21[th] International Conference on Renewable Energies and Power Quality (ICREPQ'23)
Madrid (Spain), 24[th] to 26[th] May 2023
Renewable Energy and Power Quality Journal (RE&PQJ)
ISSN 2172-038 X, Volume No.21, July 2023

Characterisation of Electrical Power Systems Based on Electrical Curves and their properties

J. Ventura[1], F. Martínez[1], L. Castro-Santos [2], A. Filgueira-Vizoso[3], A. Alcayde[1] and F. G. Montoya[1]

[1] Universidad de Almería, Escuela Superior de Ingeniería, La Cañada de San Urbano, 04120, Almería, Spain; email: jvg327@inlumine.ual.es, fmg714@ual.es, aalcayde@ual.es, pagilm@ual.es

[2] Universidade da Coruña, Campus Industrial de Ferrol, Departamento de Enxeñaría Naval e Industrial, Escola Politécnica de Enxeñaría de Ferrol, Esteiro, 15471 Ferrol, Spain; email: laura.castro.santos@udc.es

[3] Universidade da Coruña, Campus Industrial de Ferrol, Departamento de Química, Escola Politécnica de Enxeñaría de Ferrol, Esteiro, 15471 Ferrol, Spain; email: almudena.filgueira.vizoso@udc.es

Abstract. Due to the proliferation of renewable energy systems, the study of voltage and frequency stability is a crucial aspect. Recently, this problem has been approached from a purely geometrical point of view with interesting results.
The present work investigates the properties of the so-called electrical curves described by arbitrary voltage or current vectors in Euclidean spaces. Through the invariants of these curves, certain indices can be constructed to detect abnormal operation or irregular characteristics in electrical power systems. Different scenarios and examples have been solved in this work to support the proposed theory.

Keywords. Electrical curves, geometric algebra, curvature, differential geometry.

1. Introduction

The integration of renewable energy sources, such as solar and wind power, has greatly impacted the operation of electrical power systems. The intermittent and weather-dependent nature of these sources leads to significant variability and uncertainty in the power generation process, making it more difficult to maintain the stability and reliability of the power system [1]. Advanced control and monitoring systems are being developed to address this issue and mathematical methods are also being applied to study and mitigate the impact of renewable energy sources on power systems.

Voltage stability is particularly important in ensuring the proper functioning of the power system and therefore, the satisfaction of its customers. A power system is considered to be voltage stable if the voltage at all nodes in the system remains within acceptable levels under normal and abnormal operating conditions. The acceptable levels of voltage are determined by the regulatory bodies and are based on the safety and performance requirements of the system and its customers. Voltage stability is closely related to the power balance in the system, as it depends on the balance between power generation and power demand.

Frequency stability is also critical for the proper functioning of the power system. The nominal frequency of the power system is typically 50 or 60 Hz and it must be maintained within a narrow range despite changes in the power demand or generation. Frequency stability also depends on the balance between power generation and power demand. When there is an imbalance between power generation and power demand, the frequency of the power system will deviate from its nominal value. This can cause problems for the customers, such as power outages or damage to equipment.

In recent years, the study of voltage and frequency stability has benefited from the application of geometrical tools. These tools have provided a new perspective on the dynamic behaviour of these systems [2,3], enabling the representation of complex electrical systems as geometrical curves in a high-dimensional space (also known as state-space in the literature) [4]. By analysing the geometric properties and shape of these curves, it is possible to better understand an electrical system's dynamics and characteristics. This can help to improve the performance and reliability of the power system and to ensure the satisfaction of its customers.

The application of geometrical tools to the study of voltage and frequency stability is an active area of research, with many new developments and advancements being made. Some of the current research in this area focuses on developing new methods for analysing the geometric properties of the curves and for detecting abnormal operations in the power system. Other research focuses on developing new control strategies to improve the performance and reliability of the power system.

