QoS Constrained Power Minimization in the MISO Broadcast Channel with Imperfect CSI

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Abstract

In this article we consider the design of linear precoders and receivers in a *Multiple-Input Single-Output* (MISO) *Broadcast Channel* (BC). We aim to minimize the transmit power while meeting a set of per-user *Quality-of-Service* (QoS) constraints expressed in terms of per-user average rate requirements. The *Channel State Information* (CSI) is assumed to be known perfectly at the receivers but only partially at the transmitter. To solve this problem we convert the QoS constraints into *Minimum Mean Square Error* (MMSE) constraints. We then leverage MSE duality between the BC and the *Multiple Access Channel* (MAC), as well as standard interference functions in the dual MAC, to perform power minimization by means of an *Alternating Optimization* (AO) algorithm. Problem feasibility is also studied to determine whether the QoS constraints can be met or not. Finally, we present an

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algorithm to balance the average rates and manage situations that may be unfeasible, or lead to an unacceptably high transmit power.
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1. Introduction

The *Multiple-Input Single-Output* (MISO) *Broadcast Channel* (BC) is an appropriate model for the downlink of a cellular communication system in which a *Base Station* (BS) with N antennas serves a set of K single-antenna non-cooperative users. We assume signals are linearly filtered at transmission and reception to mitigate the inter-user interference. We also assume perfect *Channel State Information at the Receivers* (CSIR) but only imperfect *Channel State Information at the Transmitter* (CSIT). This is a reasonable assumption in practical setups, as receivers can accurately estimate the CSI from the incoming signals while the transmitter obtains the CSI via a feedback channel in *Frequency Division Duplex* (FDD) systems, or an estimate of the reciprocal uplink CSI in *Time Division Duplex* (TDD) systems.

Several imperfect CSI models have been considered in the literature. Some authors employ bounded uncertainty models which lead to optimization problems that can be addressed using standard convex solvers [1, 2, 3, 4]. Other authors, as in this work, model CSI uncertainty as a stochastic error whose distribution is known [5, 6, 7, 8].

Different performance metrics have been considered for BC optimization. Maximizing the *Signal to Interference–plus–Noise Ratio* (SINR) [1, 3, 9] is a common approach closely related to the maximization of the data rate.

Moreover, imperfect CSIT is considered in [9] by handling approximations for the average SINR in which the expectation is applied separately to the numerator and the denominator. The tightness of such an approximation, however, is questionable and it is unclear whether it represents an upper or a lower bound. Other metrics are based on *Mean Square Error* (MSE). Per-user MSE has been considered in [2], or in [8], in which an approximation of the average MSE based on Taylor expansion was proposed. Also, the inverse of the MSE was studied in [4]. Sum MSE [3, 5], and MSE balancing [5, 10] have also been addressed frequently. Sum MSE minimization in the BC can be converted into an equivalent minimization in the dual *Multiple Access Channel* (MAC) to perform *Alternating Optimization* (AO). Finally, weighted sum rate was studied; e.g., in [11, 12]. A common approach is to reformulate the problem as a weighted sum MSE to find solutions based on *Geometric Programing* (GP), or based on the algorithm proposed in [12]. However, sum rate optimizations may lead to unfair situations in which some of the users get low (or even zero) information rates.

Regarding optimization in the BC, some authors search for the best metric performance for the given transmit power [5, 10, 11]. Alternatively, authors in [1, 2, 3, 4] consider the minimization of the total transmit power under a set of *Quality-of-Service* (QoS) constraints, as done in this work. In particular, we ensure that users enjoy certain average rate values, which make it possible to avoid the unfair situations stated previously.

To tackle the corresponding optimization problem, average rate constraints are replaced by average MMSE requirements using Jensen's inequality. Note that, contrary to other solutions (e.g. [8, 9]), no approximations are needed to theoretically solve MSE problem formulation. Hence, we determine the MISO BC linear precoders and receivers by means of an AO process in which we resort to duality between the BC and the MAC, as done in, e.g., [10]. More specifically, we employ the MSE duality proposed in [13] according to the assumption of perfect CSIR and imperfect CSIT. In the dual MAC, power minimization can be expressed as a power allocation problem and solved using the framework proposed in [14].

This work also shows that the proposed power minimization algorithm converges if the QoS constraints can be met. Therefore, we provide a test for checking the feasibility of the average rate restrictions.

Additionally, we consider the rate balancing problem: the minimum of the average rates is maximized under a total transmit power constraint. Again, this problem is reformulated by bounding the average rates by average MMSEs. Such reformulation leads to the minimization of the maximum weighted average MSE under a total power constraint, and it can be solved by combining a bisection search with the proposed power minimization algorithm.

In recent communication systems, users are provided with more than one antenna. When we extend the system model to the MIMO scenario, two possibilities arise: single or multiple per-user streams may be considered. Considering more than one per-user stream adds more complexity to the problem, since the per-user average rate constraints have to be divided amongst all the streams allocated to the user. Discussion on such matters is not within the scope of this work. However, the methods proposed for the MISO BC directly apply in the single-stream MIMO BC, as shown in [15].

The following notation is used. Matrices and column vectors are written

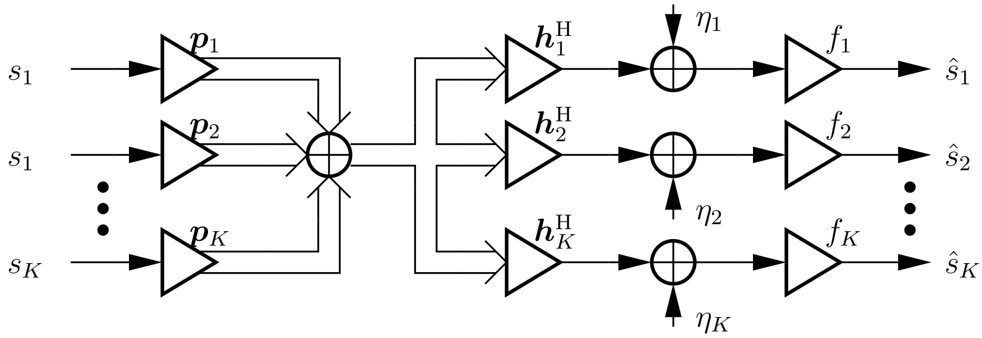


Figure 1: Sytem model of the Gaussian MISO BC.

using uppercase and lowercase boldface characters, respectively. By $[\boldsymbol{X}]_{j,k}$, we denote the element in row j and column k of matrix $\boldsymbol{X}$; $\mathrm{diag}(x_i)$ represents a diagonal matrix whose i-th diagonal element is x_i; $\mathbf{I}_N$ stands for the $N \times N$ identity matrix, and $\mathbf{1}$ represents the all ones vector. The superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote the complex conjugate, transpose, and Hermitian. $\Re\{\cdot\}$ represents the real part operator. Finally, $\mathbb{E}[\cdot]$ stands for statistical expectation, $\mathrm{tr}(\cdot)$ denotes the trace operation, and $|\cdot|$, $\|\cdot\|_2$, $\|\cdot\|_{\mathrm{F}}$ stand for the absolute value, the Euclidean norm, and the Frobenius norm, respectively.

2. System Model

Let us consider the system model of the Gaussian MISO BC depicted in Fig. 1. We assume the BS is equipped with N transmit antennas and sends the data signal $s_k \in \mathbb{C}$ to the user $k \in \{1, \ldots, K\}$. The data signal vector $\mathbf{s} = [s_1, \ldots, s_K]^T$ is assumed to be zero-mean, unit-variance, uncorrelated, and Gaussian; i.e., $\mathbf{s} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_K)$. The data signals are precoded with the linear filters $\boldsymbol{p}_k \in \mathbb{C}^N$ at the BS and propagate over the vector channels $\boldsymbol{h}_k \in \mathbb{C}^N$. At the user-ends, the received signals are linearly filtered with

$f_k \in \mathbb{C}$ to produce an estimate of the k-th user data signal

$$\hat{s}_k = f_k^* \boldsymbol{h}_k^{\mathrm{H}} \sum_{i=1}^{K} \boldsymbol{p}_i s_i + f_k^* \eta_k, \tag{1}$$

where $\eta_k \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_{\eta_k}^2)$ represents the thermal noise independent of the data signals. Note that, according to this signal model, the transmit power is $\sum_{k=1}^{K} \|\boldsymbol{p}_k\|^2$.

