



Utility and production externalities, equilibrium efficiency and leisure specification

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Abstract

This paper analyzes the implications that the specification of the leisure activity has on the equilibrium efficiency in a two-sector endogenous growth model with human capital accumulation. We consider external effects of consumption and leisure in utility, and sector-specific externalities associated to physical and human capital in production. The optimal tax policy to correct for the distortions caused by the externalities is characterized under all the typical leisure specifications considered in the literature: home production, quality time and raw time. We show that the optimal policy depends markedly on the leisure specification.

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1. Introduction

The purpose of the present paper is to analyze the implications that the specification of the leisure activity has on the efficiency of the competitive equilibrium in a two-sector endogenous growth model with human capital accumulation. We consider a relatively general framework, in which agents derive utility both from consumption and leisure, where

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individual preferences are subject to spillovers from the other agents' consumption and leisure, and production is subject to sector-specific externalities associated with physical and human capital. This will allow us to study the possible interaction between utility and production spillovers. In the presence of externalities, optimal growth paths and competitive equilibrium paths may not coincide. This paper focuses on the design of optimal policies capable of making the competitive economy replicate the optimal growth path attainable by a central planner and how these policies are affected by the specification of the leisure activity.

The presence of externalities in endogenous growth models with human capital accumulation has been considered by a number of authors. For example, Lucas (1988) and Xie (1994) consider the case where the average human capital has an external effect on the sector producing goods. Chamley (1993) and Benhabib and Perli (1994) consider instead the case where the sector producing human capital exhibits a positive external effect arising from the average learning time. Benhabib et al. (2000), Mino (2001), Ben-Gad (2003) and Nishimura and Venditti (2004), among others, introduce sector-specific externalities in both sectors, which will be the case considered in this paper.

However, they have focused on what can be categorized as production externalities, even though other types of external effects affecting the individual utility are also conceivable. Currently, consumption externalities have been widely considered in the literature (e.g., Abel, 1990; Gali, 1994; Carroll et al., 1997; Dupor and Liu, 2003; Alvarez-Cuadrado et al., 2004), but mainly restricted to the Ramsey model and the simplest Ak endogenous growth model. The presence of spillovers associated to the leisure activity is plausible as well since the satisfaction obtained from leisure usually depends on sharing activities with others. Thus, time spent in activities that involve other people represent a high fraction of total leisure time (e.g., Juster and Stafford, 1991). Empirical evidence suggests that couples arrange their work schedules to allow time for leisure that they consume jointly (e.g., Hamermesh, 2002; Hallberg, 2003; and Jenkins and Osberg, 2005). Costa (2000) argues that the compression in the length of the work day distribution over the last century could be partly explained by the increasing co-ordination of work activities within and across firms and by the increasing synchronization of leisure time activities with those of relatives and friends. Jenkins and Osberg (2005) also report estimates indicating that propensities to engage in associative activity depend on the availability of suitable leisure companions outside the household. Alesina et al. (2006) argues that leisure externalities may be an important factor for explaining differences between US and European working patterns. This evidence suggests the existence of spillovers associated to average leisure time. It is also plausible the occurrence of what could be considered as congestion effects of leisure, where an increase in average leisure time reduces the benefits of own leisure (e.g., crowded parks).

The bulk of the literature on endogenous growth has considered leisure as a nonmarket activity that requires the use of "raw time" only. Other authors considered leisure as a nonmarket activity requiring "quality time," a combination of time and human capital (e.g., Heckman, 1976; Stokey and Rebelo, 1995; Ortigueira, 2000). A recent body of literature (e.g., Greenwood and Hercowitz, 1991; Campbell and Ludvigson, 2001) has modeled leisure as a form of "home production" that uses both human and physical capital inputs, in addition to nonmarket time. The choice of the particular specification of the leisure activity has been shown to play a crucial role in the theory of endogenous growth (see Gómez, 2003b, and the references therein). Therefore, we shall analyze the implications

that the specification of the leisure activity has on the efficiency of the competitive equilibrium, and on the optimal fiscal policy designed to correct the market failure caused by the external effects. To this end, we shall consider all the typical specifications of leisure – home production, quality time and raw time – as well as the case in which labor supply is inelastic.

Related work has been recently made. [García-Castrillo and Sanso \(2000\)](#) and [Gómez \(2003a\)](#) devise optimal fiscal policies in a Lucas-type model with externalities á la [Lucas \(1988\)](#). [Gómez \(2004\)](#) shows that the competitive equilibrium is efficient in the Lucas model with sector-specific externalities associated to human capital in goods production. However, they do not examine the effect of utility externalities arising from consumption and leisure, and labor supply is assumed to be inelastic. Although a number of authors have analyzed the equilibrium efficiency in models with consumption externalities, the analysis has been mostly restricted to the Ramsey exogenous growth model (e.g., [Fisher and Hof, 2000](#); [Alonso-Carrera et al., 2004](#); [Liu and Turnovsky, 2005](#); [Gómez, 2007](#)) or the simplest one-sector endogenous growth model (e.g., [Gómez, 2006](#); and [Alonso-Carrera et al., 2005](#) and the related paper by [Hiraguchi, 2007](#)).¹ Furthermore, most of these works consider that labor supply is inelastic; an even when labor supply is assumed to be endogenous, as in [Liu and Turnovsky \(2005\)](#), the analysis is limited to the raw time specification of the leisure activity. Finally, none of these works consider the presence of spillovers associated to leisure.

Irrespective of the leisure specification, four main results are robust. First, in the presence of (only) sector-specific externalities associated to human capital in education, the long-run equilibrium growth rate is lower than the optimal one. Second, a sector-specific externality associated to physical capital in goods production provokes a distortion that can be corrected by taxing physical capital income so as to equate its social and private marginal returns. Third, a sector-specific externality associated to human capital in the educational sector causes a distortion that can be corrected by subsidizing investment on human capital. Fourth, in the absence of production externalities, the competitive equilibrium is efficient both at the steady state and off the steady state if and only if the elasticity of average consumption in utility is equal to the elasticity of average leisure.

However, the choice of the leisure function introduces important differences across leisure specifications. If leisure is modeled as “home production” or “quality time” and when labor supply is inelastic, in the absence of externalities in education, the long-run equilibrium growth rate is equal to the optimal one irrespective of the presence of sector-specific externalities in the goods sector and/or utility externalities. However, in the “raw time” model, utility externalities or sector-specific externalities in the goods sector cause the long-run equilibrium growth rate to diverge from the optimal one. If labor supply is inelastic, the competitive equilibrium is efficient in the presence of consumption externalities and/or sector-specific externalities associated to human capital in the goods sector. This extends the optimality result obtained by [Gómez \(2004\)](#). However, this result does not hold when labor supply is endogenous. Now, irrespective of the leisure specification, the presence of utility or production spillovers causes the market equilibrium to be inefficient. If leisure is raw time or quality time, there is a degree of arbitrariness in the tax policy that

¹ [Liu and Turnovsky \(2005\)](#) also examine the [Romer \(1986\)](#) endogenous growth model with consumption and production externalities.

enables the market equilibrium to replicate the first-best optimum of a centrally planned economy, as it can be achieved by taxing consumption or labor income. Alternatively, this degree of arbitrariness could be used to tax income from physical capital and labor at the same rate. This possibility disappears in the home production specification, because now there is one additional margin of choice – the allocation of physical capital between production or leisure activities – which needs to be mimicked as well. In this case, the utility externalities can be corrected by taxing consumption, and the sector-specific externalities associated to human capital in goods production can be corrected by taxing labor income so as to equate the social and private marginal returns on human capital.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibrium dynamics of the market economy, and Section 4, the optimal growth path attainable by a central planner. Section 5 compares the long-run growth rates of the market and the centrally planned economies. Section 6 studies the implications that the specification of the leisure activity has on the optimal fiscal policy. Section 7 concludes.

2. The model

Consider an economy populated by a large number of identical infinitely-lived representative agents who derive utility from the consumption of a private consumption good, c , and leisure, L . For simplicity, we assume that population is constant and normalized to one. The intertemporal utility derived by the agent is represented by

$$\int_0^{\infty} e^{-\rho t} U(c, \bar{c}, L, \bar{L}) dt, \quad \rho > 0, \quad (1a)$$

where the instantaneous utility function is

$$U(c, \bar{c}, L, \bar{L}) = \frac{[c\bar{c}^{\mu}(L\bar{L}^{\psi})^{\eta}]^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0, \quad \eta > 0, \quad \sigma + \eta(\sigma - 1) > 0. \quad (1b)$$

Here, $1/\sigma$ is the elasticity of intertemporal substitution (EIS), and η reflects preferences for leisure.² The specification (1b) entails that the agent derives positive but decreasing marginal utility from her own consumption ($U_c > 0$ and $U_{cc} < 0$) and leisure ($U_L > 0$ and $U_{LL} < 0$), and that the utility function is strictly concave in c and L .