The present work proposes a novel method for detecting abnormal operations in power systems based on the invariants of electrical curves described by voltage or current vectors in a Euclidean space. This approach is illustrated through a series of examples and scenarios, which demonstrate the effectiveness of the proposed theory. The method is based on the analysis of the geometric properties of these curves and their shape in the high-dimensional space. By identifying abnormal patterns in these curves, it is possible to detect potential issues in the system before they occur, which can prevent power outages and ensure the satisfaction of customers. Additionally, this method can also be used to optimize the operation of the system by identifying the most efficient operating points.

The rest of the paper is organised as follows. Section 2 presents the mathematical background of electrical curves and their invariants. Section 3 presents the proposed method for detecting irregular characteristics in electrical power systems based on the invariants presented in Section 2. Section 4 illustrates the proposed method through a series of examples and scenarios. Finally, Section 5 concludes the paper and discusses future directions for research.

2. Mathematical background

In this section, the mathematical background of electrical curves and their invariants is presented.
Different types of three-phase voltages have been considered in this work, including sinusoidal symmetric, sinusoidal non-symmetric, non-sinusoidal symmetric, sinusoidal with variable frequency, and sinusoidal symmetric with transients. For each case, an electrical curve is built and simulated in the time domain using the following expression.

$$\boldsymbol{u}(t) = u_a(t)\boldsymbol{e_1} + u_b(t)\boldsymbol{e_2} + u_c(t)\boldsymbol{e_3} \tag{1}$$

In Eq. (1), an orthonormal frame $(\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3})$ is used and the phase voltage u_a, u_b and u_c are used as coordinates. For simplicity, the dependency on t will be omitted from now on.
As a result, the vector $\boldsymbol{u}$ draws a curve in space over time and the geometrical properties can be analysed. A relevant feature that can be computed for a curve is the Frénet-Serret (FS) trihedron. It consists of three orthogonal and unitary vectors that travel along the curve and provide a local description of the curve's geometry, and therefore, its dynamics. They are extremely useful in characterising the curve's shape. The computation of the FS frame is as follows

$$\boldsymbol{T} = \frac{\boldsymbol{u}'}{||\boldsymbol{u}'||}$$

$$\boldsymbol{N} = \frac{\boldsymbol{u}' \times \boldsymbol{u}''}{||\boldsymbol{u}' \times \boldsymbol{u}''||} \tag{2}$$

$$\boldsymbol{B} = \frac{[\boldsymbol{u}' \times \boldsymbol{u}''] \times \boldsymbol{u}'}{||[\boldsymbol{u}' \times \boldsymbol{u}''] \times \boldsymbol{u}'||}$$

Where $\boldsymbol{T}$, $\boldsymbol{N}$ and $\boldsymbol{B}$ are the tangent, normal and binormal vectors, respectively.
Furthermore, the invariants of the curve can be computed using the FS frame. For example, the curvature κ quantifies the degree of bending of the curve, while the torsion τ quantifies its degree of twisting. These parameters provide a comprehensive global description of the curve's geometry and can be utilised to identify abnormal features within electrical power systems.

$$\kappa = \frac{||\boldsymbol{u}' \times \boldsymbol{u}''||}{||\boldsymbol{u}'||^3} \tag{3}$$

$$\tau = -\frac{\boldsymbol{u}' \cdot [\boldsymbol{u}'' \times \boldsymbol{u}''']}{||\boldsymbol{u}' \times \boldsymbol{u}''||^2} \tag{4}$$

3. Proposed Method

This section presents the proposed method for identifying abnormalities in electrical power systems, which is based on the analysis of the geometric properties of electrical curves. The invariants of these curves, such as the curvature and torsion, are used to characterise the power system and detect any deviation from the expected behaviour.

As an example, Fig. 1 depicts the electrical curve for a three-phase voltage supply with sinusoidal and symmetrical characteristics. In this scenario, the representation of the electrical curve is a circle contained in a plane embedded in a three-dimensional space. Applying Eqs. (3) and (4) result in a constant curvature along its path and zero torsion as the curve is flat. This is the ideal scenario for any three-phase power system and will be used as a reference for the absence of power quality issues.

