

Bit Error Probability and Capacity Bound of OFDM Systems in Deterministic Doubly-Selective Channels

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Abstract—Doubly-selective channels, such as those that occur when the transmitter and the receiver move relative to each other at high speeds, are a key scenario for fifth generation (5G) cellular systems, which are mostly based in the use of the orthogonal frequency-division multiplexing (OFDM) modulation. In this paper, we consider an OFDM system using quadrature amplitude modulation (QAM) symbols and we show that, when transmitting over deterministic doubly-selective channels, the inter-carrier-interference (ICI) affecting a symbol can be well approximated by a complex-valued normal distribution. Based on this, we derive a lower bound for the link capacity using the Shannon-Hartley theorem. Finally, we provide an approximation of the bit error probability (BEP) using the well-known BEP expressions for Gray-coded QAM constellations over additive white Gaussian noise (AWGN) channels, and show numerical results that confirm that the proposed BEP expression approximates accurately the bit error ratio (BER) of the OFDM system for standardized channel models. The proposed closed-form analytical expressions for the capacity and the BEP do not only allow for discarding the need of computationally-costly Monte-Carlo system simulations, but also provide a theoretical framework to optimize the system parameters directly impacting on the achievable performance.

Index Terms—Bit error probability (BEP), orthogonal frequency-division multiplexing (OFDM), doubly-selective channels, high-speed communications

I. INTRODUCTION

SOME of the most successful standards for high-speed wireless data communications, such as Wi-Fi, Long-Term Evolution (LTE), or the fifth generation (5G) cellular systems, among many others, rely on orthogonal frequency-division multiplexing (OFDM) to transmit information over the air. This widespread adoption of OFDM was prompted by its large number of advantages such as its robustness against multipath propagation and its relatively low complexity.

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This work has been funded by the Xunta de Galicia (ED431G2019/01), the Agencia Estatal de Investigación of Spain (TEC2016-75067-C4-1-R, PID2019-104958RB-C42), ERDF funds of the EU (AEI/FEDER, UE), and the National Natural Science Foundation of China (NSFC) under Grants no. 61850410529 and 61971313.

In OFDM the information is transmitted in parallel over a group of orthogonal narrowband subcarriers, employing conventional single-carrier modulation techniques such as quadrature amplitude modulation (QAM) or phase shift keying (PSK). The subcarriers used in OFDM are just harmonic complex-valued exponentials, hence OFDM systems are implemented in practice very efficiently by using the inverse fast Fourier transform (IFFT) for the modulator and the fast Fourier transform (FFT) for the demodulator [1]. OFDM systems are able to maintain the orthogonality of the subcarriers in time-invariant multipath channels, i.e., in frequency-selective channels, very easily by using a cyclic prefix (CP) [1]. In these cases, assuming that the maximum delay of the channel impulse response does not exceed the CP length, the effect of the channel for each subcarrier comes down to a multiplication by a complex number, namely the coefficient of the channel impulse response in the frequency domain over that subcarrier.

However, in time-varying channels, the multipath components of the channel vary over time, thus destroying the orthogonality of the OFDM subcarriers and introducing inter-carrier-interference (ICI) in the received signal. Hence, in these channels the performance of OFDM systems can degrade significantly. When a channel is both time- and frequency-selective it is usually referred to as a doubly-selective channel. Such channels occur in scenarios where the receiver and the transmitter move relative to each other, for example in high-speed train communications [2]–[6], vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications [7]–[11], or underwater acoustic communications [12].

One of the requirements for 5G cellular systems, as defined by the International Telecommunication Union (ITU), is to support several mobility classes [13], ranging from stationary up to high speed vehicular with a maximum speed of 500 km/h. In this regard, two important scenarios are the high speed train communications [14] and the unmanned aerial vehicles (UAV) communications [15]. The importance of such cases for 5G systems was acknowledged by the 3rd Generation Partnership Project (3GPP) which has recently approved several study and work items for both use cases [16]–[18]. Moreover, we can expect that high speed scenarios such as the aforementioned ones, and also novel ones, will be more and more important for the future sixth generation (6G) communication networks [19].

When assessing the performance of communication systems, one of the most used metrics is the bit error probability (BEP). To analyze the BEP one must first consider a suitable channel model for the desired use case being considered.

We can distinguish the following two classes of channel models [20]:

- *Stochastic channel models* which usually assume that the channel is wide-sense stationary (WSS) or WSS with uncorrelated scatterers (WSSUS), and that the statistics of some of the system functions, such as the channel impulse response, are known.
- *Deterministic channel models*¹ where the channel impulse response (CIR) is known. In practice, the CIR may come from realizations of some statistical channel model (e.g., a tapped delay line model), from measurements, or by solving Maxwell's equations (or some approximation) for a given environment.

There exist previous studies which analyze the performance of OFDM in terms of the BEP for stochastic doubly-selective channels. In this regard, the performance of OFDM was analyzed under the assumption of Gaussian ICI in [21]–[24]². However, for stochastic doubly-selective channels the probability density function (PDF) of the ICI is not Gaussian, but a weighted Gaussian mixture [24], [25]. In this case, a Gaussian assumption allows for obtaining an approximation of the actual OFDM performance, but it can actually lead to relatively large BEP estimation errors [24]. In [25], the authors approximated the actual PDF of the ICI for a stochastic doubly-selective channel by means of a truncated Gram-Charlier series representation, which allowed them to obtain a more accurate expression (albeit much more complex) for the BEP. Nevertheless, all these previous works only consider Rayleigh channel models.

Stochastic channel models are usually preferred for design and comparison of wireless systems, whereas *deterministic channel models* are more suitable for network planning and system deployment, where we must consider the site specific channel impulse responses [20]. In this regard, *deterministic channel models* are used in link [26]–[28] and system level [29], [30] simulations to determine performance metrics such as the bit error ratio (BER) or the throughput for specific deployments of wireless systems. Recall that the BER is calculated as the number of erroneous bits divided by the number of total bits transmitted. Hence, for a sufficiently large number of transmitted bits, the BER becomes an approximation of the BEP. For large complex systems, the BER can be obtained by means of Monte-Carlo simulations, but this is a very computing-intensive process. Significant computational and time savings can be obtained if analytical expressions are used to determine exact or close approximations of the BEP.

An approach considered by some of the previous works to calculate the BEP over stochastic channels is to average the BEP over several channel realizations, assuming that for each single channel realization the ICI over each symbol is Gaussian when the number of subcarriers is large due to the central limit theorem (CLT) [21]–[23]. These works showed that the Gaussian assumption for individual channel realizations can yield accurate results of the BEP when averaging

over Rayleigh channels. However, if we want to employ this approach for deterministic channels, i.e., when not considering averaging over several channel realizations, several questions and problems arise that need to be investigated. Thus, this paper presents several contributions which can be summarized in the following points:

- 1) We show in Section III that, even if we consider a large number of subcarriers, the conditions to invoke the CLT are actually not fulfilled, and the ICI is not Gaussian.
- 2) Next, we show in Section III that a Gaussian distribution may still be a good approximation for the ICI, but the goodness of such approximation will depend on the constellation order of the transmit symbols (approximation improves with larger orders) as well as the subcarrier position (approximation gets worse on the border subcarriers).
- 3) Based on the Gaussian assumption of the ICI, an approximation for the BEP for deterministic channels as well as a bound on the capacity can be easily obtained from the well-known formulas for the additive white Gaussian noise (AWGN) channel. We show this in Section IV.
- 4) The Gaussian assumption may lead to good results when averaging the BEP for Rayleigh channels. However, since the ICI is not exactly Gaussian it is not clear if good approximations can be also obtained for the case of deterministic channels, e.g., single channel realizations. Thus, we devote Section V-A to characterize the goodness of the BEP approximation for deterministic channels by means of the error quotient $\rho = \text{BER}/\text{BEP}$, where the BER is obtained by means of Monte-Carlo simulations. We show that for low signal-to-interference-plus-noise ratio (SINR) values we obtain good approximations (the error quotient is close to 1), whereas for larger SINR values, the approximation becomes worse but is always an upper bound of the BER (the error quotient is less than 1).
- 5) Finally, we calculate in Section V-B the average BEP for a stochastic channel by averaging the BER over several channel realizations, in a similar way as in previous works [21]–[23]. Different from those works, we take into account that the Gaussian approximation of the ICI depends on the constellation order and the subcarrier considered, and show results for different constellation orders and two different subcarriers, one central subcarrier where the Gaussian approximation of the ICI is better, and one border subcarrier where the Gaussian approximation of the ICI is the worst. A novel result is that even considering a border subcarrier, the mean results of the BER and the BEP match for all the constellation orders with the exception of 4-QAM for very low maximum Doppler shift values. Moreover, based on the characterization of the goodness of the BEP approximation of Section V-A we are able to explain why averaging the BER over several channel realizations yields good results.