The term \bar{c} in (1b) is the average consumption, and expresses externalities in utility arising from consumption. Following Dupor and Liu (2003), we may say that the utility function exhibits jealousy if $\mu < 0$, and admiration if $\mu > 0$ when other agents' consumption increases. When $\mu < 0$ agents are jealous because average consumption reduces agent's utility for a given level of own consumption. When $\mu > 0$ agents turn out to be admiring since average consumption increases agent's utility for a given level of own consumption. The term \bar{L} is the average level of leisure in the economy at large, and expresses externalities in utility associated to leisure. If $\psi < 0$, then average leisure reduces agent's utility for a given level of own leisure, and so, it has a negative external effect on utility. If $\psi > 0$, average leisure has a positive external effect. We shall impose the conditions $U_c + U_{\bar{c}} > 0$ and $U_{cc} + U_{c\bar{c}} < 0$, which are equivalent to $\mu + 1 > 0$ and $\sigma + \mu(\sigma - 1) > 0$, that mean that either the consumption externality augments the direct effect or, if it offsetting, it is

² The case $\eta = 0$, in which leisure does not provide utility, corresponds to the “no leisure” specification described below.

dominated by the direct effect.³ Similar restrictions are imposed on the size of the leisure externality, $U_L + U_{\bar{L}} > 0$ and $U_{LL} + U_{\bar{L}\bar{L}} < 0$, which are equivalent to $\psi + 1 > 0$ and $1 + \eta(1 + \psi)(\sigma - 1) > 0$, respectively.

The agent is endowed with one unit of time per period which can be allocated to work, u , learning, z , or leisure, l . The time constraint is then

$$1 = u + z + l. \quad (2)$$

Following Milesi-Ferretti and Roubini (1998a,b), we specify leisure in three alternative ways: “home production” (HP), “quality time” (QT) and “raw time” (RT). For comparative purposes, we also consider the case in which labor supply is inelastic; i.e., the “no leisure” (NL) case. In the home production specification, leisure is produced with a Cobb–Douglas technology that uses physical capital, k , and human capital, h , as inputs:

$$L = B[(1 - v)k]^\omega (lh)^\xi, \quad B > 0, \quad 0 < \omega < 1, \quad 0 < \xi < 1,$$

where v is the fraction of physical capital devoted to the production of goods, and $1 - v$, the fraction of physical capital devoted to the leisure activity. As Campbell and Ludvigson (2001, p. 851) argue, this specification “captures the idea that leisure is not valued for its own sake, but for what can be done with it.” In the quality time specification, leisure depends on effective time:

$$L = lh.$$

As Stokey and Rebelo (1995) point out, this specification entails that time devoted to the leisure activity is quality adjusted in the same way time devoted to working is. In the raw time specification, utility depends on pure leisure time:

$$L = l.$$

These three cases can be summarized in a single specification by defining⁴

$$L = G((1 - v)k, l, h) = B[(1 - v)k]^\omega (lh)^\phi. \quad (3)$$

The HP specification is obtained by making $\phi = 1$; the QT specifications is recovered when $B = 1$, $\omega = 0$, $\xi = 1$ and $\phi = 1$, and the RT specification is obtained by making $B = 1$, $\omega = 0$, $\xi = 1$ and $\phi = 0$. In the NL case, L and \bar{L} should be simply removed from the utility function in (1) or, equivalently, the parameter η must be set to 0.

Human capital, h , is accumulated according to the dynamic equation

$$\dot{h} = P(zh, \bar{zh}) = \delta(zh)^\varepsilon (\bar{zh})^{1-\varepsilon}, \quad \delta > 0, \quad 0 < \varepsilon \leq 1, \quad (4)$$

where \bar{zh} is the average human capital devoted to education, and expresses sector-specific externalities associated with human capital in the educational sector.

The rate of return on physical capital is denoted r , the wage rate, w , and profits, π , which we assume that are distributed back to consumers as dividends. The government taxes physical capital income at a constant rate τ_k , labor income at a constant rate τ_h , and consumption at a constant rate τ_c , and subsidizes investment on education at a

³ This is similar to conditions (i) and (ii) of assumption 1 in Liu and Turnovsky (2005). Note that their condition (iii) stems from their assumption that $U_{cL} \geq 0$. In the present model we shall not make this assumption, which would amount to restrict the EIS so that $\sigma \leq 1$ and, therefore, a condition similar to (iii) is not imposed.

⁴ I thank one anonymous referee for suggesting this specification.

constant rate s_h . In this model, the sole cost of education is foregone earnings, wzh , a fraction s_h of which is therefore financed by the government. The remaining income raised is rebated to the consumers as lump-sum transfers, s . In absence of depreciation of physical capital, the agent's budget constraint is, then,⁵

$$\dot{k} = (1 - \tau_k)rvk + (1 - \tau_h)wuh + \pi + s_hwzh - (1 + \tau_c)c + s. \tag{5}$$

Output, y , is produced with the Cobb–Douglas technology

$$y = F(vk, \overline{vk}, uh, \overline{uh}) = A(vk)^\alpha (\overline{vk})^{\alpha-\varphi} (uh)^\beta (\overline{uh})^{1-\alpha-\beta}, \quad A > 0, \quad 0 < \beta < 1, \quad 0 < \varphi < 1, \quad \varphi + \beta \leq 1, \tag{6}$$

where \overline{vk} is the average physical capital devoted to the production of goods, and \overline{uh} is the average human capital devoted to the production of goods. This specification assumes that the production of goods exhibits constant returns-to-scale at the social level but non-increasing returns-to-scale at the private level. The terms \overline{vk} and \overline{uh} express sector-specific externalities associated with physical and human capital, respectively, employed by the sector producing goods. We shall assume that $0 < \alpha < 1$ to ensure that the social marginal productivities of physical and human capital are positive.

Profit maximization by competitive firms implies that labor and capital are used up to the point at which marginal product equates marginal cost:

$$r = F_{vk} = \varphi y / (vk), \tag{7a}$$

$$w = F_{uh} = \beta y / (uh), \tag{7b}$$

where F_x denotes the partial derivate of F with respect to the variable $x = vk, uh$. If the production function exhibits decreasing returns-to-scale at the private level, the competitive firm earns positive profits,

$$\pi = (1 - \varphi - \beta)y. \tag{7c}$$

We shall assume that the government runs a balanced-budget:

$$\tau_krvk + \tau_hwuh + \tau_cc = s_hwzh + s. \tag{8}$$

3. The competitive equilibrium

A competitive equilibrium for this economy is defined as a set of market-clearing prices and quantities such that (i) the consumer's choice of c, k, h, v, u, z and l maximizes (1) subject to the constraints (2), (4) and (5), given the initial endowments of physical capital, k_0 , and human capital, h_0 , and taking as given the path of factor returns and fiscal policy variables; (ii) the firm's choice of physical capital, vk , and effective labor, uh , maximizes profits, and (iii) the government obeys its budget constraint (8). Note that in the QT and RT models, the fraction of physical capital devoted to the production of goods is $v = 1$ because physical capital is not used in any other sector, so this is not a choice variable in the consumer's problem. In the NL model, additionally, leisure time is zero, $l = 0$.

Let J be the current value Hamiltonian of the household's optimization problem, and let λ and θ be the multipliers for the constraints (5) and (4), respectively:

⁵ A tax on profits is not included since it would act as a lump-sum tax.

$$J = U(c, \bar{c}, L, \bar{L}) + \lambda[(1 - \tau_k)rvk + (1 - \tau_h)wuh + \pi + s_h w(1 - u - l)h - (1 + \tau_c)c + s] + \theta P((1 - u - l)h, \overline{(1 - u - l)h}),$$

where (2) has been used to substitute z for $1 - u - l$ and eliminate it from the problem.

Let U_x, F_x, G_x and P_x denote the partial derivate of U, F, G and P , respectively, with respect to the variable x . The first-order conditions for an interior optimum are

$$U_c = (1 + \tau_c)\lambda, \tag{9a}$$

$$U_L G_{(1-v)k} = \lambda(1 - \tau_k)r, \tag{9b}$$

$$\theta P_{zh} = \lambda(1 - \tau_h - s_h)w, \tag{9c}$$

$$U_L G_l = \lambda s_h w h + \theta P_{zh} h, \tag{9d}$$

$$\dot{\lambda} = [\rho - (1 - \tau_k)rv]\lambda - U_L G_{(1-v)k}(1 - v), \tag{9e}$$

$$\dot{\theta} = [\rho - (1 - u - l)P_{zh}]\theta - \lambda[(1 - \tau_h)wu + s_h w(1 - u - l)] - U_L G_h \tag{9f}$$

plus the usual transversality conditions. Here, the arguments of the functions have been suppressed for simplicity. In the QT and RT models, Eq. (9b) must be dropped, and in the NL model, Eq. (9d) must be eliminated too.

