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Endogenous Growth, Habit Formation and Convergence Speed*

Manuel A. Gómez

Abstract

This paper analyzes the effect on the economy dynamics of alternative formulations of habit persistence in an endogenous growth model. The focus is on the impact on the convergence speed, which is the key determinant of the local dynamics. In contrast with previous numerical results, we show that the external-habits economy may converge at a higher, lower or equal rate than the internal-habits economy depending on the specification of the utility function. We also prove that the higher the strength of habits and the lower the speed of adjustment of habits to current consumption, the lower the convergence speed.

KEYWORDS: habit formation, convergence speed, external habits, internal habits, endogenous growth

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1 Introduction

Habit persistence in consumption preferences has been introduced into dynamic equilibrium models in order to address a broad range of empirical facts that are difficult to explain under standard time-separable preferences. A partial list includes the equity premium puzzle (Abel, 1990, Constantinides, 1990, Campbell and Cochrane, 1999), time-varying expected returns (Chan and Kogan, 2002, Møller, 2009), the term structure of interest rates (Wachter, 2006, Buraschi and Jiltsov, 2007), the foreign exchange risk premium (Verdelhan, forthcoming), the relationship between savings and growth (Carroll *et al.*, 2000), some stylized facts of business cycles (Lettau and Uhlig, 2000, Boldrin *et al.*, 2001), current account dynamics (Gruber, 2004), the effects of monetary policy (Fuhrer, 2000, Walsh, 2005), and the determination of precautionary savings and the shape of the wealth distribution (Diaz *et al.*, 2003). Habit formation has also become an important feature of many dynamic stochastic general equilibrium models (e.g., Smets and Wouters, 2003, Christiano *et al.*, 2005).

In models with habit formation, individual's utility depends not only on her level of current consumption but also on how it compares to a reference level — the habits stock—. The literature distinguishes between internal habits (IH), when individual's habits depend on her own past consumption, and external habits (EH), when habits are formed from average past consumption in the economy. Furthermore, two main formulations have been proposed according to how the habits stock enters into individual's utility: subtractive habits (SH) and multiplicative habits (MH). While recent empirical evidence seems to support the habit formation hypothesis (see, e.g., Ravina, 2007, Chen and Ludvigson, 2009, Grishchenko, 2009, Korniotis, forthcoming), whether habits are formed in an internal, external or hybrid internal-external form appears to be an open question.[1] Perhaps as a result both the IH and EH specifications have been widely used in the literature.[2] An interesting

[1] There are few empirical studies estimating models that include both internal and external habits. Grishchenko (2009) finds evidence on internal habits using quarterly data, whereas Korniotis (forthcoming) reports instead evidence for external habits using annual data. While Dynan (2000), using annual data, finds no evidence on internal habits, studies that use quarterly national data report supporting evidence for internal habit formation (e.g., Ferson and Constantinides, 1991, Heaton, 1995, Grishchenko, 2009, Chen and Ludvigson, 2009). Thus, Korniotis (forthcoming) concludes that the impact of internal habits must be short-lived because it manifests itself at the quarterly frequency and it disappears at the annual frequency.

[2] Examples of internal MH models are Fuhrer (2000) and Carroll *et al.* (2000); internal SH models are Constantinides (1990) and Boldrin *et al.* (2001); external MH models are Abel (1990) and Chan and Kogan (2002), and external SH models are Campbell and Cochrane (1999) and Ljungqvist and Uhlig (2000).

issue is, therefore, to analyze the consequences of modelling habits as internal or external.

This paper examines the effect on the economy dynamics of alternative formulations of habit-forming preferences and, specially, the implications of assuming that habits are internal or externally formed. Since the focus is on the consequences of introducing habits into utility, we follow Carroll *et al.* (1997) and consider that technology is AK. This simplification allows to isolate the effect of habits on the economy dynamics because the AK model with standard time-separable utility does not exhibit transition dynamics (Rebelo, 1991). Thus, the dynamics of the economy is driven exclusively by preferences; i.e., by the presence of habits. In contrast with the simplicity of the production side of the economy, we consider a fairly general structure of preferences. Instead of assuming a specific functional form at the outset, utility depends on current consumption and the habits stock in a generic way that encompasses as particular cases the commonly-used SH and MH formulations. Furthermore, we assume a fairly general specification of the habit formation process that nests the cases of internal, external or hybrid internal-external habits. The AK model with habit persistence converges to the same steady state irrespective of whether habits are formed in an internal or external form. Hence, the different behaviour in the EH and IH models must arise along the transition path. Since the (local) dynamics are determined by the (asymptotic) speed of convergence to the steady state, the focus of the paper is on the impact of modeling habit-forming preferences on the convergence speed.

Our main result (Theorem 4) characterizes the effect on the convergence speed of switching habits from being internal to externally formed. In particular, it entails that the speed of convergence in the EH model may be higher, lower or equal than the corresponding one in the IH model depending on the specification of the utility function. When applied to the most commonly-used formulations, our results entail that the SH model features the same speed of convergence irrespective of whether habits are internal or externally formed, whereas in the MH model the speed of convergence in the EH economy is higher than the corresponding one in the IH economy. Furthermore, we provide an example of utility function for which the IH economy converges to the steady state at a faster rate than the EH economy does.

Recently, Carroll *et al.* (1997), Alvarez-Cuadrado *et al.* (2004) and Turnovsky and Monteiro (2007) have compared, by means of numerical simulations, the transition dynamics in growth models with internal and external habits that enter utility in a multiplicative form. Carroll *et al.* (1997) focus on the consequences of introducing habits into utility, and so, they consider the simplest AK model.[3] Alvarez-

[3]Ferraguto and Pagano (2003) also analyze the dynamics of the AK model under a generic utility function. However, they only consider the case in which habits are internally formed. Furthermore,

Cuadrado *et al.* (2004) consider the neoclassical model in order to examine the effect of assuming that returns to capital are diminishing rather than constant. Finally, Turnovsky and Monteiro (2007) analyze a non-scale growth model in which, additionally, they introduce the labor-leisure margin of choice. A common (numerical) finding of these works is that under multiplicative habits the speed of convergence to the steady state in the EH model is higher than the corresponding one in the IH model, a result that has been analytically proved by Gómez (2008) for the AK model. This result is justified intuitively because an individual with external habits ignores the indirect effect that increasing current consumption has in future utility through a higher stock of habits, whereas an individual with internal habits does take it into account. Since this justification seems applicable with generality, and not only to the multiplicative-habits model, it appears that the convergence speed in the EH model should be higher than the corresponding one in the IH model, irrespective of how habits enter utility. Thus, the fact that the opposite result can occur; i.e., that the IH economy may converge to its steady state at a higher rate than the EH does, casts doubts on the previous intuitive explanation and calls for a reconsideration of the determinants of the different dynamics in both economies.

We show that the relationship between the (asymptotic) convergence speeds in the EH and IH economies can be explained by the dynamic behaviour of the marginal rate of substitution (MRS) between consumption and habits around the steady state. Intuitively, a decreasing MRS as the economy evolves means that the negative effect of habits on utility becomes relatively smaller, because a lower increase in consumption would be needed to compensate a given increase in the habits stock in order to keep utility constant. An individual with internal habits, who takes into account the effect of her current consumption on the habits stock, would choose a transition path with lower consumption in the present —to keep habits relatively low— and higher consumption in the future —when the higher habits stock induced by the higher consumption is less harmful— than that chosen by an individual with external habits, who treats the habits stock as given. Thus, an individual with internal habits would be more willing to shift consumption from the present to the future than an individual with external habits. If the stable growth path requires capital accumulation —because the initial capital-habits ratio is low relative to its stationary value—, the convergence speed in the IH economy would then be higher than the corresponding one in the EH economy. The opposite happens when the marginal rate of substitution between consumption and habits is increasing, so that the negative effect of habits on utility becomes greater as the economy evolves.

they assume that the utility function is concave, which excludes the commonly-used MH model (see, e.g., Alonso-Carrera *et al.*, 2005, Hiraguchi, 2008).

This intuition is confirmed by the comparison of the marginal rates of intertemporal substitution in consumption in the EH and IH economies.

Besides the distinction between internal and external habits or between multiplicative and subtractive habits, other important features of the habit-formation process are the speed of adjustment of habits to current consumption and the strength of habits in utility. Thus, we also study analytically their impact on the speed of convergence. Specifically, we prove that the higher the speed of adjustment of the habits stock to current consumption and the lower the weight of habits in utility, the higher the speed of convergence both under subtractive or multiplicative habits, irrespective of whether habits are internal of externally formed. This provides theoretical support to previous numerical findings reported by Carroll *et al.* (1997, 2000).

