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Deployable cylindrical vaults with reciprocal linkages for emergency buildings

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ABSTRACT

Deployable bar structures are especially useful for emergency buildings. These structures require simplicity and ease of assembly and their performance can be improve by using reciprocal linkage at their bars ends. This reciprocal configuration determines singular constructive conditions that allow all types of cylindrical vaults. This article develops this new system, from a previously theoretical analysis to a test campaign that allows verifying the validity of the proposal. Comparing these results with those resulting from construction systems with straight scissor-like elements or imposed curvature, shows the adequacy of this new proposal, verifying the advantages of the reciprocal joints in cylindrical vaults configurations.

1. Introduction

The study of emergency buildings addresses an obvious social need and is therefore the subject of growing interest. According to available statistics, a total of 850 natural disasters occurred in 2018, with losses of 12,800 lives and damage of $140 billion. In addition, 289,300 people were killed in the various wars and 16.2 million people were displaced in the same year. These figures justify the need to design architectural solutions to house these groups and provide them with the necessary services. The authors are developing a research project on emergency buildings for humanitarian disaster situations, one of the main lines of research being focused on deployable structures (see Table 1).

A natural disaster can happen anytime, anywhere. This has led to numerous investigations into proposals worldwide for the design of emergency buildings that can be made operational in a short space of time. A natural emergency, such as floods, fires, earthquakes, etc. often causes serious damage to existing buildings, requiring them to be evacuated. It is necessary to be able to quickly provide solutions to assist the victims, both for their accommodation and for community spaces, which are essential to maintain collective services.

There are many different deployable systems, some consisting of bars, others of panels, with solutions based on origami. Of these, the ones that have reached a higher level are those based on three-dimensional modules formed by scissor-like-elements (SLEs).

They are pairs of bars with a through joint in the central linkage and joints at their ends. The SLE sets can be joined to form three-dimensional modules forming a resistant structure. These meshes can be folded into a very compact package. This allows them to be stored in order to have a stock in anticipation of a possible catastrophe. When required they can be easily transported in a folded position using light vehicles, and when they reach the desired location they are immediately deployed to form an enclosure. This enclosure can be used to house disaster victims [1], but their best performance is obtained in community facilities of 10 to 15 m, such as canteens, schools, medical centres, religious facilities, etc.

Deployable structures have been the subject of numerous investigations, from the first models built by the Spanish architect Emilio Pérez Piñero. This pioneer used beam modules formed by a set of 3 or 4 bars that are articulated around a central linkage [2,3]. In addition to numerous highly imaginative proposals, in 1964 this architect built the first large-scale deployable structure, the Pavilion for the 35 Years of Peace Exhibition [4]. It was deployed in Madrid and then moved to Barcelona and San Sebastian. The premature death of Pérez Piñero interrupted this research.

The next phase occurred in the 1980s, with a series of projects that focused on modules made up of triangular-based or square-based SLEs [5,6]. These structures are simpler to build than the beam structure and are more resistant and less deformable. However, they need a larger number of bars. Of these, the most useful are the square modules that make it possible to create very complex shapes, such as those proposed by García-Mora and Sánchez-Sánchez [7]. The use of square modules

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makes it possible to create grids with fewer bars than triangular grids, and above all with simpler linkages. The linkage is the most important element, as it represents a high percentage of the cost of the structure. Its optimisation is essential in the design of these structures, especially in emergency structures where it is essential to achieve the simplest and most economical linkages possible.

A large number of types of deployable structures have been analysed, both built by bundle modules and by SLE. For the purpose of this article, the typology to be studied is that of cylindrical vaults, one of the simplest and most useful. Curiously, it is one of the typologies that Pérez Piñero did not propose, since it is not possible to build a cylindrical surface with beam modules, as it is an incompatible geometry.

To design deployable cylindrical vaults it is necessary to use SLE forming triangular or square modules, simple or diagonalized (Fig. 1). By varying their lengths, different types of deployable structures can be formed. These modules began to be considered in the 1980 s, mainly on the basis of Escrig's first studies (Fig. 2) [5]. There are some previous studies, especially using flat grids, as well as some studies of triangular modules. However, Escrig significantly expanded the possibilities of using this type of structure. Escrig and Pérez-Valcárcel had already developed effective cylindrical folding vaults in their early studies [8].

Deployable grids made up of square modules have the disadvantage of a lack of transverse rigidity. In an attempt to solve this problem, several solutions were proposed. Gantes designed a system in which the square modules are braced by diagonal SLEs [9]. This is a very effective system, but it involves complicated and expensive linkage designs and an excessive number of bars. It is also difficult to apply to curved grids. The necessary bracing can be achieved with other mechanisms. Hernández proposed the design of cylindrical vaults with frustoconical ends [10]. By fixing the lower edge in place, the structure is adequately braced. The studies by Escrig and Pérez-Valcárcel, as well as the construction of various prototypes such as the ESTRAN by Hernández and Zalewsky, also show that a fabric roof provides sufficient bracing for small or medium spans [11]. For larger spans, such as the 30 m dome of San Pablo in Seville by Escrig, Sánchez and Pérez-Valcárcel, it is necessary to fit stiffeners, as the strength of the fabric would be insufficient [12].

These roof structures are all made up of square modules. However, it is also possible to design cylindrical vaults with triangular modules [13]. These vaults have an interesting property, of being bi-stable. This phenomenon, which was first noted by Gantes [9], refers to a property of many deployable structures that are geometrically compatible only when fully folded or fully deployed, but in intermediate deployment positions there are minor geometric incompatibilities. In order to deploy the structure, it is necessary to apply energy until the opening reaches a position near to the end of deployment. From this position the structure returns that energy, ending the deployment process on its own.

Bi-stability is more pronounced in triangular or diagonal structures such as Gantes modules when they are not flat. It has been the subject of several studies on triangular vaults [13] or in triangular module vaults, as proposed by Pérez-Valcárcel et al. for the roof of a stadium, which incorporate the energy balance of the structure in the deployment process [14]. Recently there have been major developments in the study of bi-stable structures, such as the works of Gantes and Konitopoulou [15], García-Mora and Sánchez-Sánchez [7] or Arnouts et al. [16,17].

The constructive simplicity of cylindrical vaults has led to interesting studies on the possibility of using rigid panel roofs that can be folded with the structure itself [18]. This is undoubtedly an interesting line of research, but this type of roof would be excessively expensive for its intended use. In emergency buildings the most suitable solutions are based on textile covers which can be folded with the bar package. This is the line that will be followed in this paper.

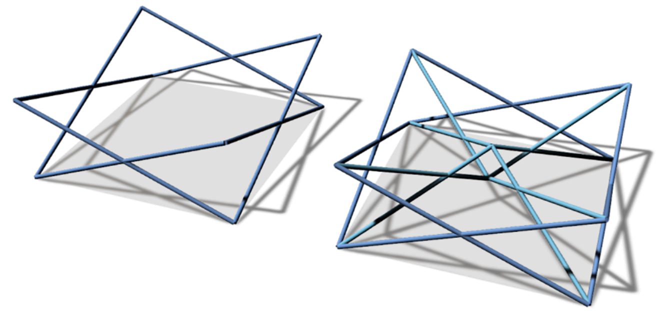

Fig. 1. Square SLE modules: simple and diagonalized module.

Cylindrical vaults are some of the simplest deployable structures to build and of greatest practical use, and have therefore been the subject of numerous studies. These include the works of Hernández [19] and those of Alegria Mira, De Temmerman, Filomeno Coelho, Koumar, Thrall, Tysmans, Roovers [20–23], in which, in addition to the studies of kinematic compatibility, they have proposed interesting optimisation mechanisms based on genetic algorithms [24]. In addition to these are the studies of Olteanu [25], Ramos [26], Ponce and Sánchez [27] and García Mora and Sánchez [28] that provide interesting solutions to vaults built with modules of variable dimensions. Freire et al. have also published interesting studies of broken guideline vaults, especially useful for roofs in rainy areas [29,30]. Less frequent are the studies on structural efficiency, such as those by Soto-Rubio and Cody [31] or experimental studies such as those by Zhanwei Zhao et al. [32]. Morales have published a recent study on the application of this type of structures to emergency shelters with complex linkage solutions [33].

In all cases, the proposed grids refer to structures whose modules are articulated at their ends. In 2018, the authors proposed a new system in which the ends of the bars are mutually supported [34]. It is a patented system that provides greater rigidity to the whole while reducing the bending stresses on the bars. The authors have also studied the resistance of the linkages, which have been extensively tested, considering both simple support and restrained support situations. The results have shown that the resistance of the linkages is in all cases higher than that of the bar, which demonstrates its higher performance [35].

In a recent article, the authors analysed the advantages of this system applied to flat grids in which it is demonstrated analytically and by means of tests that flat grids with reciprocal linkages reduce their deformation under gravitational loads of the order of 50% compared to grids with simply articulated linkages [36]. Likewise, the same authors have analyzed the effectiveness of reciprocal linkages in other types of meshes such as skew modules [37] or bundle modules, which are currently pending publication. All these studies have demonstrated the efficacy of the proposed system. This article analyzes the application of this system to deployable cylindrical meshes, considering different span height relationships, which significantly affect the overall efficiency of the system.