We also provide the source code employed to obtain the results included in this paper, which supports the reproducibility

¹We call *deterministic models* to the *site-specific models* as defined in [20].

²For simplicity, the analysis in [21] considered the symbol error probability (SEP) instead of the BEP.

of the presented results and allows other researchers to easily apply our findings to their works.

II. SYSTEM MODEL

Let us consider the transmission of OFDM symbols over a point-to-point single-antenna wireless link. The length of the OFDM symbols is $T + T_{\text{cp}}$, where T is the inverse of the subcarrier spacing, and T_{cp} is the length of the CP. The m -th transmitted OFDM symbol, starting at the time instant t_m (not taking into account the CP), can be expressed as

$$u_m(t) = \sum_{l \in \mathcal{S}} X_{m,l} e^{j2\pi l(t-t_m)/T}, \quad t_m \leq t < t_m + T \quad (1)$$

where \mathcal{S} is the set of used subcarrier indices, and $X_{m,l}$ is the symbol transmitted in the subcarrier l and OFDM symbol m . It is important for the reader to note here the difference between a symbol, i.e., $X_{m,l}$, and an OFDM symbol, i.e., $u_m(t)$. We employ this nomenclature along the rest of the paper.

We assume a doubly-selective tapped delay line channel model with a finite number of paths expressed as

$$h(t, \tau) = \sum_{i=1}^N \alpha_i e^{j2\pi \nu_i t} \delta(\tau - \tau_i) \quad (2)$$

where t and τ are the time and delay independent variables, respectively, j is the imaginary unit, $\delta(\cdot)$ is the Dirac delta function, N is the number of paths, and α_i , ν_i , and τ_i are, respectively, the complex-valued amplitude, the Doppler shift, and the delay corresponding to the i -th path. We assume that the maximum delay is less than the CP length, i.e., $\max_i(\tau_i) < T_{\text{cp}}$. Under this assumption, there is no inter-symbol-interference (ISI) affecting the signal at the receiver, and we can ignore the CP for the calculations. Thus, the m -th received OFDM symbol can be expressed as

$$\begin{aligned} y_m(t) &= h(t, \tau) * u_m(t) + \sqrt{\frac{N_0}{2}} n(t) \\ &= \sum_{i=1}^N \alpha_i e^{j2\pi \nu_i t} \sum_{l \in \mathcal{S}} X_{m,l} e^{j2\pi l(t-\tau_i-t_m)/T} + \sqrt{\frac{N_0}{2}} n(t) \end{aligned} \quad (3)$$

where $t_m \leq t < t_m + T$, N_0 is the power spectral density of the noise, $n(t)$ is a standard complex-valued Gaussian random process, and $*$ is the time-varying convolution operator defined as

$$h(t, \tau) * u_m(t) = \int_{-\infty}^{\infty} h(t, \tau) u_m(t - \tau) d\tau. \quad (4)$$

For the l -th subcarrier of the m -th OFDM symbol, the received symbol, $Y_{m,l}$, is obtained as

$$\begin{aligned} Y_{m,l} &= \frac{1}{T} \int_0^T y_m(t + t_m) e^{-j2\pi lt/T} dt \\ &= X_{m,l} H_{m,l} + \text{ICI}_{m,l} + \sqrt{\frac{N_0}{2}} N_{m,l} \end{aligned} \quad (5)$$

where $H_{m,l}$ is the corresponding channel coefficient, $\text{ICI}_{m,l}$ is the ICI coefficient, and $N_{m,l}$ is a standard complex-valued

Gaussian random variable. The channel coefficient, $H_{m,l}$, can be expressed as

$$H_{m,l} = \sum_{i=1}^N H_{m,l}(\theta_i) \quad (6)$$

where $\theta_i = [\tau_i, \nu_i, \alpha_i]$ is a vector with the i -th path parameters, and $H_{m,l}(\theta_i)$ is the i -th path contribution to the channel coefficient, defined as

$$H_{m,l}(\theta_i) = \alpha_i e^{-j2\pi l \tau_i / T} e^{j2\pi \nu_i t_m} D(\nu_i) \quad (7)$$

where the function $D(\cdot)$ is

$$D(f) = \frac{1}{T} \int_0^T e^{j2\pi f t} dt = \begin{cases} 1 & \text{if } f = 0 \\ \frac{e^{j2\pi f T} - 1}{j2\pi f T} & \text{if } f \neq 0. \end{cases} \quad (8)$$

The ICI term in (5) is defined as

$$\text{ICI}_{m,l} = \sum_{\substack{k \in \mathcal{S} \\ k \neq l}} X_{m,k} H_{m,l,k}^{\text{ICI}} \quad (9)$$

where $H_{m,l,k}^{\text{ICI}}$ is the ICI contribution of the k -th subcarrier to the l -th subcarrier during the m -th OFDM symbol. In a similar way as in (6), the ICI contribution of the k -th subcarrier to the l -th subcarrier during the m -th OFDM symbol is

$$H_{m,l,k}^{\text{ICI}} = \sum_{i=1}^N H_{m,l,k}^{\text{ICI}}(\theta_i) \quad (10)$$

where $H_{m,l,k}^{\text{ICI}}(\theta_i)$ is the ICI contribution corresponding to the i -th path, defined as

$$H_{m,l,k}^{\text{ICI}}(\theta_i) = \alpha_i e^{-j2\pi k \tau_i / T} e^{j2\pi \nu_i t_m} D\left(\frac{k-l}{T} + \nu_i\right). \quad (11)$$

III. NORMAL APPROXIMATION OF THE ICI DISTRIBUTION

We assume that the transmit symbols $X_{m,l}$ defined in the previous section are independent and identically distributed random variables which take values from a square QAM constellation of order M and have a variance of σ_x^2 . Recall that the ICI term for the l -th subcarrier and the m -th OFDM symbol, $\text{ICI}_{m,l}$ (see (9)), is expressed as a sum of products involving the transmit symbols ($X_{m,k}$) and the ICI contributions between the different subcarriers ($H_{m,l,k}^{\text{ICI}}$). Since we are considering deterministic channels, the coefficients $H_{m,l,k}^{\text{ICI}}$ are known values for a given channel realization while the transmit symbols $X_{m,k}$ are random variables. Therefore, $\text{ICI}_{m,l}$ is a weighted sum of random variables and, according to the CLT, $\text{ICI}_{m,l}$ may be approximated by a complex-valued normal distribution [21]. Note, however, that the terms of $\text{ICI}_{m,l}$ are independent but not identically distributed. Indeed, the variance for each term in $\text{ICI}_{m,l}$ is

$$\text{Var}(X_{m,k} H_{m,l,k}^{\text{ICI}}) = \sigma_x^2 |H_{m,l,k}^{\text{ICI}}|^2 \quad (12)$$

which, from (11) and (8), decays with a factor proportional to the square of the subcarrier distance, i.e., $|k-l|^2$. Thus, only a few terms will contribute significantly to $\text{ICI}_{m,l}$. Because of that, the assumptions of the CLT, or any of its variants, will not be fully satisfied, and hence the distribution of $\text{ICI}_{m,l}$ will not necessarily converge to normal as $|\mathcal{S}| \rightarrow \infty$. Nevertheless,

in the following we will experimentally show that a complex-valued normal distribution may be a good approximation to $\text{ICI}_{m,l}$. For this matter, we will use the Mardia's skewness and kurtosis measures to test for multivariate normality [31].