We assume that the receiver k has perfect knowledge of its own channel $\boldsymbol{h}_k$. Contrarily, the BS only has imperfect knowledge of the CSI which is modeled through the random variable v. The random nature of v is due to numerous sources of error (i.e., channel estimation, quantization, delay, etc.) which affect the acquisition process for the CSIT in both TDD and FDD systems. Imperfect channel knowledge is expressed through the conditional *Probability Density Functions* (PDF) $f_{\boldsymbol{h}_k|v}(\boldsymbol{h}_k|v)$, assumed to be known at the transmitter.

Recalling (1), $\hat{s}_k$ is a noisy version of the data signal s_k. The achievable instantaneous data rate in such situation is

$$R_k = \log_2(1 + \boldsymbol{p}_k^{\mathrm{H}} \boldsymbol{h}_k \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k x_k^{-1}), \tag{2}$$

where $x_k = \boldsymbol{h}_k^{\mathrm{H}} (\sum_{i \neq k} \boldsymbol{p}_i \boldsymbol{p}_i^{\mathrm{H}}) \boldsymbol{h}_k + \sigma_{\eta_k}^2$. In this work, we search for the precoders $\boldsymbol{p}_k$ that minimize the transmit power while meeting the *Quality of Service* (QoS) constraints $\mathbb{E}[R_k(v)] \geq \rho_k$, $k \in \{1, \ldots, K\}$, where $\{\rho_k\}_{k=1}^{K}$ is the set of per-user average rates to be met by the system. Note that the notation $R_k(v)$ highlights that the transmitter has access to the partial CSIT v for any channel realization $\boldsymbol{h}_k$, $\forall k$. Based on partial CSIT v, the BC precoders are determined according to the variational problem

$$\min_{\{\boldsymbol{p}_k(v)\}_{k=1}^K} \mathbb{E}\left[\sum_{k=1}^K \|\boldsymbol{p}_k(v)\|_2^2\right] \quad \text{s.t.} \quad \mathbb{E}\left[R_k(v)\right] \geq \rho_k, \, \forall k. \tag{3}$$

Note that optimization is over the maps $\boldsymbol{p}_k(v)$; i.e., with the precoders depending on the partial CSIT v. The constrained minimization problem (3) is difficult to solve in general. However, in the ensuing subsection, we exploit the relationship between the average rate and the average MMSE to express (3) in a more manageable way.

2.1. MSE Constrained Optimization

Let $\mathrm{MSE}_k^{\mathrm{BC}} = \mathbb{E}[|s_k - \hat{s}_k|^2]$ be the instantaneous MSE of the k-th user in the BC. For given channel $\boldsymbol{h}_k$,

$$\mathrm{MSE}_k^{\mathrm{BC}} = 1 - 2\Re\left\{f_k^* \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k\right\} + |f_k|^2 \left(\left|\boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k\right|^2 + x_k\right), \tag{4}$$

where x_k is as defined below (2). Note that $\boldsymbol{h}_k$ is assumed to be fixed in (4). Therefore, the partial CSIT v is likewise fixed and we drop the dependence of $\boldsymbol{p}_k$ on v for the sake of brevity. Correspondingly, the minimum MSE receive filter is expressed by

$$f_k^{\mathrm{MMSE}}(\boldsymbol{h}_k) = \left(\boldsymbol{h}_k^{\mathrm{H}} \sum_{i=1}^K \boldsymbol{p}_i \boldsymbol{p}_i^{\mathrm{H}} \boldsymbol{h}_k + \sigma_{\eta_k}^2\right)^{-1} \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k, \tag{5}$$

and the MMSE is obtained by substituting (5) into (4); i.e.,

$$\mathrm{MMSE}_k^{\mathrm{BC}} = 1 - f_k^{\mathrm{MMSE},*}(\boldsymbol{h}_k) \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k. \tag{6}$$

Finally, by applying the equality $1 - \frac{a}{b} = (1 + \frac{a}{b-a})^{-1}$ to (6) it is possible to express the k-th user rate (2) as $R_k = -\log_2(\mathrm{MMSE}_k^{\mathrm{BC}})$.

Equations (4), (5) and (6) are suitable for the BC design with perfect CSI at both ends of the communication system. However, for imperfect CSIT we consider the average MSE at the BC, $\mathbb{E}[\mathrm{MSE}_k^{\mathrm{BC}}(v)]$. Correspondingly, the average MMSE at the BC is expressed by

$$\mathbb{E}[\mathrm{MMSE}_k^{\mathrm{BC}}(v)] = \mathbb{E}\left[1 - f_k^{\mathrm{MMSE},*}(\boldsymbol{h}_k)\, \boldsymbol{h}_k^{\mathrm{H}} \boldsymbol{p}_k(v)\right],$$

where we highlight the perfect CSIR assumption by $f_k(\boldsymbol{h}_k)$.

Taking advantage of $\log_2(\cdot)$ function concavity, and employing Jensen's inequality, we arrive at the following lower bound for the average rate

$$\mathbb{E}\left[R_k(v)\right] \geq -\log_2 \mathbb{E}\left[\mathrm{MMSE}_k^{\mathrm{BC}}(v)\right] \geq -\log_2 \mathbb{E}\left[\mathrm{MSE}_k^{\mathrm{BC}}(v)\right]. \qquad (7)$$

An example of the gap between the average rate and the average MMSE lower bound is examined in Appendix A.

The constraints in (3) hold for $-\log_2 \mathbb{E}[\mathrm{MSE}_k^{\mathrm{BC}}(v)] \geq \rho_k$, and they are conservatively rewritten accordingly as

$$\mathbb{E}\left[\mathrm{MSE}_k^{\mathrm{BC}}(v)\right] \leq 2^{-\rho_k}. \qquad (8)$$

Hence, the optimization problem (3) can be reformulated as

$$\min_{\{\boldsymbol{p}_k(v), f_k(\boldsymbol{h}_k)\}_{k=1}^K} \mathbb{E}\left[\sum_{k=1}^K \|\boldsymbol{p}_k(v)\|_2^2\right] \quad \text{s.t.} \quad \mathbb{E}\left[\mathrm{MSE}_k^{\mathrm{BC}}(v)\right] \leq 2^{-\rho_k}, \forall k. \qquad (9)$$

Contrary to (3), the scalar receive filters $f_k(\boldsymbol{h}_k)$ are now involved in the optimization process. Nevertheless, in the optimum of (9), MMSE filters are used [see (5)].

Note that, by means of Bayes' rule, $\mathbb{E}[\mathrm{MSE}_k^{\mathrm{BC}}(v)] = \mathbb{E}[\mathbb{E}[\mathrm{MSE}_k^{\mathrm{BC}}(v)|\, v]]$. Thus, introducing $\overline{\mathrm{MSE}}_k^{\mathrm{BC}}(v) = \mathbb{E}[\mathrm{MSE}_k^{\mathrm{BC}}(v)|\, v]$, the variational problem of

(9) can be solved pointwise for the given v as follows

$$\min_{\{\boldsymbol{p}_k(v), f_k(\boldsymbol{h}_k)\}_{k=1}^K} \sum_{k=1}^{K} \|\boldsymbol{p}_k(v)\|_2^2 \text{ s.t. } \overline{\mathrm{MSE}}_k^{\mathrm{BC}}(v) \leq 2^{-\rho_k}, \forall k. \tag{10}$$

Note that the average transmit power resulting from (10) is larger than that obtained in (3) since the MMSE constraints in (10) are more restrictive than the rate constraints in (3). From here forward, we use $\boldsymbol{p}_k$, f_k and $\overline{\mathrm{MSE}}_k^{\mathrm{BC}}$ for the sake of notational brevity.

2.2. BC/MAC MSE Duality

It is important to note that $\overline{\mathrm{MSE}}_k^{\mathrm{BC}}$ is independent of the receive filter f_j for $j \neq k$ but depends on all precoders $\boldsymbol{p}_j$ for $j \neq k$. This means that $\boldsymbol{p}_k$ cannot be individually optimized when solving (10) but, instead, all precoders should be jointly optimized. Nevertheless, it is possible to avoid such dependence by exploiting MAC/BC MSE duality as described in [13].

