of strategies rely on the use of random codes where the codewords are assumed to be large enough.

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An alternative to this strategy is Compute-and-Forward (CoF) [10], [11] where the encoding operation at the relay node is based on lattice codes. Although CF and DF were initially developed for the single-relay channel, both strategies can be extended to scenarios with more than one relay [12]–[14].

In the context of relay-based communications, the MARC represents an interesting scenario where the use of network coding strategies helps to exploit the diversity gain in wireless networks while reducing the total number of transmission slots with respect to simple routing-based approaches [15], [16]. In this scenario, the transmit nodes send their source information to the destination node and to the relay. Next, the relay merges the information received from the transmit nodes and forwards the resulting message to the destination. Finally, the source information is recovered at the destination using the messages received directly from the transmit nodes and the relay node. Different coding strategies can be applied to transmit information over the MARC, among them extensions of the CF and DF techniques [2], [17]–[19]. These techniques lead to large delays when decoding a block of source messages since the reception of the entire block is required to recover the transmitted information, although some improvements have been proposed to reduce this delay [20]. Indeed, they are based on the use of theoretical random codes such that the computational cost of the decoding operation significantly increases with the codeword sizes. A lower complexity and delay approach is to apply AF directly for the MARC. In this case, the problem is that the receiver has to deal with the Multiple Access Interference (MAI) and, hence, it is essential to employ an appropriate distributed encoding scheme at the transmit nodes. A novel approach consists in replacing the conventional DF at the relay by lossy forwarding techniques and then applying iterative joint decoding at the destination node in order to allow a non-orthogonal transmission over the MARC [21], [22].

Another interesting alternative to the traditional network coding schemes is the use of non-linear mapping functions to implement the relaying operation. This kind of mappings has already been applied to a large number of communication scenarios including Multiple-Input and Multiple-Output (MIMO) systems [23], Multiple Access Channels (MACs) [24], [25] or Broadcast Channels (BCs) [26]. A scheme based on the combination of modulo mappings and spiral-like mappings was proposed to generate the relay message in the MARC with two source nodes [27]. It can be interpreted as an instance of DF where the spiral mapping merges the two source messages into a single one and the modulo mappings are used to exploit the side-information provided by the direct links between the transmitters and the receiver. Thus, the decoder can individually estimate each one of the source symbols using the messages received by the direct links as side information. This scheme was shown to obtain similar theoretical achievable rates to DF or CF but with an almost zero delay and a much lower complexity. The work in [27] provides a novel approach to design network coding schemes with negligible delay and low complexity, although it is mainly focused on theoretical aspects related to achievable rates and it is restricted to the case of orthogonal transmission of uncorrelated source messages.

In this paper, we design a zero-delay MARC transmission system based on the idea of using non-linear mappings to allow a non-orthogonal transmission of the source information while exploiting the presence of the relay node. Unlike [27], the transmit nodes must employ a non-orthogonal access scheme to send their symbols simultaneously, where it is essential to define an appropriate mapping function with its corresponding power allocation policy. In addition, we consider two different types of mapping functions for the relaying operation depending on whether the sources are correlated or not. At the receiver, it is required to design a decoder which integrates jointly the information from both the relay and the direct links to obtain an estimate of the source symbols. In this case, the main challenge is to express the two received messages as a function of the source symbols and integrate the resulting expressions into the receiver design. Finally, the proposed scheme is properly optimized and its performance is evaluated considering different MARC scenarios.

This scheme has been developed for the particular case of two transmit nodes and a single relay, although it can be extended to a larger number of transmitters and/or relays by adapting the non-linear mapping operations employed at the different nodes. Since a wide range of MARC scenarios could be addressed with the same design philosophy, we rather focus on a simple configuration to illustrate the main ideas. The extension to other MARC scenarios will specifically depend on the network configuration which can be different depending on the number of transmit nodes, relays or available time slots. However, in this work, we will include simple guidelines to show how the proposed design could be extended to a general MARC configuration.

The rest of the paper is organized as follows. Section II presents the zero-delay MARC scheme considered in this paper. The different blocks of the proposed MARC scheme, as well as their optimization, are described in Section III. Section IV presents the results obtained in computer simulations to illustrate the performance of this scheme in different MARC scenarios. Finally, Section V is devoted to summarize the conclusions of the paper.

A. CONTRIBUTIONS

The main contributions of the paper are summarized as follows:

- The design and performance evaluation of a practical MARC scheme with a negligible delay which is able to deal with the non-orthogonal transmission of the source information while exploiting the presence of the relay node. This scheme applies non-linear functions to encode the source information individually at each transmit node and to generate the relay message. The different parameters of the considered mappings are also optimized to improve the performance of the proposed scheme.

- The derivation of a MMSE decoder which integrates the information received at the destination directly from the source nodes and from the relay. The main hindrance to define this decoder lies in the modeling of the relay messages distortion caused by the decoding error at the relay node.

II. SYSTEM MODEL

In this paper, we focus on the design of zero-delay coding schemes for the MARC shown in Fig. 1. As observed, the communication system comprises four nodes: two transmit nodes, one relay node and the destination node. In the first channel use, the transmit nodes simultaneously broadcast their information which is received by the relay and the destination over the direct links. In the second channel use, the relay node sends a symbol to the destination while the transmit nodes are idle. The symbol generated by the relay results from the application of an appropriate transformation to the symbols received from the transmit nodes in the previous channel use. Notice that this MARC model spends the same number of channel uses as a MAC (i.e., without relay) transmitting the source symbols orthogonally over the direct links.

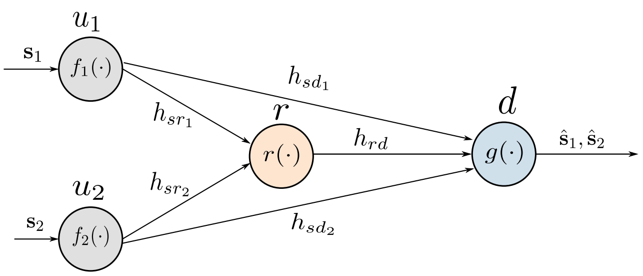

FIGURE 1. **Block diagram of the considered multiple access relay channel (MARC) with two transmit nodes, one intermediate relay node and one destination node.**

Let $\boldsymbol{s} = [s_1\ s_2]^T$ be the pair of discrete-time continuous-amplitude real-valued source symbols sent by the transmit nodes at a given time instant. The superidex ^{T} represents vector transposition. These source symbols are assumed to be generated from a zero-mean bivariate Gaussian distribution, i.e., $\boldsymbol{s} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{C_s})$, where

$$\boldsymbol{C_s} = \mathbb{E}\left[\boldsymbol{s}\boldsymbol{s}^T\right] = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \tag{1}$$

is the source covariance matrix and ρ represents the cross-correlation between the two source symbols. Sources are also assumed to be memoryless, i.e., source symbols transmitted at different time instants are statistically independent.

