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Experimental, analytical and numerical vibration analysis of long-span timber-timber composite floors in self-tensioning and non-tensioning configurations

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ABSTRACT: An experimental campaign was carried out on the dynamic behaviour of timber-timber composite pieces under different loading conditions. This study compares the behaviour of non-tensioning and self-tensioning configurations under different stages of loading. The results show that the presence of an un-bonded prestressed bar hardly alters the eigenfrequency value in comparison with non-tensioned solutions, although it significantly reduces the damping ratio. An analytical methodology is presented that makes it possible to predict dynamic behaviour in terms of eigenfrequency with great exactitude. The simplicity of factors required by the analytical technique means that this method can be used as a design tool. FEM models were developed for each test configuration, and the different load states were analysed. The results show an extremely precise match with those obtained in the test campaign, indicating that this numerical methodology is suitable for the dynamic analysis of these structural elements.

KEYWORDS: Dynamic; Tensioning; Long-span; Experimental; Analytical; Numerical.

HIGHLIGHTS:
- A test campaign was developed determing dynamic properties of timber pieces.
- The study covers non-tensioning and self tensioning timber-timber configurations.
- The study covers highly slender long span pieces under different loading conditions.
- A minimum difference between non-tensioning and self-tensioning was detected.
- A simple and precise analytical method to predict eigenfrequency is presented.
- A FEM numerical analysis was developed, with a very accurate fit with test results.

1. Introduction

All structural systems have to behave appropriately when subjected to dynamic actions that may cause vibrations that could lead to damage of some kind or affect user comfort. The first case includes minor damage to structural or non-structural elements, while the second corresponds to occupant perceptions. The suitability of structural elements is therefore an interaction of objective and subjective factors, the dynamic properties of the element and human perceptions of vibration.

Vibration is treated in all international structural and building standards as a serviceability limit for the functioning of structural members under normal use, or in terms of personal comfort, with special attention to floor behaviour. Lightweight and long span floors are potentially sensitive to vibration. A common way of approaching this issue in the design phase of structural systems is to adopt certain considerations in relation to the natural frequency of the system or structural elements. Thus the natural vibration frequency of structural members should be kept above appropriate values, which vary with the function of the building and the source of the vibration. Vibration problems increase dramatically with increasing span, and the natural frequency of elements displaces to lower values. This is even more important if we take into account the fact that humans are more sensitive to low frequencies than they are to high ones.

The sensitivity of floors to these dynamic actions becomes a serviceability-critical condition when high strength materials in efficient configurations that make long spans with minimum sections possible are used. The slenderness of these new solutions requires specific studies to determine their dynamic properties and suitability, according to codes and standard requirements that have gradually incorporated vibration in relation to human comfort.

This paper analyses the behaviour of a timber-timber composite system (TTC) with a ribbed-panel section. This T section is composed of glulam ribs rigidly connected to an upper cross-laminated timber (CLT) flange by glued perforated steel plates. A self-tensioning system was placed on both ends of the piece. This multiplier device, patented as SsS® [1], introduces an eccentric post-stressing force on the T-section, transforming the vertical loads transmitted to the supports into a horizontal tensioning force. The configuration of the device multiplies the load value, forming a mechanism that significantly improves the bending behaviour of the structural element and reduces its deflection under any load arrangement [2,3]. The effectiveness of the system allows highly slender sections and a significant reduction in self-weight in relation to maximum admissible load, which is more important in long-span situations. Vibration behaviour analysis of the floor is therefore more relevant in terms of serviceability limitations and human comfort.

2. Serviceability criteria for structural vibrations

The complexity of this problem includes factors such stiffness, mass or the damping properties of elements, different types and magnitudes of response acceleration or the nature of excitation. This means that there are no common design rules. So far the standards for floor structures only make recommendations for estimated eigenfrequency limits, depending on the structural material and the fundamental use of the building in question.

One of the most widely accepted starting points corresponds to the Ohlsson proposal [4] in relation to human action. This assumes that for natural frequencies higher than 8 Hz, the low frequency component of human walking tends to produce movements that can be considered semi-static. Controlling static deflection therefore becomes a decisive factor. On the other hand, footstep impacts excite higher frequency components, and floor response is determinate by stiffness, mass and damping ratio. In relation to these considerations, standard EN 1990:2003 [5] determines that in order to achieve satisfactory vibration behaviour of buildings and their structural members, it is necessary to adopt certain considerations in relation to their natural frequency. For timber structures, EN 1995-1-1:2004 [6] suggests considering residential floor vibrations by using three conditional verifications. First of all, the natural frequency (f_1) should be at least 8 Hz; secondly, the deflection (w) due to a single force (F) should be less than a varying value (a), where $a \geq 2.0$ shows poor performance and a $\leq$ 1.0 shows better performance; thirdly, the maximum initial value of vertical floor vibration velocity (v) caused by an ideal unit impulse (1 N) applied at the point of the floor where response is maximum must be determined by a relationship between fundamental frequency, modal damping ratio (ξ) and a parameter (b) that indicates poorer performance at values lower than 80 and better performance at values higher than 120. In any case, special research is required for residential floors with a fundamental frequency lower than 8 Hz. Nevertheless, this standard does not provide guidelines for this research.