In the *Single-Input Multiple-Output* (SIMO) MAC dual to the MISO BC, the receive and transmit filters are represented by $\boldsymbol{g}_k \in \mathbb{C}^N$ and $t_k \in \mathbb{C}$, respectively, while $\boldsymbol{\theta}_k = \boldsymbol{h}_k \sigma_{\eta_k}^{-1} \in \mathbb{C}^N$ and $\boldsymbol{n} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_N)$ represent the channel response and noise in the dual MAC. The average MSE is, thus,

$$\overline{\mathrm{MSE}}_k^{\mathrm{MAC}}(v) = 1 - 2\,\mathbb{E}\left[\Re\left\{\boldsymbol{g}_k^{\mathrm{H}}\boldsymbol{\theta}_k t_k\right\} \mid v\right] + \|\boldsymbol{g}_k\|_2^2 + \mathbb{E}\left[\sum_{i=1}^{K} |t_i|^2 \left|\boldsymbol{g}_k^{\mathrm{H}}\boldsymbol{\theta}_i\right|^2 \middle|\, v\right], \tag{11}$$

where the expectations are taken w.r.t. all channels for the given partial CSI v as in $\overline{\mathrm{MSE}}_k^{\mathrm{BC}}(v)$ from (10).

Suppose, now, that the filters in the MAC; i.e., t_k and $\boldsymbol{g}_k$, are given. Introducing the set $\{\alpha_k\}_{k=1}^K \in \mathbb{R}^+$, and the following relationships between the MAC and the BC filters

$$\boldsymbol{p}_k(v) = \alpha_k \boldsymbol{g}_k(v), \quad f_k = \alpha_k^{-1} \sigma_{\eta_k}^{-1} t_k \left(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_K \right), \tag{12}$$

it is possible to achieve identical MSEs for all the users in the BC, just as in the MAC, i.e., $\overline{\mathrm{MSE}}_k^{\mathrm{BC}} = \overline{\mathrm{MSE}}_k^{\mathrm{MAC}}\, \forall k$. Moreover, the average transmit power is preserved [13]. Note that the MAC receive filters and precoders are functions of the partial CSIT v and the channel, respectively, like the corresponding BC precoders and receive filters.

In summary, a problem in the BC based on $\overline{\mathrm{MSE}}_k^{\mathrm{BC}}$ can be equivalently reformulated in the dual MAC with $\overline{\mathrm{MSE}}_k^{\mathrm{MAC}}$, and vice-versa [13].

3. Power Minimization

We now focus on solving the power minimization problem as formulated in (10). First of all, for given BC precoders $\boldsymbol{p}_k$, the MMSE BC scalar receive filters f_k^{MMSE} are readily obtained via (5) considering perfect CSIR. Next, we convert the BC receive filters f_k to the MAC precoding weights t_k using MSE duality. Recall that t_k is a function of $\boldsymbol{h}_k$.

Let us now define the average transmit power $\xi_k = \mathbb{E}[|t_k|^2 |\, v]$ and the normalized MAC precoders $\tau_k = t_k/\sqrt{\xi_k}$ so that $\mathbb{E}[|\tau_k|^2 |\, v] = 1$. Let us also introduce the conditional expectations $\boldsymbol{\mu}_k = \mathbb{E}[\tau_k \boldsymbol{\theta}_k |\, v]$ and $\boldsymbol{\Theta}_i = \mathbb{E}[|\tau_i|^2 \boldsymbol{\theta}_i \boldsymbol{\theta}_i^{\mathrm{H}} |\, v]$. Finally, let us define $\boldsymbol{\xi} = [\xi_1, \ldots, \xi_K]^{\mathrm{T}}$ as the vector containing the average transmit powers for all users, i.e., the power allocation vector. Notice that, unlike the precoders t_k, $\boldsymbol{\xi}$ only depends on the partial CSIT v, similar to the total transmit power $\sum_{k=}^{K} \|\boldsymbol{p}_k(v)\|_2^2$ in the BC.

With these definitions, $\overline{\mathrm{MSE}}_k^{\mathrm{MAC}}$ from (11) reads as

$$\overline{\mathrm{MSE}}_k^{\mathrm{MAC}} = 1 - 2\sqrt{\xi_k}\Re\left\{\boldsymbol{g}_k^{\mathrm{H}}\boldsymbol{\mu}_k\right\} + \boldsymbol{g}_k^{\mathrm{H}}\left(\sum_{i=1}^{K}\xi_i\boldsymbol{\Theta}_i + \mathbf{I}_N\right)\boldsymbol{g}_k. \qquad (13)$$

Therefore, the equalizers minimizing the $\overline{\mathrm{MSE}}_k^{\mathrm{MAC}}$ are

$$\boldsymbol{g}_k^{\mathrm{MMSE}} = \left(\sum_{i=1}^{K}\xi_i\boldsymbol{\Theta}_i + \mathbf{I}_N\right)^{-1}\sqrt{\xi_k}\boldsymbol{\mu}_k. \qquad (14)$$

By substituting (14) into (13), we obtain the following expression

$$\overline{\mathrm{MMSE}}_k^{\mathrm{MAC}} = 1 - \xi_k\boldsymbol{\mu}_k^{\mathrm{H}}\left(\sum_{i=1}^{K}\xi_i\boldsymbol{\Theta}_i + \boldsymbol{I}_N\right)^{-1}\boldsymbol{\mu}_k. \qquad (15)$$

In order to obtain a computationally-efficient update of the power allocation, let us introduce the scalar MAC parameters r_k so that $\boldsymbol{g}_k = r_k\tilde{\boldsymbol{g}}_k$ (cf. [16, Sec. 4.1]). Thus, (13) reads as

$$\overline{\mathrm{MSE}}_k^{\mathrm{MAC}} = 1 - 2\Re\left\{r_k^*\tilde{\boldsymbol{g}}_k^{\mathrm{H}}\boldsymbol{\mu}_k\sqrt{\xi_k}\right\} + |r_k|^2\,\tilde{\boldsymbol{g}}_k^{\mathrm{H}}\left(\sum_{i=1}^{K}\xi_i\boldsymbol{\Theta}_i + \mathbf{I}_N\right)\tilde{\boldsymbol{g}}_k. \qquad (16)$$

For given $\tilde{\boldsymbol{g}}_k$, the optimal scalar filters are

$$r_k^{\mathrm{MMSE}} = \tilde{\boldsymbol{g}}_k^{\mathrm{H}}\boldsymbol{\mu}_k\sqrt{\xi_k}\left(\tilde{\boldsymbol{g}}_k^{\mathrm{H}}\left(\sum_{i=1}^{K}\xi_i\boldsymbol{\Theta}_i + \mathbf{I}_N\right)\tilde{\boldsymbol{g}}_k\right)^{-1}. \qquad (17)$$

Substituting r_k^{MMSE} into (16) yields the following average MAC MMSE

$$\Sigma_k = 1 - \xi_k\left|\tilde{\boldsymbol{g}}_k^{\mathrm{H}}\boldsymbol{\mu}_k\right|^2 y_k^{-1}, \qquad (18)$$

where $y_k = \sum_{i=1}^{K}\xi_i\tilde{\boldsymbol{g}}_k^{\mathrm{H}}\boldsymbol{\Theta}_i\tilde{\boldsymbol{g}}_k + \|\tilde{\boldsymbol{g}}_k\|_2^2$. Replacing $\tilde{\boldsymbol{g}}_k$ in (18) with $\boldsymbol{g}_k^{\mathrm{MMSE}}$ given by (14), yields (15). Therefore, (14) is the minimizer of both (13) and (18).

3.1. Power Allocation

So far, we have found the MMSE vector receivers in the MAC, $\{\boldsymbol{g}_k^{\mathrm{MMSE}}\}_{k=1}^K$, corresponding to the BC precoders $\{\boldsymbol{p}_k\}_{k=1}^K$. We now search for the optimal MAC receivers $\{\boldsymbol{g}_k\}_{k=1}^K$ and power allocation $\boldsymbol{\xi}$ that minimize the transmit power (subject to the QoS constraints $\overline{\mathrm{MMSE}}_k^{\mathrm{BC}} \leq 2^{-\rho_k}$) for given normalized precoders $\{\tau_k\}_{k=1}^K$. Due to the mutual dependence of $\{\boldsymbol{g}_k\}_{k=1}^K$ and $\boldsymbol{\xi}$, we have to jointly optimize both of them.