As commented, we seek to reduce the delay and the complexity of the overall system. Towards this aim, the source symbols s_1 and s_2 are encoded individually at the transmit nodes by means of the mapping functions $f_1(\cdot)$ and $f_2(\cdot)$, respectively. These encoding mappings are applied to a single source symbol, therefore causing the encoding delay to be negligible.

During the first channel use, the encoded symbols, $x_1 = f_1(s_1)$ and $x_2 = f_2(s_2)$, are simultaneously broadcast and received at both the relay and the destination node. On the one hand, the signal received at the relay node is

$$y_r = h_{sr_1}x_1 + h_{sr_2}x_2 + n_r, \tag{2}$$

where $h_{sr_1} \in \mathbb{R}$ represents the response of the channel between the first user and the relay node, $h_{sr_2} \in \mathbb{R}$ is the channel response for the second user, and $n_r \sim \mathcal{N}(0, \sigma^2_{n_r})$ represents the Additive White Gaussian Noise (AWGN) at the relay node. On the other hand, the signal received at the destination is

$$y_{sd} = h_{sd_1}x_1 + h_{sd_2}x_2 + n_{d_1}, \tag{3}$$

where $h_{sd_1}, h_{sd_2} \in \mathbb{R}$ represent the channel responses corresponding to the first and second users, respectively. The term $n_{d_1} \sim \mathcal{N}(0, \sigma^2_{n_d})$ is the AWGN component at the destination during the first channel use.

During the second step of the transmission procedure, the relay combines the information received from the transmitters to generate a suitable signal which is sent to the destination node. Different approaches can be considered such as AF, CF or DF. In this paper, we explore an alternative based on DF where the symbols obtained after decoding are re-encoded using a non-linear mapping function $r(\cdot)$. This mapping compresses the two symbols decoded at the relay $[\tilde{s}_1\ \tilde{s}_2]^T$ into a single coded symbol $x_r = r(\tilde{s}_1, \tilde{s}_2) \in \mathbb{R}$ which is transmitted in the next channel use. Hence, the signal received at the destination during this second channel use is

$$y_{rd} = h_{rd}x_r + n_{d_2}, \tag{4}$$

where $h_{rd} \in \mathbb{R}$ is the channel response between the relay and the destination and $n_{d_2} \sim \mathcal{N}(0, \sigma^2_{n_d})$ is the AWGN at the destination during the second channel use.

At the destination node, an estimate of the transmitted source symbols is determined from the received ones, y_{sd} and y_{rd}, using the decoding function $g(\cdot)$, i.e., $[\hat{s}_1\ \hat{s}_2]^T = g(y_{sd}, y_{rd})$. The set of encoding and decoding functions are optimized with the objective of minimizing the distortion between the source and estimated symbols according to the Mean Squared Error (MSE) metric which is computed as

$$\xi = \frac{1}{N}\sum_{n=1}^{N} |s_{1n} - \hat{s}_{1n}|^2 + |s_{2n} - \hat{s}_{2n}|^2. \tag{5}$$

where N is the total number of transmitted source symbols. Individual power constraints are considered at both the transmit and relay nodes such that $\mathbb{E}[|x_i|^2] \leq T_i\ \forall i = 1, 2$ and $\mathbb{E}[|x_r|^2] \leq T_r$. Another realistic assumption is that the channel response amplitudes corresponding to the direct links are significantly lower than those of the links involving the relay node, thus, $|h_{sd_i}| \ll |h_{sr_i}|$ and $|h_{sd_i}| \ll |h_{rd}|$. Finally, notice that the system spends two channel uses to transmit one source symbol per transmit node, i.e. the individual symbol rates are $R_1 = R_2 = 1/2$. Recall that the same rate per user results from the orthogonal transmission of the source information using only the direct links, i.e., considering a MAC with two users and without relay.

The objective is hence to design and optimize the encoding functions $f_1(\cdot)$ and $f_2(\cdot)$ at the transmit nodes, the relaying operation $r(\cdot)$, and the decoding function $g(\cdot)$ at the destination to minimize the MSE between the source and estimated symbols given by (5) for the described MARC scenario. On the one hand, the functions $f_1(\cdot)$ and $f_2(\cdot)$ should be designed to enable a non-orthogonal transmission of the source symbols to both the relay and the destination node. On the other, the relay mapping should generate a useful message to be exploited at the destination together with the direct information. Finally, the decoder must be designed to jointly integrate the two received symbols into the estimation procedure.

III. ZERO-DELAY CODING OVER THE MARC

In the ensuing subsections, we describe the encoding and decoding operations carried out by the proposed scheme in the different nodes of the MARC transmission system, and the steps required to optimize its performance.

A. ENCODING AT THE TRANSMIT NODES

As explained in the previous section, the two transmit nodes send their symbols simultaneously during the same channel use and, therefore, interfere each other at the input of the relay and destination nodes. Hence, the distributed encoding functions $f_1(\cdot)$ and $f_2(\cdot)$ should be chosen to ensure that the receiving nodes are able to separate the information corresponding to each user. Scalar Quantizer Linear Coding (SQLC) is a distributed Joint Source-Channel Coding (JSCC) mapping function which was proposed to transmit bivariate Gaussian information over a Gaussian MAC [25]. This mapping consists in sending a quantized version of the source symbols for the first node, whereas the second node transmits a scaled version of their symbols. Mathematically, this mapping function is given by

$$\begin{aligned} f_1(s_1) &= \alpha_1 \left\lceil \frac{s_1}{\Delta} - \frac{1}{2} \right\rfloor + \frac{1}{2}, \\ f_2(s_2) &= \alpha_2 s_2, \end{aligned}$$

where $\lceil\cdot\rfloor$ represents the round operation and Δ is the quantization step for the first user. The parameters α_1 and α_2 determine the power allocated to each node such that $|f_i(s_i)|^2 \leq T_i \; \forall i = 1, 2$. The performance of the Scalar Quantizer Linear Coding (SQLC) mapping essentially depends on an appropriate optimization of the parameters α_1, α_2 and Δ. We will discuss how to optimize these parameters in Section III-E.

B. DECODING AT THE RELAY

The relaying operation is of the DF type and uses non-linear analog Joint Source-Channel Coding (JSCC) mappings to generate the message to be sent to the destination node. The relay must first decode the received symbol y_r to obtain the estimates $[\tilde{s}_1, \tilde{s}_2]$ for the transmitted source symbols. Different decoding methods have been proposed for SQLC in the literature such as sequential decoding [28] or approximated MMSE estimation using sphere decoding [29]. Both approaches exhibit similar performance for the case of two users but the former has lower computational cost and it is hence preferable in the considered scenario.

Sequential decoding for SQLC mappings consists in computing an estimate of the first symbol by selecting the most likely interval of the quantized space using the received symbol and the correlation information, subtracting the decoded symbol from the received symbols, and finally computing an estimate of the uncoded symbols applying linear MMSE estimation.