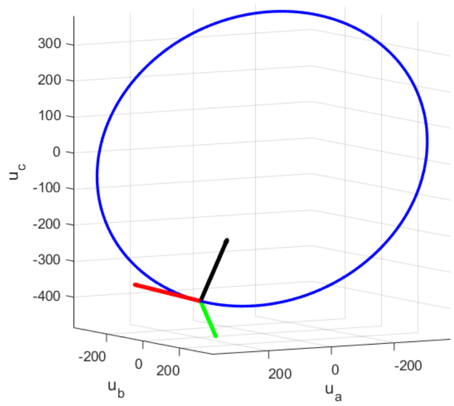

Fig. 1: Electrical curve of a balanced three-phase sinusoidal and symmetrical voltage. The Frénet-Serret trihedron is drawn on the curve for t = 62 ms.

By studying the evolution of the curvature and torsion of the electric curve, it is possible to observe how it deviates from the ideal case, and therefore, characterise the dynamics of the power system.

A significant advantage of this approach is the ability to characterise a three-phase power system using only two geometric parameters, as opposed to the need to study a large number of electrical parameters such as frequency, amplitude, phase, harmonics, etc. This simplifies the analysis process and allows for more efficient identification of abnormalities in power systems. Furthermore, working with geometric elements is much more intuitive as they can be visualised in a Euclidean space.

4. Examples and Scenarios

In this section, four specific examples and scenarios are presented to illustrate the proposed method.

A. Scenario 1: Unbalanced Sinusoidal Voltage

This scenario involves an unbalanced sinusoidal voltage applied to the three-phase electrical system and is described in Eq. (5). In this and the following examples, an amplitude of 380 volts and a frequency of 50 Hz has been taken as a reference.

$$\begin{aligned} u_a(t) &= A \cdot sin(\omega t) \\ u_b(t) &= A \cdot sin(\omega t + \phi_1) \\ u_c(t) &= A \cdot sin(\omega t + \phi_2) \end{aligned} \tag{5}$$

The angles ϕ_1 and ϕ_2 indicate different phase shifts for each voltage, with a value of 110º and 282º respectively. Fig. 2 shows the resulting curve drawn by the **u** vector using simulations for 0.1 seconds. As we can see, the curve is quite similar to the balanced sine wave, as it is still contained in a plane, but with the difference that it takes the form of an ellipse.

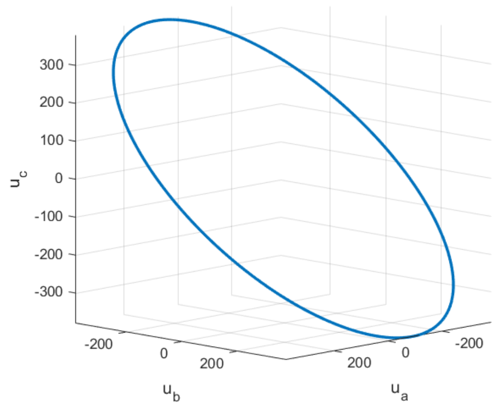

Fig. 2: Electrical curve for an unbalanced sinusoidal voltage supply.

Finally, the curvature and torsion at each instant of time have been calculated and plotted (Fig. 3). Analysing them along the curve, two observations can be made. Firstly, the torsion remains constant and zero since the binomial vector does not vary as the curve is contained in a plane in space. Secondly, the curvature varies cyclically creating a succession of peaks and valleys as the tangent vector undergoes a greater change in direction as it approaches the foci of the ellipse and less change as it moves away from them.

