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Corrigendum

Corrigendum to "Asymptotic analysis of a thin fluid layer flow between two moving surfaces" [J. Math. Anal. Appl. 507 (2022) 125735]

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Some minor errors have been detected in the equations (119), (145), (146), (B.13) and (B.14) of [1]. Equation (119) in [1] should be:

$$\begin{aligned}
\sigma_{ij}(\varepsilon) =& -\sum_{r=-2}^{\infty}\varepsilon^r p^r \delta_{ij} + \mu \sum_{r=0}^{\infty}\varepsilon^r \left[\left(\frac{\partial u_k^r}{\partial \xi_l} a_{ki} + u_k^r \frac{\partial a_{ki}}{\partial \xi_l}\right)\frac{\partial \xi_l}{\partial x_j^\varepsilon} + \left(\frac{\partial u_k^r}{\partial \xi_l} a_{kj} + u_k^r \frac{\partial a_{kj}}{\partial \xi_l}\right)\frac{\partial \xi_l}{\partial x_i^\varepsilon}\right] \\
=& -\varepsilon^{-2} p^{-2}\delta_{ij} + \varepsilon^{-1}\left\{-p^{-1}\delta_{ij} + \mu\left[\frac{a_{3j}}{h}\frac{\partial u_k^0}{\partial \xi_3} a_{ki} + \frac{a_{3i}}{h}\frac{\partial u_k^0}{\partial \xi_3} a_{kj}\right]\right\} \\
& -p^0\delta_{ij} + \mu\left[\frac{a_{3j}}{h}\frac{\partial u_k^1}{\partial \xi_3} a_{ki} + \frac{a_{3i}}{h}\frac{\partial u_k^1}{\partial \xi_3} a_{kj}\right. \\
& + \sum_{l=1}^{2}\left(\frac{\partial u_k^0}{\partial \xi_l} a_{ki} + u_k^0 \frac{\partial a_{ki}}{\partial \xi_l}\right)(\alpha_l^0 a_{1j} + \beta_l^0 a_{2j}) + \frac{\xi_3}{h}\frac{\partial u_k^0}{\partial \xi_3} a_{ki}(\alpha_3^0 a_{1j} + \beta_3^0 a_{2j}) \\
& \left. + \sum_{l=1}^{2}\left(\frac{\partial u_k^0}{\partial \xi_l} a_{kj} + u_k^0 \frac{\partial a_{kj}}{\partial \xi_l}\right)(\alpha_l^0 a_{1i} + \beta_l^0 a_{2i}) + \frac{\xi_3}{h}\frac{\partial u_k^0}{\partial \xi_3} a_{kj}(\alpha_3^0 a_{1i} + \beta_3^0 a_{2i})\right]
\end{aligned}$$

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$$
\begin{aligned}
&+\varepsilon\left\{-p^1\delta_{ij}+\mu\left[\frac{a_{3j}}{h}\frac{\partial u_k^2}{\partial\xi_3}a_{ki}+\frac{a_{3i}}{h}\frac{\partial u_k^2}{\partial\xi_3}a_{kj}\right.\right.\\
&+\sum_{l=1}^{2}\left(\frac{\partial u_k^1}{\partial\xi_l}a_{ki}+u_k^1\frac{\partial a_{ki}}{\partial\xi_l}\right)(\alpha_l^0a_{1j}+\beta_l^0a_{2j})+\frac{\xi_3}{h}\frac{\partial u_k^1}{\partial\xi_3}a_{ki}(\alpha_3^0a_{1j}+\beta_3^0a_{2j})\\
&+\sum_{l=1}^{2}\left(\frac{\partial u_k^1}{\partial\xi_l}a_{kj}+u_k^1\frac{\partial a_{kj}}{\partial\xi_l}\right)(\alpha_l^0a_{1i}+\beta_l^0a_{2i})+\frac{\xi_3}{h}\frac{\partial u_k^1}{\partial\xi_3}a_{kj}(\alpha_3^0a_{1i}+\beta_3^0a_{2i})\\
&+\xi_3h\sum_{l=1}^{2}\left(\frac{\partial u_k^0}{\partial\xi_l}a_{ki}+u_k^0\frac{\partial a_{ki}}{\partial\xi_l}\right)(\alpha_l^1a_{1j}+\beta_l^1a_{2j})+\xi_3^2\frac{\partial u_k^0}{\partial\xi_3}a_{ki}(\alpha_3^1a_{1j}+\beta_3^1a_{2j})\\
&\left.\left.+\xi_3h\sum_{l=1}^{2}\left(\frac{\partial u_k^0}{\partial\xi_l}a_{kj}+u_k^0\frac{\partial a_{kj}}{\partial\xi_l}\right)(\alpha_l^1a_{1i}+\beta_l^1a_{2i})+\xi_3^2\frac{\partial u_k^0}{\partial\xi_3}a_{kj}(\alpha_3^1a_{1i}+\beta_3^1a_{2i})\right]\right\}\\
&+\cdots
\end{aligned}
\tag{119}
$$

where, in line 3, the minus sign of the term p^{-1} was missing (now this term is in line 2).