For a random sample of size n from a p -variate population, i.e., $\mathbf{Z}_i = (Z_{1,i}, \dots, Z_{p,i})$, $i = 1, 2, \dots, n$, Mardia defined the skewness and kurtosis, respectively, as

$$b_{1,p} = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \left[(\mathbf{Z}_i - \bar{\mathbf{Z}})^T \mathbf{S}^{-1} (\mathbf{Z}_j - \bar{\mathbf{Z}}) \right]^3 \quad (13)$$

and

$$b_{2,p} = \frac{1}{n} \sum_{i=1}^n \left[(\mathbf{Z}_i - \bar{\mathbf{Z}})^T \mathbf{S}^{-1} (\mathbf{Z}_i - \bar{\mathbf{Z}}) \right]^2 \quad (14)$$

where \mathbf{S} is the sample covariance matrix and $\bar{\mathbf{Z}}$ the sample mean vector. From (13) and (14), Mardia defined two statistics to test for normality. In our case, however, $\text{ICI}_{m,l}$ does not follow exactly a normal distribution and hence the Mardia's test fails for large values of n . Thus, instead of testing for normality, we will check that the skewness and kurtosis values of $\text{ICI}_{m,l}$ are relatively close to the expected ones for a normal distribution under realistic circumstances. In our case we have $p = 2$ with $\mathbf{Z}_i = [\Re\{\text{ICI}_{m,l}\}, \Im\{\text{ICI}_{m,l}\}]$. Then, according to [31], the expected values of skewness and kurtosis for a normal distribution are $E[b_{1,2}] = 24/n$, and $E[b_{2,2}] = 8(n-1)/(n+1)$, respectively. Thus, for large n values, $E[b_{1,2}] \approx 0$ and $E[b_{2,2}] \approx 8$.

For our experiments we consider an OFDM system with parameters similar to those of the LTE 10 MHz downlink profile: the subcarrier spacing is $1/T = 15$ kHz; the CP length is $72/(15.36 \cdot 10^6)$ seconds (different CP lengths are not considered for simplicity); and 600 subcarriers are used, i.e., $\mathcal{S} = \{-300, \dots, -1, 1, \dots, 300\}$. We will consider 4-QAM, 16-QAM, and 64-QAM constellations, and the following three channel models:

- 3GPP typical urban channel model (TUx) [32, table 5.2]: 20-taps channel model, presenting a high frequency selectivity and using the Jakes Doppler spectrum for all the taps.
- 3GPP rural area channel model (RAX) [32, table 5.3]: 10-taps channel model, with a direct path, using the Jakes Doppler spectrum for the remaining taps.
- ITU-R vehicular – high antenna channel model [33, table 5]: 6-tap channel model, using the Jakes Doppler spectrum for all the taps.

We generate channel realizations by using the Clarke's model with eight sinusoids for the taps which follow the Jakes Doppler spectrum [34], and a single sinusoid for the direct paths. Random samples of $\text{ICI}_{m,l}$ are obtained by giving values to the symbols $X_{m,k}$ in (9). For the sake of brevity, we present the results for two exemplary cases: *a*) a middle subcarrier ($l = 150$), and *b*) a border subcarrier ($l = 300$), as imaged in Fig. 1. Note that the so-called middle subcarrier is surrounded by other subcarriers used for the transmission of symbols, whereas the border subcarrier only has busy subcarriers at one side. This way, the middle subcarrier is expected to be the best-case scenario since the decay of the

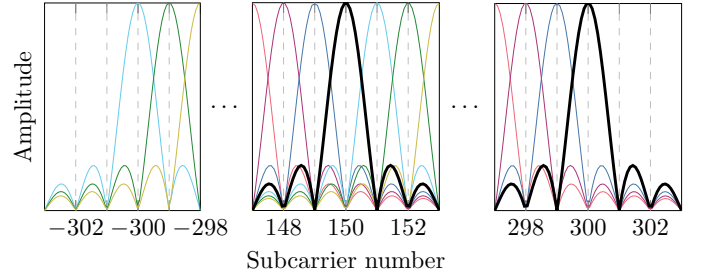


Fig. 1: Frequency spectra of the subcarriers used for the evaluations. The spectra of subcarriers $l = 150$ and $l = 300$ correspond to the thick black curves.

TABLE I: Skewness for 10^3 random samples of $\text{ICI}_{0,150}$ and $\text{ICI}_{0,300}$ over 10^3 channel realizations for $\nu_{\max}T = 0.05$

Channel model	M	$\text{ICI}_{0,150}$ skewness		$\text{ICI}_{0,300}$ skewness	
		mean	variance	mean	variance
3GPP TUx	4	0.0174	0.0002	0.0131	0.0001
	16	0.0187	0.0002	0.0154	0.0001
	64	0.0193	0.0002	0.0160	0.0001
3GPP RAX	4	0.0164	0.0001	0.0121	0.0001
	16	0.0182	0.0002	0.0140	0.0001
	64	0.0196	0.0002	0.0152	0.0001
ITU-R vehicular	4	0.0165	0.0002	0.0131	0.0001
	16	0.0182	0.0002	0.0149	0.0001
	64	0.0182	0.0002	0.0155	0.0001

variance of the ICI terms corresponding to the subcarriers at both sides of the middle-subcarrier is symmetric, according to (12). On the contrary, for the border subcarrier, only the subcarriers at one of its sides contribute to the ICI, leading to a non-symmetrical distribution of the corresponding ICI terms.

For a given channel realization and constellation order we generate $n = 10^3$ random samples of $\text{ICI}_{0,150}$ and $\text{ICI}_{0,300}$, and calculate the Mardia's skewness and kurtosis. Note that for $n = 10^3$ the expected values of skewness and kurtosis for normally distributed samples become $E[b_{1,2}] = 0.024$ and $E[b_{2,2}] = 7.984$, respectively. The previous process is performed for 10^3 channel realizations for each of the three considered channel models and for several maximum normalized Doppler shifts³ $\nu_{\max}T$. Note that for each channel realization we obtain a single value of skewness and kurtosis. Thus, with the 10^3 considered channel realizations we have a set of 10^3 values of skewness and kurtosis to be analyzed statistically.

Mean and variance results for the skewness are shown in Table I considering a maximum normalized Doppler shift of $\nu_{\max}T = 0.05$, both for $\text{ICI}_{0,150}$ and $\text{ICI}_{0,300}$. The obtained results for the skewness are close to 0 in all the cases, ranging from 0.01 to 0.02, hence indicating a high degree of symmetry of the ICI distribution. Mean and variance results for the kurtosis are shown in Table II, again considering a maximum normalized Doppler shift of $\nu_{\max}T = 0.05$ for both $\text{ICI}_{0,150}$ and $\text{ICI}_{0,300}$. In this case the kurtosis values are similar for the

³The maximum Doppler shift of a communication link is expressed as $\nu_{\max} = sf_c/c_0$, being s the relative speed between the transmitter and the receiver, f_c the carrier frequency, and c_0 the speed of light in the vacuum.

