Impact of Time-Modulated Arrays on the BER of Linear Digital Modulations

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August 1, 2016

Abstract

Time modulation provides a simple way to control the power radiation pattern of an antenna array. By appropriately selecting the parameters of a Time-Modulated Array (TMA) it is possible to obtain a reconfigurable pencil beam adequate for wireless communications. This work focuses on the impact of a TMA on the Bit Error Ratio (BER) performance of a wireless communication receiver with a linear digital modulation. We show how the BER of such a receiver is affected by the TMA synthesis variables.

Antenna arrays, digital communication, time-modulated array, linear digital modulation.

1 Introduction

Time-Modulated Arrays (TMAs) are antenna arrays whose radiated power pattern is controlled by means of periodically enabling and disabling the excitations of some of their individual elements [1, 2]. Such a periodic modulation is a non-linear operation that generates sideband radiated signals which are frequency-shifted at multiples of the time-modulation frequency. The Sideband Radiation (SR) constitutes a phenomenon that may severely reduce the TMA gain because it can represent a significant amount of the total radiated power, hence being a cause of efficiency reduction at the carrier frequency. The usual approach to face the SR problem has been to propose optimization methods capable of minimizing its relative value (e.g. [2]). Under such considerations, TMA

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synthesis has usually considered the optimum compromise between Side-Lobe Levels (SLL), SR, and efficiency. However, the SR is not necessarily harmful and sometimes can be profitably exploited, e.g. to design smart antennas in time-varying scenarios [3].

The possibility of transmitting communication signals using TMAs has been studied in [4, 5, 6]. This work goes a step further and analyzes the impact of TMAs on the system Bit Error Ratio (BER) when Linear Digital Modulation (LDM) signals are transmitted. In other words, due to the nature of TMAs, it is relevant here to link the classical array synthesis trade-off among SLL, gain, and Half-Power Beamwidth (HPBW), to a key figure of merit in digital communication systems such as the BER. It is also useful to include a less quantitative parameter, the complexity of the array feeding network.

Antenna arrays with non-uniform amplitude excitation distributions provide radiation patterns with a desired SLL value. For the case of a conventional Static Array (STA), since the dynamic range of the amplitude distribution is higher, the SLL decreases. Furthermore, the STA feeding network becomes more complex and thus more difficult to design and construct. However, the TMA philosophy is not based in amplitude control through the feeding geometric configuration. A TMA feeding network makes use of pure time control; in fact, its usual circuitry consists of simple ON/OFF periodical switching of the antenna elements [1]. Therefore, from an engineering point of view, a comparison between a TMA solution and its STA counterpart should unequivocally include the complexity of the feeding network.

In this work we take, as a design premise, a STA with a relatively low level of complexity; in other words, not very restrictive in terms of SLL. In this case, and for simplicity, we consider a real-valued amplitude distribution on a linear array. We will apply time modulation to the previous STA –thus introducing a modest extra cost in terms of complexity– in order to achieve a previously specified SLL level [1]. As a novelty, we quantify theoretically the system BER as a function of the TMA normalized pulse durations, providing closed-form expressions to compare TMAs and STAs for different scenarios in wireless communications.

2 Power Balance

Let us calculate the power balance of the wireless communication system shown in Fig. 1. Such a system uses an M-ary linear digital modulation scheme [7] with a conventional STA at the transmitter side and a TMA at the receiver side, separated from the transmitter by a distance R. Both transmit-STA and receive-TMA exhibit synthesizing pencil-beam patterns whose maximums are pointed into directions θ_T and θ_R radians off the z-axis, respectively (see Fig. 1)[1].