The new proposal transfers to deployable structures the advantages of reciprocal structures, whose main advantage is to cover an enclosure with small pieces that rest on each other, until the desired span is reached. This type of structures has demonstrated its good performance in the past, both in terms of resistance and rigidity [38–40]. The main problem they suffer from is that the failure of a single piece can cause the entire assembly to collapse, so that redundant connections are used to

Table 1
Parameters of reciprocal linkage.

Angle α	Angle β	D/d	d (bar)	D (Linkage)	$d(A_1,P_1)$	$d(A_2,P_2)$	L	G_{e1}	G_{e2}
37.287	8.764	1.76608	16	28.26	15.72	16.12	494.00	96.92%	96.84%

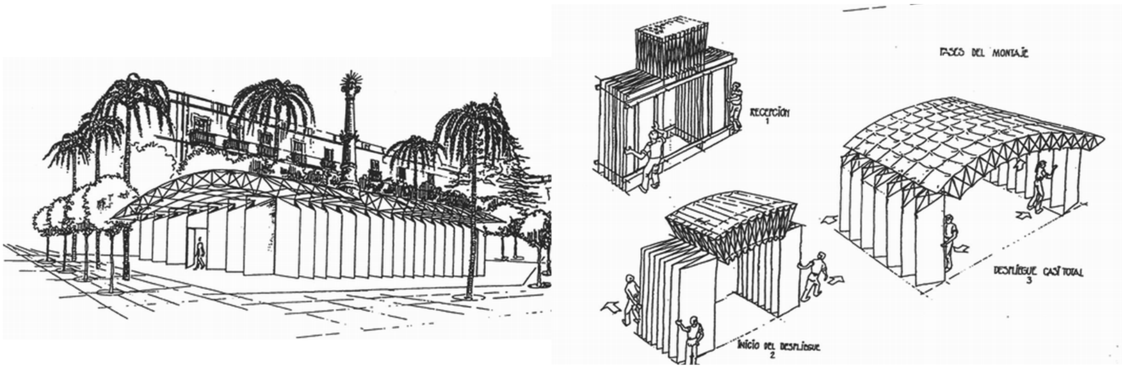

Fig. 2. Proposal for a portable exhibition hall in Almería (Escrig.).

prevent this from happening. If a part has sufficient connections, a possible failure can be compensated.

Basically, the system is composed of a hollow, circular or square section linkage, to which the bars that share the linkage are connected laterally, in such a way that when deploying, each bar rests on another, thereby forming a reciprocal structure (Fig. 3).

Using this type of linkage, the end nodes of the bars no longer function as a articulated joint and achieve a degree of embedment close to 100% [34]. The bending deformation of the bar is considerably reduced, also reducing the bending stresses. In this way, and thanks to the mechanical efficiency achieved, it is possible to create a simple and economical system, which is extremely important for the intended use as emergency buildings.

To date, and according to the data available, this system has only been developed by our team. The only possible precedent is the central linkage of Pérez Piñero for beam structures, but the external linkages were resolved with articulated joints. In subsequent works, these articulated links have always been used at the ends of the bars.

The purpose of this article is to analyse the behaviour, theoretical and experimental, of deployable cylindrical vaults with reciprocal links, in order to understand their structural response, their performance and their advantages in terms of stiffness and resistance. The originality of this article focuses on the following four aspects.

- A demonstration of the geometric and kinematic feasibility of deployable cylindrical grids formed by SLEs built with reciprocal linkages.
- Analysis of the constructive conditions of deployable structures that use this type of linkage and their application to emergency buildings.
- The comparative study of the two possible solutions for the execution of curved deployable structures with reciprocal linkages. They can be built with curved bars or with straight bars spaced at central through-linkage. This last solution is much simpler, which is a great advantage in general and specifically in emergency buildings, but it must be verified for its validation.
- A theoretical and experimental analysis of these types of structures.

The aim of the work is to evaluate the structural response of cylindrical SLE grids with reciprocal linkages and their practical applications. The reciprocal linkage system permits the design of deployable structures that are more resistant and less deformable than conventional ones. This implies continuing the line of research initiated by applying it to other types of structures of practical interest [41].

This article first analyses the geometric, constructive and resistant characteristics of the solutions proposed for emergency buildings (section 2). The materials and methods used in the models that have been tested in the research (section 3) and the results obtained (section 4) are described. In section 5, these results are analysed by making a comparative study between the theoretical and experimental results, and also by comparing the results of the tests with cylindrical grids with curved bars and with straight bars. The conclusions and perspectives are presented in section 6.

2. Description of deployable cylindrical grids with reciprocal linkages.

Deployable cylindrical grids can be built with square or triangular modules. Of these, the most suitable and useful are those formed by SLEs of square modules (Fig. 4a). Triangular vaults require a large number of bars and complex linkages (Fig. 4b). They are also bi-stable, i.e. they are only geometrically compatible in the folded position and in the fully unfolded position, while in the intermediate positions there are small incompatibilities that force the structure. This entails a certain complexity in their deployment [13]. In addition, their use with

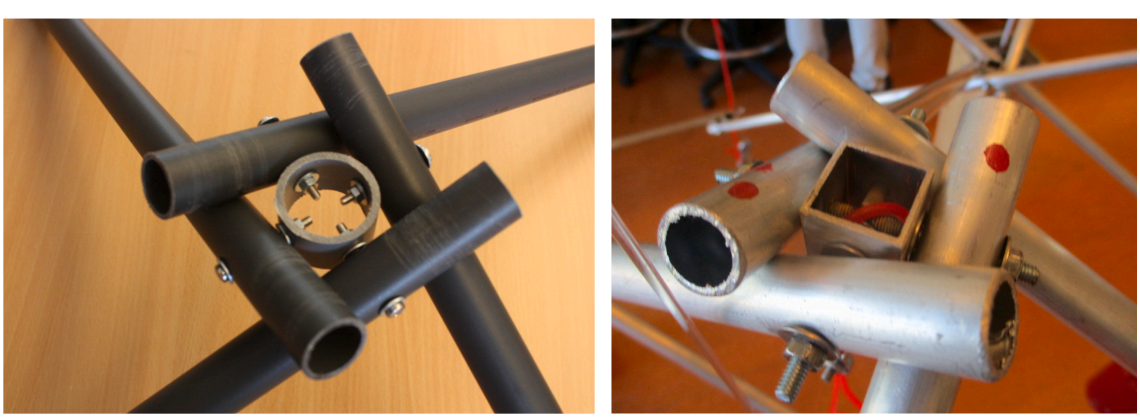

Fig. 3. Reciprocal linkages. Circular and square section.

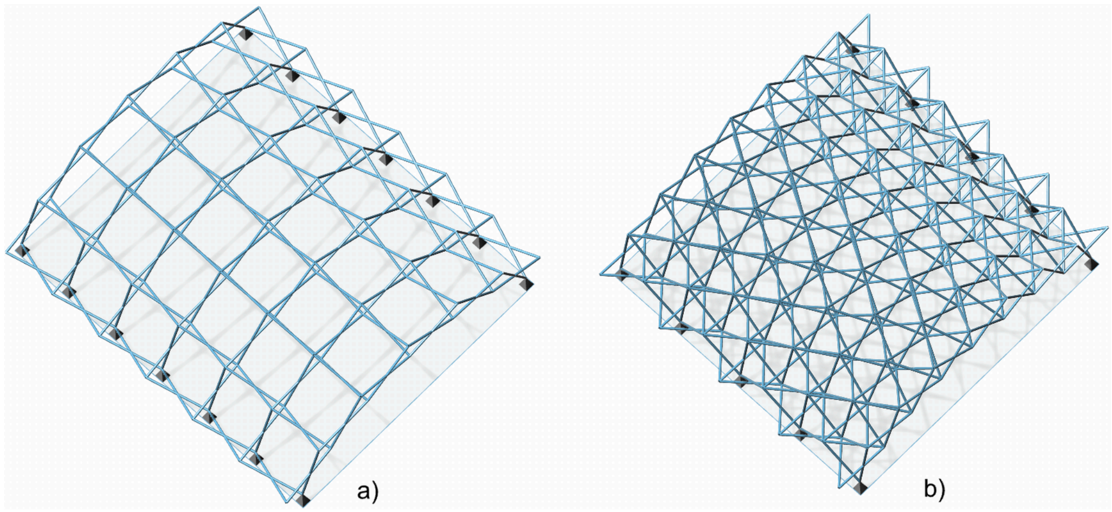

Fig. 4. Cylindrical vaults: square modules (a) and triangular modules (b).

reciprocal linkages would require large diameter linkages, as will be seen in 2.2. In addition, square module grids have fewer bars, the linkages are smaller and simpler, and the reciprocal support implies additional rigidity in the face of possible angular distortions. For this reason this type of structure will be analysed below.