A large literature studies the effect on the convergence speed of a variety of issues as, e.g., capital utilization (Chatterjee, 2005), income tax progressivity (Yamarik, 2001, Pintus, 2008), income inequality (Zhang, 2005), alternative financing of government expenditure (Gokan, 2003), international labor mobility (Rappaport, 2005) or the intertemporal and the intratemporal elasticities of substitution (Turnovsky, 2002, 2008). Another strand of the literature studies the economic consequences of alternative specifications of habit formation. Bunzel (2006) shows that the qualitative dynamic properties of SH and MH models are similar in a two-period pure exchange overlapping generations model. However, Bossi and Gomis-Porqueras (2009) argue that her results are driven by the specific choice of the utility function, and show that both models can differ in terms of concavity, homotheticity, uniqueness of equilibria and local dynamics. Wendner (2003) finds that SH and MH models may have different implications on household savings behaviour. This paper also relates and contributes to both of these strands of the literature by analyzing the theoretical implications of alternative specifications of habit-forming preferences on the speed of convergence.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the effect of specifying habits as internal or externally formed on the speed of convergence. Section 4 studies analytically the effects of the strength and the adjustment speed of habits. Section 5 concludes.

2 The model

We study a closed economy populated by N identical infinitely-lived agents that grows at the exogenous rate $\dot{N}/N = n$. The intertemporal utility derived by the agent depends both on her current consumption, C, and a reference consumption level or habits stock, H, according to

$$U(C)=\int_0^\infty u(C(t),H(t))\mathrm{e}^{-\beta t}dt, \quad \beta>0, \tag{1}$$

where u is the instantaneous utility function, and β is the rate of time preference.

The habits stock is formed as an exponentially declining average of past consumption,

$$H(t)=\rho\int_{-\infty}^{t}\mathrm{e}^{\rho(s-t)}m(C(s),\bar{C}(s))ds, \quad \rho>0, \tag{2}$$

where $\bar{C}$ denotes the economy-wide average level of consumption. Differentiating (2) with respect to time, the rate of adjustment of the habits stock is[4]

$$\dot{H}=\rho\,[m(C,\bar{C})-H]. \tag{3}$$

The parameter ρ, which governs the speed with which the habits stock adjusts to current consumption, determines the relative weight of consumption at different dates: the larger is ρ, the more important is consumption in the recent past. As $\rho\to\infty$, $H\to m(C,\bar{C})$, so that agent's utility depends only on current consumption.

In the IH model, the reference consumption stock depends only on individual's own past consumption, so that $m(C,\bar{C})=C$. In the EH model, habits arise from average past consumption in the economy, so that $m(C,\bar{C})=\bar{C}$. In the hybrid internal-external case, the habits stock is determined by both her own consumption and the economy-wide average level of consumption, which are combined by means of a continuously differentiable homogeneous mean. Thus, in this case m is a continuously differentiable function such that $m(C,C)=C$ for all $C>0$, strictly increasing in its components, and (positively) homogeneous of degree one.[5] Differentiating $m(C,C)=C$, and using its linear homogeneity, we get that $m_C(C,C)=\phi$ and $m_{\bar{C}}(C,C)=1-\phi$ for all $C>0$, with $0\le\phi\le1$. Hence, the case $\phi=1$ corresponds to the IH model, and the case $\phi=0$, to the EH model.

We assume that the instantaneous utility function u is two times continuously differentiable and satisfies that $u_C>0$, $u_{CC}<0$, $u_H\neq0$, and u_C and u_H are homogeneous of degree $-\nu<0$. This last assumption guarantees the existence of balanced growth paths for the EH and IH economies (Ferraguto and Pagano, 2003, Alonso-Carrera *et al.*, 2006). Furthermore, we assume that

$$u_C(C,\rho C/(\rho+g))+\frac{\rho}{\rho+g}u_H(C,\rho C/(\rho+g))>0, \tag{4}$$

[4] The time argument is omitted whenever there is no risk of confusion.

[5] This formulation comprises, as particular cases, the weighted geometric mean, $m(C,\bar{C})=C^{\phi}\bar{C}^{1-\phi}$, proposed by Abel (1990) and considered, among others, by Alvarez-Cuadrado *et al.* (2004), and the weighted arithmetic mean, $m(C,\bar{C})=\phi C+(1-\phi)\bar{C}$, considered, e.g., by Ravina (2007).

for all $C > 0$ and $g > 0$, which guarantees that a uniformly maintained increase in consumption along a balanced growth path will increase utility.[6]

Gross output per capita Y is determined by

$$Y = BK, \quad B > 0,$$

where K is the capital stock per capita. The single good of the economy can be either consumed or invested. The agent's budget constraint is, then,

$$\dot{K} = BK - C - (n+\delta)K = AK - C, \tag{5}$$

where δ is the rate of depreciation of capital, and we define $A = B - n - \delta$.

2.1 Solution to the agent's problem

The agent chooses C, K and H to maximize the lifetime utility (1) subject to her budget constraint (5) and the constraint on the habits stock accumulation (3), taking as given the path of economy-wide average consumption, $\bar{C}$, and the initial conditions on capital, $K(0) > 0$, and habits stock, $H(0) > 0$.

Let J be the current value Hamiltonian of the agent's maximization problem,

$$J = u(C,H) + \lambda(AK - C) + \mu\rho[m(C,\bar{C}) - H],$$

where λ and μ are the shadow values of capital and habits, respectively. The first-order conditions for an interior optimum are[7]

$$u_C(C,H) + \rho\,\mu\,m_C(C,\bar{C}) = \lambda, \tag{6}$$

$$A = \beta - \dot{\lambda}/\lambda, \tag{7}$$

$$u_H(C,H)/\mu - \rho = \beta - \dot{\mu}/\mu, \tag{8}$$

[6]Note that Eq. (14) below shows that $H = \rho C/(\rho+g)$ along a BGP, where g is the long-run growth rate. Ferraguto and Pagano (2003) assume instead that $u_C + \rho u_H/(\rho+g) > 0$ for all g>0. However, this stronger condition is not needed in the subsequent analysis, and it is not satisfied, e.g., by the commonly-used multiplicative specification. Concavity of the utility function u is not included in our set of assumptions as, e.g., in Ferraguto and Pagano (2003) and Alonso-Carrera *et al.* (2006), because u is not concave with respect to C and H in the MH specification.

[7]We assume that the first-order conditions, along with the initial and transversality conditions, characterize the interior optimal solution of the agent's problem. In the EH model, where H is exogenously given from the agent's point of view, the first-order conditions are also sufficient if the appropriate Inada conditions hold because u is concave in C. In the IH model, this is guaranteed if the utility function u is concave in (C,H) and the appropriate Inada conditions hold as, for instance, in the SH models considered by Ferraguto and Pagano (2003) and Alonso-Carrera *et al.* (2006). However, non-concavity of the utility function entails that the optimal solution, even if it exists, may not be an interior path satisfying the first order conditions. Alonso-Carrera *et al.* (2005) and Hiraguchi (2008) deal with this issue in MH models and provide conditions that guarantee that the first-order conditions characterize the interior maximum.

and the transversality condition

$$\lim_{t\to\infty} e^{-\beta t}\lambda K = \lim_{t\to\infty} e^{-\beta t}\mu H = 0. \tag{9}$$

Eq. (6) equates the marginal utility of consumption, adjusted by its effect on the future stock of habits, to the shadow price of capital. Eq. (7) equates the rate of return on capital to the rate of return on consumption. From (8) and (9), we get

$$\mu(t) = \int_t^\infty e^{-(\beta+\rho)(s-t)} u_H(C(s),H(s))ds. \tag{10}$$

This condition states that the shadow value of the habits stock is determined as the present discounted value of the stream of extra utils that would be gained (or lost) by a marginal unit of habits, which depreciates at the rate ρ. In particular, note that if $u_H < 0$, which agrees with the notion that H represents the stock of habits, Eq. (10) entails that the shadow value of the habits stock is negative, $\mu < 0$.

Let us define $c \equiv C/H$ as the ratio of consumption to habits, $h \equiv H/K$ as the ratio of habits to capital, and $q \equiv -\mu/\lambda$ as the relative shadow cost of habits. Appendix A derives the system that drives the dynamics of the economy in terms of the variables c, h and q, which are constant along a balanced growth path (BGP),

$$\dot{c} = \frac{u_C(c,1)}{u_{CC}(c,1)}\left[\beta - A + v\rho(c-1) + \frac{\rho\phi\dot{q}}{1+\rho\phi q}\right], \tag{11}$$

$$\dot{h} = h[\rho(c-1) - A + ch], \tag{12}$$

$$\dot{q} = (A+\rho)q + (1+\rho\phi q)\frac{u_H(c,1)}{u_C(c,1)}. \tag{13}$$

Examination of the dynamical system (11)–(13) shows that the dynamics of the economy is invariant to the specific homogeneous mean m chosen, aside from the implied value of the parameter ϕ. In particular, this entails that it makes no difference to the dynamics whether own and economy-wide average consumption are combined by means of a weighted geometric mean $m(C,\bar{C}) = C^\phi \bar{C}^{1-\phi}$ (e.g., Abel, 1990), or a weighted arithmetic mean $m(C,\bar{C}) = \phi C + (1-\phi)\bar{C}$ (e.g. Ravina, 2007).