Eq. (9a) equates the marginal utility of consumption to the shadow value of an additional unit of physical capital. Eq. (9b) entails that the marginal value of one additional unit of physical capital must be equal whether it is devoted to goods production or leisure. Eqs. (9c) and (9d) entail that the marginal value of one additional unit of time must be equal whether it is devoted to goods production, human capital accumulation or leisure. Eq. (9e), which can be expressed as

$$(1 - \tau_k)rv + \frac{1}{\lambda} U_L G_{(1-v)k}(1 - v) = \rho - \frac{\dot{\lambda}}{\lambda}$$

equates the return on physical capital both in goods production and the leisure activity, to the rate of return on consumption. Eq. (9f), which can be expressed as

$$(1 - u - l)P_{zh} + \frac{\lambda}{\theta} [(1 - \tau_h)wu + s_h w(1 - u - l)] + U_L G_h + \frac{\dot{\theta}}{\theta} - \frac{\dot{\lambda}}{\lambda} = \rho - \frac{\dot{\lambda}}{\lambda}$$

equates the rate of return on human capital in the goods sector, the leisure activity and the educational sector, to the rate of return on consumption. Now, the return to investing in human capital includes the rate of capital gains, $\dot{\theta}/\theta - \dot{\lambda}/\lambda$.

Hereafter, let $\gamma_x = \dot{x}/x$ denote the growth rate of the variable x . For future reference, note that using (9b) and (9e), taking into account (7a), we obtain that

$$\gamma_\lambda = \rho - (1 - \tau_k)F_{vk} \tag{10}$$

and using (9c), (9d) and (9f), and that $G_l = \xi G/l$ and $G_h = \phi \xi G/h$, we get that

$$\gamma_\theta = \rho - (1 - \tau_h)P_{zh}(1 - l + \phi l)/(1 - \tau_h - s_h). \tag{11}$$

Appendix shows that the dynamic behavior of the economy can be described by the following third order dynamical system in the variables $r, q = c/k$ and u , that are constant in the steady state:

$$\gamma_r = -\frac{(1-\alpha)}{\alpha} \left\{ (1-\tau_k)r - \frac{\varepsilon\delta(1-\tau_h)[1-l(r,q,u) + \phi l(r,q,u)]}{(1-\tau_h-s_h)} \right\}, \quad (12a)$$

$$\begin{aligned} \gamma_q = & -\frac{r}{\varphi} v(r,q,u) + \frac{(1-\tau_k)r - \rho + \eta(1+\psi)\xi(\sigma-1)(1-\phi)\delta[1-u-l(r,q,u)]}{\sigma + [\eta(1+\psi)(\xi+\omega) + \mu](\sigma-1)} \\ & - \frac{\eta(1+\psi)(\sigma-1)[\alpha(\xi+\omega) - \omega]}{(1-\alpha)\{\sigma + [\eta(1+\psi)(\xi+\omega) + \mu](\sigma-1)\}} \gamma_r + q, \end{aligned} \quad (12b)$$

$$\gamma_u = -q - \delta[1-u-l(r,q,u)] + \frac{r}{\varphi} v(r,q,u) + \frac{1-v(r,q,u)}{v(r,q,u)} (\gamma_r - \gamma_q) + \frac{1}{1-\alpha} \gamma_r, \quad (12c)$$

where l and v can be expressed as functions of r , q and u as

$$l(r,q,u) = \frac{\varphi\eta\xi(1-\tau_k)(1+\tau_c)qu}{\beta(1-\tau_h)[(1-\tau_k)r - \eta(1+\tau_c)\omega q]}, \quad (13a)$$

$$v(r,q,u) = 1 - \frac{(1+\tau_c)\omega\eta q}{(1-\tau_k)r}. \quad (13b)$$

A hat “^” over a variable will denote its steady-state value in the market economy. The steady state can be obtained by imposing the stationarity conditions $\gamma_r = \gamma_q = \gamma_u = 0$. Appendix shows that the long-run growth rate of output, physical and human capital, and consumption is

$$\begin{aligned} \hat{\gamma} = \delta(1 - \hat{u} - \hat{l}) &= \frac{(1-\tau_h)\varepsilon\delta(1 - \hat{l} + \phi\hat{l}) - \rho(1-\tau_h-s_h)}{(1-\tau_h-s_h)\{\sigma + [\eta(1+\psi)(\phi\xi+\omega) + \mu](\sigma-1)\}} = \hat{\gamma}_k \\ &= \hat{\gamma}_h = \hat{\gamma}_c = \hat{\gamma}_y. \end{aligned} \quad (14)$$

The HP model is obtained by substituting $\phi = 1$ into (12)–(14) and the QT model, by additionally setting $B = 1$, $\omega = 0$ and $\xi = 1$ which, in particular, entails that $v = 1$. The NL model is obtained by setting $\eta = 0$ which, in particular, entails that $v = 1$ and $l = 0$. In the HP, QT and NL models, the steady state can be obtained in a recursive manner. From (12a), it can be obtained \hat{r} . Then, from (12b) and taking into account (13b), it can be computed \hat{q} . Using these values and taking into account (13) and (12c) allows obtaining \hat{u} . The values of \hat{v} and \hat{l} can be computed from (13b) and (13a). We assume henceforth that parameter values are so that the steady state is feasible.⁶ Linearizing (12) around the steady state $(\hat{r}, \hat{q}, \hat{u})$ yields

$$\begin{pmatrix} \dot{\hat{r}} \\ \dot{\hat{q}} \\ \dot{\hat{u}} \end{pmatrix} = \begin{pmatrix} -(1-\alpha)(1-\tau_k)\hat{r}/\alpha & 0 & 0 \\ \cdot & \hat{q} + \eta\omega(1+\tau_c)\hat{q}/[\varphi(1-\tau_k)] & 0 \\ \cdot & \cdot & \delta(\hat{l} + \hat{u}) \end{pmatrix} \begin{pmatrix} \hat{r} - \hat{r} \\ \hat{q} - \hat{q} \\ \hat{u} - \hat{u} \end{pmatrix}, \quad (15)$$

where dots replace those elements that are irrelevant for the analysis. As the coefficient matrix is triangular, the characteristic roots are its diagonal elements. The first one is negative whereas the second and the third ones are positive if the steady state is feasible. Hence, two roots are positive and one is negative and so, the steady state is locally saddle-path stable.

⁶ Detailed derivations of the steady state and the conditions for feasibility are available upon request.

The RT model is obtained by substituting $B = 1$, $\omega = 0$, $\xi = 1$ and $\phi = 0$ into (12),(13),(14) which, in particular, entails that $v = 1$. When leisure is specified as raw time, Ladrón-de-Guevara et al. (1999) show that there exists the possibility of multiple balanced growth paths, and conclude that it seems difficult to state a set of necessary and sufficient conditions for this possibility to occur. We address to this reference for a detailed analysis of the equilibrium dynamics of this class of models.

4. The centrally planned economy

This section analyzes the equilibrium dynamics of the centrally planned economy under each of the leisure specifications considered. The central planner possesses complete information and chooses all quantities directly, taking all the relevant information into account. The central planner maximizes (1) subject to (2),

$$\dot{h} = P(zh, zh) = \delta zh, \tag{16}$$

and

$$\dot{k} = F(vk, vk, uh, uh) = A(vk)^\alpha (uh)^{1-\alpha} - c, \tag{17}$$

where leisure, L , in (1) is given by (3).

Let \bar{J} be the current value Hamiltonian of the planner's problem, and let $\bar{\lambda}$ and $\bar{\theta}$ be the multipliers for the constraints (17) and (16), respectively:

$$\bar{J} = U(c, c, L, L) + \bar{\lambda}[F(vk, vk, uh, uh) - c] + \bar{\theta}P((1 - u - l)h, (1 - u - l)h),$$

where (2) has been used to substitute z for $1 - u - l$ and eliminate it from the problem.