In the case of cylindrical grids made of square modules, there is only curvature in the transverse direction in which the angles between adjacent SLEs α are larger than in the longitudinal direction β. The parameters that allow to define the mesh are their span **b**, their height **h** and the desired height $\mathbf{h_0}$ (Fig. 5). The geometric conditions are:

$$h = r - r\cos\phi \; ; \; b = 2r\sin\phi \tag{1}$$

Which results in:

$$r = \frac{b^2/4 + h^2}{2h} \; ; \; \phi = \arcsin\frac{b\cdot h}{b^2/4 + h^2} \; ; \; \varphi = \frac{\phi}{n} \tag{2}$$

where **n** is the number of modules of the semi-arch:

First, the lengths of the bars must be calculated. The total length is calculated and subsequently the lengths of the two sections of the bar. Applying the cosine theorem to the triangle, we obtain:

$$L^2 = r^2\cdot\sin^2\varphi + (r + h_0 - r\cdot\cos\varphi)^2 \; ; L = \sqrt{r_2 + (r + h_0)\cdot(r + h_0 - 2r\cdot\cos\varphi)}$$
$$r^2 = L^2 + (r + h_0)^2 - 2L(r + h_0)\cdot\cos\alpha_1 \; ; \alpha_1 = \arccos\frac{L^2 + (r + h_0)^2 - r^2}{2L(r + h_0)} \tag{3}$$

Applying the sine theorem to the triangle ABC, the value of the angle $\alpha_{\mathbf{2}}$ is calculated:

$$\alpha_2 = arcsin\frac{h_0\cdot\sin\alpha_1}{2r\cdot sin\varphi/2} \; ; \; \alpha_4 = \frac{\pi + \varphi}{2} - \alpha_2 \; ; \; \alpha_3 = \pi - \alpha_1 - \alpha_4 \tag{4}$$

With the values calculated for these angles, the lengths of the two sections of the bar will be:

$$L_1 = \frac{L\cdot sin\,\alpha_4}{sin\,\alpha_1 + sin\,\alpha_4} ; L_2 = \frac{L\cdot sin\,\alpha_1}{sin\,\alpha_1 + sin\,\alpha_4} \tag{5}$$

And the condition for the bars of two contiguous SLEs to be in extension, as will be seen in section 2.6, will be:

$$\alpha_4 = \pi/2 \; ; \; \alpha_1 + \alpha_3 = \pi/2 \tag{6}$$

These are very important parameters in the design of the grid with the angles α and β and the separation between the SLE planes **s**:

$$\alpha = \pi/2 - \alpha_1 \; ; \; \beta = arcsin\,(h_0/L) \; ; \; s = h_0\cdot\cot\beta \tag{7}$$

In curved deployable structures for practical use, the geometrical conditions of reciprocal support depend on the position of the linkages.

In general, in curved deployable structures the conditions of reciprocal support depend on the position of the linkages. In the upper layer, the angles with the reference plane are large, which allows reciprocal support to be achieved with small linkage diameters. On the other hand, the links of the lower layer form small angles, which forces large diameters in the linkages to achieve an effective reciprocal support. The best solution is to articulate the ends of the lower linkages.

In cylindrical vaults there are two types of scissors: those corresponding to the transverse arches and those corresponding to the straight-line longitudinal scissors. The condition of folding requires that the length of the longitudinal bars is equal to the sum of the two sections of the cross bars, so that the entire structure can be folded into a compact package (Fig. 5). The condition is particularly simple:

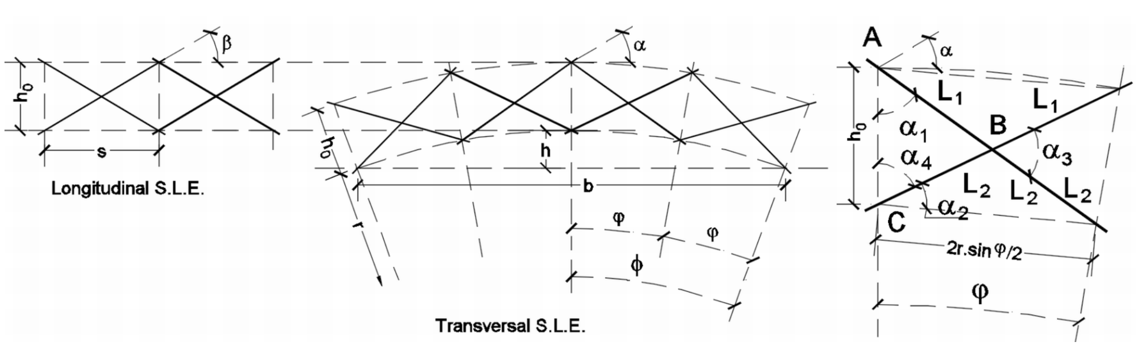

Fig. 5. Geometric conditions for cylindrical reciprocal grids.

$$L_3 = L_4 = \frac{L_1 + L_2}{2} \tag{8}$$

2.1. Support conditions

When designing deployable cylindrical vaults, external constraints must be taken into account. A vault of this type, once deployed, tends to close if the edge is not fixed in place. For this reason, the correct design of the external constraints is particularly important in this type of structure.

To do this, it is sufficient to fix a set of linkages in place that may be the upper linkages of the grid, the lower ones, or both. If the two sets of points are fixed, the structure provides optimum performance. This is important in emergency buildings, which must be designed to achieve the lightest and cheapest structures possible, so this will be the option adopted.

This is a serious design complication. It is always possible to modify the transverse SLEs to maintain the kinematic conditions of deployment, but the adjustment of the longitudinal SLE of the base is much more complex. In order to avoid angular distortions it is necessary for the distance **a** between the transversal SLEs to be kept constant (Fig. 6). The distance **d** between the support points varies, so the lengths of the horizontal SLE bars will also vary. Therefore, it is necessary to readjust the lengths of the bars so that they maintain the conditions of geometric compatibility.

This is done by taking as a parameter the distance b between the intermediate through link and the horizontal support plane. The angles α and β are defined by the grid itself (Fig. 7). The compatibility condition will be:

$$L = L_1 + L_2$$

In the triangle ABC it is possible to assign the lengths of the bar sections $\mathbf{L_1}$ and $\mathbf{L_2}$ as **a** function of **b** and the known angles α and β.

$$L = L_1 + L_2 = \frac{b}{sin\alpha} + \frac{b}{sin\beta}$$

$$d = b\cdot(ctg\alpha + ctg\beta)$$

$$L^2 = a^2 + d^2 \Rightarrow L^2 = b^2\cdot\left(\frac{1}{sin\alpha} + \frac{1}{sin\beta}\right)^2 = a^2 + b^2\cdot(ctg\alpha + ctg\beta)^2 \tag{9}$$

Which results in:

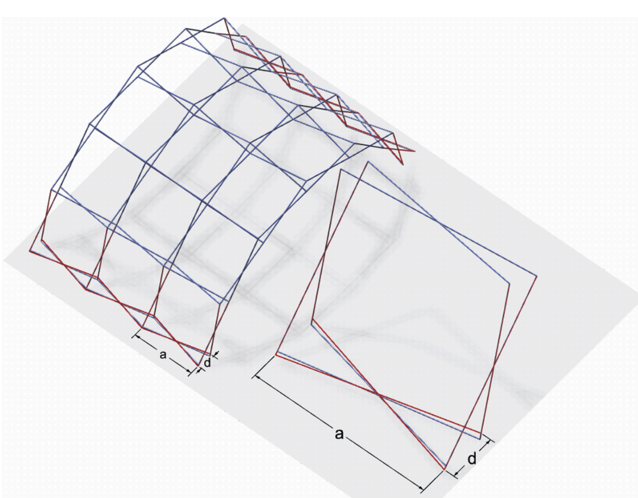

Fig. 6. Cylindrical vault with edge adjustment.

$$b = \frac{a}{\sqrt{\left(\frac{1}{sin\alpha} + \frac{1}{sin\beta}\right)^2 + (ctg\alpha + ctg\beta)^2}} \tag{10}$$

This equation makes it possible to determine the lengths of the edge bar sections that provide horizontal support.

2.2. Design of the reciprocal linkages

The novelty of the system consists in the use of reciprocal linkages formed by four bars that pivot on axes tangent to the cylindrical surface that extend from the linkage, which can be formed by a solid or hollow cylinder or prism (Fig. 3). The reciprocal support condition requires the linkage to have a minimum dimension that depends on the diameter of the bars and the desired opening angle. In cylindrical vaults, the opening angles of the bars are different α, β. In addition, these grids tend to have greater stresses in the transverse direction than in the longitudinal direction. For this reason, the diameters of the bars used are often different. For a more general formulation, it will be considered that **a** is the width of the linkage, $\mathbf{d_1}$ the diameter of the cross-section bar and $\mathbf{d_2}$ the diameter of the longitudinal section bars.

The position of the axes of the two bars is as indicated in Fig. 8. The straight lines $\mathbf{r_1}$ and $\mathbf{r_2}$, which correspond to the axes of the bars, pass through the $\mathbf{A_1}$ and $\mathbf{A_2}$ and form angles α and β with the reference plane. Their unitary vectors are:

$$\vec{u}_1\ (0,\ \cos\alpha,\ \text{sen}\ \alpha)\ \ \vec{u}_2\ (-\cos\beta,\ 0,\ \text{sen}\ \beta)$$

Their passing points are:

$$A_1\left(\frac{a + d_1}{2}, 0, 0\right)\ A_2\left(0, \frac{a + d_2}{2}, 0\right)$$

The distance between two skew lines is calculated as the mixed product, divided by the modulus of the vector product. The condition for the bars touching at contact point is that this distance is the same that the sum of the radii of the bars.

$$\frac{d_1 + d_2}{2} = \frac{\left|A_1A_2\cdot\vec{u}_1\cdot\vec{u}_2\right|}{\left|\vec{u}_1 \times \vec{u}_2\right|} = \frac{(a + d_1)\cdot cos\alpha\cdot sin\beta + (a + d_2)\cdot sin\alpha\cdot cos\beta}{2\sqrt{sen^2\alpha\cdot cos^2\beta + cos^2\alpha\cdot sen^2\beta + cos^2\alpha\cdot cos^2\beta}} \tag{11}$$

This expression does not make it possible to obtain the value of the size of the linkage a in an explicit way. Instead, it is very easy to calculate iteratively, with a simple computer program.