2.2 Balanced growth path

Now, we focus on the BGP (or steady state) at which consumption, capital and habits grow at the same rate. A hat over a variable denotes its steady state. Appendix B proves the following proposition.

Proposition 1 *The economy has a unique interior steady state with positive long-run growth*

$$\hat{c} = 1 + \frac{A-\beta}{v\rho} = \frac{\rho+\hat{g}}{\rho}, \tag{14}$$

$$\hat{h} = \frac{A-\rho(\hat{c}-1)}{\hat{c}} = \frac{A-\hat{g}}{\hat{c}}, \tag{15}$$

$$\hat{q} = -\frac{u_H(\hat{c},1)}{(A+\rho)u_C(\hat{c},1)+\rho\phi u_H(\hat{c},1)}, \tag{16}$$

where the long-run growth rate of consumption, capital and output per capita is

$$\hat{g} = \frac{A-\beta}{v}, \tag{17}$$

if and only if

$$A > \beta > (1-v)A. \tag{18}$$

Equations (14), (15) and (17) show that the steady state of c, h and g do not depend on ϕ, and so, are the same whether habits are formed in an external, internal or hybrid internal-external form. Furthermore, the unique feature of the instantaneous utility function that affects (14), (15) and (17) is the degree of homogeneity of u_C and u_H, $-v$. Hence, $\hat{c}$, $\hat{h}$ and $\hat{g}$ are also the same irrespective of the specification of the utility function —in particular, multiplicative or subtractive— provided that the degrees of homogeneity of their respective partial derivatives coincide.

Henceforth, we assume that condition (18) is fulfilled. To investigate the (local) stability of the steady state, we linearize the dynamic system (11)–(13) around the steady state (14)–(16). The linearized system is

$$\begin{pmatrix} \dot{c} \\ \dot{h} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} b_{11} & 0 & b_{13} \\ \hat{h}(\hat{h}+\rho) & \hat{c}\hat{h} & 0 \\ b_{31} & 0 & b_{33} \end{pmatrix} \times \begin{pmatrix} c-\hat{c} \\ h-\hat{h} \\ q-\hat{q} \end{pmatrix} = B \times \begin{pmatrix} c-\hat{c} \\ h-\hat{h} \\ q-\hat{q} \end{pmatrix}, \tag{19}$$

where

$$b_{11} = \frac{u_C(\hat{c},1)}{u_{CC}(\hat{c},1)}\left[v\rho + \left(\frac{\rho\phi}{1+\rho\phi\hat{q}}\right)b_{31}\right], \quad b_{13} = \frac{u_C(\hat{c},1)}{u_{CC}(\hat{c},1)}\left(\frac{\rho\phi}{1+\rho\phi\hat{q}}\right)b_{33},$$

$$b_{31} = \frac{(1+\rho\phi\hat{q})}{vu_C(\hat{c},1)^2}\hat{\Psi}, \qquad b_{33} = A+\rho+\rho\phi\frac{u_H(\hat{c},1)}{u_C(\hat{c},1)}.$$

Here, $\hat{\Psi}$ is defined as

$$\begin{aligned}\hat{\Psi} &= -v[u_{CC}(\hat{c},1)u_H(\hat{c},1) - u_{CH}(\hat{c},1)u_C(\hat{c},1)] \\ &= u_{CC}(\hat{c},1)u_{HH}(\hat{c},1) - u_{CH}(\hat{c},1)^2,\end{aligned} \tag{20}$$

where the last equality follows from the homogeneity of degree $-v$ of u_C and u_H. Therefore $\hat{\Psi}$ has the same sign as the determinant of the Hessian matrix of the instantaneous utility function, u, evaluated at the BGP.

The following proposition, which is proved in Appendix B, shows that the steady state is locally saddle-path stable irrespective of the specification of the utility function and the habit formation process.[8]

Proposition 2 *The steady state of the economy is locally saddle-path stable.*

2.3 Local dynamics

The linearized dynamics of the economy can be readily derived in an explicit manner. In what follows, we shall denote ξ as the stable root of the linearized dynamics, so that $\xi < 0$. In the present case, where the stable transitional path is one dimensional, the asymptotic convergence speed is the absolute value of the stable root of the matrix B in (19) —which is also the stable root of the matrix B_{13} defined in (B.2)— (e.g., Ortigueira and Santos, 1997, and Eicher and Turnovsky, 1999). Hence, the asymptotic speed of convergence is $|\xi| = -\xi$. An eigenvector associated to ξ is $(1, a_2, a_3)'$, with

$$a_2 = -\frac{\hat{h}(\hat{h}+\rho)}{\hat{c}\hat{h}-\xi} < 0,$$
$$a_3 = -\frac{A+\rho}{v\, b_{33}(b_{33}-\xi)}\hat{\Psi},$$

where, using that $\xi < 0$ and $b_{33} > 0$ by (B.1), we have that $\text{sign}(a_3) = -\text{sign}(\hat{\Psi})$. Hence, the linearized dynamics of the economy around the steady state is given by

$$h(t) - \hat{h} = e^{\xi t}\,[h(0) - \hat{h}], \tag{21}$$
$$c(t) - \hat{c} = \frac{1}{a_2}\,[h(t) - \hat{h}], \tag{22}$$
$$q(t) - \hat{q} = \frac{a_3}{a_2}\,[h(t) - \hat{h}]. \tag{23}$$

We now examine the local dynamics of the economy when the initial value of the habits-capital ratio is greater than its steady-state value, $h(0) > \hat{h}$, so that the

[8]Ferraguto and Pagano (2003) analyze the dynamics of the AK growth model with multiplicative internal habits. They argue that saddle-path stability of the steady state requires an additional condition, which is given by their Eq. (16). However, proceeding in a similar way as when deriving Eq. (B.1) in Appendix B, it can be shown that their condition (16) is automatically satisfied given their assumptions on the instantaneous utility function.

stock of capital is comparatively low. The case $h(0) < \hat{h}$ can be analyzed in a similar way.

If $h(0) > \hat{h}$, Eqs. (21) and (22) show that (locally) as the economy evolves, h decreases and c increases monotonically toward their respective steady-state values. Intuitively, as Carroll *et al.* (1997, p. 353) argue, the agent maintains a low level of the ratio of consumption to habits relative to its steady-state value because this allows decreasing the ratio h by fostering capital accumulation and by slowing the growth of the habits stock. As h approaches its stationary value, consumption needs to be depressed less, so c grows toward its steady state. The relative shadow cost of habits, q, will be increasing, decreasing or constant depending on whether $\hat{\Psi} < 0$, $\hat{\Psi} > 0$ or $\hat{\Psi} = 0$. In the following section, we will show that this is related to habits becoming (relatively) more, less or equal harmful for utility as the economy evolves.

Let us now compare the local dynamics of two economies "a" and "b" that converge to the same steady state, such that the convergence speed in the economy a is higher than the corresponding one in economy b; i.e., $\xi_a < \xi_b$. Comparing the evolution of the ratio of consumption to habits in both economies, using (22), we have that

$$c_a(t) - c_b(t) = \left[-\frac{\hat{c}\hat{h} - \xi_a}{\hat{h}(\hat{h}+\rho)} e^{\xi_a t} + \frac{\hat{c}\hat{h} - \xi_b}{\hat{h}(\hat{h}+\rho)} e^{\xi_b t} \right] [h(0) - \hat{h}],$$

so that at the initial time

$$c_a(0) - c_b(0) = \frac{h(0) - \hat{h}}{\hat{h}(\hat{h}+\rho)} (\xi_a - \xi_b),$$

and, therefore,

$$\text{sign}(c_a(0) - c_b(0)) = \text{sign}(C_a(0) - C_b(0)) = \text{sign}(\xi_a - \xi_b) < 0,$$

if $h(0) > \hat{h}$. Hence, the economy that features a higher speed of convergence starts off with a lower ratio of consumption to habits, c, which eventually catches up and surpasses the corresponding one to the economy with the lower convergence speed, and thereafter both converge to their common stationary value.[9] Thus, the comparative (local) dynamic behavior of two economies that converge to the same stationary solution is determined by the relationship between their respective (asymptotic) convergence speeds. This is what happens, in particular, when we compare the transition dynamics of the IH and EH economies.

[9] Furthermore, using (5) and (A.1), we can easily get that $\text{sign}(g_a^H(0) - g_b^H(0)) = -\text{sign}(g_a^K(0) - g_b^K(0)) = \text{sign}(c_a(0) - c_b(0)) < 0$, where g^K and g^H denote the growth rates of capital and the habits stock, respectively. Hence, if $h(0) > \hat{h}$, the economy that features a higher speed of convergence also starts off with a lower level of consumption, C, a lower growth rate of habits stock, g^H, and a higher growth rate of capital (and income), g^K.