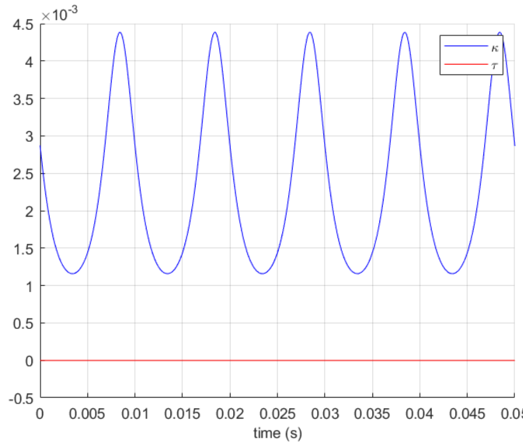

Fig. 3: Curvature and Torsion for the unbalanced sinusoidal voltage scenario.

B. Scenario 2: Non-Sinusoidal Voltage

In this scenario, the electrical system is subjected to a non-sinusoidal voltage, which is characterized by the presence of harmonic components. Eq. (6) presents the supply voltages of each phase with the addition of the third harmonic in u_a, the fifth in u_b and both in u_c.

$$\begin{aligned} u_a(t) &= A \cdot sin(\omega t) + A_3 \cdot sin(3\omega t) \\ u_b(t) &= A \cdot sin(\omega t + \phi) + A_5 \cdot sin(5\omega t + \phi) \\ u_c(t) &= A \cdot sin(\omega t - \phi) + A_3 \cdot \sin(3\omega t - \phi) \\ &\quad + A_5 \cdot \sin(5\omega t - \phi) \end{aligned} \tag{6}$$

For this case an amplitude of 41 volts has been taken for the third harmonic and 17 volts for the fifth harmonic.

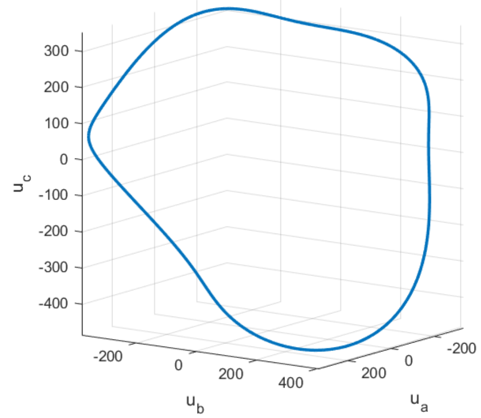

Fig. 4: Electrical curve for a non-sinusoidal voltage supply.

In this case, the electrical curve resulting from the simulation (Fig. 4) shows a large deformation compared to the previous examples due to the presence of harmonic components.

When analysing the curvature and torsion of this curve (Fig. 5), it is observed that there are major variations in the curvature and torsion values due to the fact that the normal and binormal vector are continuously varying in direction. However, it stands out mainly that the magnitude of torsion is significantly higher than that of curvature.

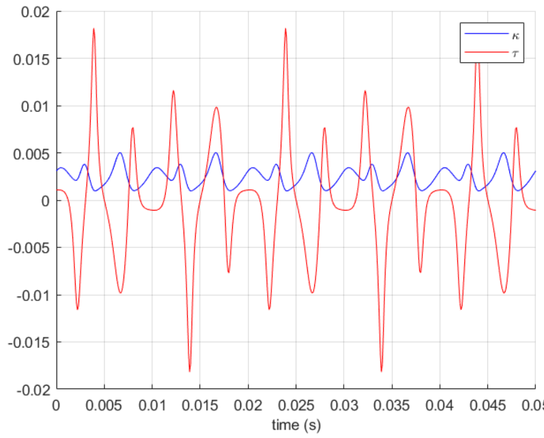

Fig. 5: Curvature and Torsion for the non-sinusoidal voltage scenario.