TABLE II: Kurtosis for 10^3 random samples of $\text{ICI}_{0,150}$ and $\text{ICI}_{0,300}$ over 10^3 channel realizations for $\nu_{\max}T = 0.05$

Channel model	M	$\text{ICI}_{0,150}$ kurtosis		$\text{ICI}_{0,300}$ kurtosis	
		mean	variance	mean	variance
3GPP TUx	4	7.2665	0.0608	6.5101	0.1882
	16	7.4881	0.0450	6.9844	0.0983
	64	7.5404	0.0510	7.0743	0.0866
3GPP RAx	4	7.1871	0.0275	6.3545	0.0181
	16	7.4474	0.0318	6.8759	0.0223
	64	7.4951	0.0378	6.9837	0.0265
ITU-R vehicular	4	7.2407	0.0500	6.5176	0.1654
	16	7.4714	0.0457	6.9840	0.0883
	64	7.5219	0.0439	7.0706	0.0841

three channel models, but there are differences with respect to the constellation order and subcarrier considered. Regarding the constellation order, the kurtosis values increase as the constellation order increases, being the biggest step the one from 4- to 16-QAM. With respect to the subcarrier considered, for $\text{ICI}_{0,150}$ the obtained mean kurtosis ranges from 7.19 to 7.54 (approx.), whereas for $\text{ICI}_{0,300}$ the obtained values are significantly lower, ranging from 6.35 to 7.07 (approx.). In all cases the mean kurtosis is less than 8, stating that the ICI does not exactly follow a normal distribution. However, values close to 8 indicate that a normal distribution may still be a good approximation. For instance, consider the Student's t -distribution, which for 30 degrees of freedom is a very good approximation to a normal distribution. A 2-dimensional multivariate Student's t -distribution with 30 degrees of freedom has a kurtosis value of approximately 8.46. Therefore, we may expect to have a good normal approximation of the ICI for middle subcarriers, whereas for border subcarriers the approximation becomes rougher.

In Fig. 2 we show exemplary joint PDFs of the ICI for subcarriers 150 and 300 for a given channel realization, considering the ITU-R vehicular channel with $\nu_{\max}T = 0.05$ and a 4-QAM constellation. The kurtosis values for the joint PDFs shown in Figs. 2a and 2b are 7.243 and 6.481, respectively. These values are similar to the mean ones for subcarriers 150 and 300 (see Table II). It can be seen that the PDF of $\text{ICI}_{0,300}$ has a squarish shape and thus is clearly non normal. On the other hand, $\text{ICI}_{0,150}$ has a larger kurtosis value and the joint PDF is much closer to a complex-valued normal distribution.

It is important to note that, when moving away from the border subcarriers, the kurtosis values increase very rapidly. This is because, as explained before, the variance of the ICI decays with a factor proportional to the square of the subcarrier distance and only the closest subcarriers contribute significantly to the ICI. In Fig. 3 we show the means of the kurtosis values, as well as their 5% and 95% percentiles, obtained for different subcarriers considering an ITU-R channel model, 10^3 channel realizations, and a normalized maximum Doppler shift of 0.05. For the sake of conciseness, we will show only results of the kurtosis for a single channel model. Results for the other channel models are similar. It can be seen that, starting at the border subcarrier, when the subcarrier number decreases by just three or four units, the mean kurtosis values are very

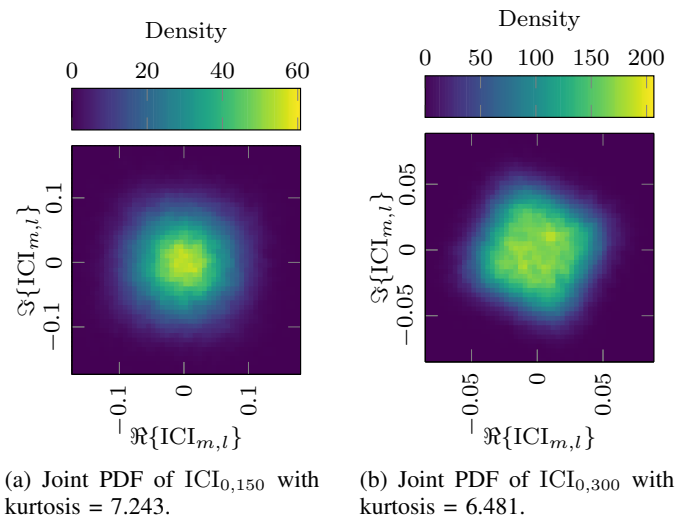


Fig. 2: Exemplary joint PDFs of the real and imaginary parts of $\text{ICI}_{0,150}$ and $\text{ICI}_{0,300}$ for 4-QAM.

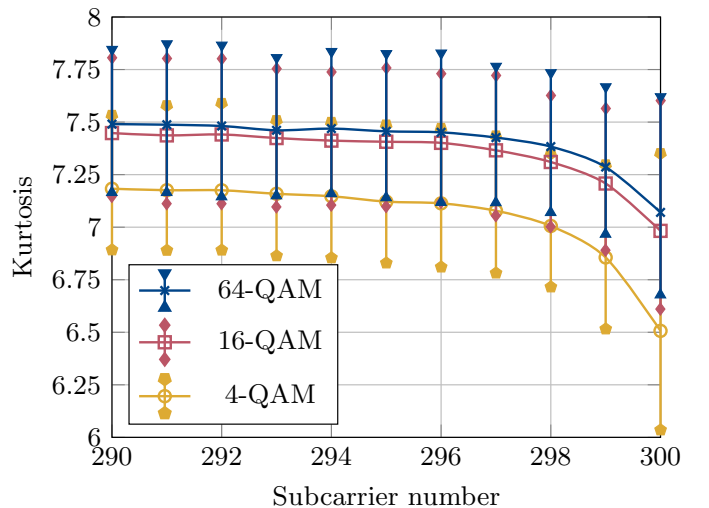


Fig. 3: Means and 5% and 95% percentiles of the kurtosis values vs subcarrier number for 10^3 realizations of the ITU-R vehicular channel model with $\nu_{\max}T = 0.05$.

close to the ones shown in Table II for subcarrier 150. Recall that, for each channel realization, the kurtosis value might be different, hence, the 5% and 95% percentiles are also plotted in Fig. 3 to show the range of the 90% central values of kurtosis obtained. It can be seen that for the 4-QAM case the kurtosis values can reach sometimes much lower values than the ones expected for a normal distribution, specially on the border subcarriers, but for higher constellation orders the values are significantly larger.

Finally, we investigate the effect of the maximum Doppler shift (i.e., the relative speed between the transmitter and the receiver) on the Mardia's skewness and kurtosis values. We find that such values do not present significant differences for any of the cases (i.e., channel model, constellation order, and subcarrier number) when the maximum Doppler shift varies. For the sake of conciseness, we only show results of the kurtosis for a single channel model and subcarrier.

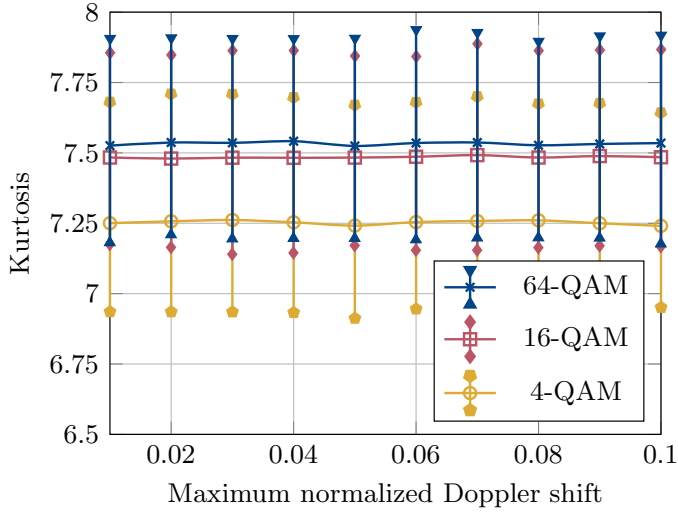


Fig. 4: Means and 5% and 95% percentiles of the kurtosis values vs maximum normalized Doppler shift for 10^3 realizations of the ITU-R vehicular channel model and subcarrier 150.