3 Internal versus external habits

In the previous section, we have shown that the convergence speed is crucial to compare the (local) dynamics of the IH and EH economies. Thus, this section studies analytically the effect on the (asymptotic) convergence speed of modeling habits as internal, external or hybrid. As in the previous section, we shall denote $\xi < 0$ as the stable root of the linearized dynamics, so that the asymptotic speed of convergence is $|\xi| = -\xi$.

3.1 Analytical results

Our first result, which is proved in Appendix B, states explicitly the convergence speed in the EH model.

Proposition 3 *The (asymptotic) convergence speed in the EH model is*

$$|\xi_{\mathrm{EH}}| = \left| \frac{v\rho u_C(\hat{c},1)}{u_{CC}(\hat{c},1)} \right|. \tag{24}$$

If $0 < \phi \leq 1$, an explicit analytical expression of ξ can also be derived. However, given its complexity it would not be of much help to perform the subsequent analysis.

The effect of the weight of own consumption in habits, ϕ, on the convergence speed is given by the following result, which is proved in Appendix B.

Theorem 4 *The (asymptotic) convergence speed satisfies that*

$$\operatorname{sign}(d\xi/d\phi) = -\operatorname{sign}(\hat{\Psi}). \tag{25}$$

As an immediate consequence, we can characterize the relationship between the speeds of convergence in the EH and IH models.

Corollary 5 *The (asymptotic) convergence speeds in the IH and EH models satisfy that*

$$\operatorname{sign}(\xi_{\mathrm{IH}} - \xi_{\mathrm{EH}}) = -\operatorname{sign}(\hat{\Psi}). \tag{26}$$

Corollary 5 entails that the following cases may arise:

- If $\hat{\Psi} > 0$, the asymptotic convergence speed of the EH economy is lower than the corresponding one in the IH economy, $|\xi_{\mathrm{EH}}| < |\xi_{\mathrm{IH}}|$,
- If $\hat{\Psi} < 0$, the asymptotic convergence speed of the EH economy is higher than the corresponding one in the IH economy, $|\xi_{\mathrm{EH}}| > |\xi_{\mathrm{IH}}|$,

- If $\hat{\Psi} = 0$, the asymptotic speeds of convergence in the EH and IH economies coincide, $|\xi_{\text{EH}}| = |\xi_{\text{IH}}|$.

We now apply the former results to the two main specifications of habits considered in the literature: multiplicative and subtractive habits.

In the multiplicative-habits (MH) model, individuals derive utility from the ratio between current consumption and the habits stock. The instantaneous utility function is given by

$$u(C,H) = w(C/H^{\gamma}), \quad 0 < \gamma < 1, \tag{27}$$

where $w' > 0$, $w'' < 0$, and w' is homogeneous of degree $-\sigma$, with $\sigma \geq 1$,[10] so that the degree of homogeneity of u_C and u_H is related to that of w' through $v = \gamma + \sigma(1-\gamma)$.[11] In this case, it can be easily shown that

$$\hat{\Psi} = \sigma v \gamma w''(\hat{c})\,\hat{c}\,w'(\hat{c}) < 0.$$

Hence, we can state the following corollary, which generalizes the one derived by Gómez (2008) in the AK model with multiplicative habits and CRRA utility function.

Corollary 6 *If habits enter utility multiplicatively according to* (27)*, then* $d\xi/d\phi > 0$*. In particular, the asymptotic convergence speed in the EH economy is higher than the corresponding one in the IH economy,* $|\xi_{\text{EH}}| > |\xi_{\text{IH}}|$*.*

In the subtractive-habits (SH) model, individuals derive utility from the difference between current consumption and the habits stock. The instantaneous utility function is given by

$$u(C,H) = w(C - \gamma H), \quad 0 < \gamma < 1, \tag{28}$$

where $w' > 0$, $w'' < 0$, and w' is homogeneous of degree $-v$. In this case, u is concave in C and H, but its hessian matrix is singular everywhere, so that $\hat{\Psi} = 0$. From Proposition 3 and Theorem 4, we can easily observe that the (asymptotic) convergence speed does not depend on the specification of the habit formation process and, therefore, it is the same in the models with internal, external or hybrid

[10] If $0 < \phi < 1$, since u is not concave in (C,H), the first-order conditions may fail to characterize the maximum. In a similar discrete-time model with 'outer' CRRA utility, $u(C,H) = [(C/H^{\gamma})^{1-\sigma} - 1]/(1-\sigma)$, Alonso-Carrera *et al.* (2005) show that the interior solution characterized by the first-order conditions will indeed be a maximum if $\sigma \geq 1$, as empirical evidence suggests.

[11] Note that $u_C(C,H) = H^{-\gamma}w'(C/H^{\gamma})$, and thus $H^{-v}u_C(c,1) = H^{-\gamma-\sigma(1-\gamma)}w'(c)$.

internal-external habits:[12]

$$|\xi| = |\xi_{\mathrm{EH}}| = |\xi_{\mathrm{IH}}| = \left| \frac{u_C(\hat{c},1)}{u_{CC}(\hat{c},1)} \nu\rho \right| \\ = \left| \frac{w'(\hat{c}-\gamma)}{w''(\hat{c}-\gamma)} \nu\rho \right| = \rho(\hat{c}-\gamma) = \hat{g} + \rho(1-\gamma). \tag{29}$$

Hence, we can state the following corollary.

Corollary 7 *If habits enter utility in a subtractive form according to* (28)*, then* $d\xi/d\phi = 0$*, and the convergence speed is* $\rho(\hat{c}-\gamma)$ *for all* ϕ*.*

Corollary 6 shows that the convergence speed in the EH economy is higher than the corresponding one in the IH model under multiplicative habits, while Corollary 7 shows that they coincide if habits are subtractive. In order to present an example in which the convergence speed in the EH economy is lower than that in the IH economy, let us consider the following generic specification of the instantaneous utility function[13]

$$u(C,H) = \frac{1}{1-\varepsilon} \left(\frac{C^{\varphi} - \gamma H^{\varphi}}{1-\gamma} \right)^{\frac{1-\varepsilon}{\varphi}}, \tag{30}$$

with $\varepsilon > 0$, $0 < \gamma < 1$, $\varphi > 0$, and $\varepsilon + \varphi \geq 1$. The utility function (30) is strictly concave in (C,H) if $\varphi > 1$, is concave but not strictly if $\varphi = 1$, and is not concave if $\varphi < 1$. If $\varphi = 1$, Eq. (30) yields the subtractive specification

$$u(C,H) = \frac{1}{1-\varepsilon} \left(\frac{C - \gamma H}{1-\gamma} \right)^{1-\varepsilon}, \tag{31}$$

whereas as $\varphi \to 0$, Eq. (30) converges to the multiplicative specification

$$u(C,H) = \frac{1}{1-\sigma} \left(C H^{-\gamma} \right)^{1-\sigma}, \tag{32}$$

with $\sigma = (\varepsilon - \gamma)/(1-\gamma)$. Recalling that

$$\hat{\Psi} = -\hat{c}^{\varphi-2} (1-\gamma)^{\frac{2(\varepsilon-1)}{\varphi}} (\hat{c}^{\varphi} - \gamma)^{-\frac{2(\varepsilon+\varphi-1)}{\varphi}} \gamma\varepsilon (1-\varphi),$$

the following corollary to Theorem 4 can be readily stated.

[12]The results in Gómez (forthcoming) entail that the dynamics in the AK model with subtractive habits are the same whether habits are formed in an internal or external form and, therefore, their convergence speeds coincide too.

[13]A similar CES function has been proposed by Dupor and Liu (2003), with H representing externalities associated to current consumption. The concavity of (30) with respect to consumption C, requires that $\varepsilon C^{\varphi} - \gamma(1-\varphi) H^{\varphi} \geq 0$ for all feasible values of (C,H); i.e., such that $C^{\varphi} - \gamma H^{\varphi} > 0$. The assumption $\varepsilon + \varphi \geq 1$ ensures strict concavity of u with respect to C.

Corollary 8 *If the utility function is given by* (30)*, then* $\text{sign}(d\xi/d\phi) = \text{sign}(1-\varphi)$ *so that, in particular,* $\text{sign}(\xi_{\text{IH}} - \xi_{\text{EH}}) = \text{sign}(1-\varphi)$.

Hence, the following cases may arise depending on the value of φ:

- if $\varphi > 1$, the IH economy converges faster than the EH economy, $|\xi_{\text{EH}}| < |\xi_{\text{IH}}|$,
- if $\varphi < 1$, the EH economy converges faster than the IH economy, $|\xi_{\text{EH}}| > |\xi_{\text{IH}}|$,
- if $\varphi = 1$, the IH and EH economies converge at the same rate, $|\xi_{\text{EH}}| = |\xi_{\text{IH}}|$.