International Standard ISO 10137:2012 [7] presents the principles for predicting vibrations at the design stage, as well as for assessing vibration acceptability in existing structures. This standard sets modelling criteria for the vibration analysis of specific structural elements such beams and floors. Vibration modes and their associated frequencies and damping values are required to predict vibration behaviour in beams and floors. This standard categorizes floors into two classes, low-frequency floors with a natural frequency (f_O) less than approximately 8 to 10 Hz, and high-frequency floors with a higher f_0. In this case, a damping value for the fundamental mode in floors depending on material composition is proposed. Wood joist floors with spans from 2 to 9 m. present a typical damping ratio (ξ) from 1.5 to 4.0 %, with an extreme range from 1.0 to 5.5% and a preliminary design value of 2.0%. In the case of human occupancy, the variable usually measured is acceleration. However, as acceptable vibration levels vary with frequency of motion, it is necessary to filter acceleration. This standard refers to ISO 2631-1:1997 [8] and ISO 2631-2:2003 [9] in which an evaluation method using root-mean-square acceleration (RMS) is provided to evaluate human comfort when exposed to whole-body vibration. The appropriate filters are given in ISO 2631-1:1997 for situations where the critical vibration direction is specified, or ISO 2631-2:2003 if the critical direction is unknown. Regarding structural damage, analysis indicates that the most common indirect method of assessing dynamic loading is the measurement of particle velocity or, less often, measuring acceleration or displacement. Correlation with observed damage then provides the necessary link between the limit state and the criterion employed. Some national structural codes such as DIN 1052 [10] or NBCC 2015 [11] specify the deflection limit due to load conditions, while others, such AS 1720.1 [12] in combination with AS 2670 [13] provide acceptability limits in the form of RMS. However, they do not provide analytical equations for floor design in terms of vibration.

In addition to the approximate rules proposed by certain standards, some researchers have proposed guidelines that take into account different analytical methodologies and weightings of the factors that intervene in the problem, with a special focus on timber solutions. Some authors, such as D. Allen and T. Murray, [14] consider peak acceleration due to walking; A. Al-Foqaha'a et al. [15] consider RMS-acceleration combined with fundamental frequency; L. J. Hu and Y. H. Chui [16] focused their proposal on deflection combined with fundamental frequency, while A. Talja et al. [17] analyzed acceleration in combination with deflection and slope. This was complemented by T. Toratti and A. Talja [18] with an acceleration criterion for low frequency floors, which they considered with a 10.0 Hz limit. P. Hamm et al. [19] consider that the frequency criterion alone is insufficient, so they add a stiffness criterion and an acceleration limit in connection with floor demands.

More recently, numerical analyses have been developed that use a finite element model (FEM) to predict fundamental frequencies and dynamic properties in timber floors and composite timber solutions. Filiatrault et al. [20] developed one of the first finite strip methods to predict modal frequencies of joisted timber floors. L. Jiang et al. [21] analyze the static and dynamic behaviour of a wood-based floor with various types of lateral reinforcement. J. Weckendorf et al. [22] proposed the use of FEM models to predict the dynamic response of joist timber flooring, in terms of modal frequencies, modal shapes and load deflections. I. Glisoivic and B. Stevanovic [23] and H. Xiong et al. [24] focused on analysing floors subjected to pedestrian induced force. New constructive proposals are analyzed by numerical methods, focusing on analysis of boundary condition and connection element transcendence. K. Lewis et al. [25] developed a model to analyze the use of a CLT for long-span flooring. M. Filippoupolitis et al. [26] developed finite element models for a solid timber floor formed from dowel-connected joists, validating them by comparison with the results of experimental modal analysis.

The first and most common analyses refer to concrete, the main material in which prestressing methodology was developed. Other materials with unbounded rods were then analyzed. In elements with continuously supported rods, Kerr [27] determined that natural frequencies are unaffected by prestressing force magnitude. New methodologies such that proposed by L. Jie [28] have been developed, showing that the natural frequency of the element would be modified with increasing pre-stress force, and that it may even in some case increase, which is contrary to mechanical theory. The proposal by T. Lunning et al. [29] also shows this. The most recent analysis by T. Bai-jian et al. [30] works in relation to prestress force, eccentricity, tendon cross-section and natural bending frequencies. Analysis has reached the same conclusion in other materials such as glass-fibre reinforced polymer composite beams [31], concluding that both experimental modal analysis and appropriate numerical modelling reveal that although the first frequency is modified in the presence of a prestressed force, this is not significant.

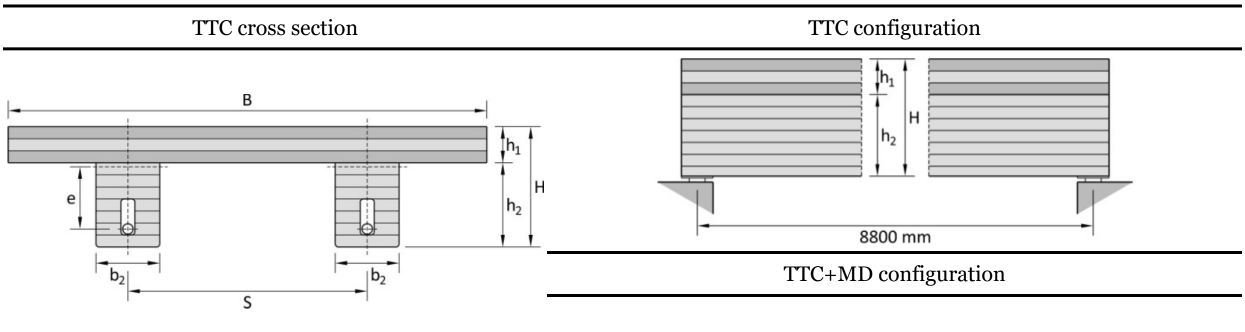
The ISO TC 165 Timber Structures Technical Committee is working on a harmonized timber floor vibration design method. International standard ISO 18324:2016 [32] specifies test procedures to measure natural frequencies, modal damping ratios and other dynamic properties under a concentrated load, in laboratory or field timber floors. More recently, ISO/TR 21136:2017 [33] provides a comprehensive procedure to establish human acceptability criteria. It uses measured or calculated response parameters and subjective evaluation rating, by means of advanced statistical analysis of a large timber floor database. This document admits that there is no general criterion, indicating that there is no general agreement among researchers and code writers on human acceptability criteria for design that prevents unpleasant floor vibration. The CEN / TC 124 Timber Structures Technical Committee is also working on test methods to determine the natural frequencies, damping, unit point, load deflection and acceleration of sawn timber floors, engineered wood products, and solid timber beam or slabs. The method proposed in EN 16929:2018 [34] also covers concrete screed solutions and timber-concrete composite floors.