In particular, the estimates for the quantized symbols are computed from

$$\tilde{s}_1 = \arg\min ||\mu(q_i) - y_r||^2, \tag{6}$$

where q_i represents the centroid of the i-th interval in the quantized space, whereas $\mu(q_i)$ is the centroid shifted due to the effect of the symbol correlation, and it is given by [28]

$$\mu(q_i) = \alpha_1 q_i (1 + \alpha_2 \rho), \tag{7}$$

with ρ the correlation factor. Note that the received symbol y_r results from the sum of the quantized symbol and the second symbol attenuated by α_2. When the source symbols are uncorrelated, the second symbol can shift the original centroid to both sides with the same probability, so it is reasonable to maintain the centroid for the received symbol in the same position. On the other side, for high correlation levels, we can predict the centroid shift statistically due to the contribution of the second symbol on the received one. Hence, the function $\mu(q_i)$ represents the more likely centroid in the superimposed domain by shifting the original centroid according to the effect of the symbol correlation and the attenuation factor.

Next, the decoded information is subtracted from the received symbol, i.e., $\tilde{y_r} = y_r - h_{sr_1}\alpha_1\tilde{s}_1$, and an estimate of the second symbol is computed with the linear MMSE estimator as

$$\tilde{s}_2 = \frac{\alpha_2 h_{sr_2}}{|\alpha_2 h_{sr_2}|^2 + \sigma_{n_r}} \tilde{y_r}. \tag{8}$$

The estimates $\tilde{s}_1$ and $\tilde{s}_2$ are then used by the relay to produce the message to be sent to the destination node.

C. ENCODING AT THE RELAY

The relay encodes the estimated symbols $[\tilde{s}_1 \; \tilde{s}_2]^T$ into $x_r = r(\tilde{s}_1, \tilde{s}_2)$, the symbol to be sent to the destination, using an appropriate encoding function $r(\cdot)$. The choice of $r(\cdot)$ and the implementation of the decoding operation at destination decisively depend on the relationship between the source and estimated symbols. For this reason, we model such a dependency as follows

$$\tilde{s}_i \approx s_i + e_i, \quad i = 1, 2, \tag{9}$$

where $e_i \sim \mathcal{N}(0, \sigma_{e_i}^2)$ is the i-th source estimation error. These errors are assumed to be independent each other. This assumption holds even when the sources are correlated. The error variances can be approximated by the i-th element in the diagonal of the covariance error matrix assuming linear MMSE estimation. This approximation becomes equality for the second user and it is a good approximation as Δ gets smaller (medium and high Signal-to-Noise Ratio (SNR) values). However, a more accurate approximation for the first user consists in calculating the quantization error assuming that the decoder guesses correctly the quantizer interval corresponding to the transmitted symbol. This can be accomplished by selecting an adequate value for the mapping parameters [29]. In such a case, the error variance is given by

$$\sigma_{e_1}^2 = 2\sum_{i=1}^{\infty}\int_{\Delta i}^{\Delta(i+1)}(s-\delta_i)^2 p(s)ds, \tag{10}$$

where δ_i is the decoded value for the i-th interval, i.e.

$$\delta_i = \int_{a_i}^{b_i} sp(s)ds = \sqrt{\frac{2}{\pi}}\frac{\exp(-a_i^2)-\exp(-b_i^2)}{Q(b_i)-Q(a_i)} \tag{11}$$

with $a_i = \Delta_k i$, $b_i = \Delta_k(i+1)$ and $Q(\cdot)$ the error function. Note that the above expression involves an addition of infinite terms, although the number of intervals with a significant weight in the addition is actually small.

On the other hand, from the approximation in (9) and considering the original correlation between the source symbols, the covariance matrix for the estimated symbols is given by

$$\boldsymbol{C}_{\tilde{s}} = \mathbb{E}[\tilde{\boldsymbol{s}}\tilde{\boldsymbol{s}}^T] = \begin{bmatrix} 1+\sigma_{e_1}^2 & \rho \\ \rho & 1+\sigma_{e_2}^2 \end{bmatrix}. \tag{12}$$

The encoding function employed at the relay will depend on the distribution of its input data. From the approximation in (9), it is clear that the estimated symbols are assumed to follow a Gaussian distribution with a covariance matrix given by (12). Different analog JSCC mappings with compression rate 2:1 have been proposed in the literature including spiral-like mappings, sine-like mappings, line-based mappings, etc. For independent sources, an encoding function based on the Archimedean spiral provides near-optimal performance for Gaussian-distributed symbols and AWGN channels [30], [31]. This spiral can parametrically be defined as

$$z(t) = \left(\mathrm{sign}(t)\frac{\delta_a}{\pi}t\sin(t), \frac{\delta_a}{\pi}t\cos(t)\right), \tag{13}$$

where $z(t)$ represents a point of the curve on the bidimensional space, δ_a is the distance between two neighboring spiral arms, and t is the angle from the origin to the point z. In this case, the mapping function searches the point on the curve defined by (13) which is closest to the point $\tilde{s} = [\tilde{s}_1, \tilde{s}_2]^T$, and provides the angle t corresponding to that point, i.e.,

$$t_o = r(\tilde{s}) = \arg\min_t \|\tilde{s} - z(t)\|^2. \tag{14}$$

A stretching function is often applied to the resulting angle t_o [32], and the resulting value is finally normalized to satisfy the power constraint at the relay node. The resulting normalized message is transmitted to the destination node.

However, the larger the correlation between the input symbols is, the worse the performance of the spiral-like mapping is. In such a situation, a mapping based on sine functions provides lower levels of distortion since it is able to exploit the input correlation efficiently [26]. The parametric expression for the sine-like mapping is given by

$$\mathbf{z}(t) = \mathbf{U}\boldsymbol{\Sigma}^{1/2}\begin{bmatrix} t - \frac{1}{2\alpha}\sin(\alpha t) \\ \delta_s\sin(\alpha t)\end{bmatrix}, \tag{15}$$

where $z(t)$ represents the point into the bidimensional space corresponding to the parameter t on the curve, and the matrices $\mathbf{U}$ and $\boldsymbol{\Sigma}$ are obtained from the eigendecomposition of the covariance matrix $\mathbf{C}_{\tilde{s}} = \mathbf{U}^H\boldsymbol{\Sigma}\mathbf{U}$. The parameters α and δ_s represent the frequency and amplitude of the sinusoidal function, respectively. Using this parametric curve, we can employ the same mapping function as in (14) which searches the closest point on the curve according to the Euclidean distance. The output t_o is then normalized to produce the symbol x_r which will be sent to the destination node.

Notice that the parameters of both mapping functions should be optimized to improve the performance of the proposed MARC scheme. We will discuss how to optimize these parameters in Section III-E.