The first-order necessary conditions for an interior solution are

$$U_c + U_{\bar{c}} = \bar{\lambda}, \tag{18a}$$

$$(U_L + U_{\bar{L}})G_{(1-v)k} = \bar{\lambda}(F_{vk} + F_{\bar{v}k}), \tag{18b}$$

$$\bar{\theta}(P_{zh} + P_{\bar{z}h}) = \bar{\lambda}(F_{uh} + F_{\bar{u}h}), \tag{18c}$$

$$(U_L + U_{\bar{L}})G_l = \bar{\theta}(P_{zh} + P_{\bar{z}h})h, \tag{18d}$$

$$\dot{\bar{\lambda}} = [\rho - (F_{vk} + F_{\bar{v}k})v]\bar{\lambda} - (U_L + U_{\bar{L}})G_{(1-v)k}(1 - v), \tag{18e}$$

$$\dot{\bar{\theta}} = [\rho - (1 - u - l)(P_{zh} + P_{\bar{z}h})]\bar{\theta} - \bar{\lambda}(F_{uh} + F_{\bar{u}h})u - (U_L + U_{\bar{L}})G_h \tag{18f}$$

plus the usual transversality conditions. In the QT and RT models, Eq. (18b) must be dropped, and in the NL model, Eq. (18d) must be eliminated too. Comparing (18) with (9), it can be observed that the main difference is that the relevant marginal utilities and rates of return in the market economy are the private ones, while in the centralized economy they are the social ones which reflect the effects of externalities.

Note for future reference that using (18b) and (18e), we get

$$\gamma_{\bar{\lambda}} = \rho - (F_{vk} + F_{\bar{v}k}) \tag{19}$$

and using (18f), (18c) and (18d), and that $G_l = \xi G/l$ and $G_h = \phi \xi G/h$, we obtain

$$\gamma_{\bar{\theta}} = \rho - \delta(1 - l + \phi l). \tag{20}$$

Proceeding in the same manner as in the case of the market economy, we can express the dynamics of the centralized economy in terms of the variables $r = F_{vk} = \phi A(vk)^{\alpha-1}(uh)^{1-\alpha}$, $q = c/k$, and u :

$$\gamma_r = -\frac{(1-\alpha)}{\phi}r + \frac{(1-\alpha)}{\alpha}\delta[1 - l(r, q, u) + \phi l(r, q, u)], \tag{21a}$$

$$\begin{aligned} \gamma_q = & -\frac{r}{\phi}v(r, q, u) + \frac{(\alpha/\phi)r - \rho + \eta(1 + \psi)\xi(\sigma - 1)(1 - \phi)\delta[1 - u - l(r, q, u)]}{\sigma + [\eta(1 + \psi)(\xi + \omega) + \mu](\sigma - 1)} \\ & - \frac{\eta(1 + \psi)(\sigma - 1)[\alpha(\xi + \omega) - \omega]}{(1 - \alpha)\{\sigma + [\eta(1 + \psi)(\xi + \omega) + \mu](\sigma - 1)\}}\gamma_r + q, \end{aligned} \tag{21b}$$

$$\gamma_u = -q - \delta[1 - u - l(r, q, u)] + \frac{r}{\phi}v(r, q, u) + \frac{1 - v(r, q, u)}{v(r, q, u)}(\gamma_r - \gamma_q) + \frac{1}{1 - \alpha}\gamma_r, \tag{21c}$$

whereas l and v are given by

$$l(r, q, u) = \frac{\alpha\eta\phi\xi(1 + \psi)qu}{(1 - \alpha)[\alpha(1 + \mu)r - \eta\omega(1 + \psi)\phi q]}, \tag{22a}$$

$$v(r, q, u) = 1 - \frac{\phi\omega\eta(1 + \psi)q}{\alpha(1 + \mu)r}. \tag{22b}$$

An asterisk “*” will denote the steady-state value of a variable in the centrally planned economy. The steady state can be obtained by imposing the stationarity conditions $\gamma_r = \gamma_q = \gamma_u = 0$. The long-run optimal growth rate of output, physical and human capital, and consumption can be determined as

$$\gamma^* = \delta(1 - u^* - l^*) = \frac{\delta(1 - l^* + \phi l^*) - \rho}{\sigma + [\eta(1 + \psi)(\phi\xi + \omega) + \mu](\sigma - 1)} = \gamma_y^* = \gamma_k^* = \gamma_h^* = \gamma_c^*. \tag{23}$$

The HP, QT and NL models can be obtained as particular cases as it is described in the previous section for the market economy, and the steady state can also be derived in a similar recursive manner. We shall assume henceforth that parameter values are so that the steady state is feasible.⁷ Linearizing (21) around the steady state (r^*, q^*, u^*) yields:

$$\begin{pmatrix} \dot{r} \\ \dot{q} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} -(1 - \alpha)r^*/\phi & 0 & 0 \\ \cdot & q^* + \eta\omega(1 + \psi)q^*/[\alpha(1 + \mu)] & 0 \\ \cdot & \cdot & \delta(l^* + u^*) \end{pmatrix} \begin{pmatrix} r - r^* \\ q - q^* \\ u - u^* \end{pmatrix}, \tag{24}$$

where dots replace those elements that are irrelevant for the analysis. As the coefficient matrix is triangular, the characteristic roots are its diagonal elements. The first one is negative whereas the second and the third ones are positive if the steady state is feasible. Hence, two roots are positive and one is negative, and so, the steady state is locally saddle-path stable.

The RT economy is obtained by substituting $B = 1$, $\omega = 0$, $\xi = 1$ and $\phi = 0$ into (21)–(23) which, in particular, entails that $v = 1$. Again, we address to [Ladron-de-Guevara et al. \(1999\)](#) for an analysis of the equilibrium dynamics of this class of models.

⁷ Detailed derivations of the steady state and the conditions for feasibility are available upon request.

5. Comparison of long-run growth rates

This section compares the long-run growth rate of the centrally planned economy and that of the market economy with no government intervention; i.e., with $\tau_k = 0$, $\tau_h = 0$, $\tau_c = 0$ and $s_h = 0$, and consequently $s = 0$. In the HP, QT and NL economies, comparing (14) with (23), it can be easily observed that the long-run equilibrium growth rate and the optimal one are related through $\hat{\gamma} = \varepsilon\gamma^* \leq \gamma^*$. Hence, we can state the following result.

Proposition 1. *In the HP, QT and NL models,*

- (i) *the presence of sector-specific production externalities in the goods sector ($\alpha - \varphi > 0$ and/or $1 - \alpha - \beta > 0$) and/or utility externalities ($\mu \neq 0$ and/or $\psi \neq 0$) has no impact in the market equilibrium growth rate, i.e. the long-run equilibrium growth rate is equal to the optimal one ($\hat{\gamma} = \gamma^*$),*
- (ii) *the presence of sector-specific externalities associated to human capital in education ($\varepsilon < 1$) results in a long-run equilibrium growth rate lower than the optimal one ($\hat{\gamma} < \gamma^*$).*

As Milesi-Ferretti and Roubini (1998a) point out, in the HP and QT models the choice between labor and leisure does not affect long-run growth because human capital is always fully employed in productive activities, as it is in the NL model as well. In the steady state, the rate of return on physical capital in goods production is equal to the rate of return on effective time in the educational sector. Hence, sector-specific externalities in goods production and utility externalities do not provoke a divergence between the competitive and the long-run optimal growth rate because the private and the social return on effective time in the educational sector, which is the constant δ , coincide. However, sector-specific externalities in education reduce the private relative to the social rate of return on effective time in the educational sector. As a result, the long-run equilibrium growth rate is lower than the optimal one.

In order to compare the competitive and the long-run optimal growth rates in the RT model, we follow a procedure similar to that used by Devereaux and Love (1994) and Milesi-Ferretti and Roubini (1998a) to examine the growth effects of taxes. Henceforth, we shall assume that there exists a unique feasible steady state, and will take into account that $\omega = 0$, $\xi = 1$ and $\phi = 0$ in the RT economy, and that $\tau_k = \tau_h = \tau_c = s_h = 0$, and consequently $s = 0$, in the absence of government intervention. Making these substitutions in (14), and using that $l = 1 - u - z$ from (2), we get the long-run growth in the RT economy with no government intervention as

$$\gamma = \frac{\varepsilon\delta(u+z) - \rho}{\sigma + \mu(\sigma - 1)} = \delta z. \quad (25)$$

Solving out for $u + z$, we find the following relationship between studying time and non-leisure time:

$$z = \chi_D(u + z), \quad (26)$$

where

$$\chi_D = \frac{\varepsilon\gamma}{[\sigma + \mu(\sigma - 1)]\gamma + \rho} < 1. \quad (27)$$

We can get two expressions relating the non-leisure time, $u + z$, and the growth rate, γ . The first one is obtained from (25) as

$$u + z = f_D(\gamma) = \frac{[\sigma + \mu(\sigma - 1)]\gamma + \rho}{\varepsilon\delta}. \tag{28a}$$