Table 1: Parameter values

A	β	ε	γ	ρ
0.09	0.05	2	0.5	0.2

In order to illustrate the former results, Figure 1 depicts the convergence speed as the externality parameter ϕ varies from zero to one —i.e., habits switch from being external to internally formed—, for different values of the parameter φ. The dashed line corresponds to the MH model ($\varphi = 0$), the dotted line to the case $\varphi = 4$, and the solid line to the SH model ($\varphi = 1$). Table 1 displays the values of the remaining parameters, which are taken from Carroll *et al.* (1997) and yield a steady-state growth rate of 2 percent. As implied by Corollary 8, the convergence speed is strictly decreasing as a function of ϕ in the MH model, is constant in the SH model, and is strictly increasing if $\varphi = 4$.

Figure 2 illustrates the different comparative (local) dynamics that may arise in the IH and EH economies depending on the relative values of their respective speeds of convergence. To this end, Figure 2 depicts the policy function for the ratio of consumption to habits $c \equiv C/H$ —a control variable— as a function of the ratio of habits to capital $h \equiv H/K$ —a state variable— in both economies when the utility function is given by (30) for $\varphi = 0$ and $\varphi = 4$. The remaining parameter values are displayed in Table 1. The filled dots (resp., empty squares) represent equally spaced points in time as the system evolves toward the steady state in the EH economy (resp., IH economy). Because both economies share the same steady state, the intersection of the two curves represents the steady state of the economy. Panel (a) of Figure 2 depicts the MH case, $\varphi = 0$. If $h(0) > \hat{h}$, the EH economy starts with a lower ratio of consumption to habits, and features a higher speed of convergence

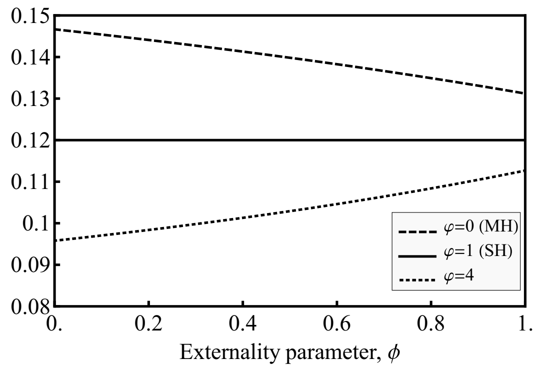
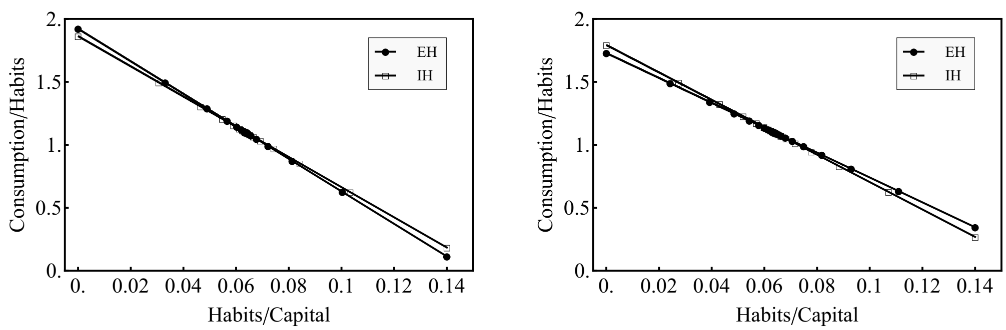


Figure 1: Effect of the externality parameter, ϕ, on the convergence speed.

than the IH economy. Panel (b) of Figure 2 depicts the case $\varphi = 4$. Now, the IH economy starts with a lower ratio of consumption to habits, and converges to its steady state at a higher rate than the EH economy does.



(a) $\varphi = 0$ (MH) $\Rightarrow |\xi_{\text{EH}}| > |\xi_{\text{IH}}|$

(b) $\varphi = 4 \Rightarrow |\xi_{\text{EH}}| < |\xi_{\text{IH}}|$

Figure 2: Optimal policy functions of $c \equiv C/H$ for the linearized dynamics.

3.2 Discussion

Carroll *et al.* (1997) find in their numerical simulations that the EH economy exhibits a higher speed of convergence than the IH economy in the AK model with multiplicative habits. They argue (Carroll *et al.*, 1997, p. 356) that an individual with internal habit formation sees a smaller increase in utility coming from an increase in consumption than that seen by an individual with external habit formation, who ignores the negative indirect effect of a higher habits stock. Hence, the utility function is effectively more curved for an individual with internal habit formation, who then acts as if she is more risk averse. Alvarez-Cuadrado *et al.* (2004, p. 60) and Turnovsky and Monteiro (2007, p. 500) provide a similar intuition in the neoclassical and non-scale growth models with multiplicative habits. They argue that, when the economy starts with a ratio $h(0) > \hat{h}$, the adjustment requires additional capital accumulation and/or a reduction in the habits stock. Since the agent in the EH economy ignores the dampening effect of consumption on habits, she reduces consumption by more than the agent in the IH economy. Under-consumption when the stable growth path requires capital accumulation accelerates the convergence process and, as a result, the EH economy converges to its steady state at a faster rate than the IH economy does.

The former justification seems applicable with generality, and not only to the MH model, suggesting that the convergence speed in the EH model should always be higher than the corresponding one in the IH model. However, Theorem 4 and Corollary 5 show that the opposite case can also occur; i.e., that the convergence speed in the EH economy may be lower than the corresponding one in the IH economy (see also Figure 1). This calls for a reconsideration on the determinants of the different transition dynamics in both economies.

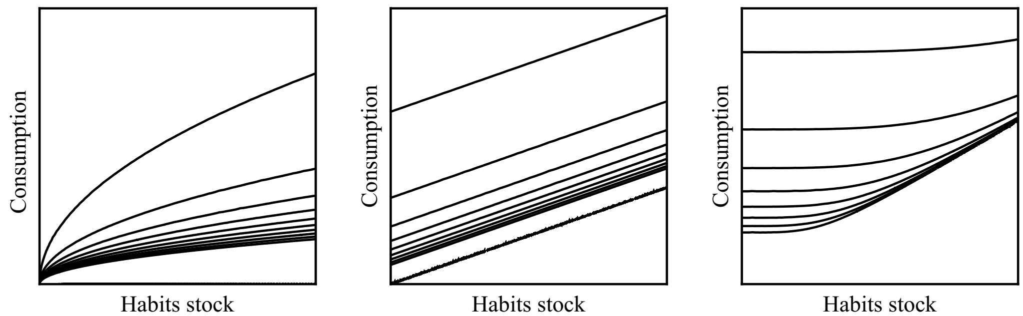
Corollary 5 shows that the crucial issue to determine the relationship between the convergence speeds in the EH and IH economies is the sign of $\hat{\Psi}$. Hence, in what follows we will try to give an insight of its implications. To this end, let us consider that $u_H < 0$, so that an increase in the habits stock, keeping current consumption constant, depresses utility. The slope of the indifference curves of the instantaneous utility function u in (H,C)–space at the BGP; i.e., the marginal rate of substitution (MRS) of consumption for habits, is given by

$$\mathrm{MRS}_{C,H} = -\frac{u_H(\hat{C},\hat{H})}{u_C(\hat{C},\hat{H})} = -\frac{u_H(\hat{c},1)}{u_C(\hat{c},1)} > 0. \tag{33}$$

Intuitively, a decreasing MRS as the economy evolves means that the negative effect of habits on utility becomes relatively smaller, because a lower increase in consumption would be needed to compensate a given increase in the habits stock in order to keep utility constant. Loosely speaking, the lower is the MRS of consumption for habits the (relatively) less harmful is the habits stock for utility. The derivative of the MRS with respect to c at the BGP is is given by

$$\frac{d\mathrm{MRS}_{C,H}}{dc} = -\frac{u_{CH}(\hat{c},1)u_C(\hat{c},1) - u_{CC}(\hat{c},1)u_H(\hat{c},1)}{u_C(\hat{c},1)^2} = -\frac{\hat{\Psi}}{\nu u_C(\hat{c},1)^2}. \tag{34}$$

Hence, the sign of $\hat{\Psi}$ determines whether the MRS is decreasing, increasing or constant or, loosely speaking, whether the habits stock becomes less, more or equally



(a) $\varphi = 0$ (MH) $\Rightarrow \hat{\Psi} < 0$. $|\xi_{\text{EH}}| > |\xi_{\text{IH}}|$

(b) $\varphi = 1$ (SH) $\Rightarrow \hat{\Psi} = 0$. $|\xi_{\text{EH}}| = |\xi_{\text{IH}}|$

(c) $\varphi = 4 \Rightarrow \hat{\Psi} > 0$. $|\xi_{\text{EH}}| < |\xi_{\text{IH}}|$

Figure 3: Indifference map of $u(C,H) = \frac{1}{1-\varepsilon}\left(\frac{C^{\varphi}-\gamma H^{\varphi}}{1-\gamma}\right)^{(1-\varepsilon)/\varphi}$ for $\varphi = 0,1,4$.

harmful for utility, as c increases. In order to illustrate the different behaviour of the MRS that may arise, Figure 3 depicts the indifference map of the utility function u defined in (30) for $\varphi = 0,1,4$. The remaining parameter values are taken from Table 1.