D. DECODING AT THE DESTINATION NODE

At the destination node, the receiver determines an estimate of the transmitted source symbols $\hat{\boldsymbol{s}} = [\hat{s}_1\ \hat{s}_2]^T$ from y_{sd} and y_{rd}. Since the objective of the communication system is to minimize the MSE distortion between the source and estimated symbols, the MMSE decoder is optimal. The MMSE estimates are computed as

$$\hat{\boldsymbol{s}} = \mathbb{E}\left[\boldsymbol{s}|\boldsymbol{y}\right] = \frac{\int \boldsymbol{s}p(\boldsymbol{y}|\boldsymbol{s})p(\boldsymbol{s})d\boldsymbol{s}}{\int p(\boldsymbol{y}|\boldsymbol{s})p(\boldsymbol{s})d\boldsymbol{s}}, \tag{16}$$

where $\boldsymbol{y} = [y_{sd}\ y_{rd}]^T$ and $p(\boldsymbol{s})$ corresponds to the pdf of a zero-mean Gaussian with covariance $\boldsymbol{C}_{\boldsymbol{s}}$. The conditional probability is given by

$$p(\boldsymbol{y}|\boldsymbol{s}) = \frac{1}{2\pi|\boldsymbol{C}_n|^{1/2}}\exp\left(-\frac{1}{2}[\boldsymbol{y}-\boldsymbol{m}(\boldsymbol{s})]^T\boldsymbol{C}_n^{-1}[\boldsymbol{y}-\boldsymbol{m}(\boldsymbol{s})]\right),$$

where $\boldsymbol{C}_n$ represents the noise covariance at destination and the vector $\boldsymbol{m}$ is

$$\boldsymbol{m}(\boldsymbol{s}) = \begin{bmatrix}\boldsymbol{h}_{sd}^T\boldsymbol{f}(\boldsymbol{s}) \\ h_{rd}r(\boldsymbol{s})\end{bmatrix}, \tag{17}$$

with $\boldsymbol{h}_{sd} = [h_{sd_1}\ h_{sd_2}]^T$ and $\boldsymbol{f}(\boldsymbol{s}) = [f_1(s_1)\, f_2(s_2)]^T$. Assuming that the noise components on the received signals are uncorrelated, $\boldsymbol{C}_n$ will be a diagonal matrix whose first coefficient is directly the variance of the noise component n_1, i.e., $[\boldsymbol{C}_n]_{1,1} = \sigma_{nd}^2$.

From the approximation in (9), it is possible to observe that we have an additional source of distortion on the signal received over the link involving the relay node: the decoding error at the relaying operation. In such a case, we can use the first-order Taylor polynomial to approximate the result of encoding a noisy input with a given mapping function $r(\cdot)$ as

$$r(\tilde{\boldsymbol{s}}) \approx r(\boldsymbol{s}+\boldsymbol{e}) \approx r(\boldsymbol{s}) + \frac{\mathrm{d}r(t)}{\mathrm{d}\boldsymbol{s}}\boldsymbol{e}, \tag{18}$$

where the right term represents the distortion component on the relay message as a consequence of the SQLC decoding errors, and $\boldsymbol{e} = [e_1\ e_2]^T$. Notice that the two-dimensional derivative

$$\frac{\mathrm{d}r(t)}{\mathrm{d}\boldsymbol{s}} = \left[\frac{\mathrm{d}r(t)}{\mathrm{d}s_1}\ \frac{\mathrm{d}r(t)}{\mathrm{d}s_2}\right]$$

measures the impact of varying the input symbols s_1 and s_2 on the resulting encoded value t_o. Unfortunately, the derivative of the mapping function $r(\cdot)$ in (14) is not trivial because it results from a minimization problem and there is not a closed-form expression for the solution. However, we can circumvent this problem by using an implicit expression for the optimal parameter t_o given a pair of input symbols, and then applying implicit differentiation. On the one hand, we rewrite the function to be minimized as

$$c(t) = \|\boldsymbol{s} - \boldsymbol{z}(t)\|^2 = \boldsymbol{s}^T\boldsymbol{s} + \boldsymbol{z}(t)^T\boldsymbol{z}(t) - 2\boldsymbol{s}^T\boldsymbol{z}(t). \tag{19}$$

whose derivative is given by

$$\frac{\mathrm{d}c(t)}{\mathrm{d}t} = (s_1 - z_x(t))z_x'(t) + (s_2 - z_y(t))z_y'(t), \tag{20}$$

with $\boldsymbol{z}(t) = [z_x(t)\ z_y(t)]^T$, and $z_x'(t)$ and $z_y'(t)$ the corresponding derivatives of $\boldsymbol{z}(t)$ in the components x and y of the bidimensional space. An implicit expression can be obtained for the resulting mapped symbol, t_o, considering that the previous derivative must be zero at that point, i.e.,

$$(s_1 - z_x(t_o))z_x'(t_o) + (s_2 - z_y(t_o))z_y'(t_o) = 0. \tag{21}$$

Applying implicit differentiation to the above expression with respect to s_1 and s_2, we obtain

$$\begin{aligned} z_x'(t_o) &+ \frac{\mathrm{d}t_o}{\mathrm{d}s_1}\left(z_x''(t_o)(s_1 - z_x(t_o)) - z_x'(t_o)^2\right) \\ &+ \frac{\mathrm{d}t_o}{\mathrm{d}s_1}\left(z_y''(t_o)(s_2 - z_y(t_o)) - z_y'(t_o)^2\right) = 0 \\ &\times z_y'(t_o) + \frac{\mathrm{d}t_o}{\mathrm{d}s_2}\left(z_y''(t_o)(s_2 - z_y(t_o)) - z_y'(t_o)^2\right) \\ &+ \frac{\mathrm{d}t_o}{\mathrm{d}s_2}\left(z_x''(t_o)(s_1 - z_x(t_o)) - z_x'(t_o)^2\right) = 0, \end{aligned}$$

where $z_x''(t)$ and $z_y''(t)$ represent the second derivatives of $\boldsymbol{z}(t)$. From the above equations, we can solve for $\frac{\mathrm{d}t_o}{\mathrm{d}s_1}$ and $\frac{\mathrm{d}t_o}{\mathrm{d}s_2}$, and define

$$\frac{\mathrm{d}r(t)}{\mathrm{d}\boldsymbol{s}} = \left[\frac{\mathrm{d}r(t)}{\mathrm{d}s_1}\ \frac{\mathrm{d}r(t)}{\mathrm{d}s_2}\right] = \left[\frac{\mathrm{d}t_o}{\mathrm{d}s_1}\ \ \frac{\mathrm{d}t_o}{\mathrm{d}s_2}\right]. \tag{22}$$