2.3. Fabric covering

One extremely important aspect in the construction of this type of structure is the design of self-folding systems for the textile roof. If this is not provided for, the roof may fold irregularly, which could cause the blades to close, causing a shearing effect and tearing the fabric. The system which has proved to be the most effective is the simple positioning of a band which is attached to the top knots and to the centre of each frame of the fabric, so that when the structure is folded, the top knots pull the fabric upwards and force it to fold in the centre of the folded module. Naturally, when the fabric is placed on the upper side the webbing is attached from the lower points of the grid.

The obligatory condition is that the sum of the lengths of the webbing and the fabric is equal to that of the folded package (Fig. 9). This is a simple geometrical condition that for regular modules is met provided that $d_1 + L_5 = L_1 + L_2$. The fabric section corresponding to the intermediate length $\mathbf{L_6}$ must be suspended in the folded position. This was done on the roof of San Paulo Pool in Seville and enabled compact packing without any particular difficulties [12] (Fig. 10). For irregular

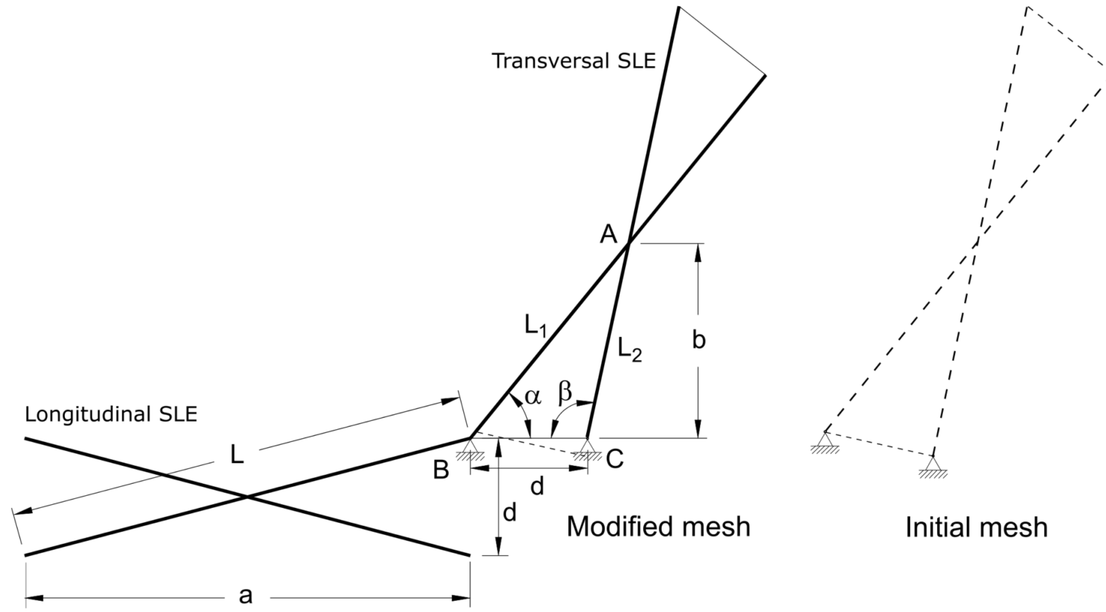

Fig. 7. Diagram of the edge adjustment.

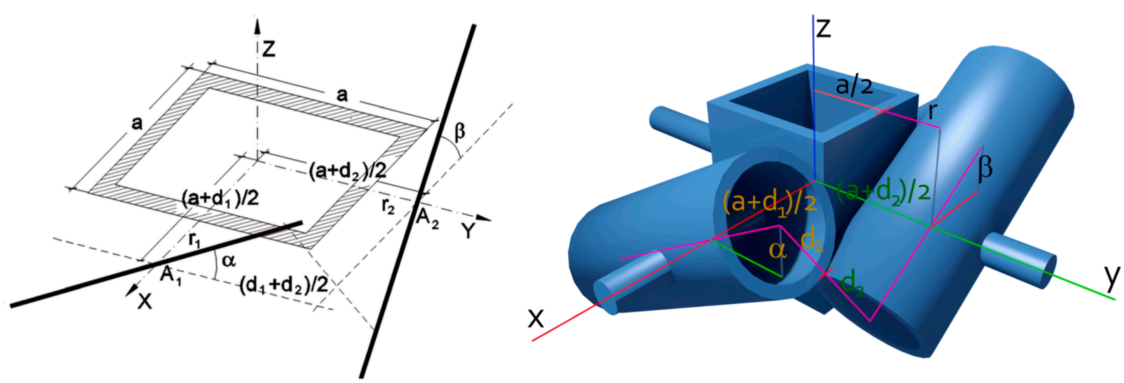

Fig. 8. Reciprocity condition for square grids.

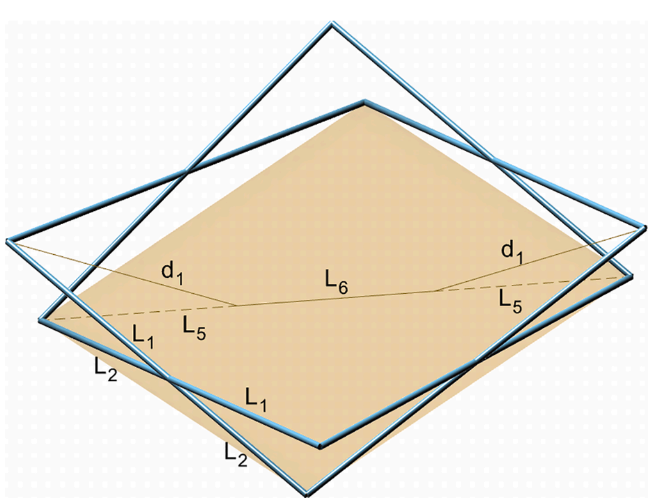

Fig. 9. Diagram of the textile folding system.

Fig. 10. Roof of San Pablo Pool folded.

modules it is easier to determine the lengths of the webbing by trial and error.

2.4. Calculation of grids with reciprocal linkages

This structure was calculated with software developed for this purpose. The program is called Despleg 19.1 and performs a matrix structural analysis [6,28,29]. It has been working well since 1990 and allows to consider non-linearity effects as well as joint eccentricity.

The degree of embedment can be calculated using the scheme in Fig. 11 by the Mohrs' theorem. We consider the bending of the support bar to be negligible, as it occurs very close to the linkage. We use **e** to refer to the distance from the bolt to the support point on the adjacent bar (equation [11]) and **L** as the length of the bar section (see Fig. 12).

Applying the turning conditions, the degree of embedding will be:

$$g_e = \frac{L}{L + e} \tag{12}$$

In a deployable structure, the value of e is much less than the length of the bar, which means very high degrees of embedding, of around 95%. Under these conditions, the equilibrium of the bar in space is defined by the forces in the three linkages, the bending moments due to the condition of reciprocity, and by the position of the local axes with respect to the global axes (Fig. 9). The full development of the rigidity matrix has been published in a recent article [33]. The results obtained for the deployable vaults are presented below.

Considering the existing loads, the bar stiffness equation in local coordinates results in where:

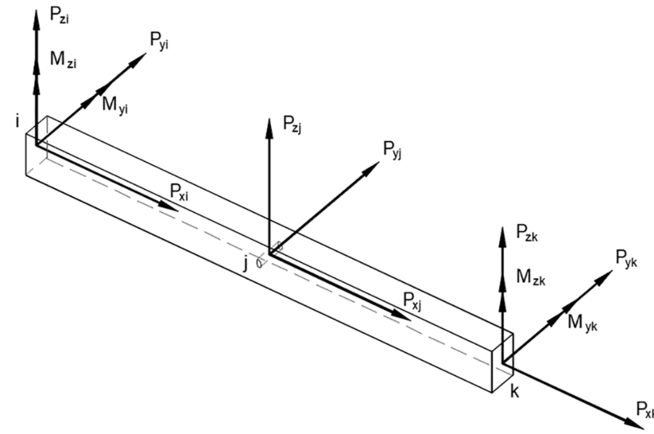

Fig. 12. Balance of a bar with three joints in space, with angled linkages at its ends.

The compatibility matrix $\widetilde{A}$ is simply the rotation matrix that corresponds to the change of axes. By applying the equilibrium equation, we obtain this system of equations to solve in order to calculate the loads and displacements of the structure.

The software Despleg 19.1 uses these values for the stiffness matrix. In section 3.1 it is indicated with which experimental models the numerical calculations have been validated.