Let us first examine the case in which $\hat{\Psi} > 0$ (panel (c) in Figure 3). If $h(0) > \hat{h}$, in Section 2.3 we have shown that as the economy evolves, h decreases and c increases toward their respective steady-state values. The intuition is that the agent maintains a low level of the ratio of consumption to habits relative to its stationary value because this allows decreasing the ratio $h \equiv H/K$ by fostering capital accumulation and by slowing the growth of the habits stock. As h approaches its stationary value, consumption needs to be depressed less, so c grows toward its steady state. As c increases, the MRS is (locally) decreasing, so that habits become less harmful for utility. An individual with internal habits, who takes into account the effect of her current consumption on the habits stock, would choose a transitional path with a lower consumption in the present —to keep habits relatively low— and a higher consumption in the future —when the higher habits induced by the higher consumption are less harmful— than that chosen by an individual with external habits, who treats the habits stock as given. In other words, an individual with internal habits would be more willing to shift consumption from the present to the future than an individual with external habits. This results in a higher growth rate of capital (and income) and, since the stable growth path requires capital accumulation, the IH economy converges to the steady state at a higher rate than the EH economy.

The comparison of the marginal rates of intertemporal substitution (MRIS) in consumption in the IH and EH economies confirms the former intuition. Following

Ryder and Heal (1973), the MRIS between consumption at times t_1 and t_2, with $t_2 > t_1$ without loss of generality, can be defined as

$$\begin{aligned}\text{MRIS}_{t_1,t_2} &= \frac{\partial U(C)/\partial C(t_2)}{\partial U(C)/\partial C(t_1)}\\ &= \frac{u_C(C(t_2),H(t_2))e^{-\beta t_2} + \rho\phi e^{\rho t_2}\int_{t_2}^{\infty} e^{-(\beta+\rho)t}u_H(C(t),H(t))dt}{u_C(C(t_1),H(t_1))e^{-\beta t_1} + \rho\phi e^{\rho t_1}\int_{t_1}^{\infty} e^{-(\beta+\rho)t}u_H(C(t),H(t))dt},\end{aligned}$$

where $\partial U(C)/\partial C(t)$ denotes the Volterra derivative of $U(C)$ —defined in (1)— with respect to consumption at time t. Using (10), and also (A.2) and (A.3) in Appendix A, the MRIS can be rewritten as

$$\text{MRIS}_{t_1,t_2} = \frac{u_C(C(t_2),H(t_2))(1+\rho\phi q(t_1))}{u_C(C(t_1),H(t_1))(1+\rho\phi q(t_2))}e^{-\beta(t_2-t_1)}. \tag{35}$$

Eq. (23) shows that, if $h(0) > \hat{h}$, as the economy evolves the relative shadow cost of habits, q, decreases toward its stationary value, reflecting that habits become less harmful for utility as the economy evolves. Hence, the MRIS in consumption in the EH economy would be lower than that in the IH economy, $\text{MRIS}^{\text{EH}}_{t_1,t_2} < \text{MRIS}^{\text{IH}}_{t_1,t_2}$, when evaluated along the transition path of the IH economy. A lower value of the MRIS means that an agent in the EH economy is less willing to substitute present by future consumption than an agent in the IH economy.

Why does the standard justification fail in explaining this case in which the IH converges at a faster rate than the EH economy? The standard explanation relies on the fact that an EH agent overstates the effect on utility of increasing consumption, because she ignores its negative indirect effect through a higher habits stock. However, to make her consumption/saving decision, the agent compares the benefit of consuming today against the benefit of saving and enjoy greater consumption in the future, and an EH agent overstates both benefits. So, the relevant issue would be the relationship between the marginal rates of intertemporal substitution in consumption in the IH and EH economies.

If $\hat{\Psi} < 0$ (panel (a) in Figure 3), the MRS of consumption for habits increases and, therefore, the habits stock becomes more harmful for utility as the economy evolves with an increasing ratio of consumption to habits, c. A symmetric argument to that made above entails that an individual with internal habits would be less willing to postpone consumption than an individual with external habits, which results in a lower speed of convergence in the IH economy. In this case, Eq. (23) shows that the relative shadow cost of habits, q, is increasing —reflecting the fact that habits become more harmful— and, therefore, $\text{MRIS}^{\text{EH}}_{t_1,t_2} > \text{MRIS}^{\text{IH}}_{t_1,t_2}$ when evaluated along the transition path of the IH economy. Thus, the desire to substitute intertemporally in the IH economy would be effectively lower than that in the EH economy.

Finally, if $\hat{\Psi} = 0$ (panel (b) in Figure 3), the MRS is constant, and so, an individual with internal habits faces no incentive to depart from the consumption path chosen by an individual with external habits. Hence, the speed of convergence is the same in both economies. Actually, in this case the MRIS in consumption of the EH and IH economies coincide, $\text{MRIS}^{\text{EH}}_{t_1,t_2} = \text{MRIS}^{\text{IH}}_{t_1,t_2}$.

4 Strength and adjustment speed of habits

Besides the distinction between internal and external habits or between multiplicative and subtractive habits, other important features of the habit-formation process are the speed of adjustment of habits to current consumption and the strength of habits in utility. In this section, we study analytically their impact on the speed of convergence in the most commonly-used MH and SH specifications.

4.1 The strength of habits

Our first analytical result, which is proved in Appendix B, proves that the lower the strength of habits —the lower γ— the higher is the speed of convergence in the MH model.

Proposition 9 *If habits enter utility in a multiplicative form according to* (27)*, then* $d\xi/d\gamma \geq 0$*, with equality if and only if* $\sigma = 1$ *and* $\phi = 0$*.*

It should be noted that if $\sigma \neq 1$, since $v = \gamma + \sigma(1-\gamma)$, the steady state and the long-run growth rate of the MH economy is affected by changes in γ, as shown by Eqs. (14), (15) and (17). Hence, Proposition 9 refers to the convergence speed to different steady states as the value of γ changes. We can also examine the effect of an increase in γ compensated by a change in σ so that the degree of homogeneity, $-v$, remains unchanged. Thus, the steady state of the real variables and the long-run growth rate to which the economy converges would be the same. Appendix B proves the following result.

Proposition 10 *If habits enter utility in a multiplicative form according to* (27)*, then* $d\xi/d\gamma\,|_{v=\text{constant}} \geq 0$*, with equality if and only if* $\sigma = 1$ *and* $\phi = 0$*.*

A similar result can be readily obtained in the SH model. Differentiating (29) with respect to γ, we get that $d\xi/d\gamma = \rho > 0$ (or, equivalently, $d|\xi|/d\gamma = -\rho < 0$). So, we can state the following result that states that the convergence speed is strictly decreasing in the strength of habits, γ.

Proposition 11 *If habits enter utility in a subtractive form according to* (28)*, then* $d\xi/d\gamma > 0$*.*

Propositions 9, 10 and 11 provide analytical support to the numerical findings of Carroll *et al.* (2000), who report that the lower the strength of habits —the lower γ— the higher is the speed of convergence in the AK model with multiplicative internal habits. In doing their simulations, the value of the parameter σ is adjusted so as to hold the steady-state growth rate constant; i.e., so as to maintain constant the degree of homogeneity, $-\nu$ (as in Proposition 10). As Carroll *et al.* (2000) argue, the intuition of this result is that habits tend to pull consumption toward the habits stock and away from the stationary value of the ratio of consumption to income. In an economy with no habit formation there is no pull of consumption toward the habits stock; thus the gap between C and H is larger and so H will adapt to C faster. The more habits matter the stronger the pull on consumption toward habits, and so, the slower the speed of convergence.

4.2 The rate of adjustment of habits to consumption

Our first analytical result, which is proved in Appendix B, proves that the convergence speed is strictly increasing in the rate of adjustment of habits to current consumption, ρ, in the MH model.

Proposition 12 *If habits enter utility in a multiplicative form according to* (27)*, then* $d\xi/d\rho < 0$*.*

A similar result can be derived in the SH model. Differentiating (29) with respect to ρ, we can readily find that $d\xi/d\rho = -(1-\gamma) < 0$ (or, equivalently, $d|\xi|/d\rho = (1-\gamma) > 0$). Thus, we can state the following result.