Finally, using the approximation in (18), the received symbol from the relay can be expressed as

$$y_{rd} = h_{rd}\left(r(\boldsymbol{s}) + \frac{\mathrm{d}r(t)}{\mathrm{d}\boldsymbol{s}}\boldsymbol{e}\right) + n_2, \tag{23}$$

and, hence, the variance of the noise component corresponding to y_{rd} will be

$$[\boldsymbol{C}_n]_{2,2} = h_{rd}^2\left(\frac{\mathrm{d}r(t)^2}{\mathrm{d}\boldsymbol{s}}\boldsymbol{\sigma}_e^2\right) + \sigma_{n_d}^2. \tag{24}$$

with $\boldsymbol{\sigma}_e^2 = [\sigma_{e_1}^2\ \sigma_{e_2}^2]^T$. Notice that the normalization factor has been omitted for simplicity but it can be integrated easily into the previous expression. The last step consists in computing the MMSE estimates for the source symbols by solving the integrals in (16) with Monte Carlo techniques and using the appropriate noise covariance given by

$$\boldsymbol{C_n} = \begin{bmatrix} \sigma_{n_d}^2 & 0 \\ 0 & h_{rd}^2\left(\frac{\mathrm{d}r(t)^2}{\mathrm{d}\boldsymbol{s}}\boldsymbol{\sigma}_e^2\right) + \sigma_{n_d}^2 \end{bmatrix}. \tag{25}$$

E. ENCODING OPTIMIZATION

As explained previously, the optimization of the encoding functions employed at both the transmit and relay nodes is a fundamental issue.

The optimization of the SQLC parameters can be carried out using the algorithm presented in [29]. This optimization procedure determines the optimal values for Δ, α_1 and α_2 depending on the source correlation, the channel gains and the noise variance. On the one hand, the algorithm selects the minimum value for the quantization step Δ and the corresponding attenuation factor for the second user, α_2, which ensure that the decoding operation does not fail while minimizing the sum-distortion. This is not a trivial decision because decreasing Δ and increasing α_2 would lead to lower individual distortion, but the network nodes actually receive a single symbol which results from the addition of the two transmitted symbols together with the noise component. In such a case, the SQLC decoder will break down when the summation of the second symbols and the noise causes the first symbol crosses to an adjacent quantization interval. Hence, it is essential to set the attenuation factor α_2 to a value which minimizes the probability of a crossing event. Regarding the power allocation factor α_1, it is chosen to ensure that the first user transmits its symbols with all the available power, and its value will directly depend on the quantization step Δ.

Note that in the case of the MARC, the set of SQLC parameters can be optimized for the direct links or for the source-relay link. Taking into account the assumption $|h_{sd_i}| \ll |h_{sr_i}|$ and $|h_{sd_i}| \ll |h_{rd}|$, the latter alternative will usually be preferable although, in the general case, we should choose those paths which provide the lowest MSE distortion.

Assuming that the SQLC parameters Δ, α_1 and α_2, are appropriately optimized in such a manner that the SQLC decoder is able to guess correctly the quantization interval, an upper bound of the distortion for the direct links is given by [29]

$$\phi_{\mathrm{DL}} = \sigma_{e_1}^2(\Delta^{\mathrm{DL}}, \alpha_1^{\mathrm{DL}}) + \sigma_{e_2}^2(\alpha_2^{\mathrm{DL}}), \tag{26}$$

TABLE 1. Optimization steps for the proposed MARC scheme.

Node	Step	Procedure
Tx Nodes	1. Optimize SQLC parameters $\Delta, \alpha_1, \alpha_2$ for DL and RL (see [29])	Solving a minimization problem with an adequate method
	2. Compute overall distortion for DL and RL using (26) and (27)	Closed-form expressions
	3. Select the set of parameters corresponding to the best path (28)	Closed-form expressions
Relay Node	1. Select the mapping from the estimated covariance matrix in (12)	Closed-form expressions
	2. Optimize the parameter δ_a for the Archimedean spiral (or)	Off-line exhaustive search
	Optimize the parameters δ_s, α for the sine-like mapping (see [33])	Solving a minimization problem with an adequate method

where $\sigma_{e_1}^2(\Delta^{\mathrm{DL}}, \alpha_1^{\mathrm{DL}})$ is the average distortion for the quantized symbols given by (10), and $\sigma_{e_2}^2(\alpha_2^{\mathrm{DL}})$ is the average distortion for the uncoded symbols considering linear MMSE estimation. Notice that the above upper bound implicitly depends on α_1^{DL} since this parameter must be chosen also to ensure a correct decoding of the quantized symbols.

A similar distortion bound can be computed for the path from the sources to the destination via the relay node. We again assume that the SQLC parameters are optimized to guarantee the SQLC decoder is able to guess correctly the quantization interval but, in this case, this optimization is accomplished considering the source-relay channel responses, h_{sr_1} and h_{sr_2}, instead of the responses corresponding to the direct channels. We now distinguish three different contributions to the overall distortion: the approximation error of the mapping function $r(\cdot)$, the channel errors due to the relay-destination link, and an additional distortion induced by the estimation errors of the relay decoding. The two first sources of distortion can be approximated using the expressions presented in [34] for the mapping based on the Archimedean spiral, and with the expressions obtained in [26] for the sine-like mappings. Finally, the distortion due to the SQLC decoding errors can be approximated by

$$\phi_{de} = h_{rd}^2 \left(\sigma_{e_1}^2(\Delta^{\mathrm{RL}}, \alpha_1^{\mathrm{RL}}) \mathbb{E}\left[\frac{\mathrm{d}r(t)^2}{\mathrm{d}s_1} \right] + \sigma_{e_2}^2(\alpha_2^{\mathrm{RL}}) \mathbb{E}\left[\frac{\mathrm{d}r(t)^2}{\mathrm{d}s_2} \right] \right)$$

which can be deduced directly from (23) considering the approximation to the decoding errors given by (18). Thus, the distortion expected at the destination when the SQLC parameters are optimized for the source-relay links is given by

$$\phi_{\mathrm{RL}} = \phi_{ap} + \phi_{ch} + \phi_{de}, \tag{27}$$

where ϕ_{ap} and ϕ_{ch} represent the approximation and the channel distortion, respectively, and they both depend on the parameters of the relay mapping $r(\cdot)$. Hence, we select the set of SQLC parameters corresponding to the path with lower distortion, i.e.

$$[\Delta, \alpha_1, \alpha_2] = \begin{cases} [\Delta^{\mathrm{DL}}, \alpha_1^{\mathrm{DL}}, \alpha_2^{\mathrm{DL}}] & \phi_{\mathrm{DL}} \leq \phi_{\mathrm{RL}} \\ [\Delta^{\mathrm{RL}}, \alpha_1^{\mathrm{RL}}, \alpha_2^{\mathrm{RL}}] & \phi_{\mathrm{DL}} > \phi_{\mathrm{RL}}. \end{cases} \tag{28}$$

In the case of the mapping function employed for the relay to encode its message, the parameters can be optimized following two approaches: an exhaustive search or an optimization procedure using the analytical expressions for the approximation and channel distortions. In general, the former strategy provides a slight performance gain at the expense of a considerable increase in the computational cost of the optimization. However, the optimal parameters of the mapping function can be obtained off-line for a range of SNRs, and then the parameter values are selected from this set of optimal values according to the channel response h_{rd}. In any case, an important feature of non-linear continuous mappings is their graceful degradation when using suboptimal parameters as we will show in the ensuing section.