Equations (145)-(146) should be:

$$
\begin{aligned}
&\varepsilon^{-1}\frac{\mu}{h}\left(E\frac{\partial u_1^0}{\partial\xi_3}+F\frac{\partial u_2^0}{\partial\xi_3}\right)+\mu\left[\frac{E}{h}\frac{\partial u_1^1}{\partial\xi_3}+\frac{F}{h}\frac{\partial u_2^1}{\partial\xi_3}+\frac{\partial u_3^0}{\partial\xi_1}+eu_1^0+fu_2^0\right]\\
&\quad+\varepsilon\mu\left[\frac{E}{h}\frac{\partial u_1^2}{\partial\xi_3}+\frac{F}{h}\frac{\partial u_2^2}{\partial\xi_3}+\frac{\partial u_3^1}{\partial\xi_1}+eu_1^1+fu_2^1-\frac{\xi_3}{h}\frac{\partial u_3^1}{\partial\xi_3}\frac{\partial h}{\partial\xi_1}+\xi_3h\sum_{l=1}^{2}\left(\frac{\partial u_3^0}{\partial\xi_l}+u_k^0\frac{\partial\vec{a}_k}{\partial\xi_l}\cdot\vec{a}_3\right)(\alpha_l^1E+\beta_l^1F)\right]\\
&\quad+\cdots=-s_0\left(\varepsilon\vec{f}_{R0}^1+\varepsilon^2\vec{f}_{R0}^2+\cdots\right)\cdot\vec{a}_1\text{ on }\xi_3=0
\end{aligned}
\tag{145}
$$

$$
\begin{aligned}
&\varepsilon^{-1}\frac{\mu}{h}\left(F\frac{\partial u_1^0}{\partial\xi_3}+G\frac{\partial u_2^0}{\partial\xi_3}\right)+\mu\left[\frac{F}{h}\frac{\partial u_1^1}{\partial\xi_3}+\frac{G}{h}\frac{\partial u_2^1}{\partial\xi_3}+\frac{\partial u_3^0}{\partial\xi_2}+fu_1^0+gu_2^0\right]\\
&\quad+\varepsilon\mu\left[\frac{F}{h}\frac{\partial u_1^2}{\partial\xi_3}+\frac{G}{h}\frac{\partial u_2^2}{\partial\xi_3}+\frac{\partial u_3^1}{\partial\xi_2}+fu_1^1+gu_2^1-\frac{\xi_3}{h}\frac{\partial u_3^1}{\partial\xi_3}\frac{\partial h}{\partial\xi_2}+\xi_3h\sum_{l=1}^{2}\left(\frac{\partial u_3^0}{\partial\xi_l}+u_k^0\frac{\partial\vec{a}_k}{\partial\xi_l}\cdot\vec{a}_3\right)(\alpha_l^1F+\beta_l^1G)\right]\\
&\quad+\cdots=-s_0\left(\varepsilon\vec{f}_{R0}^1+\varepsilon^2\vec{f}_{R0}^2+\cdots\right)\cdot\vec{a}_2\text{ on }\xi_3=0
\end{aligned}
\tag{146}
$$

where a minus sign was missing after the equal signs (last line of each equation).

Equation (B.13) (Appendix B, Remark 8) should be:

$$
H_{il3}^0=-\frac{B_{il}}{A^0}\quad(i,l=1,2)
\tag{B.13}
$$

where a minus sign was missing after the equal sign.

Finally, coefficient $F_i^0(h)$ was not properly defined, and equation (B.14) should be

$$
F_i^0(h)=\int_0^1 f_i^0\,d\xi_3+\frac{s_0}{\rho h}(\vec{f}_{R_1}^1+\vec{f}_{R_0}^1)\cdot\left(\alpha_i^0\vec{a}_1+\beta_i^0\vec{a}_2\right)\quad(i=1,2)
\tag{B.14}
$$

References

[1] J.M. Rodríguez, R. Taboada-Vázquez, Asymptotic analysis of a thin fluid layer flow between two moving surfaces, J. Math. Anal. Appl. 507 (1) (2022) 125735, https://doi.org/10.1016/j.jmaa.2021.125735.