All these design methods and the different proposals conclude that there is a close relationship between static deflection and fundamental natural frequency, and these factors are therefore a design variable for lightweight timber floors [35]. As static deflection can be simply measured with a certain degree of accuracy, it has become a fundamental predictor of floor vibration behaviour. The most recent works in the search for a unified criterion therefore accept static deflection under a concentrated load applied in the middle of a span as an indirect means of vibration control. A simple analytical methodology is developed in this paper that takes this relationship into account, and it can be applied in lightweight timber joist floors under any load condition.

3. Test procedure and results

3.1. Model configuration. Materials

Two different models with identical timber cross sections and material composition were tested (Fig. 1). In the first model, a simply supported Timber-Timber composite (TTC configuration) was tested in a ribbed-panel configuration with an 8.80 m. span between supports. In the second model the multiplier device SsS® was placed in both ends of the ribs (TTC+MD configuration). The geometry of this multiplier device increases the span between supports to 9.00 m.



Geometry [mm]					
S	600	h1	90		
b2	160	h2	210		
B	1200	H	300	e	159.88

Fig. 1. Model Configurations

The lower Glulam ribs were made of Picea Abies, with a strength class of GL28 h [36], while the CLT flanges, CLT90S L3S [37], were 90 mm thick and were composed of three sheets of 30 mm Picea abies C24 [38] (Table 1).

The connection between the webs and flanges of the T-sections consisted of 410×80×4 mm perforated plates in S235JR [39] hot-dipped galvanized steel. The circular drilled holes in the plates had a diameter of 10 mm and were spaced at 5 mm. The plates were glued to the wooden specimens with a 2-component polyurethane adhesive.

Glulam Ribs GL28h				CLT Flanges (CLT90S L3S)			
Bending strength	$f_{m,k}$	28.0	MPa	Bending strength	$f_{m,k}$	24.0	MPa
Tensile strength ‖ to the grain	$f_{t,0,k}$	19.5	MPa	Tensile strength ‖ to the grain	$f_{t,0,k}$	14.0	MPa
Tensile strength ⊥ to the grain	$f_{t,90,k}$	0.45	MPa	Tensile strength ⊥ to the grain	$f_{t,90,k}$	0.12	MPa
Compressive strength ‖ to the grain	$f_{c,0,k}$	26.5	MPa	Compressive strength ‖ to the grain	$f_{c,0,k}$	21.0	MPa
Compressive strength ⊥ to the grain	$f_{c,90,k}$	3.0	MPa	Compressive strength ⊥ to the grain	$f_{c,90,k}$	2.5	MPa
Shear strength	$f_{v,k}$	3.2	MPa	Shear strength	$f_{v,k}$	32.5	MPa
Elasticity Modulus ‖ to the grain *	$E_{0,g,m}$	10663	MPa	Elasticity Modulus ‖ to the grain *	$E_{0,g,m}$	11979	MPa
Shear Modulus	$G_{g,m}$	850	MPa	Shear Modulus	$G_{g,m}$	460	MPa
Density	ρ_k	460	kg/m³	Density	ρ_k	500	kg/m³
* Value obtained in test (See references [1], [2], [3])				* Value obtained in test (See references [1], [2], [3])			

Table 1. Timber component properties

The specimens were tensioned using threaded Y1100H [40] 26.5 mm diameter steel bars, with a cross-section of 570 mm², a characteristic density of 7850 kg/m³, an elastic limit of f_{pk} = 900 MPa and a tensile strength of $f_{pmax,k}$ = 1100 MPa.

The test was performed in a similar way in the two different configurations, with an incremental loading sequence according to Fig. 2. The effectiveness of the multiplier device depends on the value of the load and the way in which the whole load is transferred to the supports. The test was carried out with an incremental load process, from an initial stage without any load (ST-0) to a final stage of maximum load (ST-4). The test was completed with three subsequent stages (ST-1, ST-2, ST-3) in a partial distributed separated load configuration (Fig. 2), using blocks with an individual mass of 65 kg., in an equivalent distribution of 270 kg/m².