Recall now that the radiated field of a linear TMA composed of N isotropic

[1]It can be seen from that figure that the main axes of the antennas are considered to be both parallel to the z-axis of a global coordinate system.

elements distributed along the z axis is given by [1, 4, 5]

$$F(\theta,t) = \mathrm{e}^{j\omega_c t}\sum_{n=0}^{N-1}\sum_{q=-\infty}^{\infty} G_{nq}\mathrm{e}^{jq\omega_0 t} I_n \mathrm{e}^{jkz_n\cos\theta} = \sum_{q=-\infty}^{\infty} F_q(\theta,t), \tag{1}$$

where G_{nq} are the Fourier series expansion coefficients of periodic functions (usually rectangular pulses with period $T_0 = 2\pi/\omega_0$) applied to the array n-th element; $I_n = |I_n|\mathrm{e}^{j\phi_n}$ and z_n are, respectively, the complex current excitation and the position on the z-axis for such an element; the θ variable corresponds to the angle with respect to the z-axis; and $k = 2\pi/\lambda$ is the wavenumber for a carrier wavelength $\lambda = c/f_c$.

The maximum gain of the receive-TMA is given by [5]:

$$G_{\mathrm{TMA}} = \eta_s 4\pi |F_0(\theta,t)|^2_{\max}/P_{\mathrm{rad}}, \tag{2}$$

where η_s corresponds to the efficiency of the switches of the feeding network including the power absorption of their off-state; $F_0(\theta,t)$ is $F_q(\theta,t)$ given in Eq. (1) for $q = 0$; and P_{rad} is the average radiated power by the TMA when a sinusoidal carrier is transmitted and is given by [6]:

$$P_{\mathrm{rad}} = 4\pi\sum_{q=-\infty}^{\infty}\left\{\sum_{n=0}^{N-1}|I_n|^2|G_{nq}|^2 + 2\sum_{n=0}^{N-1}\sum_{\substack{m=0\\ m\neq n}}^{N-1}\mathrm{Re}\{I_m I_n^* G_{mq} G_{nq}^*\}\,\mathrm{sinc}(k(z_m - z_n))\right\} \tag{3}$$

A simplified expression for Eq. (2) is obtained when the inter-element spacing is set to $\lambda/2$, the element excitations are real-valued $I_n \in \Re$ (thus $I_n = |I_n|$), and the pulse modulations $g_n(t)$ are (e.g. in [4]) even rectangular pulses. This last hypothesis allows for the representation of $g_n(t)$ by means of a Fourier series expansion with real coefficients of the form

$$G_{nq} = \xi_n\,\mathrm{sinc}(q\pi\xi_n), \tag{4}$$

where ξ_n is the normalized pulse time duration of the n-th element. In such a case, the maximum squared amplitude can be simplified to

$$|F_0(\theta,t)|^2_{\max} = [F_0(\theta,t)F_0^*(\theta,t)]_{\theta=\frac{\pi}{2}} = \left[\sum_{n=0}^{N-1} I_n\xi_n\right]^2 \tag{5}$$

We now plug Eqs. (3) to (5) into Eq. (2) to rewrite the receive-TMA gain as

$$G_{\mathrm{TMA}} = \frac{\eta_s\left[\sum_{n=0}^{N-1} I_n\xi_n\right]^2}{\sum_{n=0}^{N-1} I_n^2\xi_n^2 + \sum_{n=0}^{N-1}\left[\sum_{\substack{q=-\infty\\ q\neq 0}}^{\infty} I_n^2\xi_n^2\,\mathrm{sinc}^2\left(q\pi\xi_n\right)\right]}, \tag{6}$$

Having in mind that for all $\xi_n \in (0,1)$ the sinc-squared infinite series converges to $1/\xi_n$, then

$$\sum_{\substack{q=-\infty \\ q\neq 0}}^{\infty} \operatorname{sinc}^2(q\pi\xi_n) = \frac{1}{\xi_n} - 1, \tag{7}$$

and we can simplify Eq. (6) to obtain

$$G_{\mathrm{TMA}} = \frac{\eta_s \left[\sum_{n=0}^{N-1} I_n \xi_n\right]^2}{\sum_{n=0}^{N-1} I_n^2 \xi_n} = \frac{\eta_s \left[\sum_{n=0}^{N-1} I_{\mathrm{TMA}_n}\right]^2}{\sum_{n=0}^{N-1} I_n^2 \xi_n}, \tag{8}$$

where $I_{\mathrm{TMA}_n} = I_n \xi_n$ is the n-th dynamic excitation [4].