To that end, we rely on standard interference functions [14, 17]. They concisely describe the framework of the system requirements depending on power allocation as the vector inequality $\boldsymbol{\xi} \geq \boldsymbol{f}(\boldsymbol{\xi})$. To ensure that the fixed point iteration $\boldsymbol{\xi}^{(n+1)} = \boldsymbol{f}(\boldsymbol{\xi}^{(n)})$ converges to the optimal solution for $\boldsymbol{\xi}$, the function $\boldsymbol{f}(\cdot)$ must satisfy $\boldsymbol{f}(\boldsymbol{\xi}) > \mathbf{0}$ (positivity), $a\boldsymbol{f}(\boldsymbol{\xi}) > \boldsymbol{f}(a\boldsymbol{\xi})$ $\forall a > 1$ (scalability), and $\boldsymbol{f}(\boldsymbol{\xi}) \geq \boldsymbol{f}(\boldsymbol{\xi}')$, $\boldsymbol{\xi} \geq \boldsymbol{\xi}'$ (monotonicity).

We now define $I_k(\boldsymbol{\xi}) = \xi_k \Sigma_k$, which can be interpreted as the interference for user k. Applying the equality $1 - \frac{a}{b} = (1 + \frac{a}{b-a})^{-1}$ to (18) yields

$$I_k(\boldsymbol{\xi}) = \left(\frac{1}{\xi_k} + \left|\tilde{\boldsymbol{g}}_k^{\mathrm{H}} \boldsymbol{\mu}_k\right|^2 \left(y_k - \xi_k \left|\tilde{\boldsymbol{g}}_k^{\mathrm{H}} \boldsymbol{\mu}_k\right|^2 \right)^{-1} \right)^{-1}. \tag{19}$$

We subsequently collect all of these functions into the vector $\boldsymbol{I}(\boldsymbol{\xi}) = [I_1(\boldsymbol{\xi}), \ldots, I_K(\boldsymbol{\xi})]$, meeting the conditions of a standard interference function.

Note that, due to BC/MAC average MSE duality, the QoS constraints can be equivalently expressed as $\overline{\mathrm{MSE}}_k^{\mathrm{MAC}} \leq 2^{-\rho_k}$. Furthermore, since $\Sigma_k = \frac{I_k(\boldsymbol{\xi})}{\xi_k}$, we reformulate the power minimization problem (10) in the dual MAC for a given set of normalized precoders $\{\tau_k\}_{k=1}^K$ as

$$\min_{\{\xi_k, \tilde{\boldsymbol{g}}_k\}_{k=1}^K} \sum_{i=1}^K \xi_i \quad \text{s.t.} \quad \frac{I_k(\boldsymbol{\xi})}{\xi_k} \leq 2^{-\rho_k}, \ \forall k. \tag{20}$$

As shown in [14], since $\boldsymbol{I}(\boldsymbol{\xi})$ is a standard interference function, the iteration $\xi_k^{(n)} = 2^{\rho_k} I_k(\boldsymbol{\xi}^{(n-1)})$ converges to ξ_k^{opt} for given $\{\tilde{\boldsymbol{g}}_k\}_{k=1}^K$.

Moreover, the aforementioned iteration can also be used to jointly find the $\{\xi_k, \tilde{\boldsymbol{g}}_k\}_{k=1}^K$ that solve the power minimization problem (20). Indeed, let $I_k(\boldsymbol{\xi}, \tilde{\boldsymbol{g}}_k) = \xi_k \Sigma_k$ be the same function explicitly highlighting the dependence on $\tilde{\boldsymbol{g}}_k$. Similarly, we rewrite the interference function as $\boldsymbol{I}(\boldsymbol{\xi}, \tilde{\boldsymbol{G}}) = [I_1(\boldsymbol{\xi}, \tilde{\boldsymbol{g}}_1), \ldots, I_K(\boldsymbol{\xi}, \tilde{\boldsymbol{g}}_K)]^{\mathrm{T}}$, with $\tilde{\boldsymbol{G}} = [\tilde{\boldsymbol{g}}_1, \ldots, \tilde{\boldsymbol{g}}_K]$. Since $\boldsymbol{I}(\boldsymbol{\xi}, \tilde{\boldsymbol{G}})$ is standard for any $\tilde{\boldsymbol{G}}$, so is $\min_{\tilde{\boldsymbol{G}}} \boldsymbol{I}(\boldsymbol{\xi}, \tilde{\boldsymbol{G}})$, in which minimization is performed element-wise. As a consequence, the *Alternating Optimization* (AO) iteration

$$\begin{aligned} \tilde{\boldsymbol{g}}_k^{(n)} &\leftarrow \left(\sum_{i=1}^{K} \xi_i^{(n-1)} \boldsymbol{\Theta}_i + \mathbf{I}_N \right)^{-1} \sqrt{\xi_k^{(n-1)}} \boldsymbol{\mu}_k \forall k, \\ \xi_k^{(n)} &\leftarrow 2^{\rho_k} I_k \left(\boldsymbol{\xi}^{(n-1)}, \tilde{\boldsymbol{g}}_k^{(n)} \right) \forall k, \end{aligned} \tag{21}$$

converges to the global optimum of (20), as shown in [17].

Finally, the dual MAC equalizers obtained can be converted into the BC precoders by applying BC/MAC average MSE duality [see (12)].

3.2. Power Minimization Algorithm

Algorithm 1 presents the steps to solve the optimization problem (10) according to the ideas presented so far. Recall that we assume that v and $f_{\boldsymbol{h}_k|v}(\boldsymbol{h}_k|v)$ are known at the transmitter, according to the imperfect CSIT model. Since closed-form expressions of the expectations in (20) are not known for general channel models, we evaluate them by using a Monte Carlo method. To that end, we generate M channel realizations $\boldsymbol{h}_k^{(m)} \sim f_{\boldsymbol{h}_k|v}(\boldsymbol{h}_k|v)$, $m = 1, \ldots, M$, and introduce the matrix $\boldsymbol{H}_k = \sigma_{\eta_k}^{-1}[\boldsymbol{h}_k^{(1)}, \ldots, \boldsymbol{h}_k^{(M)}]$ to collect the M dual MAC channel realizations. We also define $t_k^{(m)}$ as the k-th user scalar MAC precoder for given channel realization $\boldsymbol{h}_k^{(m)}$. By collecting the $t_k^{(m)}$ we get the normalized diagonal precoder

$$\boldsymbol{T}_k = \frac{1}{\sqrt{\xi_k}} \operatorname{diag}\left(t_k^{(1)}, \ldots, t_k^{(M)}\right), \tag{22}$$

where $\xi_k = \frac{1}{M}\sum_{m=1}^{M} |t_k^{(m)}|^2$ is the k-th user average transmit power for given v. Therefore, we calculate the expectations as $\boldsymbol{\mu}_k = \frac{1}{M}\boldsymbol{H}_k\boldsymbol{T}_k\mathbf{1}$ and $\boldsymbol{\Theta}_k = \frac{1}{M}\boldsymbol{H}_k\boldsymbol{T}_k\boldsymbol{T}_k^{\mathrm{H}}\boldsymbol{H}_k^{\mathrm{H}}$.

We start with an initial set of BC random precoders $\{\boldsymbol{p}_k^{(0)}\}_{k=1}^{K}$ (line 1). We next calculate the M BC receivers $f_k^{\mathrm{MMSE},(m)}$ corresponding to the channel realizations $\boldsymbol{h}_k^{(m)}$ (line 5). Applying BC/MAC duality, we determine the M dual MAC precoders (line 7). The normalized matrix of MAC precoders is obtained after lines 8 and 9 are executed.

The following two steps (lines 10 and 11) perform iteration (21) to update the power allocation and the dual MAC receivers. Observe, however, that we do not include the loop arising from the optimization in (21). The reason is to avoid convergence problems -which may occur even when the problem constraints are feasible- caused by the lack of feasibility of the power minimization problem for given MAC precoders $\boldsymbol{T}_k^{(\ell)}$ at the ℓ-th iteration (cf. (20)). Therefore, considering a single loop, we avoid this undesirable effect, as can be seen in our simulation experiments (cf. [16, 15]).