Fig. 4 plots the means of the kurtosis values, as well as their 5% and 95% percentiles, obtained for different values of the maximum normalized Doppler shift, ranging from 0.01 to 0.1. The values are calculated for the subcarrier 150 using 10^3 channel realizations of the ITU-R vehicular channel model for each value of maximum normalized Doppler shift. It can be seen that neither the means nor the percentile values experience significant variations for any of the considered maximum normalized Doppler shift values.

IV. CAPACITY BOUND AND BIT ERROR PROBABILITY

In the previous section we have shown that, although $\text{ICI}_{m,l}$ does not follow a normal distribution, the normal distribution is still a good approximation to model the ICI in practical cases. More specifically, we have seen that the approximation is only noticeably worse for the subcarriers near the edges of the effectively used bandwidth. This way, assuming that the ICI distribution is well-approximated by a complex-valued normal distribution, the ICI plus the noise for a given symbol can be also approximated by a complex-valued normal distribution. The variance of the distribution of the ICI plus the noise for the symbol at time m and subcarrier l is

$$\sigma_{\text{ICI}_{m,l}}^2 = \text{Var}(\text{ICI}_{m,l}) + N_0 \quad (15)$$

where, from (9) and considering that the transmit symbols are mutually independent, the variance of $\text{ICI}_{m,l}$ is

$$\text{Var}(\text{ICI}_{m,l}) = \sigma_x^2 \sum_{\substack{k \in S \\ k \neq l}} |H_{m,l,k}^{\text{ICI}}|^2. \quad (16)$$

The transmitted energy per bit, considering a square QAM constellation of order M , is defined as

$$E_b^{\text{TX}} = \sigma_x^2 / \log_2 M. \quad (17)$$

Based on this definition, the received energy per bit for the symbol at time m and subcarrier l is defined as

$$E_{b_{m,l}}^{\text{RX}} = E_b^{\text{TX}} |H_{m,l}|^2. \quad (18)$$

Then, the ratio of the received energy per bit to the noise plus interference power spectral density is

$$r_{m,l} = \frac{E_{b_{m,l}}^{\text{RX}}}{\sigma_{\text{ICI}_{m,l}}^2} = \frac{\sigma_x^2 |H_{m,l}|^2}{\sigma_{\text{ICI}_{m,l}}^2 \log_2 M}. \quad (19)$$

The capacity⁴ can be bounded from (19) by using the Shannon-Hartley theorem. This is because the capacity of a channel with additive non-Gaussian noise is greater than or equal to the capacity of a channel with additive Gaussian noise, assuming that the noise covariances are the same [35]. Therefore, the capacity for the symbol at time m and subcarrier l , with units of bits/s/Hz, can be expressed as the following lower bound

$$C_{m,l} \geq \frac{T}{T + T_{\text{cp}}} \log_2 (1 + r_{m,l} \log_2 M) \quad (20)$$

where the factor $T/(T + T_{\text{cp}})$ accounts for the spectral efficiency loss due to the CP, and $r_{m,l} \log_2 M$ is the ratio of the received energy per symbol to the noise plus interference power spectral density.

In the same way, by using (19) and considering the normal approximation, the BEP for a given symbol can be approximated analytically. First, note that the exact closed-form BEP expression for a communication system employing an M -ary square QAM constellation with Gray mapping over an AWGN channel is [36]

$$P_M(r) = \frac{1}{\log_2 \sqrt{M}} \sum_{j=1}^{\log_2 \sqrt{M}} P_{M,j}(r) \quad (21)$$

where r is the ratio of the energy per bit to the noise power spectral density, and the $P_{M,j}(r)$ is the error probability of the j -th bit for the M -QAM constellation, defined as

$$P_{M,j}(r) = \frac{1}{\sqrt{M}} \sum_{k=0}^{(1-2^{-j}) \cdot \sqrt{M}-1} \left[(-1)^{\lfloor k \cdot 2^{j-1} / \sqrt{M} \rfloor} \cdot \left(2^{j-1} - \left\lfloor \frac{k \cdot 2^{j-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \cdot \text{erfc} \left((2k+1) \sqrt{\frac{3 \log_2 M \cdot r}{2(M-1)}} \right) \right] \quad (22)$$

where $\lfloor \cdot \rfloor$ is the floor function and $\text{erfc}(\cdot)$ is the complementary error function [37]. Hence, the analytical BEP approximation when using an M -ary square QAM constellation for the symbol at time m and subcarrier l is simply obtained as

$$\text{BEP}_{m,l} = P_M(r_{m,l}). \quad (23)$$

However, note that, although the capacity expression in (20) is a lower bound, this does not imply that the analytical BEP approximation in (23) is an upper bound, because the receiver is optimal for Gaussian noise but now we have Gaussian noise plus non-Gaussian interference, which can make the receiver suboptimal [38].

⁴We define the capacity as the maximum rate at which we can transmit information over a given channel with our system (i.e., the achievable throughput per Hertz).

V. RESULTS

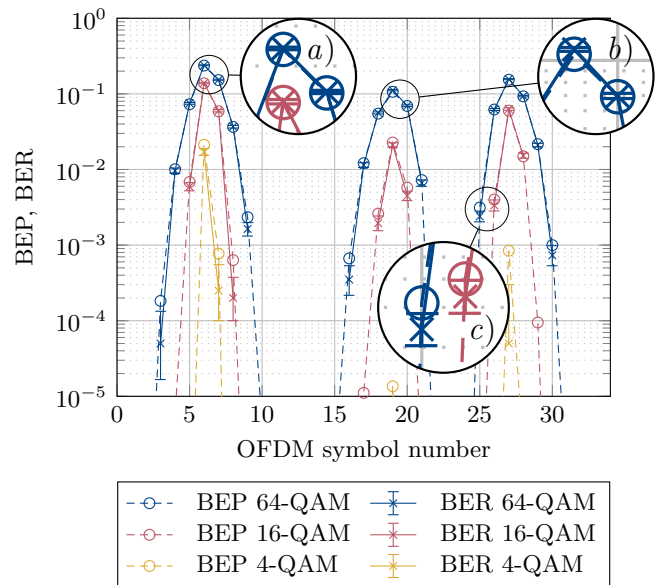
In the following we show results of the analytical BEP approximation as per (23) and results of BER obtained by means of Monte-Carlo computer simulations. Note that we will not show experimental results of the capacity due to the complexity of the mathematical expressions and algorithms required to estimate the capacity for the non-Gaussian case.

For the results we consider $\sigma_x = 1$, and that the average channel gain is also 1. For the simulations, several iterations are performed over each given channel realization to obtain averaged BER results. Assuming that we want to calculate the BER for a symbol at time m and subcarrier l , an iteration of the simulation involves the following steps:

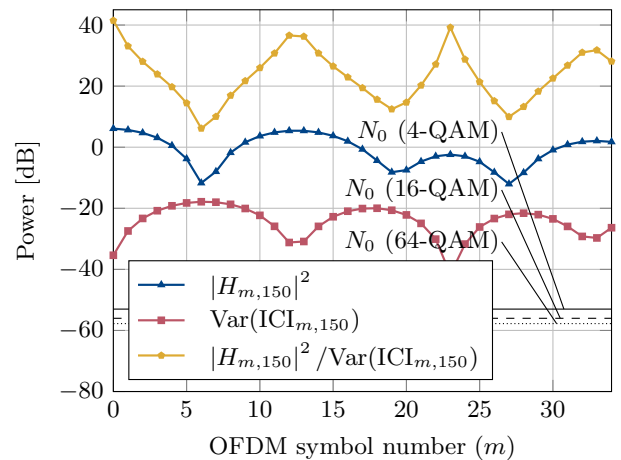
- 1) Generate a sequence of random bits.
- 2) Encode the bits to the M -QAM transmit symbols $X_{m,k}$ using Gray coding. Note that even though we only want to obtain the BER for the subcarrier l , we must consider the transmission of symbols over all subcarriers.
- 3) Apply the channel response and add the noise to the transmit symbols to obtain the receive symbol $Y_{m,l}$ as per (5).
- 4) Obtain an estimate of the transmit symbol $X_{m,l}$. We consider perfect channel knowledge at the receiver and we employ a zero forcing estimation. Hence, the estimated receive symbol is obtained as $\hat{X}_{m,l} = Y_{m,l}/H_{m,l}$.
- 5) Decode the estimated symbol $\hat{X}_{m,l}$ using Gray decoding to obtain the corresponding sequence of bits.
- 6) Compare the received bits with the corresponding transmitted bits to obtain the BER for the iteration.