The second one is given by

$$u + z = g_D(\gamma) = \frac{1}{1 + \frac{1}{\varepsilon\beta}\eta(1 - \chi_D)(\varepsilon - \varphi\chi_D)}, \tag{28b}$$

where χ_D is given by (27). Note that the fulfilment of the feasibility condition $u + z < 1$ entails that $\varepsilon - \varphi\chi_D > 0$. Eq. (28b) is obtained as follows. Replacing r by (7a), q by c/k , and l by $1 - u - z$ from (2), Eq. (13a) can be rewritten as⁸

$$u + z = 1 - \frac{\eta}{\beta} \frac{c}{y} u = 1 - \frac{\eta}{\beta} \frac{c}{y} (1 - \chi_D)(u + z), \tag{29}$$

where the last equality follows from the fact that (26) implies that $u = (1 - \chi_D)(u + z)$. In the steady state, the resource's constraint (A.8) can be rewritten as

$$c/y = 1 - \gamma(k/y). \tag{30}$$

The Euler Eq. (A.4) entails that $[\sigma + \mu(\sigma - 1)]\gamma + \rho = r = \varphi(y/k)$. Hence, (27) can be rewritten as $\chi_D = \varepsilon\gamma/r = \varepsilon\gamma(k/y)/\varphi$, and thus, $\gamma(k/y) = \varphi\chi_D/\varepsilon$, which substituted into (30) yields $c/y = (\varepsilon - \varphi\chi_D)/\varepsilon$. Substituting c/y for this expression into (29) and solving out for $u + z$ we finally get (28b). The intersection between f_D and g_D determines the long-run growth rate and the stationary non-leisure time in the market economy. It can be easily shown that both f_D and g_D are strictly increasing in γ . It should be noted that since g_D is always below one for feasible values of the growth rate, it has to intersect f_D from above as long as the steady state is unique.

Proceeding in a similar manner as in the case of the market economy, we can get two expressions relating the non-leisure time, $u + z$, and the growth rate, γ , in the centrally planned economy:

$$u + z = f_C(\gamma) = \frac{[\sigma + \mu(\sigma - 1)]\gamma + \rho}{\delta}, \tag{31a}$$

$$u + z = g_C(\gamma) = \frac{1}{1 + \frac{(1+\psi)}{(1-\alpha)(1+\mu)}\eta(1 - \chi_C)(1 - \alpha\chi_C)}, \tag{31b}$$

where

$$\chi_C = \frac{\gamma}{[\sigma + \mu(\sigma - 1)]\gamma + \rho} < 1.$$

The intersection between f_C and g_C determines the long-run growth rate and the stationary non-leisure time in the centralized economy. It can be easily shown that both f_C and g_C are strictly increasing in γ . Since g_C is always below one for feasible values of the growth rate, it has to intersect f_C from above as long as the steady state is unique.

⁸ Note that in the RT economy with no government intervention, Eq. (13a) simplifies to $l = \varphi\eta qu/(\beta r)$.

Comparing f_D with f_C , we see that $f_D(\gamma) \geq f_C(\gamma)$, where the equality $f_D(\gamma) = f_C(\gamma)$ holds if and only if $\varepsilon = 1$. In the absence of production externalities ($\varepsilon = 1$, $\varphi = \alpha$ and $\beta = 1 - \alpha$), comparing the denominators of (28b) and (31b), we obtain that

$$\begin{aligned} & \frac{1}{\varepsilon\beta} \eta(1 - \chi_D)(\varepsilon - \varphi\chi_D) - \frac{(1 + \psi)}{(1 - \alpha)(1 + \mu)} \eta(1 - \chi_C)(1 - \alpha\chi_C) \\ &= \frac{\eta(\mu - \psi)(1 - \chi_C)(1 - \alpha\chi_C)}{(1 - \alpha)(1 + \mu)}, \end{aligned}$$

which implies that $\text{sgn}[g_D(\gamma) - g_C(\gamma)] = -\text{sgn}(\mu - \psi)$. In the absence of utility externalities ($\mu = 0$ and $\psi = 0$), since $1 - \chi_D \geq 1 - \chi_D/\varepsilon = 1 - \chi_C$, $\varepsilon - \varphi\chi_D \geq \varepsilon - \alpha\chi_D = \varepsilon(1 - \alpha\chi_C)$, and $1/\beta \geq 1/(1 - \alpha)$, we get that $g_D(\gamma) \leq g_C(\gamma)$, where the equality holds if and only if there are no production externalities.

Now, we can determine the effects of externalities on the relationship between the long-run equilibrium growth rate and the optimal one. In the absence of production externalities ($\varepsilon = 1$, $\varphi = \alpha$, $\beta = 1 - \alpha$), we have that $f_D(\gamma) = f_C(\gamma)$. If $\mu > \psi$, then $g_D(\gamma) < g_C(\gamma)$ and thus, the intersection between f_D and g_D must lie to the left of the intersection between f_C and g_C . Hence, the long-run equilibrium growth rate is lower than the optimal one. If $\mu < \psi$, then $g_D(\gamma) > g_C(\gamma)$ and thus, the intersection between f_D and g_D must lie to the right of the intersection between f_C and g_C . As a result, the long-run equilibrium growth rate is higher than the optimal one. If $\mu = \psi$, then $g_D(\gamma) = g_C(\gamma)$ and thus, the long-run equilibrium growth rate coincides with the optimal one. In the absence of utility externalities ($\mu = 0$, $\psi = 0$), $f_D(\gamma) \geq f_C(\gamma)$ and $g_D(\gamma) \leq g_C(\gamma)$, where at least one of the inequalities holds strictly in the presence of production externalities. In any case, the intersection between f_D and g_D must lie to the left of the intersection between f_C and g_C , and thus, the long-run equilibrium growth rate is lower than the optimal one. In the presence of both utility and production externalities, this relationship becomes ambiguous in the case in which the presence of utility externalities entails that the equilibrium growth rate is higher than the optimal one. We can state the following proposition.

Proposition 2. *Let a unique interior steady state exist for both the market and the centrally planned economies in the RT model. Then,*

- (i) *in the absence of production externalities ($\varepsilon = 1$, $\varphi = \alpha$, $\beta = 1 - \alpha$) and if $\mu < \psi$, the long-run equilibrium growth rate is higher than the optimal one ($\hat{\gamma} > \gamma^*$); if $\mu > \psi$, the long-run equilibrium growth rate is lower than the optimal one ($\hat{\gamma} < \gamma^*$), and if $\mu = \psi$, the competitive and the long-run optimal growth rates coincide ($\hat{\gamma} = \gamma^*$).*
- (ii) *in the absence of utility externalities ($\mu = 0$, $\psi = 0$), production externalities cause the long-run equilibrium growth rate to be lower than the optimal one ($\hat{\gamma} < \gamma^*$).*

As Milesi-Ferretti and Roubini (1998a) argue, when leisure is modeled as raw time the fraction of human capital corresponding to the fraction of time devoted to leisure is unemployed, and the more human capital is unemployed the lower are the incentives to accumulate human capital. Hence, the relationship between the long-run equilibrium growth rate and the optimal one depends on the leisure time in the market and the centralized economies. Let us first consider the case in which there are only utility externalities

($\varepsilon = 1$, $\varphi = \alpha$, $\beta = 1 - \alpha$). In the market economy, using (9a), (9c) and (9d) to eliminate λ and θ , and using (7b), the competitive marginal rate of substitution (MRS) between consumption and leisure satisfies that

$$\frac{U_L}{U_c} = F_{uh}h. \quad (32a)$$

In contrast, using (18a), (18c) and (18d), the efficient MRS satisfies that

$$\frac{U_L + U_{\bar{L}}}{U_c + U_{\bar{c}}} = F_{uh}h = \frac{(1 + \psi)U_L}{(1 + \mu)U_c}. \quad (32b)$$

If the elasticity of average consumption in utility, μ , is higher (lower) than the elasticity of average leisure time, ψ , the long-run equilibrium growth rate is lower (higher) than its long-run optimal value. The intuition is simple: The efficient MRS between consumption and leisure would be lower (higher) than the competitive one, as can be deduced from (32a) and (32b) and, as a result, leisure time in the market economy would be higher (lower) than that in the centralized economy. If the elasticities of average consumption and average leisure are equal, the efficient and the competitive MRS between consumption and leisure coincide and, therefore, so do the long-run equilibrium growth rate and the optimal one.

Let us now consider the case in which there are only production externalities ($\mu = 0$, $\psi = 0$). Sector-specific externalities in goods production reduce the private net return on physical and/or human capital in goods production relative to the social return. Sector-specific externalities in human capital accumulation reduce the private return to human capital in the educational sector relative to the social one. As a result, in the market economy working and/or studying is less preferable relative to enjoying leisure than in the centrally planned economy, and thus, the long-run equilibrium growth rate is lower than the optimal one.