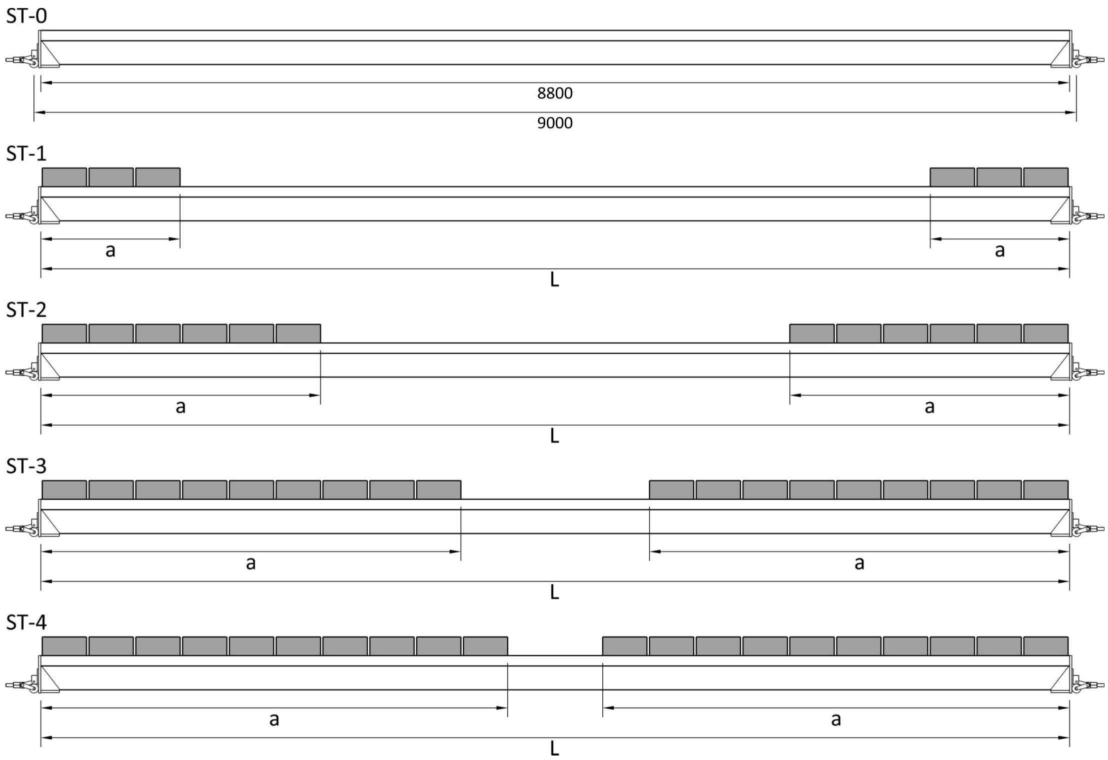


Fig. 2. Load sequence

An experimental modal analysis (EMA) was developed from these two formal configurations, obtaining results for each load stage. This methodology involves mechanically exciting the system under test by an impulse achieved by striking it with an instrumented hammer, in this case a model 8210 Brüel&Kjaer. The vibratory response was measured with a MEM DC model 3711B112 PCB Piezotronics INC accelerometer. The relative magnitude and phase lag between the response acceleration and the input excitation impulse were computed for all frequencies to produce Frequency Response Function (FRF) spectra. The data acquisition system for electromechanical testing was an IMC Cronos SL-2. The frequency response functions through the Fast Fourier Transform (FFT) was applied with IMC Famos Software, and they were analysed by LMS Test.Lab Modal Analysis Siemens PLM Software, obtaining the natural frequencies, vibration mode shapes and damping ratio.

Fig. 3. Test process (Loading Stage ST-3)

3.2. Test results

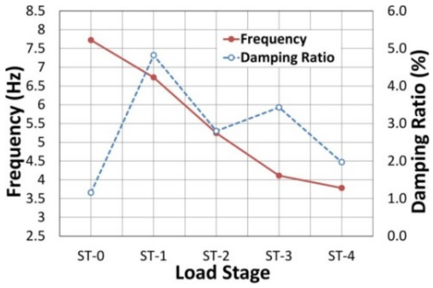
Test results are shown in Fig. 4 (TCC Configuration) and Fig. 5 (TTC+MD configuration), for all five load steps. The results are shown in terms of frequency and damping ratio, as well as the graphs corresponding to frequency response functions (amplitude vs. frequency).

The tests took place at a span that is unusual in wooden floors (9.00 m.). The results therefore show relatively low frequency values, as may be expected in a very slender low mass system. The solutions show poor vibration performance, and this deteriorates with increasing mass in an invariable stiffness configuration.

The self-prestressing system induces an eccentric force that hardly modifies system behaviour. Very slight variations were observed, leading to the conclusion that the presence of an unbonded prestress bar does not alter the results in comparison with those for a non-prestressed solution in frequency terms. The differences are minimum at all load states in all cases, except for the unloaded stage (ST-0) of the TTC+MD configuration. This has a slightly higher value, with a maximum difference of 3.0% in the ST-2 load stage and a minimum of 0.64% in the final ST-4 load stage.

TTC configuration

Load sequence	Frequency fo [Hz]	Damping ratio ξ [%]
ST-0	7.724	1.16
ST-1	6.729	4.82
ST-2	5.245	2.80
ST-3	4.111	3.43
ST-4	3.783	1.97



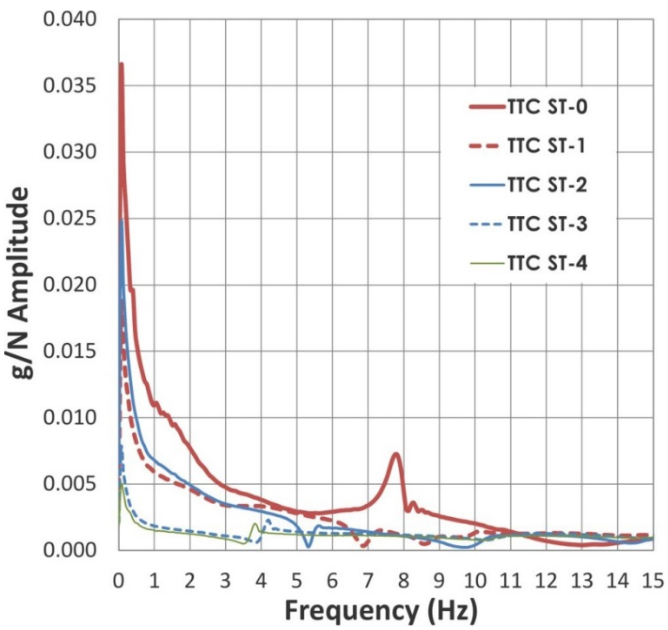
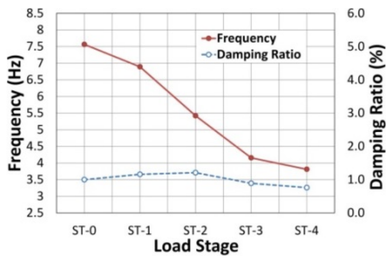