Reciprocal links have the advantage of reducing the displacements of the structure by increasing the stiffness of the joints. Another advantage is that rigid connections reduce the bending moments in the bars, therefore the diameter of the bars can be reduced. This reduction in the values of the bending moments is shown in the attached figure. In 13a the bending moments in a bar with hinged ends have been drawn. Fig. 13b shows the effect of the moments generated by the reciprocal connections. It will therefore happen that thanks to the reciprocal links, some bending moments M_1 and M_2 will appear in the direction that has the largest values (vertical direction Z, or moments M_y), which means

$$K = \begin{pmatrix}
\frac{E\cdot A}{l_1} & 0 & 0 & -\frac{E\cdot A}{l_1} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{3\cdot E\cdot I_1}{\phi_1\cdot l_1^2\cdot L\cdot k_M} & 0 & 0 & \frac{3\cdot E\cdot I_1}{\phi_1\cdot l_1^2\cdot l_2\cdot k_M} & 0 & 0 & \frac{3\cdot E\cdot I_1}{\phi_1\cdot L\cdot l_1\cdot l_2\cdot k_M} & 0 \\
0 & 0 & \frac{3\cdot E\cdot I_2}{\phi_2\cdot l_1^2\cdot L} & 0 & 0 & \frac{3\cdot E\cdot I_2}{\phi_2\cdot l_1^2\cdot l_2} & 0 & 0 & \frac{3\cdot E\cdot I_2}{\phi_2\cdot L\cdot l_1\cdot l_2} \\
-\frac{E\cdot A}{l_1} & 0 & 0 & \frac{E\cdot A}{l_1}+\frac{E\cdot A}{l_2} & 0 & 0 & -\frac{E\cdot A}{l_2} & 0 & 0 \\
0 & -\frac{3\cdot E\cdot I_1}{\phi_1\cdot l_1^2\cdot l_2\cdot k_M} & 0 & 0 & \frac{3\cdot L\cdot E\cdot I_1}{\phi_1\cdot l_1^2\cdot l_2^2\cdot k_M} & 0 & 0 & -\frac{3\cdot E\cdot I_1}{\phi_1\cdot l_1\cdot l_2^2\cdot k_M} & 0 \\
0 & 0 & -\frac{3\cdot E\cdot I_2}{\phi_2\cdot l_1^2\cdot l_2\cdot k_M} & 0 & 0 & \frac{3\cdot L\cdot E\cdot I_2}{\phi_2\cdot l_1^2\cdot l_2^2\cdot k_M} & 0 & 0 & -\frac{3\cdot E\cdot I_2}{\phi_2\cdot l_1\cdot l_2^2\cdot k_M} \\
0 & 0 & 0 & -\frac{E\cdot A}{l_2} & 0 & 0 & \frac{E\cdot A}{l_2} & 0 & 0 \\
0 & \frac{3\cdot E\cdot I_1}{\phi_1\cdot L\cdot l_1\cdot l_2} & 0 & 0 & -\frac{3\cdot E\cdot I_1}{\phi_1\cdot l_1\cdot l_2^2} & 0 & 0 & \frac{3\cdot E\cdot I_1}{\phi_1\cdot l_2^2\cdot L} & 0 \\
0 & 0 & \frac{3\cdot E\cdot I_2}{\phi_2\cdot L\cdot l_1\cdot l_2\cdot k_M} & 0 & 0 & -\frac{3\cdot E\cdot I_2}{\phi_2\cdot l_1\cdot l_2^2\cdot k_M} & 0 & 0 & \frac{3\cdot E\cdot I_2}{\phi_2\cdot l_2^2\cdot L\cdot k_M}
\end{pmatrix} \tag{13}$$

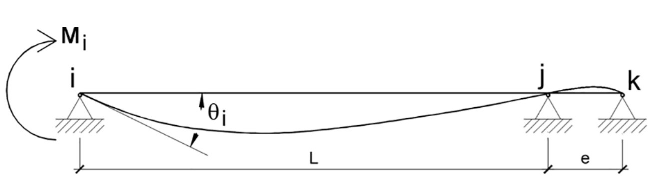

Fig. 11. Stiffness of the bar with reciprocal linkages.

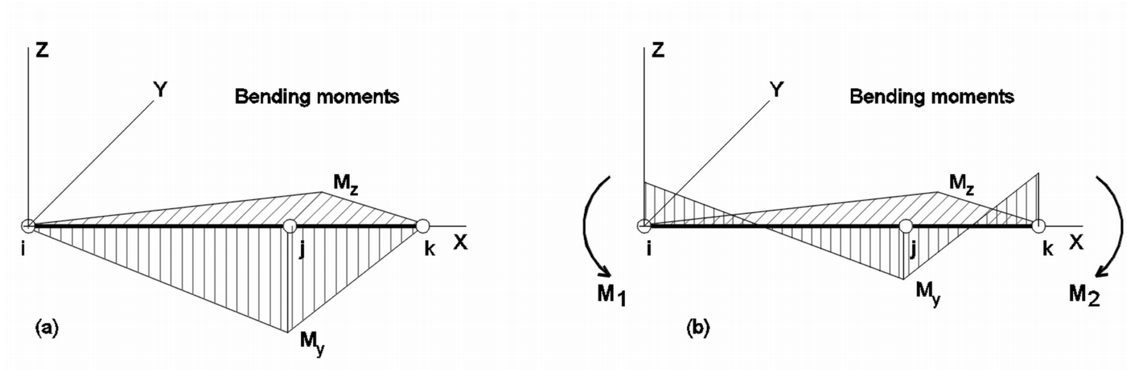

Fig. 13. Diagrams of bending moments for articulated and reciprocal linkages.

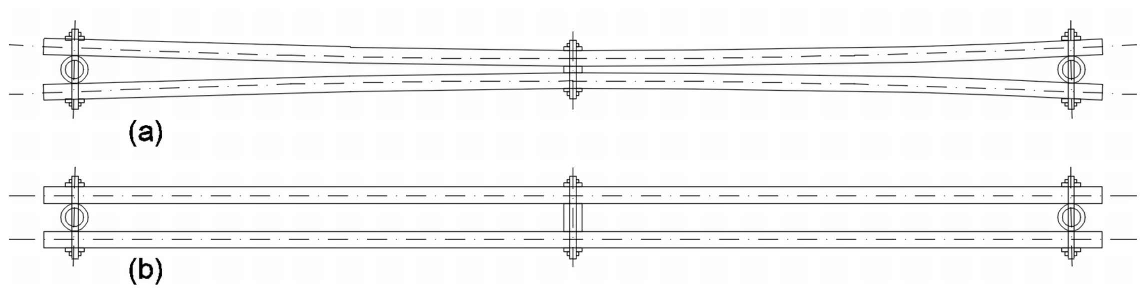

Fig. 14. Imposed curvature effect on SLE modules with reciprocal linkages.

that their values can be reduced in the order of half. The M_z moments do not vary.

2.5. Effect of the central through-linkages.

The condition of reciprocity in the linkage can be achieved with different types of linkages, currently under study, but the simplest is undoubtedly the one described in section 2.2. This is the type of linkage used in emergency structures, as it is particularly economic, which is an essential condition in this type of structure.

Its main problem is that the size of the linkage forces a displacement to be applied to the central through linkage, which causes bending in the bars (Fig. 14a). In order to solve this problem, the calculation model applies transversal forces that cause the adjustment of both bars. The method was proposed for the solution of triangular cylindrical vaults, which suffer from the same problem [13].

From a practical point of view it is preferable to place a separator of the same size as the linkage (Fig. 14b). This avoids bending, but instead introduces an eccentricity in the central linkage. In order to evaluate these effects, tests have been carried out with a cylindrical grid using flanged bars and with a central spacer, in order to be able to compare the results. These tests are described in section 4.

The construction of a real grid with imposed curvatures entails practical difficulties that are not apparent in the models. In a 1/h scale model the forces required to bend the bar are small, but in the real structure they are h^2 greater, as predicted by the laws of similarity, and assuming that the scale is respected in all elements and the material is the same. On the contrary, if these scale patterns are maintained, the previous stress is the same in the model as in the real structure. The use of bars with imposed curvatures does not pose a major problem in terms of resistance, but it does pose a serious construction obstacle.

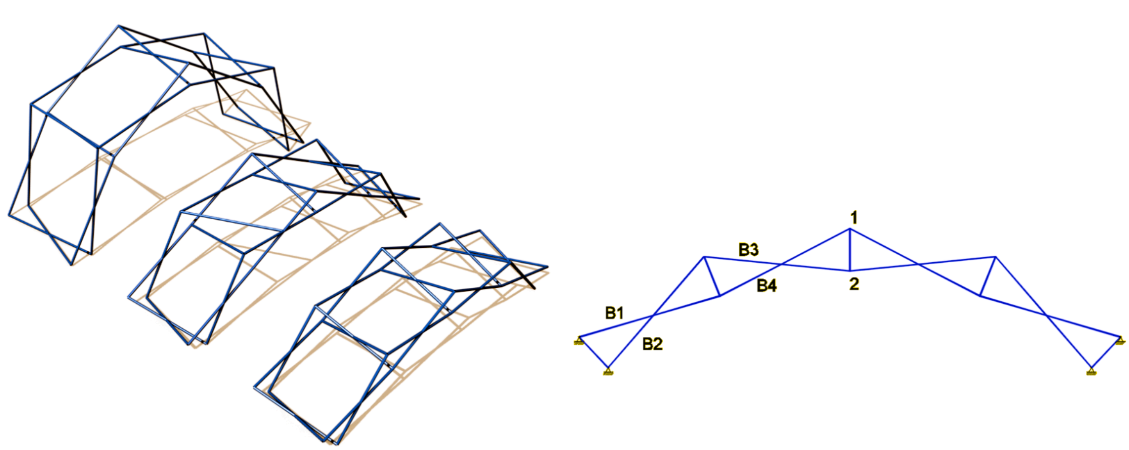

Fig. 15. Models with cylindrical vaults of the same span and variable height.