6. Optimal fiscal policy

This section designs a fiscal policy capable of making the market economy replicate the first-best optimum attainable by a central planner. To this end, the tax and subsidy rates must be chosen so as to the time paths of c , k , h , v , l and u in the market economy replicate those of the centrally planned economy. Hereafter, we shall take into account that in the market economy, $r = F_{vk}$ and $w = F_{uh}$ from (7a) and (7b) when considering the expressions (9a)–(9f).

One way to derive the optimal fiscal policy would be to compute the tax and subsidy rates such that system (12) that drives the dynamics of the market economy replicates that of the centralized economy given by (21), and the expressions (13a) and (13b) for l and v in the market economy replicate those of the centralized economy given by (22a) and (22b), respectively, for each of the four leisure specifications considered in this paper. However, in order to get some intuition on the optimal policy, we shall compare the first-order conditions (9) for the market economy with those of the centralized economy (18).

Irrespective of the leisure specification, comparison of (9) with (18) reveals that replication of the optimal growth path requires that $\lambda = a\bar{\lambda}$ and $\theta = b\bar{\theta}$, where a and b are arbitrary constants; i.e., $\gamma_\lambda = \gamma_{\bar{\lambda}}$ and $\gamma_\theta = \gamma_{\bar{\theta}}$. It should be noticed that Liu and Turnovsky (2005) obtain the same condition $\lambda = a\bar{\lambda}$. However, and because the present paper

considers a two-sector model, there is an extra distortion that needs to be accounted for and thus an extra condition is required; i.e., $\theta = b\theta$.

Comparing (10) with its counterpart (19) in the centrally planned economy we find that the tax on physical capital income must be set according to

$$1 - \tau_k = (F_{vk} + F_{vk}^-) / F_{vk} = \alpha / \varphi, \tag{33}$$

which satisfies that $\tau_k < 1$. Inefficiency arises from the discrepancy between the before-tax marginal return of physical capital in the competitive economy and the corresponding marginal return in the centrally planned economy. This inefficiency can be corrected if τ_k is set so as to equate the private and social marginal returns to physical capital. The optimal τ_k would be negative or positive according to whether the production externality is positive or negative. In the absence of such a production externality, physical capital income should be free of taxation. A similar result has been obtained by Liu and Turnovsky (2005).

Comparing (11) with its counterpart (20) in the centrally planned economy we find that the labor income tax and the subsidy to investment on education should be set according to

$$\frac{1 - \tau_h}{1 - \tau_h - s_h} P_{zh} = P_{zh} + P_{zh}^-, \tag{34}$$

or equivalently,

$$s_h = (1 - \tau_h) \frac{P_{zh}^-}{P_{zh} + P_{zh}^-} = (1 - \varepsilon)(1 - \tau_h). \tag{35}$$

Thus, it is corrected the inefficiency that arises from the divergence between the private and social marginal returns of human capital devoted to education. It should be noted that (34) entails that this inefficiency cannot be corrected by making use solely of a labor income tax.

Now, we focus on the differences that arise across leisure specifications.

6.1. Leisure as “home production”

Recall that the HP economy is obtained when $\phi = 1$. In the market economy, using (9a) and (9b) to eliminate λ , we find that the competitive marginal rate of substitution (MRS) equals the marginal rate of transformation (MRT) between physical capital employed in the leisure activity and consumption:

$$\frac{U_c}{U_L G_{(1-v)k}} = \frac{1 + \tau_c}{(1 - \tau_k) F_{vk}}. \tag{36}$$

Using (18a) and (18b), we get that a similar condition must hold in the centrally planned HP economy, but replacing competitive with socially optimal marginal rates:

$$\frac{U_c + U_{\bar{c}}}{(U_L + U_{\bar{L}}) G_{(1-v)k}} = \frac{1}{F_{vk} + F_{vk}^-}. \tag{37}$$

Inefficiency may arise because of the divergence between the before-tax MRS and MRT of the competitive economy and the corresponding ones of the centrally planned economy. As shown above, the optimal tax on physical capital income corrects for the wedge

between the social and private marginal productivities of physical capital. Therefore, making use of the result derived in expression (33) for $1 - \tau_k$, Eqs. (36) and (37) entail that the consumption tax must be set so as to equate the competitive and socially optimal MRS between physical capital devoted to the leisure activity and consumption; so that

$$1 + \tau_c = \frac{U_c}{(U_c + U_{\bar{c}})} \frac{(U_L + U_{\bar{L}})}{U_L} = \frac{1 + \psi}{1 + \mu}. \quad (38)$$

If the MRS of the market economy evaluated along the optimal growth path is larger than the MRS of the centralized economy, the individuals' willingness to substitute physical capital for consumption would be too high. In this case, the optimal growth path can be reached by the market economy through a consumption tax that increases the price of consumption. On the contrary, if the MRS of the market economy along the optimal growth path turns out to be smaller than that of the centralized economy, then consumption should be subsidized. The optimal τ_c is zero in the absence of consumption and leisure externalities ($\mu = \psi = 0$). When there are no external effects from leisure ($\psi = 0$), the consumption tax, $\tau_c = -\mu/(1 + \mu)$, is positive (negative) if utility exhibits jealousy (admiration) about consumption. When there are no consumption externalities ($\mu = 0$), the consumption tax, $\tau_c = \psi$, is positive (negative) if the leisure externality is positive (negative).

In the HP market economy, using that $G_l = G_{lh}h$, (9c) and (9d) entail that

$$\lambda(1 - \tau_h)F_{uh} = U_L G_{lh} = \theta \varepsilon \delta (1 - \tau_h) / (1 - \tau_h - s_h). \quad (39)$$

Combining this condition with (9b), to eliminate λ and θ , we find that the MRT between human and physical capital in leisure production must equal the MRT in goods production:

$$\frac{G_{(1-v)k}}{G_{lh}} = \frac{(1 - \tau_k)F_{vk}}{(1 - \tau_h)F_{uh}}. \quad (40)$$

In the centrally planned economy, (18b), (18c) and (18d) entail that a similar condition must hold but replacing competitive with socially optimal rates:

$$\frac{G_{(1-v)k}}{G_{lh}} = \frac{F_{vk} + F_{v\bar{k}}}{F_{uh} + F_{u\bar{h}}}. \quad (41)$$

Inefficiency may arise because of the divergence between the before-tax MRT of the competitive economy in goods production and the corresponding one of the centrally planned economy. Given that τ_k must be set so as to equate the social and private marginal productivities of physical capital, the labor income tax must be set so as to equate the social and private marginal productivities of human capital:

$$1 - \tau_h = \frac{F_{uh} + F_{u\bar{h}}}{F_{uh}} = \frac{1 - \alpha}{\beta}. \quad (42)$$

In the absence of sector-specific externalities associated to human capital in goods production ($\beta = 1 - \alpha$), labor income should be untaxed. The optimal τ_h is positive or negative according to whether there are negative or positive externalities. Note that (35) and (42) entail that the subsidy to investment on education is given by

$$s_h = \frac{(F_{uh} + F_{\bar{uh}})}{F_{uh}} \frac{P_{\bar{zh}}}{(P_{zh} + P_{\bar{zh}})} = \frac{(1 - \varepsilon)(1 - \alpha)}{\beta}. \quad (43)$$

The following Proposition summarizes the former results.

Proposition 3. *The market economy can attain the first-best equilibrium when leisure is “home production” if physical capital income is taxed at a rate $\tau_k = -(\alpha - \varphi)/\varphi$, labor income is taxed at a rate $\tau_h = -(1 - \alpha - \beta)/\beta$, consumption is taxed at a rate $\tau_c = (\psi - \mu)/(1 + \mu)$, and investment on education is subsidized at a rate $s_h = (1 - \varepsilon)(1 - \alpha)/\beta$.*

The sizes of the tax rates required to correct the distortions caused by external effects are likely to be significant. To give some idea of their numerical magnitudes, assume that the (private) elasticities of physical and human capital in goods production are $\varphi = 0.3$ and $\beta = 0.5$, respectively. Assuming small externalities associated to physical and human capital in goods production of $\alpha - \varphi = 0.1$ and $1 - \alpha - \beta = 0.1$, physical capital income should be subsidized at a rate of about 33%, and labor income, at a rate of 20%. Assuming a small externality of $1 - \varepsilon = 0.1$ in the educational sector, investment in education should be subsidized at a rate of 12%. Since people care about their relative position as well as their absolute levels of income and consumption, the normal expectation is the presence of negative consumption externalities; i.e., $\mu < 0$. The evidence reported in the introduction suggests the presence of positive leisure externalities; i.e., $\psi > 0$. A small negative consumption externality, $\mu = -0.1$, combined with a positive leisure externality of $\psi = 0.1$ entails an optimal consumption tax of about 22%.