In Fig. 5a we show some exemplary results of the BEP and BER versus the OFDM symbol number for a single realization of the ITU-R vehicular channel. We considered the subcarrier 150, 35 consecutive OFDM symbols, a maximum normalized Doppler shift of 0.05, and 10^4 iterations for the BER results. The ratio of the energy per bit on transmission to the noise spectral power density, i.e., E_b^{TX}/N_0 , is set to 50 dB. Note that E_b^{TX}/N_0 is employed to obtain the noise value in the results, except those shown in Figs. 6 and 7, since the energy per bit at reception, as well as the power of the ICI, depend on the OFDM symbol, the subcarrier, and the channel realization.

We also show in Fig. 5a the 95% confidence intervals (CIs) of the BER results obtained by means of a bootstrapping technique [39]. It can be seen that the BEP presents large variations over the time, due to the variations of the channel response and the ICI powers. In Fig. 5b we show the corresponding values of the channel and ICI power, as well as their ratio, versus the OFDM symbol number. Note that since we are considering a constant symbol energy, $\sigma_x = 1$, the bit energy depends on the constellation order, and thus the noise power N_0 also depends on the constellation order, satisfying $E_b^{\text{TX}}/N_0 = 50$ dB. The corresponding values of the noise power N_0 for the different constellation orders are also shown in Fig. 5b. It can be seen that the E_b^{TX}/N_0 value of 50 dB is large enough so that the noise is significantly lower than the ICI for all the OFDM symbols. Therefore, the BEP and BER values shown in Fig. 5a are mostly due to the ICI and not the noise.



(a) BER (including the 95% confidence intervals) and BEP results.



(b) Channel power, ICI power, and ratio of the channel power to the ICI power.

Fig. 5: Exemplary results for subcarrier 150, 35 OFDM symbols, and a single realization of the ITU-R vehicular channel model with $\nu_{\max}T = 0.05$.

Even though the results in Fig. 5a correspond to a single channel realization, they show already several interesting features that we comment below

- For some symbols, the ratio of the received energy per bit to the noise plus interference is very large, which means that for these symbols we obtained a BER of zero. This is notorious in the 4-QAM case.
- For the largest BER values, the BEP matches the BER very well, e.g., see magnification *a*) in Fig. 5a. In some cases the BER may be slightly larger than the BEP, e.g., see magnification *b*) in Fig. 5a.
- For lower BER values, the BEP values are larger, and in some cases even larger than the upper 95% CI. Nevertheless, most of the BER values are still close to the BEP values, e.g., see magnification *c*) in Fig. 5a.

A. Instantaneous BEP and BER

Based on the previous remarks, it can be seen that the goodness of the BEP approximation, i.e., how close is the BEP as defined in (23) to the BER, may depend on the ratio of the received energy per bit to the noise plus interference. To characterize this behavior, we define the approximation error factor metric for a given symbol as

$$\rho = \frac{\text{BER}}{\text{BEP}}. \quad (24)$$

We calculate the error factors for the same three constellation orders and three channel models considered in Section III, for subcarrier 150, 10 consecutive OFDM symbols, a maximum Doppler shift of 0.05, and 100 different channel realizations. As in the results in Fig. 5a, we also consider $E_b^{\text{TX}}/N_0 = 50$ dB. For each transmitted symbol the BEP is obtained as per (23), and the BER is estimated using 10^6 iterations (each iteration is specified at the beginning of Section V). Moreover, to avoid possible outliers and to reduce the variance of the results shown, we discard the BER estimates that lead to less than 10 erroneous bits. Finally, the ρ value corresponding to the BER estimate is computed according to (24). Note that here we are considering the “instantaneous” results, i.e., the values are obtained for single symbols and no averaging between symbols is performed.

The results obtained are shown in Fig. 6 as a cloud of points of the the error factors ρ versus the ratio of the received energy per bit to the noise plus interference, i.e., $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$, for each of the symbols. Moreover, the BEP curve as per (23) is also shown for the considered range of $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$. Note that, as shown in Fig. 4, the goodness of approximation of the ICI by a Gaussian distribution is independent of the maximum normalized Doppler shift. Thus, the results shown in Figs. 6 and 7 only depend on $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$. It can be seen that the results obtained agree with the aforementioned remarks made for the results in Fig. 5a. We can extract the following conclusions from the results shown in Fig. 6:

- There is no significant difference in the error factors obtained for the different channel models considered. This is the expected result since the goodness of the normal approximation of the ICI also does not exhibit significant changes for the different channel models, as shown in Section III. Moreover, as shown in Fig. 4, the goodness of the normal approximation of the ICI does not depend on the maximum normalized Doppler shift. Thus, the results shown in Fig. 6 will also be the same for different values of the maximum normalized Doppler shift.
- For low $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ values, ρ is close to 1. Thus, the BEP results approximate well the BER.
- It can be seen that for 16-QAM and 64-QAM, $1.1 < \rho < 0.8$ when $\text{BEP} < 10^{-2}$. For 4-QAM, due to the worse normal approximation of the ICI, $1.1 < \rho < 0.5$ when $\text{BEP} < 10^{-2}$.
- For higher $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ values, i.e., when $\text{BEP} > 10^{-2}$, ρ is always less than one and monotonically decreases as $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ increases. Thus, the BER