After the updates in lines 10 and 11, the new MAC transmit filters are determined in line 13. Finally, we switch back to the BC in line 15. Due to the existence of a unique minimum in (10), and to the fact that every step in the algorithm either reduces the average MMSEs or the total transmit power, the convergence of the algorithm is guaranteed when the QoS constraints

Algorithm 1 Power Minimization trough AO

1: $\ell \leftarrow 0$, initialize $\boldsymbol{p}_i^{(0)}$, $\forall i$
2: **repeat**
3: $\quad \ell \leftarrow \ell + 1$, execute commands for all $k \in \{1, \dots, K\}$
4: $\quad$ **for** $m = 1$ to M **do**
5: $\quad\quad f_k^{(\ell,m)} \leftarrow f_k^{\mathrm{MMSE},(\ell,m)}$ [see (5)]
6: $\quad$ **end for**
7: $\quad t_k^{(\ell,m)} \leftarrow$ BC-to-MAC conversion [see Sec. 2.2]
8: $\quad \xi_k^{(\ell-1)} \leftarrow \frac{1}{M} \sum_{m=1}^{M} |t_k^{(\ell,m)}|^2$
9: $\quad \boldsymbol{T}_k^{(\ell)} \leftarrow \frac{1}{\sqrt{\xi_k^{(\ell-1)}}} \operatorname{diag}(t_k^{(\ell,1)}, \dots, t_k^{(\ell,M)})$
10: $\quad \xi_k^{(\ell)} \leftarrow 2^{\rho_k} I_k(\boldsymbol{\xi}^{(\ell-1)})$ [power update]
11: $\quad \boldsymbol{g}_k^{(\ell)} \leftarrow$ update MAC receiver [see (14)]
12: $\quad$ **for** $m = 1$ to M **do**
13: $\quad\quad t_k^{(\ell,m)} \leftarrow \sqrt{\xi_k^{(\ell)}} [\boldsymbol{T}_k^{(\ell)}]_{m,m}$ [include power allocation]
14: $\quad$ **end for**
15: $\quad \boldsymbol{p}_k^{(\ell)} \leftarrow$ MAC to BC conversion [see Sec. 2.2]
16: **until** $||\boldsymbol{\xi}^{(\ell)} - \boldsymbol{\xi}^{(\ell-1)}||_1 \leq \delta$

are feasible (see Section 4). To check whether we have reached the desired accuracy or not, we set a threshold δ (line 16).

The computational complexity is approximately linear in the number of channel realizations, $\mathcal{O}(M)$, since the sizes of the matrices to be inverted in lines 7, 15, and 11 are small compared to M, i.e. $K \ll M$ and $N \ll M$.

4. Problem Feasibility

In this section, we analyze the feasibility of the power minimization problem (10). Due to the imperfect CSI assumption, interferences cannot be completely removed in the BC. Consequently, increasing the total transmit power does not necessarily lead to a reduction of the MMSEs for all the users because, although this increases the received power, it also increases the power of the interferences. In certain scenarios, the QoS constraints may require that some users achieve low MMSE values that may be unfeasible even though the transmit power is increased unlimitedly. Below, we present a feasibility test to determine whether it is possible or not to accomplish the QoS constraints $\overline{\mathrm{MMSE}}_k^{\mathrm{MAC}} = 2^{-\rho_k}$.

Let us start by considering the average MMSE in the MAC

$$\overline{\mathrm{MMSE}}_k^{\mathrm{MAC}} = 1 - \bar{\boldsymbol{\theta}}_k^{\mathrm{H}} \left(\sigma^2 \boldsymbol{I}_N + \sum_{i=1}^{K} \mathbb{E}[|t_i|^2 \, \boldsymbol{\theta}_i \boldsymbol{\theta}_i^{\mathrm{H}} | \, v] \right)^{-1} \bar{\boldsymbol{\theta}}_k, \qquad (23)$$

where $\bar{\boldsymbol{\theta}}_k = \mathbb{E}[\boldsymbol{\theta}_k t_k | \, v]$ and σ^2 is the thermal noise variance in the dual MAC. We define $\boldsymbol{\Upsilon} = [\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_K] \, \mathrm{diag}(t_1, \ldots, t_K)$ and rewrite (23) as follows

$$\overline{\mathrm{MMSE}}_k^{\mathrm{MAC}} = 1 - \left[\mathbb{E}[\boldsymbol{\Upsilon}^{\mathrm{H}} | v] \left(\mathbb{E}[\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^{\mathrm{H}} | v] + \sigma^2 \mathbf{I}_N \right)^{-1} \mathbb{E}[\boldsymbol{\Upsilon} | v] \right]_{k,k}. \qquad (24)$$

Hence, the sum average MMSE is

$$\sum_{i=1}^{K} \overline{\mathrm{MMSE}}_i^{\mathrm{MAC}} = K - \mathrm{tr} \left(\mathbb{E}[\boldsymbol{\Upsilon}^{\mathrm{H}} | v] \left(\mathbb{E}[\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^{\mathrm{H}} | v] + \sigma^2 \mathbf{I}_N \right)^{-1} \mathbb{E}[\boldsymbol{\Upsilon} | v] \right). \qquad (25)$$

When $K \geq N$ and the channel knowledge is perfect on both sides, (25) can be made arbitrarily small [18]. However, due to the imperfect CSI at the MAC receiver, we cannot reduce the average MMSE as much as desired.

Expression (25) allows us to determine the region where the feasible average MMSEs lie. Indeed, by setting the MAC thermal noise variance to zero (i.e., $\sigma^2 = 0$) we obtain the following lower bound for the sum average MMSE for any finite total average power allocation

$$\sum_{i=1}^{K} \overline{\mathrm{MMSE}}_i^{\mathrm{MAC}} > K - \mathrm{tr}\{\boldsymbol{X}\}, \tag{26}$$

where $\boldsymbol{X} = \mathbb{E}[\boldsymbol{\Upsilon}^{\mathrm{H}}|v](\mathbb{E}[\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^{\mathrm{H}}|v])^{-1}\,\mathbb{E}[\boldsymbol{\Upsilon}|v]$. The bound is asymptotically achieved when the powers for all users reach infinity. Therefore, we can formulate a necessary condition for the feasibility of QoS targets: any power allocation with finite sum power achieves a MMSE tuple $\{\overline{\mathrm{MMSE}}_i^{\mathrm{MAC}}\}_{i=1}^{K}$ inside the polytope

$$\mathcal{P} = \left\{ \{\overline{\mathrm{MMSE}}_i^{\mathrm{MAC}}\}_{i=1}^{K} \,|\, \sum_{i=1}^{K} \overline{\mathrm{MMSE}}_i^{\mathrm{MAC}} \geq K - \mathrm{tr}\left\{\boldsymbol{X}\right\} \right\}. \tag{27}$$

Moreover, for each MMSE tuple in $\mathcal{P}$, there is a power allocation vector $\boldsymbol{\xi}$ (see [15] for the complete proof).

5. Rate Balancing

So far we have considered the design of the filters in a MISO BC to minimize the transmit power while meeting certain QoS constraints. However, when the QoS constraints are rather stringent, the problem may be unfeasible. We now address a different problem, referred to as rate balancing in the literature, in which the per-user average rate constraints $\{\rho_k\}_{k=1}^{K}$ are scaled by a common factor $\varsigma \in \mathbb{R}^+$, and a power restriction P_{tx} is imposed. Observe that, unlike the power minimization formulation, we can relax the per-user requirement so that the problem is always feasible. For such a formulation, we propose to jointly optimize the balance level ς together with the precoders and receivers for given transmit power P_{tx}.

Using the lower bound (7), the problem formulation reads as

$$\max_{\{\varsigma(v),\boldsymbol{p}_k(v),f_k(\boldsymbol{h}_k)\}_{k=1}^K} \mathbb{E}\left[\varsigma(v)\right] \text{ s.t. } \mathbb{E}\left[\sum_{i=1}^K \|\boldsymbol{p}_i(v)\|_2^2\right] \leq P_{\text{tx}},$$
$$\text{and} \quad \mathbb{E}\left[\text{MSE}_k^{\text{BC}}\right] \leq 2^{-\mathbb{E}[\varsigma(v)]\rho_k}, \forall k. \tag{28}$$

Following a line of argumentation similar to the one presented in Sections 2 and 3, the problem (28) can be solved pointwise for each v using the MSE duality and interference functions. Hence, we rewrite (28) as

$$\max_{\{\varsigma,\xi_k,\boldsymbol{g}_k\}_{k=1}^K} \varsigma \quad \text{s.t.} \quad \frac{I_k\left(\boldsymbol{\xi}\right)}{\xi_k} \leq 2^{-\varsigma\rho_k}, \text{ and} \sum_{i=1}^K \xi_i \leq P_{\text{tx}}, \tag{29}$$

where $\boldsymbol{\xi} = [\xi_1, \ldots, \xi_K]^T$ is the power allocation vector, $\boldsymbol{g}_k$ are the dual MAC receivers, and $I_k\left(\boldsymbol{\xi}\right)$ are the interference functions [cf.(19)]. Similarly to (20), this formulation considers given MAC precoders. Algorithm 1 can be used to determine optimum filters for given ς, but it does not provide the optimum ς. Our proposal is to combine it with a bisection search to solve (29).