C. Scenario 3: Variable Frequency Voltage

An example has now been implemented with balanced sine voltages, where the frequencies of each phase voltage vary sine-wise and independently around the fundamental frequency around the fundamental frequency as indicated in Eq. (7).

$$\begin{array}{c} u_a(t) = A \cdot sin(\omega_1(t)t) \\ \hline u_b(t) = A \cdot sin(\omega_2(t)t + \phi) \\ u_c(t) = A \cdot sin(\omega_3(t)t - \phi) \end{array} \quad (7)$$

Eq. (8) demonstrates how the frequency of each voltage depends on time, exhibiting a sine variation of specific amplitude and frequency.

$$\begin{aligned} \omega_1(t) &= \omega_0 + 2\pi \cdot sin(2\pi 10t) \\ \omega_2(t) &= \omega_0 + 2\pi \frac{1}{2} \cdot sin(2\pi 3t) \\ \omega_3(t) &= \omega_0 + 2\pi \frac{1}{4} \cdot sin(2\pi 5t) \end{aligned} \quad (8)$$

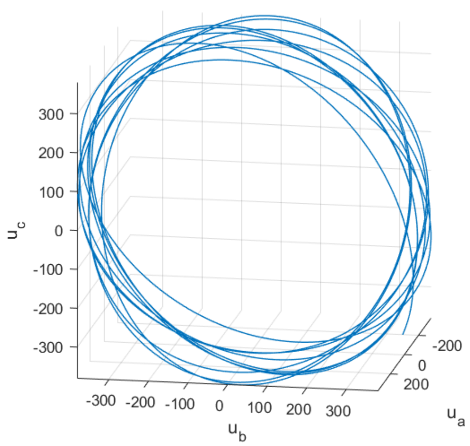

Fig. 6: Electrical curve for a variable frequency voltage supply.

As depicted in Fig. 6, the resulting electrical curve for this scenario appears to follow a circular trajectory despite constantly changing the plane in which it is drawn. At first glance, it may be challenging to determine the type of voltage supply causing the curve. However, by analysing Fig. 7, it is possible to extract additional information. It can be seen that both the torsion and curvature exhibit an oscillating pattern over time, and these variations in amplitude and frequency can be used for identification purposes.

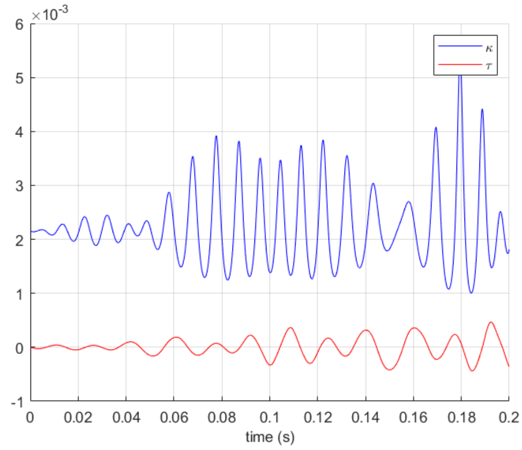

Fig. 7: Curvature and Torsion for the variable frequency voltage scenario.

D. Scenario 4: Overvoltage Transient

In the final scenario presented, an overvoltage transient was introduced into the three-phase electrical power system. This was achieved by simulating a sudden increase in voltage within one phase of the system. To accomplish this, a Gaussian function was applied to the first phase with an amplitude of 500 volts. The timing of the event was centred at 24 milliseconds, with a standard deviation of 0.0004. For the purpose of analysis, the electrical power system was initially designed as a balanced sinusoidal system, allowing for a point of reference to compare the effects of the transient event.

This event can be clearly seen in Fig. 8, which presents the electrical curve of the system. The curve shows a clear peak, or bulge, protruding from the circumference of the curve, which serves as a clear visual representation of the overvoltage transient. Furthermore, when the curve returns to its normal trajectory, it can be observed that the effect of the transient event is relatively short-lived, lasting only a few milliseconds.