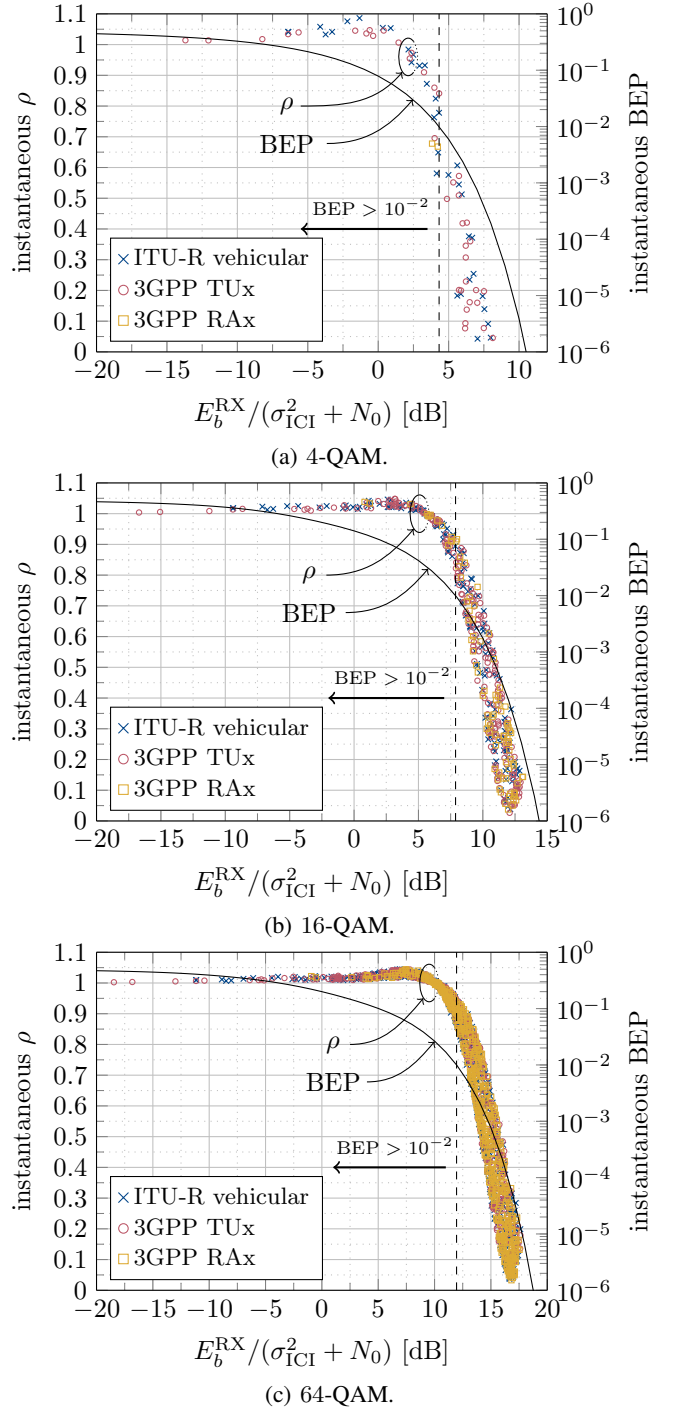


Fig. 6: Instantaneous error factors, $\rho = \text{BER}/\text{BEP}$, and instantaneous BEP versus the ratio of the received energy per bit to the noise plus ICI for subcarrier 150, $\nu_{\max}T = 0.05$, and 100 channel realizations for each of the considered channel models.

is always lower than the BEP, i.e., the BEP becomes an upper bound of the BER.

In Fig. 7 we show results of error factors for subcarrier 300 and the ITU-R vehicular channel model. Results for the other channel models are similar and we do not show them for conciseness. We can see that these results

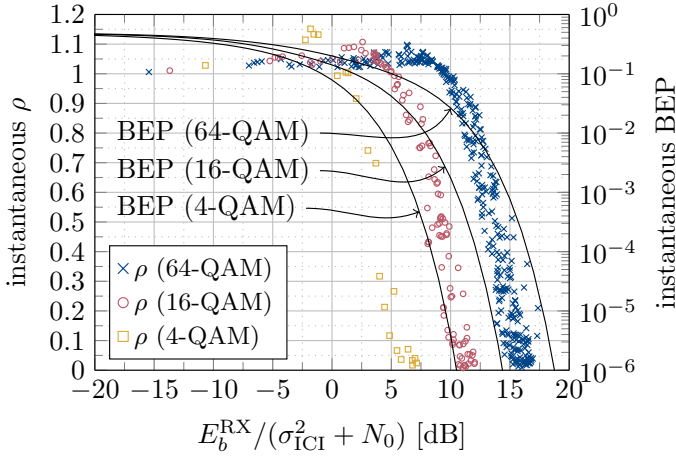


Fig. 7: Instantaneous error factors, $\rho = \text{BER}/\text{BEP}$, and instantaneous BEP versus the ratio of the received energy per bit to the noise plus ICI for subcarrier 300, $\nu_{\max}T = 0.05$, and 100 channel realizations of the ITU-R vehicular channel.

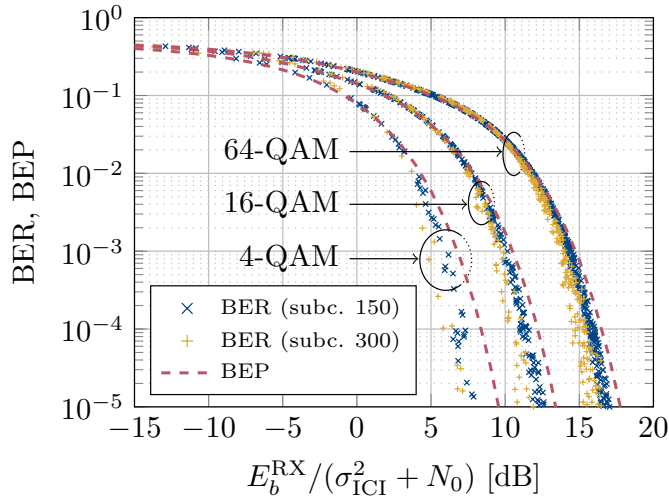


Fig. 8: Instantaneous BER and BEP versus the ratio of the received energy per bit to the noise plus ICI for subcarriers 150 and 300, $\nu_{\max}T = 0.05$, and 100 channel realizations of the ITU-R vehicular channel.

are similar to the ones shown in Fig. 6 for subcarrier 150. However, for all the constellation orders it can be seen that for low $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ values the error factors obtained are slightly higher than those in Fig. 6. Also, for the highest $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ values the error factors are lower than for subcarrier 150, which indicates that the BEP approximation is worse in this case.

In Fig. 8 we show the results of the instantaneous BER and BEP for subcarriers 150 and 300 and the ITU-R vehicular channel model. These results correspond to the ones shown in Figs. 6a and 7. Fig. 8 allows us to better appreciate the differences between the BEP and BER results obtained. We can see clearly that, as the $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ increases, the BEP becomes an upper bound of the BER. We can also see that the BEP approximation for the subcarrier 300 is clearly worse than for the subcarrier 150.

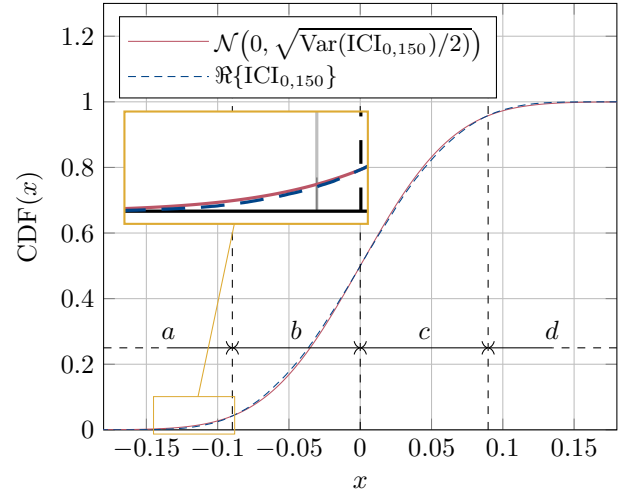


Fig. 9: CDFs of the ICI and the corresponding normal approximation for subcarrier 150 and a single channel realization of the ITU-R vehicular channel.

The behavior of the BEP approximation shown in the previous figures is due to the non-Gaussianity of the ICI distribution, more specifically because the ICI distribution is slightly platykurtic, i.e., its kurtosis is slightly lower than that of the Gaussian distribution. This can be clearly seen if we compare the cumulative density functions (CDFs) of the ICI and the Gaussian distributions. In Fig. 9 we show the CDF for the real part of the ICI for subcarrier 150 and a single channel realization of the ITU-R vehicular channel. The channel realization corresponds to the one shown also in Fig. 2a. We also show the CDF of the normal distribution which would be used to obtain the BEP approximation. We define $\text{CDF}_{\text{ICI}}(\cdot)$ as the CDF of the ICI and $\text{CDF}_{\mathcal{N}}(\cdot)$ as the CDF of the normal distribution shown in Fig. 9. Due to the inferior kurtosis of the ICI distribution, intervals a , b , c , and d can be found such that:

$$\text{CDF}_{\text{ICI}}(x) \leq \text{CDF}_{\mathcal{N}}(x) \quad \forall x \in a \vee x \in c \quad (25)$$

$$\text{CDF}_{\text{ICI}}(x) \geq \text{CDF}_{\mathcal{N}}(x) \quad \forall x \in b \vee x \in d \quad (26)$$

Thus, from Eqs. (25) and (26) we can clearly see that for low values of $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ the BEP approximation will be an upper bound, whereas for higher values of $E_b^{\text{RX}}/(\sigma_{\text{ICI}}^2 + N_0)$ the BEP approximation will be slightly higher than the BER, as shown in the previous results.