Proposition 13 *If habits enter utility in a subtractive form according to* (28)*, then* $d\xi/d\rho < 0$*.*

Propositions 12 and 13 provide analytical support to the numerical results of Carroll *et al.* (1997), who find that the convergence speed increases as the speed of adjustment of the habits stock increases in the AK model with multiplicative habits. Intuitively, the parameter ρ determines the speed with which the reference stock adjust to current consumption. As the dynamics of the AK model with habit formation is driven exclusively by preferences; i.e., by the presence of habits, the more rapidly habits adjust to consumption, the faster the economy converges to its steady state.

5 Conclusion

This paper examines the effect on the economy dynamics of alternative formulations of habit persistence. The focus is on the impact on the speed of convergence, which is the key determinant of the (local) dynamics of the economy. Our main result characterizes the effect on the convergence speed of switching habits from being internal to externally formed. One important implication is that the EH economy may converge to its steady sate at a higher, lower or equal rate than the IH economy, depending on the specification of the utility function. We also prove that the higher the strength of habits and the lower the speed of adjustment of the habits stock to current consumption, the lower the speed of convergence both in the multiplicative- and subtractive-habits models. This provides analytical support to previous numerical findings reported in the literature.

Our results also shed new light on the determinants of the different dynamic behavior of economies with internal and external habits. Previous works found in their numerical simulations that the EH economy converges at a faster rate than the IH economy. This was justified intuitively because an agent in the EH economy ignores the indirect effect that increasing her current consumption has in future utility through a higher habits stock, whereas an agent in the IH economy does take it into account. However, the fact that the convergence speed in the EH economy can be lower than that in the IH economy calls for a re-consideration of the former justification. Thus, we show that the relationship between the speeds of convergence in both models can be explained by the behavior of the marginal rate of substitution between consumption and habits as the economy evolves which, in turn, affects the agent's willingness to substitute intertemporally in the IH relative to the EH economy.

Appendix

A Derivation of the dynamic system (11)–(13)

Henceforth, we use that $\bar{C} = C$ because all agents are identical. As $m(C,C) = C$ and $m_C(C,C) = \phi$, Eqs. (3) and (6) can be rewritten, respectively, as

$$\dot{H} = \rho(C - H), \tag{A.1}$$

$$u_C(C,H) + \rho\phi\mu = \lambda. \tag{A.2}$$

Since $q \equiv -\mu/\lambda$, from (A.2) we get

$$\lambda = u_C(C,H)/(1+\rho\phi q), \tag{A.3}$$
$$\mu = -qu_C(C,H)/(1+\rho\phi q). \tag{A.4}$$

Differentiating (A.2) with respect to time, using (7) and (A.3), we get

$$u_{CC}(C,H)\dot{C} + u_{CH}(C,H)\dot{H} = \dot{\lambda}(1+\rho\phi q) + \lambda\rho\phi\dot{q} = \left(\beta - A + \frac{\rho\phi\dot{q}}{1+\rho\phi q}\right)u_C(C,H),$$

which, rearranging terms, can be rewritten as

$$\dot{C} = \frac{u_C(C,H)}{u_{CC}(C,H)}\left(\beta - A + \frac{\rho\phi\dot{q}}{1+\rho\phi q} - \frac{u_{CH}(C,H)}{u_C(C,H)}\dot{H}\right).$$

As $\dot{c}/c = \dot{C}/C - \dot{H}/H$, using the homogeneity of degree $-v$ of u_C, we get

$$\frac{\dot{c}}{c} = \frac{u_C(c,1)}{c\,u_{CC}(c,1)}\left(\beta - A + \frac{\rho\phi\dot{q}}{1+\rho\phi q} + v\frac{\dot{H}}{H}\right).$$

Using (A.1), we get (11). Since $\dot{h}/h = \dot{H}/H - \dot{K}/K$, (12) results from (A.1) and (5). Since $\dot{q}/q = \dot{\mu}/\mu - \dot{\lambda}/\lambda$, (13) results from (7) and (8), taking into account (A.3) and (A.4).

B Proofs

Proof of Proposition 1. Equating (11)–(13) to zero, we get (14)–(16). Eq. (17) is obtained from $\hat{g} = \rho(\hat{c}-1)$. We have $\hat{g} > 0$ if and only if $A > \beta$, which yields $\hat{c} > 1$, and $\hat{h} > 0$ if and only if $(v-1)A + \beta > 0$. The transversality condition (9) is met, as it is equivalent to $-\beta + \dot{\hat{\lambda}}/\hat{\lambda} + \dot{\hat{K}}/\hat{K} = -\beta + \dot{\hat{\mu}}/\hat{\mu} + \dot{\hat{H}}/\hat{H} = -A + \hat{g} < 0$.

Proof of Proposition 2. Note first that

$$b_{33} = A + \rho + \rho\phi\frac{u_H(\hat{c},1)}{u_C(\hat{c},1)} > 0. \tag{B.1}$$

Condition (B.1) is trivially satisfied if $u_H(\hat{c},1) \geq 0$. It also holds if $u_H(\hat{c},1) < 0$ because, using that $A > \hat{g}$ if condition (18) is met and recalling (4), we have that

$$(A+\rho)u_C(\hat{c},1) + \rho\phi u_H(\hat{c},1) > (\hat{g}+\rho)\left[u_C(\hat{c},1) + \left(\frac{\rho}{\hat{g}+\rho}\right)u_H(\hat{c},1)\right] > 0.$$

Given the structure of the matrix B, its second diagonal element, $\hat{c}\hat{h}$, is an unstable root. The other two roots are those of the submatrix

$$B_{13} = \begin{pmatrix} b_{11} & b_{13} \\ b_{31} & b_{33} \end{pmatrix}. \tag{B.2}$$

The determinant of B_{13} is

$$\det = \frac{v\rho u_C(\hat{c},1)}{u_{CC}(\hat{c},1)} b_{33} = \frac{v\rho\left[(A+\rho)u_C(\hat{c},1)+\rho\phi u_H(\hat{c},1)\right]}{u_{CC}(\hat{c},1)} < 0, \tag{B.3}$$

where the sign follows from (B.1). Hence, B has one negative (real) root and two positive (real) roots. The steady state is locally saddle-path stable, since system (19) features one predetermined variable, h.

Proof of Proposition 3. In the EH model, $\phi = 0$, we have $b_{13} = 0$, and so, $\xi = b_{11}$.

Proof of Theorem 4. The characteristic polynomial of B_{13} is

$$p(x) = x^2 - \mathrm{tr} \cdot x + \det, \tag{B.4}$$

where det is given by (B.3), and the trace is

$$\mathrm{tr} = \mathrm{tr}\,(B_{13}) = b_{11} + b_{33} = A + \rho + \frac{\rho[v u_C(\hat{c},1) + \phi u_{CH}(\hat{c},1)]}{u_{CC}(\hat{c},1)}.$$

Let ξ_ϕ denote the stable root of B_{13} for any given value of ϕ. Differentiating $p(\xi_\phi) = 0$ in (B.4) with respect to ϕ, using the implicit function theorem, we get

$$\frac{d\xi_\phi}{d\phi} = \frac{\rho[v\rho u_H(\hat{c},1) - \xi_\phi u_{CH}(\hat{c},1)]}{v\rho u_C(\hat{c},1) + \rho\phi u_{CH}(\hat{c},1) + (A+\rho-2\xi_\phi)u_{CC}(\hat{c},1)} = \frac{N}{D}. \tag{B.5}$$

Here, N and D denote the numerator and the denominator of (B.5), respectively. The following Lemma B.1 shows that $D < 0$, and Lemma B.2 shows that $\mathrm{sign}(N) = \mathrm{sign}(\hat{\Psi})$. Hence, we can conclude that $\mathrm{sign}(d\xi_\phi/d\phi) = -\mathrm{sign}(\hat{\Psi})$.

Lemma B.1 *The denominator D of* (B.5) *is negative, $D < 0$.*

Proof of Lemma B.1. Let us define

$$\Omega = v\rho u_C(\hat{c},1) + \rho\phi u_{CH}(\hat{c},1) + (A+\rho)u_{CC}(\hat{c},1),$$

so that $D = \Omega - 2\xi_\phi u_{CC}(\hat{c},1)$. If $\Omega \leq 0$, then $D < 0$. If $\Omega > 0$, let us define

$$\hat{\xi} = \frac{\Omega}{2u_{CC}(\hat{c},1)} = \frac{v\rho u_C(\hat{c},1) + \rho\phi u_{CH}(\hat{c},1) + (A+\rho)u_{CC}(\hat{c},1)}{2u_{CC}(\hat{c},1)} < 0.$$

Inserting $\hat{\xi}$ into (B.4), after simplification and using (B.1), we get

$$p(\hat{\xi}) = -\frac{\Omega^2 - 4v\rho u_{CC}(\hat{c},1)[(A+\rho)u_C(\hat{c},1) + \rho\phi u_H(\hat{c},1)]}{4u_{CC}(\hat{c},1)^2} < 0 = p(\xi_\phi).$$

This entails that $\xi_\phi < \hat{\xi} < 0$ or, equivalently, $D = \Omega - 2\xi_\phi u_{CC}(\hat{c},1) < 0$. Hence, we can conclude that $D < 0$ in any case.