Table 1 summarizes the set of steps required at the transmit and relay nodes to optimize the proposed scheme.

F. EXTENSION TO OTHER MARC SCENARIOS

In this section, we discuss how the system design explained in the previous sections could be extended to other MARC configurations with a larger number of users, relays or available time slots.

At the transmit nodes, the following encoding strategy can be carried out:

- We can consider Distributed Quantizer Linear Coding (DQLC mappings which are the natural extension of the SQLC ones for an arbitrary number of users [25], [29]. In such a case, all the transmitters would send their symbols simultaneously in the same time slot.
- In the general case, if the number of nodes is K and we have S_t available time slots to send the source symbols, we could define S_t clusters with K/S_t nodes each one, apply Distributed Quantizer Linear Coding (DQLC) to encode the K/S_t source symbols at each cluster and transmit the encoded symbols of each cluster in a time slot.

At the relay nodes, the symbols received from each cluster are individually decoded using the strategy proposed in [29] for DQLC mappings and any number of users. After the decoding phase, we obtain an estimate of the K source symbols at each relay. The next step is the modeling of the covariance matrix for the estimated symbols and the decoding errors following the same approach described in Section III-C.

The encoding operation at the relays will again depend on the number of available time slots and the type of channel access strategy. For example, assuming orthogonal transmission and S_r slots for each relay, it would be required to employ a mapping function $r_i : \mathbb{R}^K \rightarrow \mathbb{R}^{S_r}$ at the i-th relay node. In this case, the extensions for the Archimedean spiral [32] and for the sine-like mappings [35] could be considered to achieve higher compression degrees.

Finally, at the destination node, the proposed MMSE estimator could be extended by defining the noise covariance matrices corresponding to each possible path from the sources to the destination. In general, these derivations will be more cumbersome for relaying functions of higher dimensions, but the procedure described in Section III-D which involves the use of implicit differentiation would be applicable for any differentiable mapping function.

It is worth remarking that the number of potential MARC scenarios is very extensive depending on the specific configuration regarding the number of transmitters, relays and time slots employed at each transmission phase. We have sketched out the main ideas to extend the proposed approach to a general MARC scenario, but the specific details of the system design will depend on the particular network configuration.

IV. PERFORMANCE EVALUATION

In this section, the results of several computer simulations are presented to illustrate the performance of the proposed communication scheme in different MARC scenarios. In the computer experiments, a matrix of $2 \times N$ source symbols is first generated from a bivariate Gaussian distribution with zero-mean and covariance matrix given by (1). Each pair of symbols is encoded by using a SQLC mapping where the parameters are optimized as explained in Section III-E. The resulting encoded symbols are broadcast to the destination and to the relay in the first channel use. At the relay node, the received symbol is decoded to obtain an estimate of the transmitted symbols. Then, the obtained estimates are re-encoded using either the Archimedean spiral mapping or the sine-like mapping, depending on the correlation level between the estimates. The symbol produced by the relaying operation is sent to the receiver over the relay-destination link in the second channel use. Finally, the receiver obtains an MMSE estimate of the source symbols using the information received from the direct links and the relay, and computes the MSE for the N pairs of source symbols.

Without loss of generality, we assume the different noise components have unit variance and the power constraints at both the transmit nodes and the relay are also equal to 1, i.e., $T_1 = T_2 = T_r = 1$. Hence, the relevant parameters in the simulations are the average gains of the different links between the system nodes. Let us denote $\bar{h}_{rd} = 10\log_{10}(\sigma_{rd}^2)$ as the average power of the relay-destination link in decibels. In this case, the different realizations for the relay-destination channel are generated from a Rayleigh distribution such that the average power is σ_{rd}^2. For the remaining links, we will use a similar notation and procedure to generate the channel realizations. For simplicity, we focus on the symmetric case such that $\bar{h}_{sd} = \bar{h}_{sd_1} = \bar{h}_{sd_2}$ and $\bar{h}_{sr} = \bar{h}_{sr_1} = \bar{h}_{sr_2}$, but an asymmetric configuration could be considered without modifying the proposed scheme.

In general, the figures of this section will compare the performance of the proposed scheme to that of other suitable communication strategies for the MARC. Since we are interested in the transmission of discrete-time continuous-amplitude symbols, the performance is measured in terms of the Signal-to-Distortion Ratio (SDR), which is computed as

$$\mathrm{SDR} = 10\log_{10}\left(\frac{1}{\xi}\right), \tag{29}$$

where ξ is the MSE between the source symbols and their corresponding estimates at the destination, computed as in (5).

A. PERFORMANCE BOUND FOR CF

The proposed DF-like relaying operation is based on the use of non-linear continuous mappings which individually re-encode each pair of estimated symbols obtained after the decoding phase. The complexity and delay of this strategy are significantly lower than that of other well-known relaying approaches such as DF or CF which are based on theoretical codes using large blocks of symbols. However, it might be interesting to consider these strategies as a benchmark for the performance of the proposed scheme.

The theoretical performance bound for CF-based systems is computed assuming source-channel separation, i.e., the rate distortion function of the source is equated to the sum-rate achievable with CF in the considered scenario. This bound represents the best performance achievable for any communication scheme designed according to the separation principle and based on CF. Although the source-channel separation is not optimal for the considered scenario, the resulting bound is a suitable benchmark to evaluate the performance of the proposed scheme considering that CF is able to provide the theoretical optimal rate for separation and it is widely employed in the literature. In addition, the minimum achievable distortion region for the MARC is in general unknown.

On the one hand, the rate distortion function for multivariate Gaussian sources can be represented parametrically as [36]

$$D(\theta) = \frac{1}{M}\sum_{i=1}^{M} \min[\theta, \lambda_i], \tag{30}$$

$$R(\theta) = \frac{1}{M}\sum_{i=1}^{M} \max\left[0, \frac{1}{2}\log\left(\frac{\lambda_i}{\theta}\right)\right], \tag{31}$$

where $D(\theta)$ is the distortion function, λ_i represents the eigenvalues of the covariance matrix C_s, and M is given by the number of eigenvalues larger than zero. For the particular case of a bivariate Gaussian distribution, $M = 2$, $\lambda_1 = 1 + \rho$ and $\lambda_2 = 1 - \rho$.