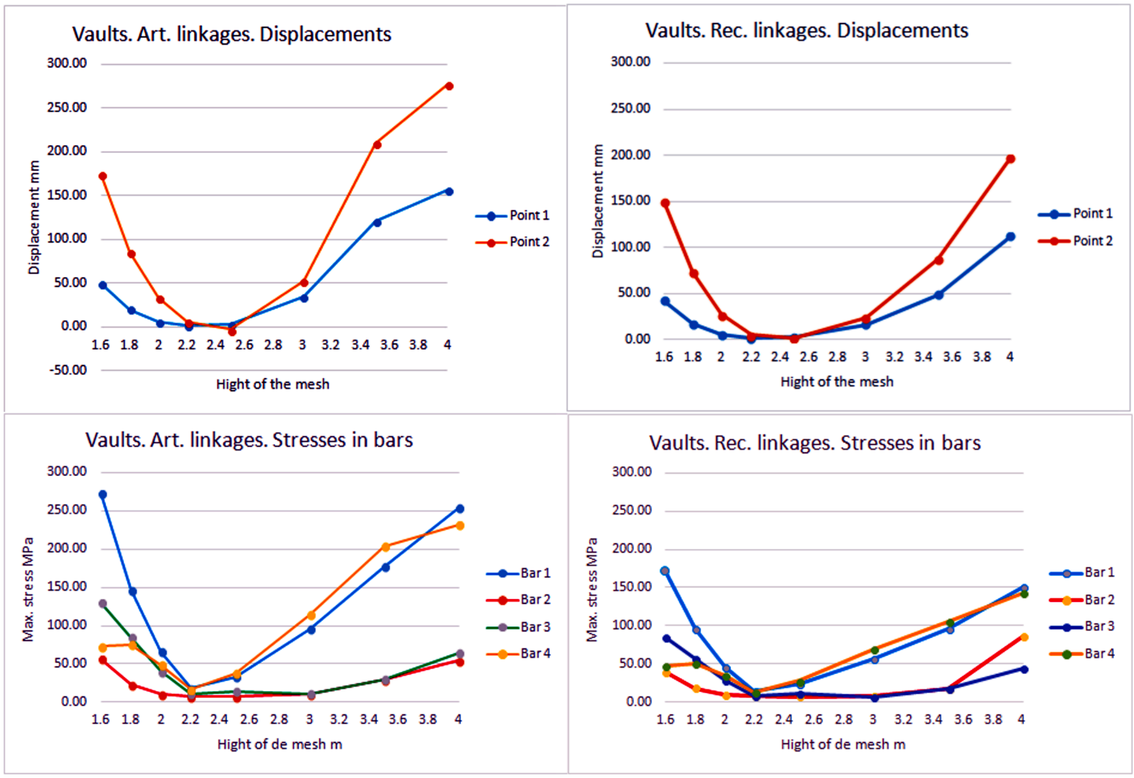

Fig. 16. Displacements and maximum stresses in the bars analyzed. Comparative between articulated linkages and reciprocal linkages.

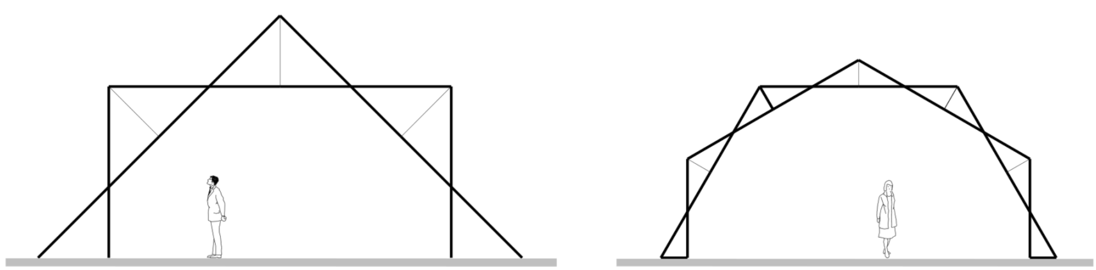

Fig. 17. Cylindrical grids with extension bars.

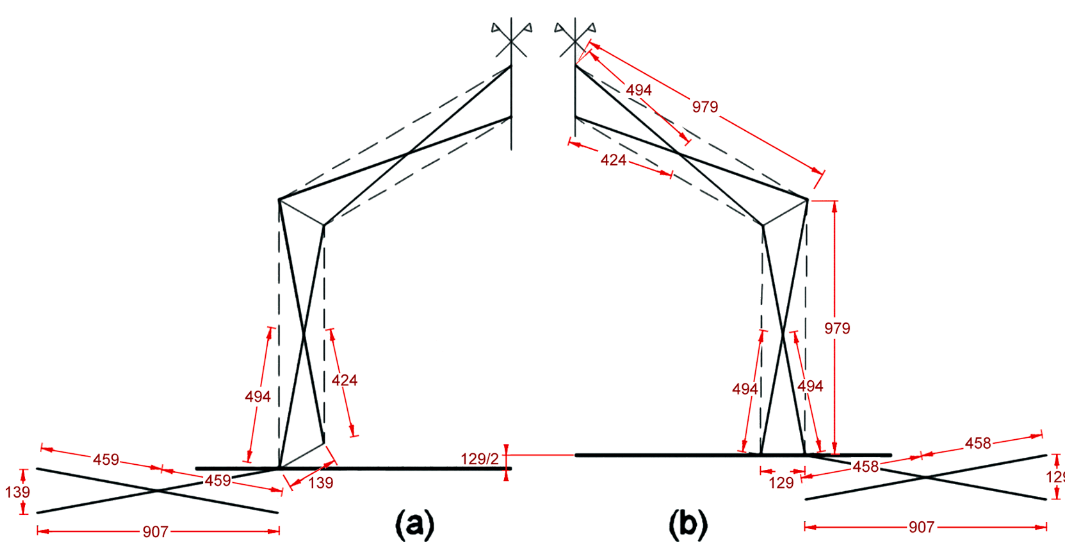

Fig. 18. Deployable structure with gable roof. Hexagonal pattern.

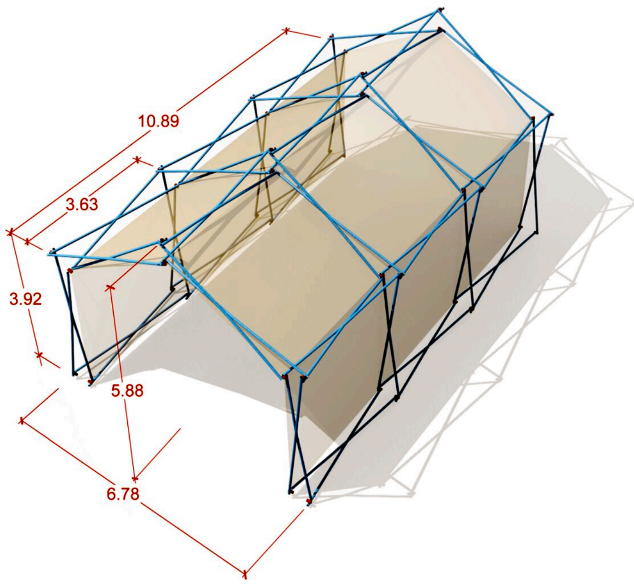

Fig. 19. Model of the designed structure with gable roof. Units in meters.

2.6. *Application to emergency structures.*

The use of cylindrical grids in emergency buildings has some special features. As mentioned above, this typology has been the subject of numerous studies, but many of the solutions proposed are unsuitable for this use. The main characteristics that a vault should have for an emergency building are:

- The general characteristics of deployable structures, i.e. lightness and foldability.
- Minimising the number of linkages. The linkages are the most expensive elements of the assembly and contribute a considerable percentage of the total weight. The best solutions for these buildings are obtained with larger modules, even if they require larger sections. The most suitable solutions involve bar lengths of between 3 and 6 m.
- The enclosure should be included in the bar package so that the structure is quickly usable.
- It must allow for the simple installation of a floor, which if possible serves as a resistant element against horizontal forces, so that it only transmits vertical forces to the foundation.
- The foundation must be as simple as possible, as it has to be built on site. It is therefore highly recommended that the vault starts from ground level.

In general, the best performance of folding cylindrical vaults is achieved when the edge linkages are fixed to the foundation. To analyse which structures are the most efficient, a study was carried out for cylindrical vaults with 8 m spans and variable heights (Fig. 15). For all cases, 6060-T5 aluminium tubes of 40.4 were considered. The displacements and maximum stresses in the bars indicated in Fig. 15 were calculated assuming articulated linkages and reciprocal linkages (Fig. 16).