Indeed, let us start setting two feasible rate balancing values ς^{L} and ς^{H} so that $\varsigma^{\text{L}} \leq \varsigma^{\text{opt}} \leq \varsigma^{\text{H}}$. Let $\boldsymbol{\xi}^{\text{L}}$ and $\boldsymbol{\xi}^{\text{H}}$ be the optimum power allocation vectors corresponding to ς^{L} and ς^{H}, respectively. Such optimal power allocation vectors satisfy on the one hand $\frac{I_k(\boldsymbol{\xi}^{\text{L}})}{\xi_k^{\text{L}}} = 2^{-\varsigma^{\text{L}}\rho_k}$ and $\frac{I_k(\boldsymbol{\xi}^{\text{H}})}{\xi_k^{\text{H}}} = 2^{-\varsigma^{\text{H}}\rho_k}$, and on the other $\sum_{i=1}^K \xi_i^{\text{L}} \leq \sum_{i=1}^K \xi_i^{\text{opt}} \leq \sum_{i=1}^K \xi_i^{\text{H}}$, as shown below.

We now introduce the average MMSE balancing factors $\epsilon_k = \frac{2^{-\varsigma\rho_k}}{2^{-\rho_k}} = 2^{-\rho_k(\varsigma-1)}$. Note that increasing the balance level ς, decreases the scaling factors ϵ_k, $\forall k$. Let ϵ_k^{L} and ϵ_k^{H} be the MSE scaling factors corresponding to ς^{L} and ς^{H}, respectively. Note that $\epsilon_k^{\text{L}} \geq \epsilon_k^{\text{opt}} \geq \epsilon_k^{\text{H}}$.

To prove that a bisection search can be performed, we consider $\epsilon_k^{\mathrm{L}} = a\epsilon_k^{\mathrm{opt}}$, with $a > 1$. The constraints in (29) are met with equality when $\epsilon_k = \epsilon_k^{\mathrm{opt}}$ and $\boldsymbol{\xi} = \boldsymbol{\xi}^{\mathrm{opt}}$. Hence, $a\epsilon_k^{\mathrm{opt}} 2^{-\rho_k} = a\frac{I_k(\boldsymbol{\xi}^{\mathrm{opt}})}{\xi_k^{\mathrm{opt}}}$ means that increasing the MSE targets results in a decrease in the transmit power (i.e. $\xi_k = a^{-1}\xi_k^{\mathrm{opt}}$, $\forall k$) when we keep the interference constant. Moreover, notice that keeping the interference constant sets an upper bound for the interference with the reduced transmit powers $I_k(a^{-1}\boldsymbol{\xi}^{\mathrm{opt}}) < I_k(\boldsymbol{\xi}^{\mathrm{opt}})$. Therefore, the power needed to meet the constraint with equality is lower than $a^{-1}\boldsymbol{\xi}^{\mathrm{opt}}$, and $\mathbf{1}^{\mathrm{T}}\boldsymbol{\xi}^{\mathrm{L}} < a^{-1}\mathbf{1}^{\mathrm{T}}\boldsymbol{\xi}^{\mathrm{opt}} < P_{\mathrm{tx}}$ holds.

We now prove the relationship in the reverse direction; that is, a power reduction translates into larger scaling factors ϵ_k. Let us consider the power reduction $\boldsymbol{A}\boldsymbol{\xi}^{\mathrm{opt}}$ with $\boldsymbol{A} = \operatorname{diag}(a_1, \cdots, a_K) < \mathbf{I}$, which yields a certain average MSE scaling factor $\tilde{\epsilon}_k$ for any user k, i.e., $\tilde{\epsilon}_k 2^{-\rho_k} = \frac{1}{a_k\xi_k^{\mathrm{opt}}} I_k(\boldsymbol{A}\boldsymbol{\xi}^{\mathrm{opt}})$. Since no assumptions about user k have been made, we can focus on user k' so that $a_{k'} \leq a_k \, \forall k$. Consequently,

$$\tilde{\epsilon}_{k'} 2^{-\rho_{k'}} = \frac{I_{k'}\left(\boldsymbol{A}\boldsymbol{\xi}^{\mathrm{opt}}\right)}{a_{k'}\xi_{k'}^{\mathrm{opt}}} \geq \frac{I_{k'}\left(a_{k'}\boldsymbol{\xi}^{\mathrm{opt}}\right)}{a_{k'}\xi_{k'}^{\mathrm{opt}}} > \frac{I_{k'}\left(\boldsymbol{\xi}^{\mathrm{opt}}\right)}{\xi_{k'}^{\mathrm{opt}}} = \epsilon_{k'}^{\mathrm{opt}} 2^{-\rho_{k'}}. \tag{30}$$

Therefore, $\tilde{\epsilon}_{k'} > \epsilon_{k'}^{\mathrm{opt}}$ for $\boldsymbol{\xi}^{\mathrm{opt}} > \boldsymbol{A}\boldsymbol{\xi}^{\mathrm{opt}}$. We have previously shown that relaxing the balancing level $\epsilon_k^{\mathrm{opt}}$ implies a power reduction with respect to $\boldsymbol{\xi}^{\mathrm{opt}}$. Hence, we conclude that a power reduction entails a lower balancing level ς, and vice-versa, when the precoders, receive filters, and power allocation vectors are optimum for every balancing level.

Finally, reducing the gap between ς^{L} and ς^{H} results in the optimum balancing level ς^{opt} for the total average transmit power $\mathbf{1}^{\mathrm{T}}\boldsymbol{\xi}^{\mathrm{opt}} = P_{\mathrm{tx}}$.

5.1. Rate Balancing Algorithm

Algorithm 2 presents the steps to solve the optimization problem (29). The algorithm is initialized with two balancing levels $\varsigma^{\mathrm{L},(0)}$ and $\varsigma^{\mathrm{H},(0)}$ (line 1). Next, their corresponding vector power allocation vectors, $\boldsymbol{\xi}^{\mathrm{H},(0)}$ and $\boldsymbol{\xi}^{\mathrm{L},(0)}$, are computed via Algorithm 1 (line 2). Observe that the optimum lies between the initial balancing levels. Next, the algorithm enters a loop that first computes a new balancing level as the geometric mean of the balancing levels obtained in the previous iteration (line 5). Then, the power allocation vector for this new balancing level is computed via Algorithm 1 (line 6). Next, we check whether the power obtained is lower than the power constraint or not (line 7) and update the balancing levels accordingly (lines 8 and 10). Lastly, we test if the current power has the desired accuracy (line 12).

The convergence of Algorithm 2 depends on the feasibility of the initial average MSE targets $2^{-\varsigma^{\mathrm{H},(0)}\rho_k}\,\forall k$. Indeed, recall that the feasibility region is described in Section 4 as a bounded polytope and that the initial balancing levels $\varsigma^{\mathrm{L},(0)}$ and $\varsigma^{\mathrm{H},(0)}$ are chosen so that $\varsigma^{\mathrm{L},(0)} \leq \varsigma^{\mathrm{opt}} \leq \varsigma^{\mathrm{H},(0)}$. Hence, if $2^{-\varsigma^{\mathrm{H},(0)}\rho_k}\,\forall k$ lies inside the polytope, $2^{-a\varsigma^{\mathrm{H},(0)}\rho_k}\,\forall k$ likewise lies inside for $0 \leq a < 1$. Considering that the average MMSE given by $\frac{1}{\xi_k^{(\ell)}} I_k(\boldsymbol{\xi}^{(\ell)})$ is monotonically decreasing in $\boldsymbol{\xi}^{(\ell)}$, the bisection procedure reduces gaps $(\varsigma^{\mathrm{H},(\ell)} - \varsigma^{\mathrm{L},(\ell)})$ and $|\mathbf{1}^{\mathrm{T}}\boldsymbol{\xi}^{(\ell)} - P_{\mathrm{tx}}|$ at every iteration.