On the other hand, the sum-rate obtained with a comparable CF scheme can be derived directly from [37, Proposition 3] for the considered scenario assuming that the

CF system spends two channel uses to transmit each pair of source symbols, and that the transmit nodes only send their information during the first channel use. In that particular case, the achievable sum-rate is given by

$$R_{\mathrm{CF}}^{\mathrm{sum}} = \frac{1}{8}\log_2\left(1 + h_{sd_1}^2 + \frac{h_{sr}^2}{1+\sigma_{w_1}^2}\right) \tag{32}$$

$$+\frac{1}{8}\log_2\left(1 + h_{sd_2}^2 + \frac{h_{sr}^2}{1+\sigma_{w_2}^2}\right), \tag{33}$$

with

$$\sigma_{w_i}^2 = \frac{1 + h_{sd_i}^2 + h_{sr}^2}{h_{rd_i}^2\left(1 + h_{sd_i}\right)}.$$

Equation (33) determines the maximum number of bits which can be transmitted at each channel use with an arbitrarily low probability of error, whereas (31) determines the minimum number of bits required to achieve the distortion given by the parameter θ. Since we are transmitting two source symbols in two channel uses, (31) and (33) can be equated directly, searching for the parameter θ which satisfies such equality. The resulting distortion is finally determined by replacing the resulting θ into the distortion function (30).

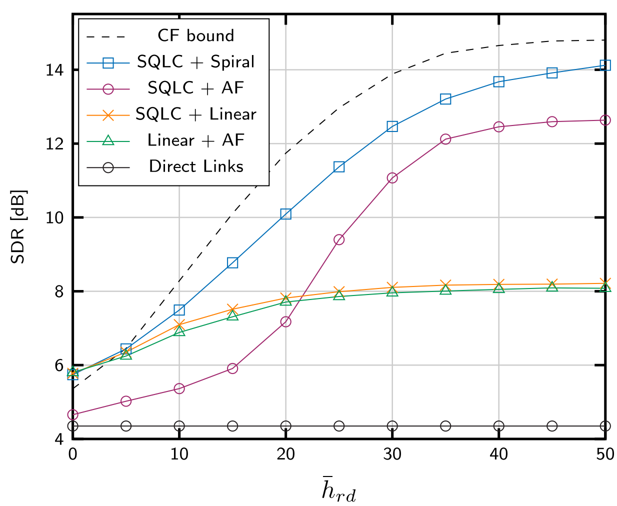

FIGURE 2. **Performance of different transmission schemes for independent sources over the MARC with $\bar{h}_{sr}$ = 30 dB, $\bar{h}_{sd}$ = 5 dB, and different channel gains for $\bar{h}_{rd}$.**

B. SIMULATION RESULTS

In the first simulation experiment, we measure the SDR obtained for the proposed scheme in a MARC with $\bar{h}_{sd}$ = 5 dB, $\bar{h}_{sr}$ = 30 dB and $\bar{h}_{rd}$ ranging from 0 to 50 dB. The source symbols are assumed to be independent ($\rho = 0$). Fig. 2 compares the performance of the proposed scheme to that of four alternative schemes: 1) SQLC + AF, 2) SQLC + DF based on linear mappings, 3) Linear encoding + AF, and 4) Orthogonal transmission over the direct links.

The two first approaches have in common the use of SQLC mappings to encode the source information, while they differ in the type of relaying operation. In the former approach we simply apply AF to the received symbol at the relay, whereas in the case of DF based on linear encoding, an estimate of the source symbols is first computed at the relay using the SQLC decoder explained in Section III-B. Those estimates are then merged into one single message by summing them and scaling the resulting symbol to satisfy the relay power constraint. At the destination node, linear MMSE estimation is applied considering the approximation for the decoding error in (9) to determine the noise covariance matrix. In the third approach, the transmit nodes send a scaled version of their source symbols, and the relay node simply forwards the received symbol according to its power constraint. Finally, the last strategy consists in sending the two source symbols only over the direct links and using two different channel uses (orthogonal transmission), i.e., ignoring the relay. Notice that the symbol rate of all these schemes is the same: two symbols are transmitted in two channel uses. The theoretical performance of a CF-based scheme is computed following the steps explained in Section IV-A, and it is also included in Fig. 2.

As observed, the best performance is achieved by the proposed scheme based on the combination of the SQLC to encode the source information and non-linear continuous mappings for the relaying operation. On the one hand, the intrinsic non-linearity of SQLC mappings provides certain gain with respect to applying linear coding at the transmit nodes, especially as the quality of the relay-destination link gets higher. On the other hand, the benefits of the proposed relaying operation can also be observed in Fig. 2 since the performance of the other two schemes with SQLC is inferior in terms of SDR. The relaying operation based on linear mappings clearly saturates for $\bar{h}_{rd} > 20$ dB, whereas SQLC + AF provides a good performance for medium and high values of $\bar{h}_{rd}$, although its performance is rather poor for lower values. This degradation is more significant when the quality of the relay-destination link is very low because the AF operation is not able to deal with high levels of noise. In this case, the destination node basically receives a noisy version of the SQLC symbols which is not able to decode. In addition, the performance of this strategy ends up saturating for high values of $\bar{h}_{rd}$ since it is limited by the quality of source-relay links. This behavior is not observed when using non-linear DF with the spiral-like mapping because the information is first decoded to filter the noise introduced in the source-relay transmission, and then re-encoded to exploit the good quality of the relay-destination link. Notice that this improvement is achieved at the expense of increasing the computational cost of the relaying operation.

Another interesting conclusion from Fig. 2 is that the different strategies considered for the MARC provide better performance than the case of an orthogonal transmission of the source symbols only over the direct links. Thus, the incorporation of the relay node is justified in this scenario because of the low quality of the direct links, $\bar{h}_{sd}$ = 5 dB. Finally, the proposed scheme approaches the theoretical performance of the CF for all the range of values considered for $\bar{h}_{rd}$, and even it is able to exceed this theoretical limit when the quality of the relay-destination link is very poor. At this point, it is worth paying attention to two important aspects of the CF bound: 1) it is computed assuming the use of theoretical codes with infinite block size; and 2) the CF scheme is accommodated to work in a similar way to the proposed scheme, which is not necessarily the best we can do with CF.

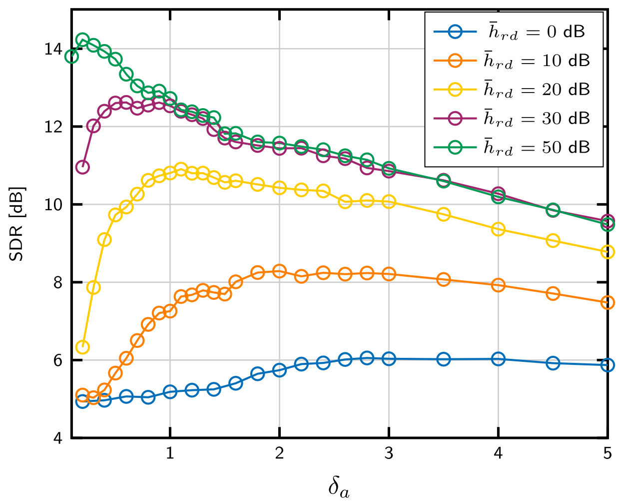

FIGURE 3. Performance of the proposed scheme based on SQLC and spiral mappings depending on the δ_a parameter for $\bar{h}_{sr} = 30$ dB, $\bar{h}_{sd} = 5$ dB, and different gains $\bar{h}_{rd}$.