These results show that reciprocal linkages provide an improvement in both movement and effort on the bars, which confirms previous studies with other typologies [33]. The graphs show a very clear minimum when the bars of two contiguous SLE are in extension. This is the

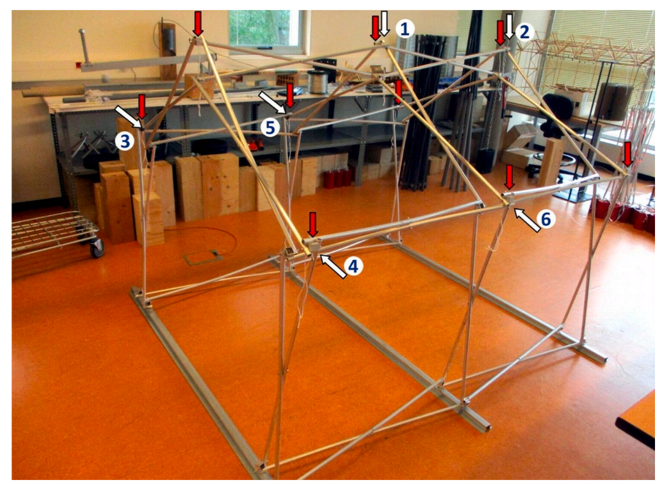

Fig. 21. Test model with the position of the displacement sensors (white arrows) and the applied loads (red ones). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

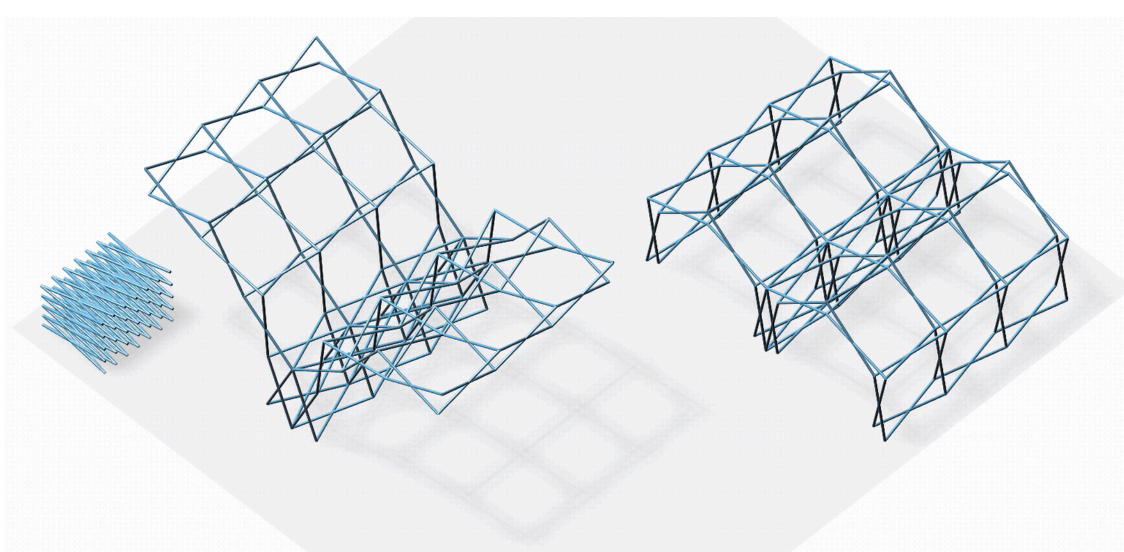

Fig. 20. Combination of two modules and deployment process.

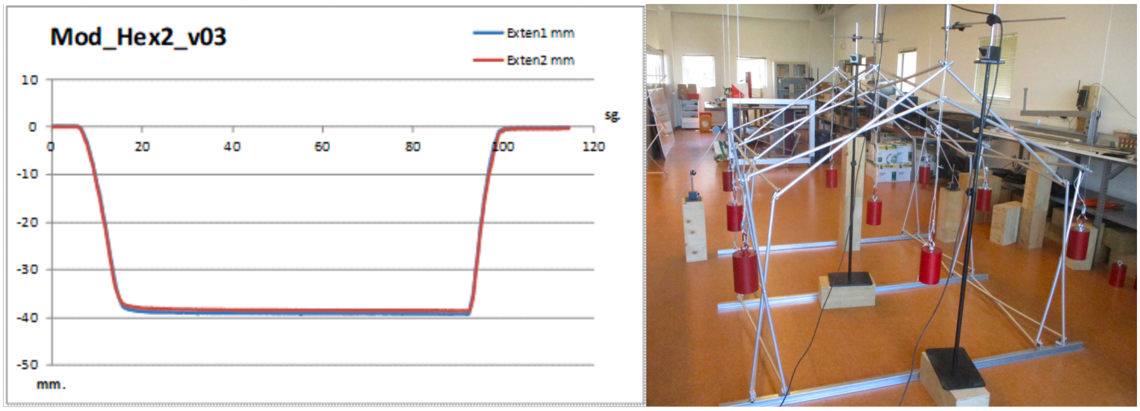

Fig. 22. Testing a curved grid made of SLEs showing the position of the displacement sensors.

optimum opening for the mesh, but is not necessarily the best solution. It is very often necessary to go beyond this optimal point to achieve a larger habitable space (Fig. 17).

One interesting solution that will be analysed in detail in this article is the grid described in Figs. 18 and 19. This grid can be opened to the point where the lower bars go beyond the point of extension between them, forming an enclosure that integrates the walls and roof in a single folding grid. As the upper linkages are angled, they cannot go beyond this position and the mesh can only be folded in the correct direction. This was a serious disadvantage in folding structures with articulated linkages, which the proposed linkage easily solves.

If all the bars are the same, the proposed structure would have the profile of a regular hexagon, so the edge would be inclined 30° (Fig. 18a). It is possible, but it would force a more complex base solution. It is more convenient to modify the lengths of the lower SLEs to ensure that the two linkages are supported by the foundation at the ground level, maintaining the folding conditions following the procedure described in 2.1 (Fig. 18b). In the aforementioned figure the dimensions of the tested model are indicated, which is a 1/4 scale model of the proposed prototype.

The proposed system allows two structures to be combined so that they can be deployed together. This makes it possible to resolve a complex detail such as the roof valley. It is possible to attach new modules without problems, but then this detail must be solved on site. The proposed grouping allows for areas of considerable lightness and strength.

As mentioned above, it is very important to provide a simple system for constructing the floor of the building. In a cylindrical vault, the edge supports are usually on two levels, which means that more complicated foundation solutions are required. The best solution is to ensure that the supports are all at ground level. The problem can be easily solved as indicated in 2.1. This involves adding a new condition to the design of the grid, but allows the support elements to be used to hold floor panels (Fig. 21).

This model was built on a scale of 1:4 of the actual design. The supports are omega profiles to which the lower linkages of the grid are fixed and which serve as a support to install the panels that form the floor of the building between them. The textile cover is placed inside the grid, which allows for a very simple solution for the linkage between the folding structure and the fabric (see Fig. 22).

3. Materials and methods.

3.1. Test elements

Reciprocal linkages have been tested for flat meshes [36] or oblique modulus meshes [37] with good performance. In this case, it is a question of checking the effectiveness of the reciprocal linkages for deployable cylindrical vaults. A vault based on the hexagonal module has been chosen for its usefulness to be used as an emergency shelter, despite not being the most structurally effective. For this, the model shown in Fig. 21 was carried out and tested. Two types of tests have been carried out. Firstly, the model was built with the internal through linkage forcing the imposed curvature of the bars of each SLE (Fig. 14a). Washers were then placed between the bars to prevent the imposed bending and the tests were repeated (Fig. 14b). The results allowed both solutions to be contrasted.

These modules were tested on a load bench in order to measure the displacements. Loading-unloading cycles were carried out, and the results were compared with the theoretical ones in order to validate the calculation model.

3.2. Materials

The bars of the test model are aluminium tubes type 6060-(aluminium - magnesium - silicon) state T5 with Ø16 mm and a thickness of 1.9 mm (Fig. 20). They have a specific weight of 2700 kN/m^3, an elastic modulus 69500 N/mm^2, an elastic limit of 185 N/mm^2 and a breaking load of 220 N/mm^2. The aluminium used corresponds to a guaranteed product, so the data is provided by the manufacturer. However, control tests have been carried out that have confirmed these parameters.

The linkages are made of aluminium hollow tube sections (SHS) of the same quality and performance. They have a lateral measurement of 20 mm sides, 1 mm thickness and a height of 20 mm. The pivots are made of 4 mm threaded steel rods that are welded in the middle. The screws and threaded rods are made of 5.6 grade steel according to ISO 898–1. They have an elastic modulus of 200,000 N/mm2, an elastic limit of 300 N/mm^2 and a breaking load of 500 N/mm2 with an elongation of 20%.

3.3. Manufacturing of the component elements

To verify the validity of the proposed links, a 1:4 scale model was constructed with the bars and links described. The length of the bars must be carefully designed, especially if the edges of the grid are to be at ground level. The ratio between the diameter of the linkage and the diameter of the D/d bar is always the same, so all the linkages are equal. For all these reasons, their manufacture and assembly are relatively simple.

The model tested was composed of eight square modules formed by SLEs. The distances between the rotation axes were 424 and 494 mm in the transverse SLEs, and 458 mm in both sections of the longitudinal SLE. In the case of reciprocal linkages, the bar was prolonged by 30 mm from the extreme pivot axis. The overall dimensions of the model were 1696 (width) × 2722 (deep) mm, and its maximum height was 1469 mm. The linkages measured 20 mm and the bars had reciprocal support in the upper layer and articulated support in the lower layer. With the dimensions indicated, the angles that converge in the linkage are 37.287° in the transverse direction and 8.764° in the longitudinal direction. This means that the relationship between the width of the linkage and that of the bar is a/d = 1.766, giving a figure of 28 mm. In this case, a 20 mm linkage was used and the distance was adjusted with washers. Firstly, this model was built by applying a curvature imposed on the central linkage to the SLEs. Subsequently, spacers were added to the model on these links to carry out further tests.