6.2. Leisure as “quality time”

Notice that the QT economy is obtained when $\phi = 1$, $B = 1$, $\xi = 1$ and $\omega = 0$. Now, Eqs. (9b) and (18b) should be dropped. In the market QT economy, using that $G_l = G_{lh}$, (9a), (9c) and (9d) entail that the MRS must equal the MRT between consumption and effective time:

$$\frac{U_L G_{lh}}{U_c} = \frac{(1 - \tau_h) F_{uh}}{1 + \tau_c}. \quad (44)$$

It should be noted that, unlike the case of the HP model, the former expression shows that labor income and consumption taxation affect the same margin of choice.

In the centrally planned economy, (18a), (18c) and (18d) entail that a similar condition must hold but replacing competitive with socially optimal rates:

$$\frac{(U_L + U_{\bar{L}}) G_{lh}}{U_c + U_{\bar{c}}} = F_{uh} + F_{\bar{uh}}. \quad (45)$$

Therefore, the consumption and labor income taxes should be set so as to the competitive economy replicate this condition, according to

$$\frac{(1 - \tau_h)}{(1 + \tau_c)} F_{uh} \frac{U_c}{U_L} = (F_{uh} + F_{\bar{uh}}) \frac{U_c + U_{\bar{c}}}{U_L + U_{\bar{L}}}, \quad (46)$$

or, equivalently,

$$(1 - \tau_h) \frac{\beta}{1 - \alpha} = (1 + \tau_c)(1 + \mu) \frac{1}{1 + \psi}. \quad (47)$$

It should be noted that this expression collapses to the result obtained in Liu and Turnovsky (2005) as long as there is no leisure externalities and no spillovers associated to human capital in goods production.

Eq. (47) shows that a single labor income or consumption tax can correct for the distortions caused by consumption externalities, leisure externalities, and sector-specific externalities associated to human capital in production. In particular, this entails that income from physical and human capital could be treated uniformly, i.e., taxed at identical rates according to (33), and then the consumption tax rate be set according to (47). As a general rule, the number of instruments required to eliminate a distortion equals the number of relationships subject to distortions. As shown above, in the QT economy both the consumption tax and the labor income tax affect the same margin of choice and thus, one of them can be set arbitrarily. A similar result has been obtained by Liu and Turnovsky (2005) in a Ramsey model with leisure as “raw time”. It should be noticed that the fact that one tax can be arbitrarily chosen entails that there is a degree of arbitrariness that is absent in the HP economy. This degree of freedom follows from the fact that in the QT model there is one less margin of choice than in the HP model: the allocation of physical capital between production and leisure.

6.3. Leisure as “raw time”

Recall that the RT economy is obtained when $\phi = 0$, $B = 1$, $\xi = 1$, $\omega = 0$, so that $L = l$. Now, Eqs. (9b) and (18b) should be dropped. In the RT market economy, (9a), (9c) and (9d) entail that the MRS must equal the MRT between consumption and time:

$$\frac{U_L}{U_c} = \frac{(1 - \tau_h)F_{uh}h}{1 + \tau_c}. \quad (48)$$

In the centrally planned economy, (18a), (18c) and (18d) entail that a similar condition must hold but replacing competitive with socially optimal rates:

$$\frac{U_L + U_{\bar{L}}}{U_c + U_{\bar{c}}} = (F_{uh} + F_{\bar{uh}})h. \quad (49)$$

Therefore, consumption and labor income tax rates should be set so as to the market economy replicate this condition, according to (47). As in the QT model, the extra degree of freedom relative to the case of the HP model follows from the fact that in the RT model there is one less margin of choice than in the HP model: the allocation of physical capital between production and leisure. The following Proposition summarizes the findings of this and the previous section.

Proposition 4. *The market economy can attain the first-best equilibrium when leisure is “quality time” or “raw time” if physical capital income is taxed at a rate $\tau_k = -(\alpha - \varphi)/\varphi$, the (constant) tax rates on labor income and consumption are set so as to satisfy the condition*

$$(1 - \tau_h) \frac{\beta}{1 - \alpha} = (1 + \tau_c) \frac{1 + \mu}{1 + \psi},$$

and investment on human capital is subsidized at a rate $s_h = (1 - \varepsilon)(1 - \tau_h)$.

As discussed above, the optimal growth path may be replicated by the market equilibrium by means of an income tax, a consumption tax, and a subsidy to education. If income from physical and human capital is taxed at the same rate τ_y , the optimal fiscal policy is summarized in the following Proposition.

Proposition 5. *The market economy can attain the first-best equilibrium when leisure is “quality time” or “raw time” if income is taxed at a rate $\tau_y = -(\alpha - \varphi)/\varphi$, consumption, at a rate*

$$1 + \tau_c = \frac{\alpha}{\varphi} \frac{\beta}{(1 - \alpha)} \frac{(1 + \psi)}{(1 + \mu)},$$

and investment on human capital is subsidized at a rate $s_h = (1 - \varepsilon)\alpha/\varphi$.

Considering the same parameter values as in Section 6.1, we get again that physical capital income should be subsidized at a rate of about 33%. Now, if the labor income tax is set to zero, consumption should be taxed at a rate of 1.85%, and investment in education should be subsidized at a rate of 10%. If the consumption tax is set to zero, labor income should be taxed at a rate of 1.82%, and investment in education should be subsidized at a rate of 9.81%. If physical and human capital incomes are taxed at the same rate, they should be subsidized at a rate of 33.33%, consumption should be subsidized at a rate of 32.10% and investment on education, at a rate of 6.67%.

Gómez (2004) finds that, when labor supply is inelastic, the competitive equilibrium is efficient in the Lucas (1988) model with sector-specific externalities associated to human capital in the goods sector. Setting $\mu = \psi = 0$, $\alpha = \varphi$, and $\varepsilon = 1$, Propositions 3 and 4 show that Gómez’s optimality result does not hold when labor supply is elastic, regardless of whether leisure is specified as home production, quality time or raw time. In any case, such externalities provoke a market failure that needs the government intervention to correct it. Gómez argues that these sector-specific externalities are non-distorting because they change in the same proportion the returns to and the opportunity cost of investment on human capital, and so, the time allocation decision of the representative agent is the same as that of the central planner. However, this argument does not hold when labor supply is elastic because now there is one additional margin of choice: the allocation of time between labor-education and leisure.

Recalling Propositions 3 and 4, it can be observed that, in the absence of production externalities (i.e., $\alpha = \varphi$, $\beta = 1 - \alpha$, and $\varepsilon = 1$), the competitive equilibrium is efficient both at the steady state and off the steady state if and only if the elasticity of average consumption in utility, μ , and the elasticity of average leisure, ψ , coincide. Intuitively, the efficient and the competitive MRS between consumption and leisure would also be equal and, therefore, the presence of utility externalities does not cause any distortion. The following Proposition summarizes this result.

Proposition 6. *In the absence of production externalities, the competitive equilibrium is efficient both at the steady state and off the steady state when leisure is “home production”, “quality time” or “raw time” if and only if $\mu = \psi$.*

6.4. “No leisure”

The NL economy is obtained when $\eta = 0$. Now, Eqs. (9b), (9d), (18b) and (18d) must be dropped. In this case, taxes on labor income and consumption can be set arbitrarily and, therefore, we can therefore state the following Proposition.

Proposition 7. *The market economy can attain the first-best equilibrium when labor supply is inelastic if physical capital income is taxed at a rate $\tau_k = -(\alpha - \varphi)/\varphi$, and investment on human capital is subsidized at a rate $s_h = (1 - \varepsilon)(1 - \tau_h)$. Consumption and labor income tax rates can be set arbitrarily.*

When labor supply is inelastic, there is additionally one less margin of choice than in the QT and RT models: the allocation of time between labor-education and leisure. Therefore, consumption externalities and sector-specific externalities associated to human capital in the goods sector do not provoke a market failure, and so, do not need to be corrected.⁹ The only distorting externalities are those associated to physical capital in goods production and to human capital in the educational sector. Thus, we can extend the Gómez (2004) optimality result to allow for consumption externalities as well.