Fig. 4. FRF´s (TTC configuration). Test Results

TTC+MD configuration

Load sequence	Frequency fo [Hz]	Damping ratio ξ [%]
ST-0	7.565	1.00
ST-1	6.892	1.17
ST-2	5.420	1.21
ST-3	4.156	0.89
ST-4	3.811	0.76



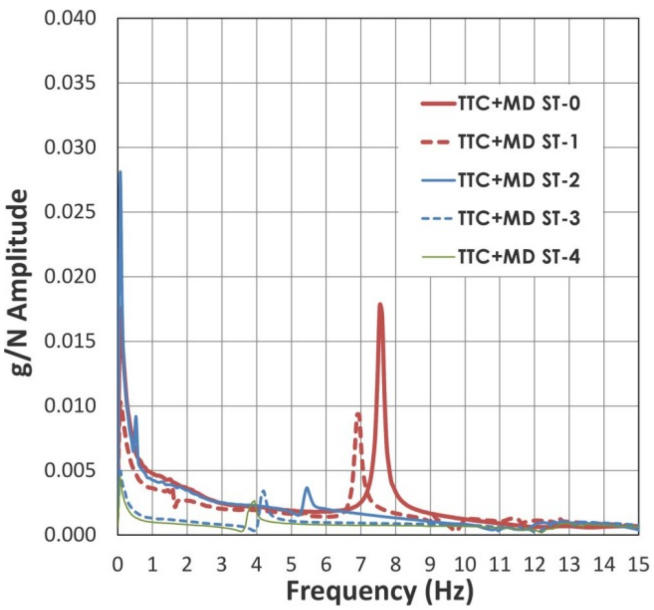


Fig. 5. FRF´s (TTC+MD configuration). Test Results

A highly significant difference was found in the relationship with the damping ratio, as in all cases the TTC+MD configuration showed much lower values. In the first stage (ST-0) this difference was only 13.36 %, but in the intermediate stages it was 75.8%, 56.8% and 73.9%, while at the final maximum load stage (ST-4) it was 61.4 %.

4. Analytical proposal

The earliest proposal to limit timber floor vibration in serviceability design used a static deflection limitation, i.e. ensuring sufficient floor stiffness. The fundamental frequency, alone or in combination with root mean square acceleration and deflection under concentrated load at the centre of the span, was proposed in standards and developed by different authors, such as those mentioned in Section 2 above. Analytical development proposals have resulted in expressions that require the use of many factors, so that many criteria require information that is not available at an initial design stage, restricting their use.

A range of methodologies have been developed for orthotropic plates. After the original Timoshenko and Woinowsky plates theory [41] and previous studies of transverse vibrations by Leissa [42], Chui [43] proposed an expression (1) to calculate the natural frequency of rectangular joisted floors simply supported along all edges, and a complementary expression for static displacement caused by a concentrated load, indicating the extreme relationship between the factors that determine both values.

$$f = \frac{\pi}{2\sqrt{\rho}}\sqrt{D_x\left(\frac{1}{L}\right)^4 + 2D_{xy}\left(\frac{1}{LB}\right)^2 + D_y\left(\frac{1}{B}\right)^4} \quad (1)$$

Where D_x, D_y and D_{xy} represent longitudinal, transverse and torsion rigidity of the floor, depending on factors such as gaps between subfloor elements or partial composite action between subflooring and joist; L is the length; B the width of the floor, and ρ is mass per unit of floor area.

Torrati and Talja [44] proposed a simpler expression in relation to orthotropic plate theory. This takes into account longitudinal rigidity (D_x), length (L) and mass (m), with an analytically derived expression to predict deflection.

$$f = \frac{\pi}{2\sqrt{m}}\sqrt{D_x\left(\frac{1}{L}\right)^4} \quad (2)$$

A simplified expression for orthotropic plates was proposed by Ohlsson [45], under the hypothesis that D_{xy} is approximately equal to D_y (3). This expression can be rewritten (4) in relation to the stiffness properties of the floor in the main direction (EI_x) and the transverse direction (EI_y), span length (L), width (B) and mass (m).

$$f = \frac{\pi}{2}\sqrt{\frac{D_x}{\rho L^4}}\sqrt{1 + \left[2\left(\frac{L}{B}\right)^2 + \left(\frac{L}{B}\right)^4\right]\frac{D_y}{D_x}} \quad (3)$$

$$f = \frac{\pi}{2}\sqrt{\frac{EI_x}{mL^4}}\sqrt{1 + \left[2\left(\frac{L}{B}\right)^2 + \left(\frac{L}{B}\right)^4\right]\frac{EI_y}{EI_x}} \quad (4)$$

Smith and Chui [46] proposed a new simplified expression to calculate the natural frequency of lightweight timber floors. This expression takes into account the number of joists (n_j), joist width and depth (b,h), decking thickness (t), MOE and the second moment of joist area (E_j, I_j), joist density (ρ_j) and that of the upper decking (ρ_s), in relation to length (L).

$$f = \frac{\pi}{2L^2}\sqrt{\frac{E_j I_j (n_J - 1)}{\rho_s tB + \rho_j bh(n_J - 1)}} \quad (5)$$