The tests were carried out in the structures laboratory of the School of Architecture in A Coruña with a test bench of our own design. Nine 10 kgf (98.1 N) loads were applied to the top layer linkages in the linkages pointed out in Fig. 21. The displacements were measured with Schreiber's Sm407.100.2.T inductive displacement sensors with a linearity of <0.25% and a deviation of <0.01%/°C. The data collection was completed with Y103 digital extensometers with an accuracy of ±0.1 mm. The displacements were measured in the centre of the mesh and on one of its edges in the upper and lower linkages.

The model was fixed on omega profiles to which the lower linkages of the inner and upper layers were bolted. These profiles allow horizontal forces to be balanced, so that only vertical loads are transferred to the ground. In the tests it was not necessary to fix the structure to the ground.

The point loads applied are equivalent to a distributed load of 30 kN /m^2. These loads are similar to those of a real structure of these characteristics, but the aim of the test is to validate the calculation model, so any load could be used. It is only necessary that the same loads are applied in the calculation model, in order to be able to compare the results.

3.4. Organisation of tests

Firstly, model 1 was tested to verify the grid with reciprocal linkages with imposed curvatures. Model 2 was then tested by fitting a washer of sufficient width to keep the bars straight.

Like mobile structures, deployable structures have tolerances to allow the unfolding process. Therefore, when they are subjected to loads, they have a certain degree of readjustment that is not due to the deformation of the material, but to these tolerances. For this reason, a preload process was previously performed on all tests that helped consolidate these preset movements.

Otherwise, the adjustment movements would overlap with the elastic deformations, which would distort the measurements taken. For this reason, it is considered very important to perform this preloading process before performing any type of motion control.

4. Results

4.1. Theoretical model

The models tested have been verified using the Despleg19.1 software, which uses the structural analysis matrix. In the case of the grid with reciprocal linkages, the programme assigns to the upper end links of the bars a degree of embedding that is entered as data. The lower linkages are articulated so that the degree of embedding is zero. For the central linkages of the rods, the programme assumes a through linkage: the movements of both points connected to the linkage are equal, but the bars can rotate freely in the plane of the SLE.

It has been calculated that in the tested models the reciprocal connections provide a degree of embedment of 96.92%. But this effect will only occur when the bar finds reciprocal support. In the case of links with four bars, this value can be applied to all the bars, which in the built models only happens in the internal linkages of the upper layer. In the case of edges and corners, reciprocal support only applies to those bars that are supported by others. There is also no reciprocal support in any bars at the joints of the inner layer. In all these cases a degree of embedment of 0% was considered.

As already indicated, these structures require certain tolerances. In the models used, the diameter of the bolt is 4 mm, while the diameter of the hole of the bar is 4.2 mm, which determines a tolerance of 0.2 mm that is considered as the initial movement of the bars, without causing any effort.

Constraints are applied in order to consider the effect of the lower support bars. The loads are those of the real model and the results refer to the application points of the sensors.

4.2. Experimental results

The tests were carried out with the loads arranged in the nodes of the upper layer and with the displacement sensors at the points indicated in Fig. 21. A progressive load input was carried out for about 15 s. The load was maintained for a sufficient period to stabilise the displacements between 20 and 95 s, and then progressively discharged for 5 s. Two tests were carried out for each of the cases.

In summary, the results of the test carried out are as indicated below:

The theoretical results have been compared with the experimental ones. The calculation program considers the eccentricities at the three nodes of the structure so it can easily calculate the model with straight bars. It can also calculate models with imposed curvatures, such as those used in triangulated cylindrical vaults [13]. However, it is designed for bars that bend on deployment and not those that are already curved initially, so these results would not be accurate. On the other hand, the difference in these bars, which have a reduced section, is very small, which has been confirmed with the experimental results.

5. Discussion

Considering the minimal deviation of the results measured in the tests, it was considered unnecessary to carry out more trials. In addition, two other test trials with very similar results had previously been carried out, although as they were previous trials they were not recorded.

In the tests carried out, the vertical displacements at ridge links 1 and 2 and the horizontal displacements at points 3, 4, 5 and 6 were measured.

The efficiency of the reciprocal linkages has already been analysed in a theoretical and experimental study by the authors [35] and also its application to different structural types such as flat grids [36] or skew meshes [37]. The main objective of the trials carried out was to check different types of cylindrical deployable grids with different sizes and proportions and to check whether the modification of the intermediate through linkage using curved or straight bars, has a significant influence on the results of the tests. If it could be confirmed that it has little influence, it would allow for a simpler construction and assembly of this type of structure, as it would not be necessary to introduce prior bending moments in the grid.

As can be seen in Table 2, the differences are very small. In the central zone of the grid, the displacements of the model with bars without initial curvature are slightly greater, 0.19% in the vertical displacement of point 1 and slightly less, 8.12% in the horizontal displacement (points 5–6). The pattern is inverted in the edge plane, in which the vertical displacement decreases by 5.17% (point 2), while the horizontal displacement increases by 8.07% (point 3–4).

In both cases the differences are small, which confirms the validity of the constructive simplification that involves building the structure with straight bars without forcing the bars with imposed curvatures. In these tests, the displacements of the links in the upper layer are very precisely adjusted to the expected specifications. They are precisely the linkages where the loads have been applied. On the contrary, the linkages in the lower layer have displacements with greater dispersion, which we consider to be due to the fact that the movement of the linkage is not so closely adjusted because it is not loaded.

Table 2
Results of test.

Curved bars	Point 1 Displ. mm	Point 2 Displ. mm	Point 3 Displ. mm	Point 4 Displ. mm	Point 5 Displ. mm	Point 6 Displ. mm	Displ. 3–4 Displ. mm	Displ. 5–6 Displ. mm
Test 1	38.45	39.33	11.12	14.61	17.00	12.99	25.73	29.99
Test 2	39.25	41.38	11.41	16.28	17.79	14.44	27.69	32.23
Averaged	38.85	40.36	11.27	15.44	17.40	13.72	26.71	31.11
Straight bars	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Displ. 3–4	Displ. 5–6
Test 1	39.11	38.45	10.15	18.67	13.47	15.34	28.82	28.81
Test 2	38.74	38.09	10.10	18.81	13.12	15.24	28.91	28.36
Averaged	38.93	38.27	10.13	18.74	13.30	15.29	28.87	28.59
Increase	0.19%	−5.17%					8.07%	−8.12%

Table 3
Comparison between theoretical and experimental results.

Straight bars	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6	Displ. 3–4	Displ. 5–6
Averaged	38.93	38.27	10.13	18.74	13.3	15.29	28.87	28.59
Theoretical	34.18	34.11	19.13	19.16	19.13	19.16	38.29	38.29
Ratio	113.90%	112.20%					75.40%	74.67%

In Table 3 it can be seen that the comparison between experimental and theoretical results shows reasonable agreement. The vertical displacements in the upper nodes are greater in the tests than in the theoretical model, while the horizontal displacements are smaller. This is attributed to the natural imperfections of the model that cause the stiffness of the reciprocal nodes of the ridge to be overestimated and that of the lateral nodes to be underestimated. In the calculation model, it had been considered that in these lateral nodes the reciprocal support only affects in the vertical direction, but it has been observed that it contributes a not negligible, but difficult to quantify, stiffness to the lateral movements. This has suggested the realization of new tests to assess this horizontal stiffness, which is of great importance to resist the effects of the wind, which are being carried out.

6. Conclusions

The results (both analytical and experimental) show us that the proposed system of creating reciprocal linkages at the ends of the bars is effectively applicable to cylindrical vaults. This system is also clearly more efficient than the usual articulated linkages since it reduces the stresses and deformations of the structure. Another added advantage, and no less important, is that the deployable structures present a stable final position, without the need to add auxiliary reinforcing elements.

The construction of this type of grid requires a curvature to be imposed on the central link or a separation element to be installed which introduces a certain eccentricity into the linkage. The experimental results show that the difference in terms of resistance between the two options is minimal. This makes it possible to effectively construct these cylindrical grids with straight bars, which implies greater simplicity and economy.

Their structural behaviour is excellent, as shown by the results obtained. The diagrams are practically linear and there is hardly any residual deformation. Furthermore, this residual deformation is congruent with the readjustment process of a deployable structure under load. In all the tests, previous load steps were carried out so that this readjustment would not distort the measured results.

The reciprocity of the end linkages determines a degree of embedding that exceeds 90% for the usual dimensions, which allows for less deformation of the whole. In addition, this high degree of embedding at the ends of the bars improves the bending behaviour, which is a determining factor in the dimensioning, so that it is possible to reduce the section of the bars. The result is a simpler, more economical and better performing structure.

The use of reciprocal linkages prevents the angle between the bars from exceeding the angle of extension of the bars. This means that the structure can only be folded in the correct direction, thereby facilitating the process of folding or deploying the system. On the other hand, the tests carried out with the models of deployable cylindrical vaults show that the reciprocal node provides a very useful rigidity against horizontal displacements in this type of structures, which with articulated nodes are excessively deformable and require brace. This significantly improves its performance.

The proposed structures are valid solutions for emergency buildings. They make it possible to build stronger and especially more rigid vaults with smaller sections, which are consequently lighter and easier to transport. They therefore represent a very appreciable advantage in the design of this type of building.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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