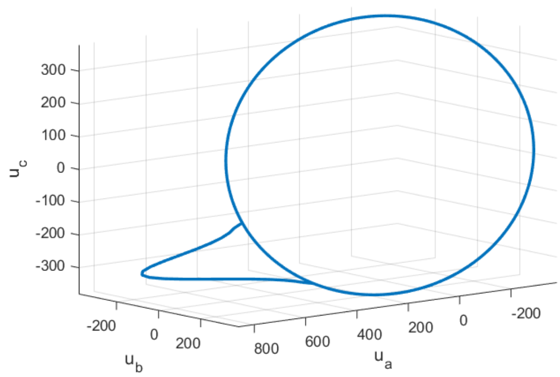

Fig. 8: Electrical curve for a balanced sinusoidal voltage supply with an overvoltage transient.

It can be seen from Fig. 9 that the curvature and torsion values undergo a sudden change at the time of the transient. In the case of the curvature this event becomes much more noticeable with null zones when the overvoltage rises or falls following a straight trajectory and high peaks referring to the closed turns. This sudden spike in values can serve as an indicator of the overvoltage transient and can be used to identify and mitigate any potential problems caused by this event.

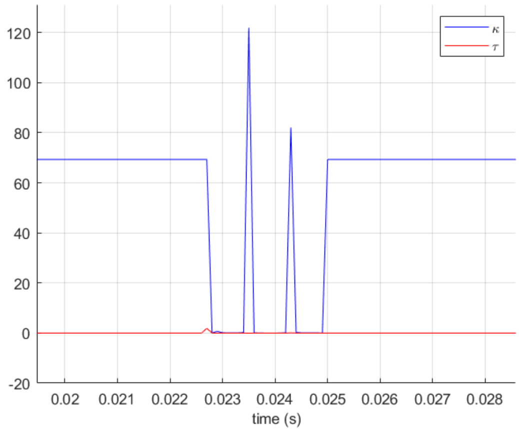

Fig. 9: Curvature and Torsion for the overvoltage transient scenario.

As we can see, each scenario results in characteristic curvature and torsion values which are also correlated with the forms of the curves in a Euclidean space. Closed and sudden turns in the curve path result in notable changes in curvature, as they produce a significant variation in the direction of the tangent vector. On the other hand, when the curve moves twisting and deviating from the plane it is contained in at that moment, large variations in the direction of the binormal vector and consequently in torsion occur. These results demonstrate that the proposed method of analysing the curvature and torsion of electrical curves can effectively characterise an electrical power system.

5. Conclusion

In this work, we have explored the use of geometrical tools to analyse the stability and reliability of electrical power systems impacted by the increasing adoption of renewable energy sources. We proposed a method for detecting abnormal operations in these systems based on the invariants of electrical curves described by voltage or current vectors in a Euclidean space. Our proposed approach was illustrated through a series of examples and scenarios, demonstrating its effectiveness in identifying irregular characteristics in power systems. The properties of electrical curves presented in this work have potential applications in various areas of power systems analysis, such as fault detection, control, and optimization. For example, the abnormal operation detection approach proposed in this work can be integrated into real-time monitoring systems to enhance the reliability and stability of power grids. Additionally, the electrical curve invariants can be used to optimize the placement and sizing of renewable energy sources in power systems. Further investigation is needed to explore these potential applications in depth.

References

[1] Kundur, P. S., & Malik, O. P. (2022). Power system stability and control. McGraw-Hill Education.
[2] J. M. Aller, A. Bueno, and T. Paga, “Power system analysis using space-vector transformation,” IEEE Transactions on power systems, vol. 17, no. 4, pp. 957–965, 2002.
[3] F. Milano, “A geometrical interpretation of frequency,” IEEE Transactions on Power Systems, vol. 37, no. 1, pp. 816–819, 2022.
[4] A. H. Eid and F. G. Montoya, “A systematic and comprehensive geometric framework for multiphase power systems analysis and computing in time domain,” IEEE Access, vol. 10, pp. 132 725–132 741, 2022.