Lemma B.2 *The numerator N of* (B.5) *satisfies that* $\text{sign}(N) = \text{sign}(\hat{\Psi})$.

Proof of Lemma B.2. If $u_{CH}(\hat{c},1) = 0$, it follows from (20) that $\text{sign}(u_H(\hat{c},1)) = \text{sign}(\hat{\Psi})$ and, therefore, $\text{sign}(N) = \text{sign}(u_H(\hat{c},1)) = \text{sign}(\hat{\Psi})$. If $u_H(\hat{c},1) = 0$, Eq. (20) entails that $\text{sign}(u_{CH}(\hat{c},1)) = \text{sign}(\hat{\Psi})$ and, therefore, $\text{sign}(N) = \text{sign}(u_{CH}(\hat{c},1)) = \text{sign}(\hat{\Psi})$. Let us now consider the remaining case in which $u_{CH}(\hat{c},1) \neq 0$ and $u_H(\hat{c},1) \neq 0$. If $\text{sign}(u_H(\hat{c},1)) = \text{sign}(u_{CH}(\hat{c},1))$, we readily get that $\text{sign}(N) = \text{sign}(u_H(\hat{c},1)) = \text{sign}(\hat{\Psi})$. If $\text{sign}(u_H(\hat{c},1)) \neq \text{sign}(u_{CH}(\hat{c},1))$, let us define

$$\bar{\xi} = \frac{v\rho u_H(\hat{c},1)}{u_{CH}(\hat{c},1)} < 0. \tag{B.6}$$

Note that

$$\xi_\phi - \bar{\xi} = -\frac{1}{u_{CH}(\hat{c},1)}[v\rho u_H(\hat{c},1) - \xi_\phi u_{CH}(\hat{c},1)] = -\frac{N}{\rho u_{CH}(\hat{c},1)},$$

so that

$$\text{sign}(\xi_\phi - \bar{\xi}) = -\text{sign}(u_{CH}(\hat{c},1)) \times \text{sign}(N). \tag{B.7}$$

Inserting $\bar{\xi}$ into (B.4), we get

$$p(\bar{\xi}) = -\frac{\rho[v\rho u_H(\hat{c},1) - (A+\rho)u_{CH}(\hat{c},1)]\hat{\Psi}}{u_{CH}(\hat{c},1)^2 u_{CC}(\hat{c},1)} = -\frac{\rho}{u_{CH}(\hat{c},1)^2 u_{CC}(\hat{c},1)}\hat{\Psi}\Theta,$$

where $\hat{\Psi}$ is defined in (20) and $\Theta = v\rho u_H(\hat{c},1) - (A+\rho)u_{CH}(\hat{c},1)$. Given that $\text{sign}(u_H(\hat{c},1)) \neq \text{sign}(u_{CH}(\hat{c},1))$ entails that $\text{sign}(\Theta) = -\text{sign}(u_{CH}(\hat{c},1))$, we obtain

$$\text{sign}(p(\bar{\xi})) = \text{sign}(\Theta) \times \text{sign}(\hat{\Psi}) = -\text{sign}(u_{CH}(\hat{c},1)) \times \text{sign}(\hat{\Psi}). \tag{B.8}$$

Since $p(\xi_\phi) = 0$ and $\bar{\xi} < 0$, using (B.7), we have that

$$\operatorname{sign}(p(\bar{\xi})) = \operatorname{sign}(\xi_\phi - \bar{\xi}) = -\operatorname{sign}(u_{CH}(\hat{c}, 1)) \times \operatorname{sign}(N). \tag{B.9}$$

Comparing (B.8) and (B.9), we get that $\operatorname{sign}(N) = \operatorname{sign}(\hat{\Psi})$.

Proof of Proposition 9. Let ξ_γ denote the stable root of B_{13} for any given value of γ. Implicit differentiation of $p(\xi_\gamma) = 0$ in (B.4) with respect to γ yields, after simplification,

$$\frac{d\xi_\gamma}{d\gamma} = \frac{\rho(\Pi - (\sigma - 1)\xi_\gamma \Gamma)}{\sigma(A - \hat{g}) + \rho\hat{c}\gamma(\sigma - 1)(1 - \phi) - 2\sigma\xi_\gamma}, \tag{B.10}$$

where

$$\Pi = [A - \hat{g} + \rho\hat{c}(1 - \phi) + \rho\phi\hat{c}(\hat{c} - \gamma)](\sigma - 1) + \rho\phi\hat{c}(v + \hat{c} - 1) \geq 0,$$

with equality if and only if $\sigma = 1$ and $\phi = 0$, and

$$\Gamma = \hat{c}(1 - \phi) - \frac{(\hat{c} - 1)[v + (\sigma - 1)\gamma\phi]}{v}.$$

The denominator of (B.10) is positive. If $\sigma = 1$ and $\phi = 0$ then $d\xi_\gamma/d\gamma = 0$. Otherwise, we have that $\Pi > 0$. Now, if $\Gamma \geq 0$ then $d\xi_\gamma/d\gamma > 0$. Let us suppose now that $\Gamma < 0$. Corollary 6 shows that $d\xi_\phi/d\phi > 0$, for any given value of γ. Hence, we have that $\xi_\gamma\big|_{\phi=0} = -v\rho\hat{c}/\sigma \leq \xi_\gamma < 0$ for all ϕ, where $\xi_\gamma\big|_{\phi=0}$ stands for the value of ξ_γ when $\phi = 0$. Therefore, after simplification, we obtain that

$$\begin{aligned}
&\Pi - (\sigma - 1)\xi_\gamma\Gamma \geq \Pi + (\sigma - 1)\frac{v\rho\hat{c}}{\sigma}\Gamma \\
&= (A - \hat{g})(\sigma - 1) + \rho\phi\hat{c}^2 + \frac{\rho\hat{c}}{\sigma}(\sigma - 1)[(\sigma - 1)(1 - \gamma) + (\sigma + 1)(1 - \phi\gamma)] > 0.
\end{aligned}$$

Hence, the numerator of (B.10) is strictly positive, and so, $d\xi_\gamma/d\gamma > 0$.

Proof of Proposition 10. Let ξ_γ denote the stable root of B_{13} for any given value of γ. Differentiating $p(\xi_\gamma) = 0$ in (B.4) with respect to γ, keeping v constant, we get

$$\begin{aligned}
&\left.\frac{d\xi_\gamma}{d\gamma}\right|_{v=\text{constant}} \\
&\quad = \frac{(\sigma - 1)[A - \hat{g} + \rho\hat{c}(1 - \gamma\phi) - (1 - \phi)\xi_\gamma] + \rho\phi\sigma\hat{c}(1 - \gamma)}{(1 - \gamma)\sigma[(A - \hat{g})\sigma + \rho\hat{c}\gamma(\sigma - 1)(1 - \phi) - 2\sigma\xi_\gamma]} v\rho\hat{c} \geq 0,
\end{aligned}$$

with equality if and only if $\sigma = 1$ and $\phi = 0$.

Proof of Proposition 12. Let ξ_ρ denote the stable root of B_{13} for any given value of ρ. Differentiating $p(\xi_\rho) = 0$ in (B.4) with respect to ρ, we get

$$\frac{d\xi_\rho}{d\rho} = -\frac{v[A-\hat{g}+2(\hat{g}+\rho)(1-\gamma\phi)]+(\sigma-1)\gamma(1-\phi)\xi_\rho}{(\sigma-1)(\hat{g}+\rho)\gamma(1-\phi)+\sigma(A-\hat{g}-2\xi_\rho)}. \tag{B.11}$$

The denominator of (B.11) is positive. Corollary 6 shows that $d\xi_\phi/d\phi > 0$, for any given value of ρ. So, we have that $\xi_\rho\big|_{\phi=0} = -v\rho\hat{c}/\sigma = -v(\rho+\hat{g})/\sigma \le \xi_\rho < 0$ for all ϕ, where $\xi_\rho\big|_{\phi=0}$ stands for the value of ξ_ρ when $\phi = 0$. Hence, after simplification, we obtain that

$$\begin{aligned} & v[A-\hat{g}+2(\hat{g}+\rho)(1-\gamma\phi)]+(\sigma-1)\gamma(1-\phi)\xi_\rho \\ & \qquad \ge v[A-\hat{g}+2(\hat{g}+\rho)(1-\gamma\phi)]-(\sigma-1)\gamma(1-\phi)\frac{v(\rho+\hat{g})}{\sigma} \\ & \qquad\qquad = v\left\{A-\hat{g}+\frac{(\hat{g}+\rho)}{\sigma}[(\sigma+1)\gamma(1-\phi)+2\sigma(1-\gamma)]\right\} > 0. \end{aligned}$$

Hence, the numerator of (B.11) is strictly positive, and so, $d\xi_\rho/d\rho < 0$.

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