Proposition 8. *The competitive equilibrium is optimal in the Lucas model with consumption externalities and sector-specific externalities associated to human capital in the goods sector when labor supply is inelastic.*

7. Conclusions

This paper has analyzed the implications that the choice of the leisure function has on the equilibrium efficiency in a two-sector endogenous growth model with human capital accumulation in the presence of consumption and leisure externalities in utility, sector-specific externalities associated to physical and human capital in goods production, and sector-specific externalities associated to human capital in the educational sector. To address this issue we have considered the three main leisure specifications that have been proposed in the literature: leisure as home production, quality time and raw time, and the case in which labor supply is inelastic. The optimal fiscal policy to correct for the distortions caused by the externalities has been characterized for every specification of the leisure activity.

Several results are robust to the specification of the leisure activity: (i) externalities in the educational sector cause the long-run equilibrium growth rate to be lower than the optimal one; (ii) the distortion caused by sector-specific externalities associated to physical capital in goods production can be corrected by taxing physical capital income; (iii) sector-specific externalities in the educational sector provoke a distortion that can be corrected by subsidizing investment on human capital, and (iv) in the absence of production externalities, the competitive equilibrium is efficient only in the knife-edge case in which the elasticity of average consumption in utility is equal to the elasticity of average leisure.

However, the choice of the leisure function introduces important differences across specifications. In the home production, quality time and “no leisure” models, the long-run equilibrium growth rate coincides with the optimal one irrespective of the presence of sector-specific externalities in goods production and/or utility externalities. However, in the raw time model, such spillovers cause the long-run equilibrium growth rate to diverge from the optimal one. If labor supply is inelastic, the competitive equilibrium is

⁹ The fact that consumption externalities are not necessarily a source of inefficiency has already been shown, e.g., by Fisher and Hof (2000) in the Ramsey exogenous growth model, Liu and Turnovsky (2005), in the Romer (1986) model, and Alonso-Carrera et al. (2006), in the AK endogenous growth model.

efficient in the presence of consumption externalities and/or sector-specific externalities associated to human capital in the goods sector. This result does not hold when labor supply is endogenous. If leisure is raw time or quality time, there is a degree of arbitrariness in the optimal policy, as the consumption or the labor income tax can be set arbitrarily. Alternatively, this degree of arbitrariness could be used to tax income from physical and human capital at the same rate. This possibility does not exist in the home production model. In this case, utility externalities must be corrected by taxing consumption, and sector-specific externalities associated to human capital in the goods sector must be corrected by taxing labor income.

Appendix

Derivation of the competitive equilibrium

The first-order conditions (9a)–(9f) for an interior optimum can be rewritten as

$$c^{-\sigma} \bar{c}^{\mu(1-\sigma)} L^{(1-\sigma)\eta} \bar{L}^{(1-\sigma)\eta\psi} = (1 + \tau_c)\lambda, \quad (\text{A.1a})$$

$$\frac{\omega\eta c^{1-\sigma} \bar{c}^{\mu(1-\sigma)} L^{(1-\sigma)\eta} \bar{L}^{(1-\sigma)\eta\psi}}{1-v} = \lambda(1 - \tau_k)rk, \quad (\text{A.1b})$$

$$\theta\varepsilon\delta h[(1-u-l)h]^{\varepsilon-1} [(1-u-l)\bar{h}]^{1-\varepsilon} = \lambda(1 - \tau_h - s_h)wh, \quad (\text{A.1c})$$

$$\frac{\xi\eta c^{1-\sigma} \bar{c}^{\mu(1-\sigma)} L^{(1-\sigma)\eta} \bar{L}^{(1-\sigma)\eta\psi}}{l} = \lambda s_h wh + \theta\varepsilon\delta h[(1-u-l)h]^{\varepsilon-1} [(1-u-l)\bar{h}]^{1-\varepsilon}, \quad (\text{A.1d})$$

$$\dot{\lambda} = (\rho - (1 - \tau_k)rv)\lambda - \frac{\omega\eta c^{1-\sigma} \bar{c}^{\mu(1-\sigma)} L^{(1-\sigma)\eta} \bar{L}^{(1-\sigma)\eta\psi}}{k}, \quad (\text{A.1e})$$

$$\begin{aligned} \dot{\theta} = & (\rho - \varepsilon\delta[(1-u-l)h]^{\varepsilon-1} [(1-u-l)\bar{h}]^{1-\varepsilon} (1-u-l))\theta \\ & - \lambda[(1 - \tau_h)wu + s_h w(1-u-l)] - \frac{\phi\xi\eta c^{1-\sigma} \bar{c}^{\mu(1-\sigma)} L^{(1-\sigma)\eta} \bar{L}^{(1-\sigma)\eta\psi}}{h}. \end{aligned} \quad (\text{A.1f})$$

In what follows, the symmetry conditions $\bar{v}k = vk$, $\bar{u}h = uh$, $\bar{c} = c$, $\bar{L} = L$, and $\frac{\bar{h}}{(1-u-l)h} = (1-u-l)h$ will be imposed, and the expressions 7a, 7b and 7c for r , w and π , respectively, will be taken into account. Eqs. (10) and (11) can be rewritten as

$$\gamma_\lambda = \rho - (1 - \tau_k)r, \quad (\text{A.2})$$

$$\gamma_\theta = \rho - (1 - \tau_h)\varepsilon\delta(1-l + \phi l)/(1 - \tau_h - s_h). \quad (\text{A.3})$$

Using (A.1a) and (A.1b), we obtain (13b). From (A.1b), (A.1c) and (A.1d), using (13b), we get (13a). Log-differentiating (A.1a), using (A.2), we find the Euler equation

$$\begin{aligned} & -[\sigma + \mu(\sigma - 1)]\gamma_c + \eta(1 - \sigma)(1 + \psi)\{\omega[\gamma_k - v\gamma_v/(1-v)] + \xi(\gamma_l + \phi\gamma_h)\} \\ & = \rho - (1 - \tau_k)r. \end{aligned} \quad (\text{A.4})$$

Denoting $\zeta = \lambda/\theta$, (A.2) and (A.3) imply that

$$\gamma_\zeta = \gamma_\lambda - \gamma_\theta = (1 - \tau_h)\varepsilon\delta(1-l + \phi l)/(1 - \tau_h - s_h) - (1 - \tau_k)r. \quad (\text{A.5})$$

Log-differentiating (7a) and (A.1c) we obtain, respectively,

$$\gamma_r = (1 - \alpha)(\gamma_u + \gamma_h - \gamma_v - \gamma_k), \quad (\text{A.6})$$

$$\gamma_\zeta = \alpha(\gamma_u + \gamma_h - \gamma_v - \gamma_k). \quad (\text{A.7})$$

Using the government's budget constraint (8) and (7), Eq. (5) can be expressed as

$$\gamma_k = y/k - c/k = rv/\varphi - q. \quad (\text{A.8})$$

The growth rate of human capital (4) can be rewritten as

$$\gamma_h = \delta(1 - u - l). \quad (\text{A.9})$$

Log-differentiating $q = c/k$, (13b) and (13a) with respect to time yields, respectively,

$$\gamma_q = \gamma_c - \gamma_k. \quad (\text{A.10})$$

$$\gamma_v = \frac{\eta\omega(1 + \tau_c)q(\gamma_r - \gamma_q)}{(1 - \tau_k)r - \eta\omega(1 + \tau_c)q} = \frac{(1 - v)}{v}(\gamma_r - \gamma_q), \quad (\text{A.11})$$

$$\gamma_u = \gamma_l + \frac{(1 - \tau_k)r(\gamma_r - \gamma_q)}{(1 - \tau_k)r - \eta\omega(1 + \tau_c)q} = \gamma_l + \frac{1}{v}(\gamma_r - \gamma_q). \quad (\text{A.12})$$

Using (A.6) and (A.7) we get that $\gamma_r = (1 - \alpha)\gamma_\zeta/\alpha$ which, using (A.5), yields (12a). Eq. (12c) is obtained from (A.6) after substituting γ_k for (A.8), γ_h for (A.9) and γ_v for (A.11). Substituting γ_k , γ_h , γ_v , and γ_l for (A.8), (A.9), (A.11) and (A.12), respectively, into (A.4), and then substituting γ_u for (12c) and $\gamma_q = \gamma_c - \gamma_k$ into the resulting expression, we can get an expression for γ_c as a function of γ_r and l . Substituting γ_c for the ensuing expression and γ_k for (A.8) into (A.10), we obtain (12b).

At the steady state, physical and human capital and consumption grow at the same constant rate, $\hat{\gamma}_k = \hat{\gamma}_h = \hat{\gamma}_c$, and leisure time and the fraction of physical capital devoted to leisure are constant, $\hat{\gamma}_l = \hat{\gamma}_v = 0$. From (A.4), (A.2) and (A.3), taking into account that $\hat{\gamma}_\lambda = \hat{\gamma}_\theta$ at the steady state, we get the long-run growth rate (14).

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