A comparative analysis of different frequency prediction models was developed by R. Rijal [47] in an equivalent T Beam composition. Simpler expressions are used to calculate natural frequency, considering the equivalent plate bending stiffness of the floor along span direction (EI)L. Eurocode 5 [6] proposes a direct approach (6) that takes into account this consideration and the presence of uniformly distributed mass per unit surface (m). Similar considerations are assumed by other standards, such the Canadian NBBC [11] for composite timber floors, in equivalence to simply supported T-Beam analysis (7), where EI_{eff} represents the effective composite bending stiffness in the joist span direction.

$$f = \frac{\pi}{2L^2}\sqrt{\frac{(EI)_L}{m}} \tag{6}$$

$$f = \frac{\pi}{2L^2}\sqrt{\frac{EI_{eff}}{m}} \tag{7}$$

In all of these expressions, the presence of a load condition is considered that corresponds to continuous uniform distribution. The usability of the expression is therefore limited, as it cannot be used for other load configurations. A simplified methodology is proposed to solve this problem. The close relationship between deflection and frequency is recovered, so the results can be transferred to any load condition, even in the presence of a self-tensioning force. The starting point is the consideration of a simply supported single span beam, with a static mass equal to floor self-weight (m_{sw}) and any extra superimposed mass (m_{si}), considering it to be uniformly distributed. In this case the natural frequency can be determined by the expression (8) in terms of mass (m), length of the floor (L) and bending stiffness in the joist span direction (EI).

$$f = \frac{\pi}{2L^2}\cdot\sqrt{\frac{EI}{m}} = \frac{\pi}{2L^2}\sqrt{\frac{EI}{m_{sw}+m_{si}}} \tag{8}$$

This expression can be modified (9), considering a stiffness factor (K), adopting the relationship between mass and uniformly distributed load corresponding to self-weight (q_{sw}) and a superimposed uniformly distributed load (q_{si}).

$$f = \frac{\pi}{2L}\cdot\sqrt{\frac{K}{(q_{sw}\cdot L)+(q_{si}\cdot L)}} \tag{9}$$

For any other load distribution it is possible to define a general expression that adopts the following form (10).

$$f = \frac{\pi}{2L}\cdot\sqrt{\frac{K}{m_{sw}+\alpha\cdot m_{si}}} \tag{10}$$

$K = \frac{EI}{L}$ bending stiffness in the joist span direction, in kNmm
E represents modulus of elasticity, in kN/mm²
I represents the second moment of the area, in mm4
L represents the span, in mm
α load distribution coefficient

The value of "α" is directly determined by the relationship with the deflection expression corresponding to a uniformly distributed load and the particular load distribution. This is the same as considering that in the case of a uniformly distributed load, $\alpha = 1$. From this consideration it is possible to determine α, establishing the relationship between deflection due to a specific load configuration and uniformly distributed load that causes the same deflection. In the specific case of the tests that were carried out, corresponding to a partial distributed load, the α coefficient adopts the next expression (11), where a represents load distribution (Fig. 3).

$$\alpha = \frac{4}{5}\frac{a\left(3L^2 - 2a^2\right)}{L^3} \tag{11}$$

Using this approach it is also possible to apply the expression to a single load (P) at the centre, in which case the coefficient α adopts the value of (8/5) and m_{si} represents the equivalent mass of the applied single load. In the particular case of 1 kN single load, the general expression that also takes self-weight mass into account is simple (12), considering standard acceleration due to gravity (g).

$$f = \frac{\pi}{2L}\cdot\sqrt{\frac{K}{m_{sw} + \alpha \cdot m_{si}}} = \frac{\pi}{2L}\cdot\sqrt{\frac{K}{m_{sw} + \frac{8}{5}\cdot\left(\frac{1000}{g}\right)}} = \frac{\pi}{2L}\cdot\sqrt{\frac{K}{m_{sw} + 163.5}} \tag{12}$$

The self-tensioning effect due to the multiplier device can be considered an eccentrically prestressed beam with a straight unbounded tendon and attached to the beam only at its ends, so the force could be treated as external axial force. It is therefore possible to consider an equivalent effect, transferring the tensioning force as the superposition of an axial force and a moment coupled at the centre of the cross section. From the well-known expression [13] that defines the natural frequency of a beam affected by N prestressing force, it is possible to evaluate the effect of this force in terms of deformation and therefore apply the same considerations as the general expression. The positive deflection effect of the eccentric force is determined by the presence of self-weight (m_{sw}), the rest of the super imposed load (m_{si}), the eccentricity (e) with which it is applied and the multiplier effect (M) of the device. The general expression therefore has the following form (14), in which both components of opposite deflection participate.

$$f = \frac{\pi}{2L}\cdot\sqrt{\frac{EI}{m} - \frac{N}{m}} \tag{13}$$

$$f = \frac{\pi}{2L}\cdot\sqrt{\frac{K}{m_{sw} + \alpha \cdot m_{si}} - \left(\frac{16}{L}\cdot\frac{K}{M \cdot e \cdot \left(m_{sw} + 4 \cdot m_{si}\right)}\right)} \tag{14}$$

Static tests performed previously [1, 2, 3] showed behaviour very similar to fully composite action, with minimum slip at the interfaces. In this case the transformed-section method is applicable, determining the neutral axis and the section properties of the composite section. Thus bending stiffness factor can be determined as K = 791790.135 kNmm. These previous tests showed a multiplier effect due to the MD device of M = 3.65. The analytical proposal referring to expressions (10), (11) and (14) was applied in both test configurations, and the results are shown in Table 2 and compared with test results.

5. Numerical Analysis

A direct approach with a 3D FEM model was developed using Ansys Workbench Academic Research® V.18. Numerical analysis aims to evaluate the ability of the model to predict the behaviour obtained in laboratory tests.

