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Computation of Resonance Modes in Open Cavities with Perfectly Matched Layers †

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Abstract: During the last decade, several authors have addressed that the Perfectly Matched Layers (PML) technique can be used not only for the computation of the near-field in time-dependent and time-harmonic scattering problems, but also to compute numerically the resonances in open cavities. Despite such complex resonances are not natural eigen-frequencies of the physical system, the numerical determination of this kind of eigenvalues provides information about the model, what can be used in further applications. The present work will be focused on two main specific goals—firstly, the mathematical analysis of the frequency-dependent highly non-linear eigenvalue problem associated to the computation of resonances with the standard PML technique. Second, the implementation of a robust numerical method to approximate resonances in open cavities.

Keywords: exterior acoustics; perfectly matched layers (PML); open cavities

1. Introduction

The calculation of resonances has been always relevant in applied acoustics as well as in the research field. Its application is not only very extensive but critical, for instance, in the study of many mechanical systems. Moreover, the solution to this kind of problems leads to some singularities that keep different research lines open in this field. More specifically, the eigenvalue problem in exterior acoustics has been deeply studied during the last years, due to its difficulty compared to its application over bounded domains (see for instance [1,2]). For instance, the computation of leaky modes in an open waveguide can be used as the spectral basis for a modal decomposition technique [3].

There are two main approaches to this type of problem. The first one includes the boundary element method (BEM), based on the solution by surface integration over an interior boundary. This technique has certain computational advantages due to the dimensional reduction of the computational domain, but requires very sophisticated numerical integration techniques, as analyzed in Reference [4]. Another known group of methods that has become relevant in the last years is based on the discretization of the infinite domain. Examples of this type of approach are the combination of Finite Elements Method (FEM) with Infinite Elements Method (IFEM) [5,6], as well as with Absorbing Boundary Conditions (ABCs) [7] or Perfectly Matched Layers (PML) [8–10].

All these techniques are based on the division of the problem into two different domains, thus being able to work with an interior and an exterior one. The exterior one simulates the conditions of an infinite domain around the region of interest, contained in the interior one. The main difference between these types of methods lies in the way that this second domain is handled. This work will be focused on the use of PMLs, which consists in surrounding the interior region of interest with some sponge layers which couple perfectly without generating any spurious reflections. The key ingredient of the PML technique is the definition of an adequate absorption profile which allows to damp the outgoing waves travelling at free field regime from the interior region of interest [8]. So, this type of exterior PML domain is characterized by its thickness and the associated absorption profile. In [10] a variety of scenarios are analyzed, showing that the numerical performance of the PML technique depends clearly on the data set, the mesh and the geometry of the problem.

2. Expected Results

The proposed work will be devoted to two main goals. First, the mathematical analysis of the PML technique involving non-integrable absorption profiles will be faced, to analyze whether the computed eigenvalues are independent of the thickness of the PML, as shown in Reference [9] for source problems. Secondly, a robust numerical method will be implemented in order to avoid the instabilities presented by the use of finite element methods, as shown in Reference [1], for resonance calculation in open cavities. The proposed method will try to avoid dependencies with respect to the PML thickness, as well as the position of the inner PML boundary, which is shared with the physical interior region of interest. More precisely, a finite element discretization (or other high order methods) will be implemented to illustrate the numerical behavior associated with different discretizations used in the computation of resonances. The observed numerical instabilities in the different discrete methods will be analyzed, taking into account that the linear matrices involved in the discretization are not hermitian. Once the origin of such instabilities are identified, the current state-of-the-art non-linear eigenvalue techniques will be used to design a robust methodology.

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References

1. Hein, S.; Hohage, T.; Koch, W.; Schöberl, J. Acoustic resonances in a high-lift configuration. *J. Fluid Mech.* **2007**, *582*, 179.
2. Aguilar, J.; Combes, J.M. A class of analytic perturbations for one-body Schrödinger Hamiltonians. *Commun. Math. Phys.* **1971**, *22*, 269–279.
3. Dhia, A.B.B.; Goursaud, B.; Hazard, C.; Prieto, A. Finite element computation of leaky modes in stratified waveguides. In *Ultrasonic Wave Propagation in Non Homogeneous Media*; Springer: Berlin/Heidelberg, Germany, 2009; Volume 128, pp. 73–86.
4. Kress, R. Boundary integral equations in time-harmonic acoustic scattering. *Math. Comput. Model.* **1991**, *15*, 229–243.
5. Astley, R. Infinite elements for wave problems: a review of current formulations and an assessment of accuracy. *Int. J. Numer. Methods Eng.* **2000**, *49*, 951–976.
6. Moheit, L.; Marburg, S. Convergence of modes in exterior acoustics problems with infinite elements. In Proceedings of the 45th International Congress and Expositionon Noise Control Engineering, Hamburg, Germany, 21–24 August 2016.
7. Turkel, E. Introduction to the special issue on absorbing boundary conditions. *Appl. Numer. Math.* **1998**, *4*, 327–329.
8. Berenger, J.P. A perfectly matched layer for the absorption of electromagnetic waves. *J. Comput. Phys.* **1994**, *114*, 185–200.
9. Bermúdez, A.; Hervella-Nieto, L.; Prieto, A.; Rodrı, R. An optimal perfectly matched layer with unbounded absorbing function for time-harmonic acoustic scattering problems. *J. Comput. Phys.* **2007**, *223*, 469–488.
10. Bermúdez, A.; Hervella-Nieto, L.; Prieto, A.; Rodríguez, R. An exact bounded perfectly matched layer for time-harmonic scattering problems. *SIAM J. Sci. Comput.* **2008**, *30*, 312–338.