Fig. 3 shows the performance obtained for the proposed scheme with SQLC + Archimedean spiral when varying the spiral parameter δ_a, considering the same scenario as in the previous experiment, optimal parameters for SQLC mapping, and different $\bar{h}_{rd}$ channel gains for the relay-destination link. As expected, the obtained results show that small values of δ_a are recommendable when the link quality is high, while larger values for this parameter provide better performance as the link quality gets worse. This is motivated because the shape of the Archimedean spiral must be designed to prevent that the symbols mapped to a given branch of the spiral cross to a different branch due to the effect of the channel noise (threshold effect). If the channel quality is good enough, the spiral curve can be made denser or, equivalently, the value of the parameter δ_a can be decreased. The opposite happens when the link quality is poor and small values of δ_a would imply a high probability of branch crossing. In fact, almost the same optimal values are obtained in this scenario as in the case of point-to-point AWGN channels [23]. However, the most interesting conclusion of this experiment is the slight impact of using suboptimal values for the parameter δ_a on the overall performance of the MARC scheme. As observed, when choosing δ_a equal to 1, the proposed scheme virtually obtains the optimal performance for most channel gains, although a small loss (about 1.5 dB) is observed when $\bar{h}_{rd} =$ 50 dB.

The third computer experiment aims at evaluating the impact of varying the quality of the source-destination link,

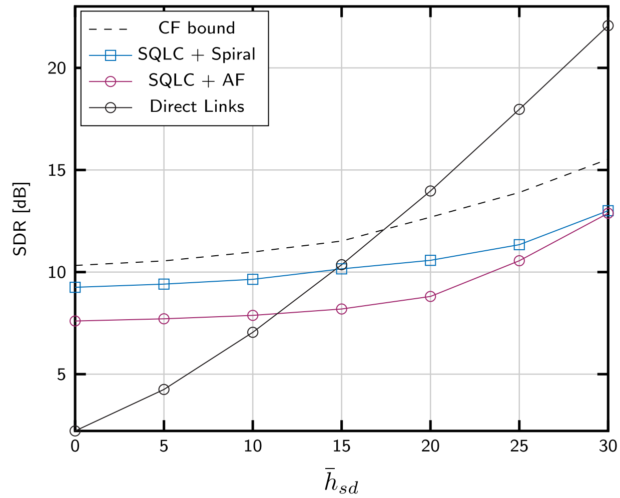

FIGURE 4. Performance of different transmission schemes for independent sources over the MARC with $\bar{h}_{sr} = 20$ dB, $\bar{h}_{rd} = 30$ dB, and different channel gains for $\bar{h}_{sd}$.

while the channel gains for the other two links remain constant. Fig. 4 compares the performance of the two strategies which provide better results in the previous scenario: SQLC + AF and SQLC + Archimedean spiral. The theoretical performance achieved with CF and the SDR curve for the orthogonal transmission over the direct links are also included in Fig. 4 for comparison. The source symbols are assumed to be uncorrelated ($\rho = 0$), $\bar{h}_{sr}$ is set to 20 dB, $\bar{h}_{rd}$ is set to 30 dB, and $\bar{h}_{sd}$ ranges from 0 to 30 dB. From the obtained results, it is clear that the use of the relay node does not make sense beyond 15 dB, since the direct transmission of the information provides better results and the performance gain significantly grows as the quality of the direct links improves. The problem of the SQLC-based systems is that their performance is limited by the quality of the source-relay link. When the source-destination links have enough quality, it is preferable to disregard the relay path and to spend the two channel uses on sending each source symbol orthogonally instead of wasting one channel use on sending the two symbols simultaneously to the relay which results in a larger distortion. This is an interesting conclusion although the scenario considered in this experiment is not common in practice. In any case, it will be easy to determine the point from which we can ignore the relay, considering that the expected distortion of the orthogonal transmission is well-known. Hence, we can compare this expected distortion to the distortion bound given by (27) for the proposed scheme.

Finally, Fig. 5 compares the same schemes as in the first experiment for the same MARC scheme, but now considering correlated sources. In particular, the correlation factor is $\rho =$ 0.9, the channel gain for the relay-destination link ranges from 0 to 50 dB, and the qualities for the other two links are $\bar{h}_{sr} = 30$ dB and $\bar{h}_{sd} = 5$ dB. As observed, results similar to the case of uncorrelated sources are obtained, although the performance gain of non-linear mappings with respect to the

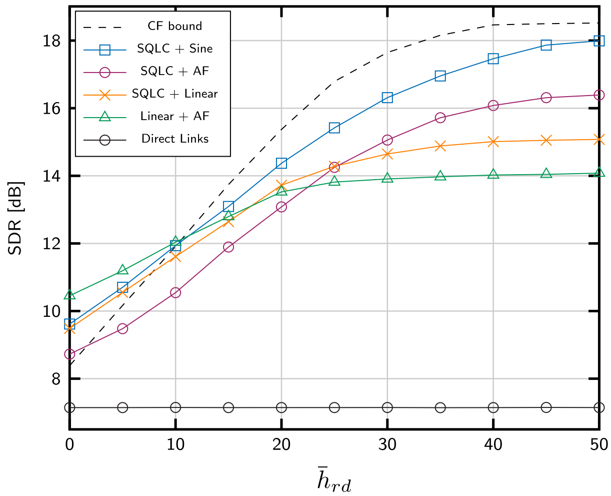

FIGURE 5. **Performance of different transmission schemes for correlated sources over the MARC with $\rho = 0.9$, $\bar{h}_{sr} = 30$ dB, $\bar{h}_{sd} = 5$ dB, and different channel gains for $\bar{h}_{rd}$.**

strategies which apply linear coding at the source or relay nodes is less significant. In fact, linear transmission at the sources with AF relaying is able to obtain a slight gain with respect to the proposed scheme at the low Signal-to-Noise Ratio (SNR) regime (namely for $\bar{h}_{sd} \leq 10$ dB). This behavior is reasonable since it is well-known that linear coding is optimal for those SNR values lower than a given threshold when sending correlated sources [38]. Moreover, this threshold is increasingly larger as the correlation between the source information increases. In practice, this implies that the linear strategies saturate for larger SNR values and the effect of using non-linear mappings is more visible in that regime. Finally, it is interesting to highlight that the proposed scheme continues to closely approach the CF-based bound and it significantly improves the strategy disregarding the relay node and transmitting only over the direct links.

V. CONCLUSIONS

The problem of transmitting discrete-time continuous-amplitude symbols over a MARC using practical zero-delay encoding schemes has been considered. The proposed solution consists in combining the use of non-linear mappings to encode the information at the transmit and relay nodes, and an approximation to the optimal MMSE decoder at the final destination node. This solution has been shown to provide better performance than other techniques with similar characteristics, and to closely approach the performance of other theoretical strategies which require a significantly higher delay and computational cost. The paper focuses on the case of two transmit nodes, although it can be extended to consider a larger number of transmitters by adapting the non-linear mapping operations or expanding the number of available time slots.

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