The lower ribs were modelled with 3D-volume elements and mesh with the 20-node SOLID186-elements, with three degrees of freedom per node. The CLT deck was modelled by taking each layer into account, with a rigid coupling applied between layers, disregarding the flexibility of the glue and using the same SOLID186-elements. Steel elements combine SOLID186-elements with SOLID187-elements defined by 10 nodes having three grades of freedom, which are suitable for modelling irregular meshes.

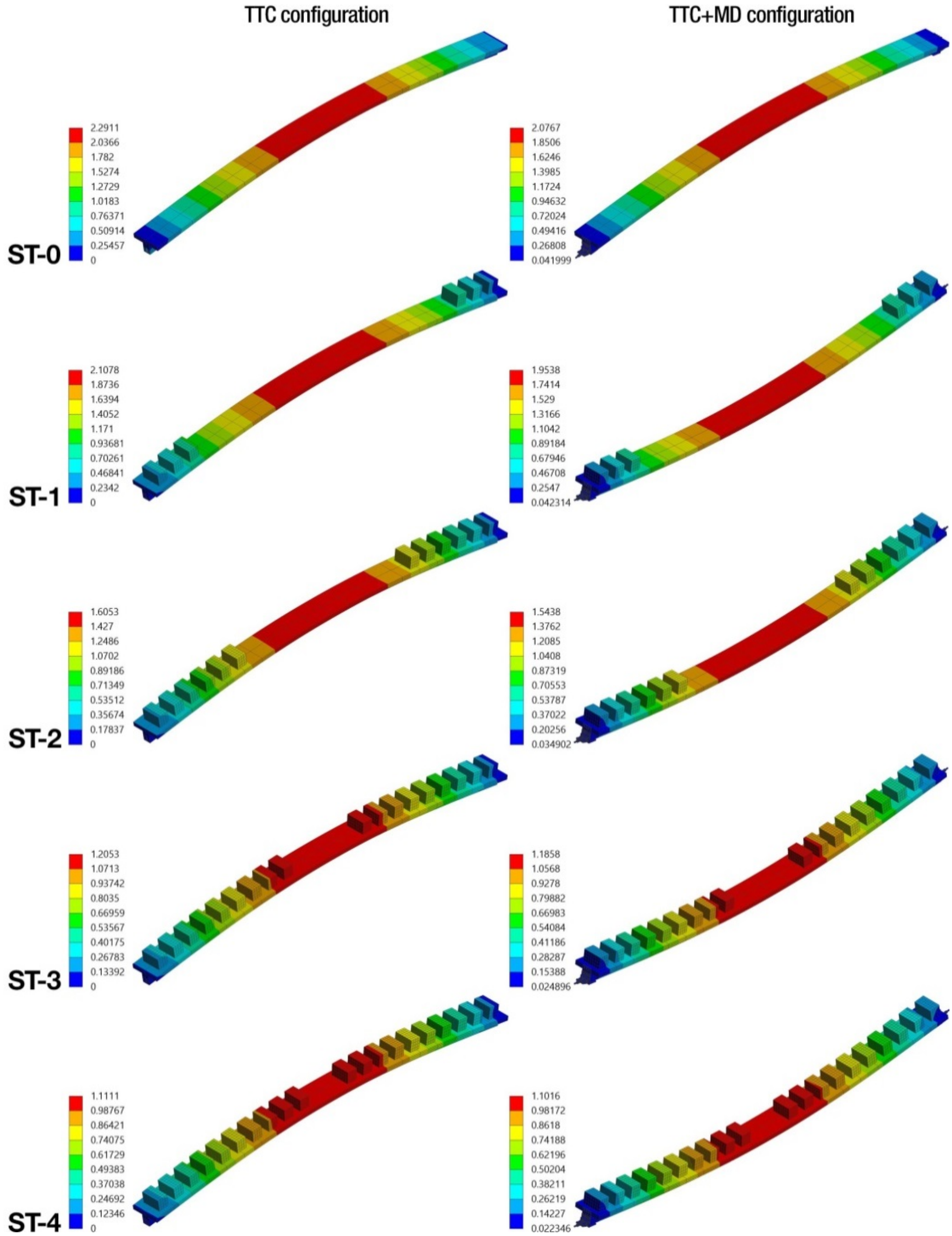


Fig. 6. First mode (mm). TTC and TTC+MD configuration. FEM Numerical Analysis

The multiplier device is composed of a series of interconnected mobile parts. The model reproduces the dynamic behaviour of a system of interconnected bodies with joint elements that allow suitable kinematic constraint of the relative motion between the bodies forming the joint. The model uses MPC184 joint elements, defined by two nodes with 6 degrees of freedom at each node. Kinematic constraints in the joint elements are imposed using the Lagrange multiplier method. Previous analysis showed performance very similar to fully composite action, with minimum slip at the interfaces between ribs and deck. The model combines the said joints with CONTA174 to represent contact between 3D target surfaces (TARGE170), limiting contact restrictions according to the relative behaviour of the bodies.

Modal analysis was developed using the Block-Lanzcos eigensolver. This is recommended when the model consists of solid and shell elements, and it is especially suitable for large matrix problems. Rayleigh damping was applied to the models with the modal damping ratio defined in EC 5 as 1%. Analysis determined behaviour in any load stage in accordance with the test procedure, predicting the first mode and frequency.

The first mode is shown graphically in Fig. 6 for each configuration and for each loading stage, in accordance with the test procedure.

6. Results and discussion

Table 2 summarizes the set of results obtained in the test campaign compared to the prediction obtained by applying the analytical proposal presented and the numerical model. It also shows the percentage of difference (Δ%) between them.