B. Average BEP and BER

Some works [21]–[23] have previously shown that a good approximation of the BEP can be obtained for stochastic channel models by averaging the BER over several channel realizations. However, those works only considered the BEP over a large number of subcarriers. It should be noted that the Gaussian approximation of the ICI depends on the constellation order and the subcarrier considered. Thus we will show in this section results for the different constellation orders and subcarriers considered in previous sections, i.e., the subcarrier 150 where the Gaussian approximation is better,

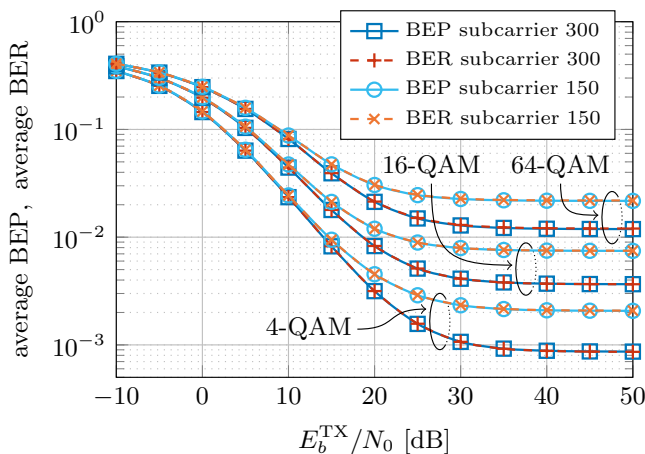


Fig. 10: Average BEP and average BER versus E_b^{TX}/N_0 for a symbol in subcarriers 150 and 300 considering 10^4 realizations of the ITU-R vehicular channel model with $\nu_{\max}T = 0.05$.

and the subcarrier 300 where the Gaussian approximation is the worst.

Fig. 10 shows the average BEP and BER results versus the ratio of transmitted energy per bit to the noise power spectral density for 10^4 realizations of the ITU-R vehicular channel model considering a single symbol for subcarriers 150 and 300, and a maximum normalized Doppler shift of 0.05. The BER is calculated using 10^4 iterations (each iteration is specified at the beginning of Section V). Fig. 10 shows that both the analytical and the simulation results match for the three constellation orders and the two considered subcarriers for the entire range of E_b^{TX}/N_0 values considered. As expected, the BEP results obtained for subcarrier 300 are lower than those of subcarrier 150 because the ICI affecting the border subcarriers is also lower.

Fig. 11 shows the average BEP and BER versus the maximum normalized Doppler shift for 10^4 realizations of the ITU-R vehicular channel model considering a single symbol for subcarriers 150 and 300, and $E_b^{\text{TX}}/N_0 = 50$ dB. As before, the BER was calculated using 10^4 iterations. Again, Fig. 11 shows that the analytical and the simulation results match for the three constellation orders and the two considered subcarriers for all the range of maximum normalized Doppler shifts considered. The only exception is the case of the minimum normalized Doppler shift considered (0.01) for subcarrier 300 and 4-QAM, where the BER is slightly lower than the BEP. In these results we also observe that, for each maximum normalized Doppler shift value, the BEP results obtained for subcarrier 300 are always lower than those of subcarrier 150, because the ICI is also lower.

Hence, the results show that, even for border subcarriers, the obtained BEP values are a good approximation of the BER. This can be justified because in the presence of Rayleigh fading the main contribution to the average BEP, as can be seen in Fig. 5a, is due to a few symbols with a relatively low SINR, and thus a high BEP. As shown in Section V-A, we obtain good approximations of the BER for deterministic channels and low SINR values. Therefore when we consider

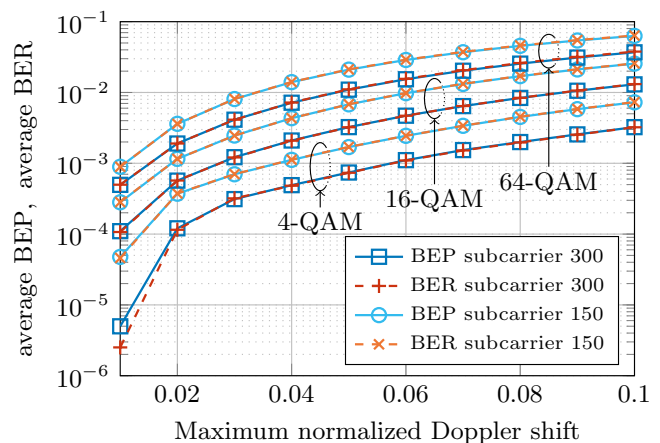


Fig. 11: Average BEP and average BER versus maximum normalized Doppler shift for a symbol in subcarriers 150 and 300 considering 10^4 realizations of the ITU-R vehicular channel model and $E_b^{\text{TX}}/N_0 = 50$ dB.

the average of the BEP, we obtain good approximations of the BER because the main contribution to the average is due to symbols with low SINR values. As shown in Fig. 11, the results do not match only for the case of the border subcarrier with 4-QAM and a low maximum Doppler shift value. This is because the symbols contributing to the average BER have higher SINR values where the BEP approximation is worse as shown in Section V-A.

VI. CONCLUSIONS

In this paper we proposed and studied an analytical approximation of the BEP and provided a lower capacity bound for OFDM systems when transmitting random Gray-coded QAM symbols over deterministic doubly-selective channels. Assuming a general tapped delay channel model we firstly calculated the expressions for the channel response and ICI coefficients affecting the OFDM symbols and showed that the ICI is not normally-distributed. However, considering deterministic channel realizations, i.e., that the channel response is previously known, we showed by means of Monte-Carlo simulations on three different standardized channel models (3GPP typical urban, 3GPP rural area, and ITU-R vehicular) that the ICI is suitably approximated by a complex-valued normal distribution in realistic scenarios. The goodness of the approximation was found to be better as the constellation order increases, specially from 4-QAM to 16-QAM, whereas from 16-QAM to 64-QAM the improvement is not so large. It was also shown that for the border subcarriers (the ones closer to the edge of the effective used bandwidth) the normal approximation may be poor, but the goodness increases rapidly as we move away from the border subcarrier, reaching values near the maximum in just 4 or 5 subcarriers.

Next, we analyzed the capacity and the BEP of the OFDM systems considered. Firstly, we obtained an analytical expression for a lower bound of the capacity by means of the Shannon-Hartley theorem, assuming that the noise plus ICI is Gaussian. Secondly, we showed that the BEP can be

approximated also by considering that the noise plus ICI is Gaussian and using well-known BEP formulas for Gray-coded QAM symbols transmitted over AWGN channels. To characterize the goodness of this approximation, we defined the metric of the error factor, expressed as the ratio of the BER (which we calculated by means of Monte-Carlo simulations) to the BEP (the proposed analytical approximation). Results of the instantaneous error factor metric showed that the instantaneous BEP approximates well the instantaneous BER for low SINR values, whereas for larger SINR values, the instantaneous BEP was shown to be always lower than the instantaneous BER, i.e., the BEP is an upper bound of the BER. Finally, we showed results of the averaged BEP and BER for several channel realizations versus the ratio of transmitted bit energy to noise spectral power density and versus the maximum normalized Doppler shift. Results were shown for two subcarriers, one at the middle and one on the edge where the normal approximation of the ICI was shown to be worse. In these results the average BEP matched almost exactly the average BER, even for the border subcarrier. Thus, we conclude that the proposed BEP approximation is indeed a good approximation of the average BER in OFDM systems.

The results obtained in this paper represent a great contribution to the research and industrial societies working on the performance evaluation of different deployments and network architectures for wireless communications in doubly-selective channels, which are a key scenario for 5G cellular systems. Different from previous related works in the literature, the performed study is not constrained to a specific kind of channel, but applicable to a general tapped delay one. Moreover, the proposed closed-form analytical expressions for the capacity and BEP do not only allow for noticeably improving the efficiency of the evaluations (by discarding the need of computationally-costly Monte-Carlo simulations), but also provide a theoretical framework to optimize the parameters of the OFDM communication systems directly impacting on the achievable performance. Finally, in order to help other researchers to easily use our results, we have made all the source code employed to obtain the results in this paper freely available.

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