6. Results and Discussion

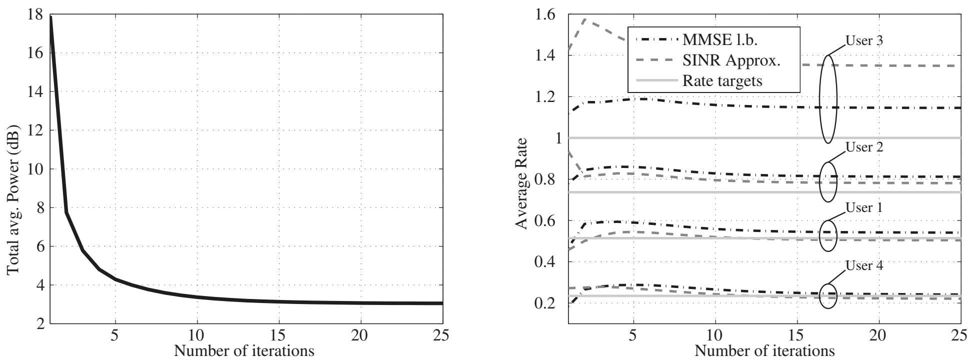
In this section, we present the results of several simulation experiments. First, let us introduce the error model corresponding to the imperfect CSIT

$$\boldsymbol{h}_k = \bar{\boldsymbol{h}}_k + \tilde{\boldsymbol{h}}_k, \tag{31}$$

Algorithm 2 Rate Balancing

$\ell \leftarrow 0$, initialize $\varsigma^{\mathrm{L},(0)}, \varsigma^{\mathrm{H},(0)}$
find $\boldsymbol{\xi}^{\mathrm{H},(0)} \leq \boldsymbol{\xi}^{\mathrm{L},(0)}$ via Alg. 1 [power min.]
repeat
  $\ell \leftarrow \ell + 1$
  $\varsigma^{(\ell)} \leftarrow \sqrt{\varsigma^{\mathrm{L},(\ell-1)}\varsigma^{\mathrm{H},(\ell-1)}}$ [new candidate]
  find $\boldsymbol{\xi}^{(\ell)}$ for $\varsigma^{(\ell)}$ via Alg. 1 [power min.]
  if $\sum_{i=1}^{K} \xi_i^{(\ell)} < P_{\mathrm{tx}}$ **then**
    $\varsigma^{\mathrm{H},(\ell)} \leftarrow \varsigma^{(\ell)}$, $\varsigma^{\mathrm{L},(\ell)} \leftarrow \varsigma^{\mathrm{L},(\ell-1)}$ [weights update]
  else
    $\varsigma^{\mathrm{L},(\ell)} \leftarrow \varsigma^{(\ell)}$, $\varsigma^{\mathrm{H},(\ell)} \leftarrow \varsigma^{\mathrm{H},(\ell-1)}$ [weights update]
  end if
until $|\sum_{i=1}^{K} \xi_i^{(\ell)} - P_{\mathrm{tx}}| < \delta$

where $\bar{\boldsymbol{h}}_k = \mathbb{E}[\boldsymbol{h}_k | \, v]$ and $\tilde{\boldsymbol{h}}_k$ is the error. This flexible model can represent, for example, the errors as result of calibration in TDD systems or the quantization and estimation errors in FDD systems. We assume that the imperfect CSI error is zero-mean Gaussian, i.e. $\tilde{\boldsymbol{h}}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{C}_k)$ where $\boldsymbol{C}_k = \mathbb{E}[(\boldsymbol{h}_k - \bar{\boldsymbol{h}}_k)(\boldsymbol{h}_k - \bar{\boldsymbol{h}}_k)^{\mathrm{H}} | \, v]$ is the k-th user CSI error covariance matrix. Recall that v and $f_{\boldsymbol{h}_k|v}(\boldsymbol{h}_k|v)$ are known at the transmitter, although the specific realizations of $\boldsymbol{h}_k$ and $\tilde{\boldsymbol{h}}_k$ are not. According to that assumption, it is possible to generate the channel realizations $\boldsymbol{h}_k^{(m)} = \bar{\boldsymbol{h}}_k + \tilde{\boldsymbol{h}}_k^{(m)}$ for $k = \{1, \ldots, K\}$ and $m = \{1, \ldots, M\}$, with $\bar{\boldsymbol{h}}_k = \mathbb{E}[\boldsymbol{h}_k|v]$ and $\tilde{\boldsymbol{h}}_k^{(m)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{C}_k)$. In our scenario, the number of users and transmit antennas were $K = 4$ and $N = 4$, respectively. We generated $M = 1000$ channel realizations considering $\boldsymbol{C}_k = \mathbf{I}_N$,



(a) (b)

Figure 2: Performance of Algorithm 1 for power minimization. (a) Total average power vs. Number of iterations. (b) Per-user average rates vs. Number of iterations.

and $\bar{\boldsymbol{h}}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I}_N), \forall k$. We also considered $\sigma^2_{\boldsymbol{\eta}_k} = 1$, $\forall k$.

6.1. Power Minimization

In this subsection, Algorithm 1, which solves optimization problem (3), is considered. We choose users with different rate requirements, viz., $\rho_1 = 0.5146$, $\rho_2 = 0.737$, $\rho_3 = 1$, and $\rho_4 = 0.2345$ bits per channel use, respectively. These requirements correspond to the following targets in the MMSE domain: $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.6$, $\varepsilon_3 = 0.5$, and $\varepsilon_4 = 0.85$. The threshold in Algorithm 1 is set to $\delta = 10^{-2}$. Initial precoders are random.

Fig. 2a shows the evolution of the total average power over the iterations. Starting above 16 dB, it gradually reduces until convergence is reached at 3 dB. The total average power is dramatically reduced during the first five iterations whereas the improvement is marginal after iteration 15.

Fig. 2b shows the corresponding average rates throughout the iterations. Recall from (7) that the actual average rates are lower bounded by the

MMSE-based targets ε_k; i.e., $\mathbb{E}[R_k|\, v] \geq -\log_2(\varepsilon_k)$, as discussed in Section 2. Since the problem is feasible, the constraints in (20) are met with equality at the optimum. As can be seen, the first steps go in the direction of meeting the requirements and the average rates increase. Nevertheless, the subsequent iterations reduce the rates until the MMSE-based targets are reached for all users. The gap between the average rates obtained with Algorithm 1 and the average rate targets corresponding to the QoS constraints can be also observed. Moreover, we also include the rates obtained by using the widely-employed SINR approximation proposed in [9]. This approach determines the average rates as $\log_2(1 + \overline{\mathrm{SINR}}_k)$, in which $\overline{\mathrm{SINR}}_k$ reads as

$$\overline{\mathrm{SINR}}_k = \frac{\boldsymbol{p}_k^{\mathrm{H}}\,\mathbb{E}\left[\boldsymbol{h}_k\boldsymbol{h}_k^{\mathrm{H}}|\, v\right]\boldsymbol{p}_k}{\sigma^2_{\boldsymbol{\eta}_k} + \sum_{i\neq k}\boldsymbol{p}_i^{\mathrm{H}}\,\mathbb{E}\left[\boldsymbol{h}_k\boldsymbol{h}_k^{\mathrm{H}}|\, v\right]\boldsymbol{p}_i}. \tag{32}$$

Note that the average rates for the SINR approximation are larger than the true average rates for users 2 and 3, but smaller for users 1 and 4. Hence, it is not possible to guarantee the QoS restrictions. Contrary to this, the MMSE-based targets ensure the rate targets.

6.2. Rate Balancing

This subsection focuses on the performance of Algorithm 2. The rate targets employed in Subsection 6.1 are also used in this section. We scale them with a common factor to obtain the rate targets. The threshold to check convergence is set to $\delta = 10^{-2}$.