Load Stage	TTC configuration						TTC+MD configuration					
	Test	ξ	Analytical	Δ %	Numerical	Δ %	Test	ξ	Analytical	Δ %	Numerical	Δ %
ST-0	7.724	1.16	7.818	1.22	7.532	2.49	7.565	1.00	7.807	3.10	7.507	0.76
ST-1	6.729	4.82	6.828	1.47	6.994	3.94	6.892	1.17	6.826	0.96	7.116	3.26
ST-2	5.245	2.80	5.274	0.55	5.442	3.76	5.420	1.21	5.272	2.78	5.747	6.05
ST-3	4.111	3.43	4.168	1.39	4.097	0.35	4.156	0.89	4.167	0.26	4.236	1.93
ST-4	3.783	1.97	3.901	3.12	3.781	0.06	3.811	0.76	3.899	2.29	4.004	5.08

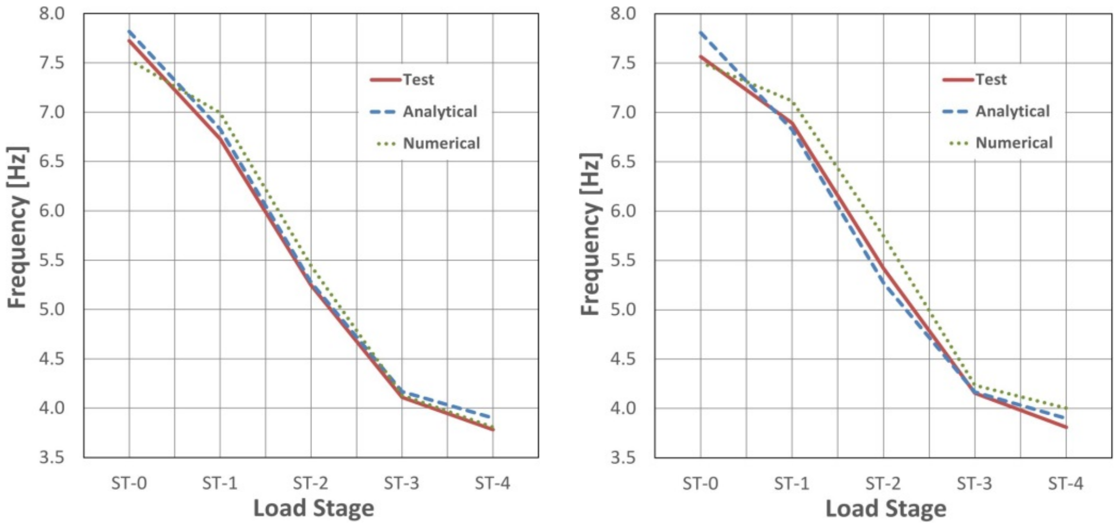


Table 2. Comparative results (TTC and TTC+MD configuration)

Test results in frequency terms show minimum prestressing force repercussion, regardless of its value. TTC+MD configuration showed a slightly higher value in all stages, with the exception of the initial stage (ST-0), in which the self-tensioning force was due solely to the self-weight of the model. The maximum difference was found for load stage ST-2, at 3.32 %.

The test results show a great difference in terms of damping ratio, as in all cases the value obtained in the TCC configuration was far higher than the one measured in the TTC+MD configuration, with a maximum difference of 73.9 % in load stage ST-3.

The predictive values obtained by application of the analytical proposal are extremely similar to test values, especially in the case of TTC configuration, with a maximum difference of Δ 3.12%, corresponding to load stage ST-4. The presence of moving parts in the multiplier device and the various factors that determine the effective transmission of the eccentric axial load, lead to slightly less accurate predictive values in the TTC+MD device, with a maximum difference of Δ 3.10 %, corresponding to load stage ST-0.

A relative discrepancy was found in the numerical models. In the TTC model configuration the greatest difference arises in load stage ST-1 (Δ=3.94%), coinciding with the highest damping ratio measured in the test (ξ=4.82). On the other hand, the largest difference in the TTC+MD model configuration corresponds to load stage ST-2 (Δ=6.05%), in which the maximum damping ratio (ξ=1.21) was measured.

The results obtained show that the analytical model is highly reliable, with values that are extremely close to the test results in both configurations. Its simplicity of use means that it can be considered a design tool that is able to immediately predict the fundamental frequency of a simple system under any load condition, even when there is an axial prentensing force.

The numerical models developed show a similar degree of precision, indicating that the finite elements method is an effective tool that is able to characterise the dynamic behaviour of singular systems, even ones in which moving parts interact.

The simplicity of the proposed analysis in terms of the values necessary to use it and the precision it achieves in different load configurations have the great advantage of allowing it to be used as a design tool. To use this expression for other load situations only requires adjusting the load distribution coefficient "α" to fit the associated deformation values.

7. Conclusions

Tests of the different configurations showed a minimum difference in terms of natural frequency between the untensioning and self-tensioning configurations under all load conditions. The presence of an unbonded prestress bar is therefore not decisive in frequency terms. The discrepancy in terms of damping ratio is not conclusive, since the support conditions of the multiplier device involve the presence of moving parts.

An analytical model is presented that makes it possible to predict the behaviour of the system under any type of load. The simplicity of this proposal and its versatility also make it possible to consider an eccentric pretensioning force. The precision of this method means that it can be used as a design tool to immediately predict the fundamental frequency of a system, based on knowledge of its basic properties and load configuration.

The numerical model developed using the finite elements method makes it possible to establish the dynamic behaviour of both configurations in terms of mode and frequency. Analysis was translated to all load states, together with implementation of the presence of partial restrictions that determine the geometrical configuration of the multiplier device in relation to its moving parts. The precision of the results obtained indicates that this methodology is valid to analyse dynamic behaviour, although it requires considerably more calculation power than the proposed analytical model.

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