Taking into account the numerical results obtained in Subsection 6.1, we consider a total average transmit power of 3 dB, yielding an expected balancing level of approximately one. Therefore, we pick $\varsigma^{\mathrm{L},(0)} = 0.6$ and $\varsigma^{\mathrm{H},(0)} = 1.3$, from which $\varsigma^{\mathrm{opt}} \in [0.6, 1.3]$. Fig. 3 plots the average power versus

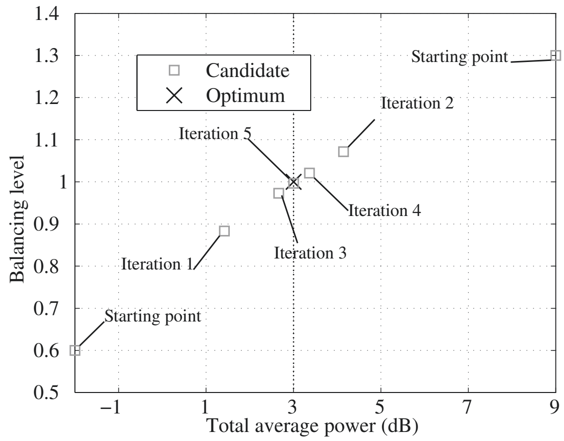


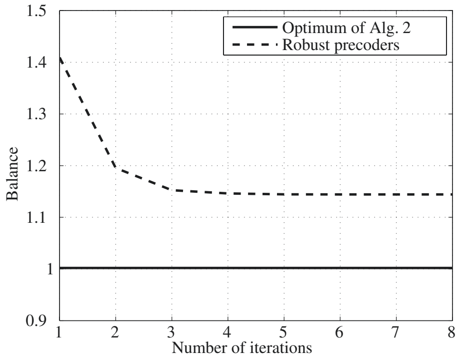
Figure 3: MMSE balancing. Balancing level candidates vs. Total average power.

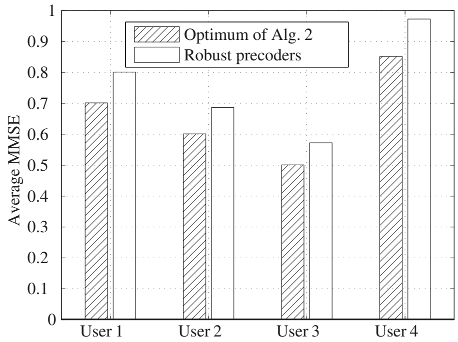
the balancing level for the different iterations of the bisection algorithm. The two initial values correspond to the points located on the left and the right vertical axis in the figure. Note that the searching interval reduces as the algorithm progresses until it converges after five iterations to the point $\varsigma^{\mathrm{opt}} = 0.99659$ and $P_{\mathrm{tx}} = 3.0072$ dB. This is in accordance with the experimental results obtained in Subsection 6.1.

We also performed a computer experiment to compare our approach with robust precoding as presented in [10], in which the same CSI for the users and the BS is considered. More specifically, we focused on the following weighted MSE Min-Max problem (see Section V.B of [10])

$$\min_{\{\boldsymbol{p}_k, \boldsymbol{f}_k\}_{k=1}^{K}} \max_{i} \frac{\overline{\mathrm{MSE}}_i^{\mathrm{BC}}}{w_i} \quad \text{s.t.} \quad \sum_{j=1}^{K} \|\boldsymbol{p}_j\|_2^2 \leq P_{\mathrm{tx}}, \tag{33}$$

where w_i is the weight for the i-th user. Robust precoders and filters are designed via an AO process, and power allocation is calculated by solving an eigen-system. The optimum of (33) is obtained after a few iterations, and





(a) (b)

Figure 4: (a) Robust transceiver. Balancing level vs. Number of iterations. (b) Average MMSEs for proposed Algorithm 2 vs. Robust transceivers.

meets $\sum_{i=1}^{K} \|\boldsymbol{p}_i\|_2^2 = P_{\text{tx}}$ and $\overline{\text{MMSE}}_k^{\text{BC}}/w_k = w^{\text{opt}}$, $\forall k$ (see Fig. 4a). The error precision for min max ratio w^{opt} is 10^{-4}.

This min max problem can be seen as a balancing problem with $w_i = \varepsilon_i$. Thus, Fig. 4b represents the comparison between the solutions which use robust transceivers and the one proposed in this work. As can be seen in the figure, our proposed Algorithm 2 performs better because $\varsigma^{\text{opt}} = 0.99659$ is closer than $w^{\text{opt}} = 1.1442$ to 1. Note that robust precoders are more conservative since their performance depends on the radius of the uncertainty region, i.e., on the estimation error variance. However, the robust filters from [10] are designed using a computationally cheaper algorithm.

7. Conclusions

We focused on the design of linear precoders and receivers to minimize the transmit power in a MISO BC while meeting a set of per-user QoS constraints given as per-user average rate requirements. Likewise, we explained that QoS constraints can be substituted by more manageable restrictions based on the average MMSE. We later exploited MSE BC/MAC duality to jointly determine the optimum transmit and receive filters by means of an AO algorithm and optimum power allocation was found trough the so-called standard interference functions. Furthermore, we analyzed feasibility of the problem to ensure convergence of the proposed algorithm and we addressed the balancing problem by combining the proposed algorithm with a search. Lastly, we carried out simulation experiments to show the performance of the proposed methods and compare them with previous solutions.

8. Acknowledgment

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Appendix A. Average-MMSE-Based Lower Bound Gap

In this appendix, we study the accuracy of the lower bound (7). To that end, we approximate the MMSE *Cumulative Distribution Function* (CDF) by a beta CDF. We focus on the setup: $K = 1$, $N = 4$, and *Maximum Ratio Transmission* (MRT). Then, considering $\boldsymbol{h} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \mathbf{I})$ and $\eta \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$, the average MMSE is $\mathbb{E}[\text{MMSE}] = \mathbb{E}\left[\frac{\sigma^2}{|\boldsymbol{h}^{\mathrm{H}}\boldsymbol{p}|^2+\sigma^2}\right]$. By using $\boldsymbol{p} = \frac{\sqrt{P}}{\|\boldsymbol{h}\|_2}\boldsymbol{h}$ with $P = 1$, and $\sigma^2 = 10$, the corresponding CDF parameters are $\alpha = 14.3$ and $\beta = 5.3$. Fig. A.5 shows the comparison between the two CDFs.

We next introduce the PDF $f_\varepsilon(\text{MMSE})$ and the variable $\varepsilon = \text{MMSE}$. Now, the expectation of the logarithm of ε is $\mathbb{E}[\ln(\varepsilon)] = \int_0^1 f_\varepsilon(\varepsilon)\ln(\varepsilon)dx$. Considering ε has a beta PDF, the logarithm of the geometric mean reads

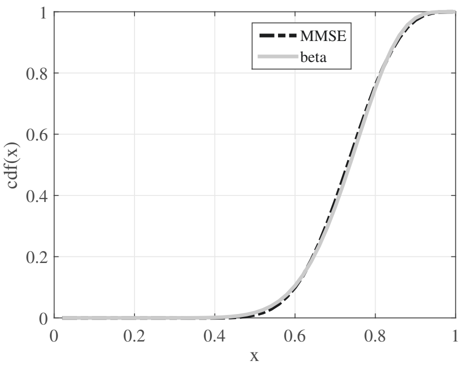

Figure A.5: MMSE Cumulative Distribution vs. Beta Cumulative Distribution.

as $\mathbb{E}[\ln(\varepsilon)] = \psi(\alpha) - \psi\left(\alpha+\beta\right)$, where $\psi(x)$ is the digamma function. Such a function can be approximated as $\psi(x) \approx \ln(x+\frac{1}{2})$ for $x > 1$. Then, we approach the lower bound as $-\mathbb{E}[\log_2(\varepsilon)] \approx \frac{1}{\ln(2)}\log_2\left(1+\frac{\beta}{\alpha-\frac{1}{2}}\right)$.

The expectation of the beta distribution is $\mathbb{E}[\varepsilon] = \frac{\alpha}{\alpha+\beta}$. Accordingly, the average MMSE lower bound is $-\log_2(\mathbb{E}[\varepsilon]) = \frac{1}{\ln(2)}\ln(1+\frac{\beta}{\alpha})$. Hence, the gap between the average rate $\mathbb{E}[R]$ and the lower bound is

$$\mathbb{E}\left[R\right] - \left[-\log_2\left(\mathbb{E}\left[\mathrm{MMSE}\right]\right)\right] \approx \frac{1}{\ln(2)}\log_2\left(1+\frac{\beta}{\alpha-\frac{1}{2}}\right) - \frac{1}{\ln(2)}\ln\left(1+\frac{\beta}{\alpha}\right)$$
$$= \frac{1}{\ln(2)}\left[\ln\left(1+\frac{\beta}{\alpha-\frac{1}{2}}\right) - \ln\left(1+\frac{\beta}{\alpha}\right)\right] = \log_2\left(1+\frac{\frac{\beta}{2}}{\left(\alpha-\frac{1}{2}\right)\left(\alpha+\beta\right)}\right).$$

In our example, this yields $\log_2(1+0.0098) = 0.0141$.

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