Since the transmitting antenna is considered to be a conventional N-element STA with a $\lambda/2$ inter-element spacing and the same static excitations I_n, we can find the maximum gain of the transmit-STA in a completely analogous way. We will then arrive to

$$G_{\mathrm{STA}} = \frac{\left[\sum_{n=0}^{N-1} I_n\right]^2}{\sum_{n=0}^{N-1} I_n^2}. \tag{9}$$

Finally, we apply the well-known Friis Transmission Equation [8] to determine the received-to-transmitted power ratio,

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{\mathrm{STA}} G_{\mathrm{TMA}}, \tag{10}$$

where the transmit and receive antennas are considered to be perfectly aligned[2], i.e., $\theta_T = \theta_R = \pi/2$.

3 Discrete-Time Receiver: System BER

Figure 2 shows the block diagram of a M-ary LDM receiver with an embedded TMA. Such a receiver is able to detect, with minimum error probability,[3] the waveform sent at every symbol interval T_s from a set of M possible transmit waveforms, with M being the order of the LDM signal constellation. We also assume that the received signal is distorted by Additive White Gaussian Noise (AWGN). Therefore, after down-conversion of the signal received by the TMA

[2]In this case, as the array elements are isotropic, the polarization matching factor is tacitly regarded as unity.

[3]The minimum error probability (P_e) criterion [9] is applied in Detection Theory when a multiple hypothesis test H_i is present ($H_i \in \{\text{True, False}\}$ being True if s_i was sent and False otherwise), with $i \in \Psi = \{0, 1, \cdots, M-1\}$ for an M-ary scheme. In addition, both a priory probabilities $p(H_i)$ and probability density functions $p(x/H_i)$ are assumed to be known for the indices $i \in \Psi$. The detector chooses the H_i that maximizes $p(H_i/x)$ which is equivalent to minimize $P_e = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \delta_{ij} p(H_i/H_j) p(H_j)$, where δ_{ij} is the Kronecker delta.

(which, depending on the receiver architecture, involves operations such as band-pass filtering, I/Q demodulation or low-pass filtering) at every T_s seconds, the following signal is available at the input of the LDM detector (see Fig. 2):

$$x(t) = s_i(t) + w(t), \text{ with } i \in \Psi = \{0, 1, \ldots M-1\}. \tag{11}$$

If the frequencies of both the TMA and the signal satisfy [4]:

$$T_0 \gg 1/f_c \quad \text{and} \quad \omega_0 > B_\omega, \tag{12}$$

with ω_0 the angular frequency of the TMA periodic pulses, f_c the carrier frequency of the LDM signal, and B_ω its bandwidth, then the signals $s_i(t)$, $i \in \Psi$ (see Fig. 2) will be deterministic. On the other hand, $w(t)$ represents AWGN with zero mean and variance σ^2. Under these circumstances, the BER expressions for the system in Fig. 2 can be obtained through the derivations found in [7]. All BER expressions can be obtained as particular cases of the following closed-form expression with a common structure [7]:

$$\text{BER}_{\text{mod}}(E_N) = f_{\text{mod}}(M) Q\Big(\sqrt{g_{\text{mod}}(M) E_N}\Big), \tag{13}$$

where f_{mod} and g_{mod} are two scalar parameters that depend on the constellation size M and on the linear modulation type (ASK, PSK, and QAM); $Q(\cdot)$ is the well-known Gaussian error function; and $E_N = E_b/N_0$ is the ratio between energy per bit and power spectral density of the thermal noise at the receiver, being given by [7] $E_N = B_\omega \text{SNR}/(2\pi f_s \log_2 M)$ with SNR, B_ω, and f_s the Signal-to-Noise Ratio, the bandwidth, and the symbol rate of the received LDM signal.

Since our aim is to compare the performance of the system when incorporating a receive-TMA with respect to its STA counterpart, we introduce the following TMA cost function (see Eq. (8) and Eq. (9)):

$$\alpha = \alpha(I_n, \xi_n) = \frac{G_{\text{TMA}}}{G_{\text{STA}}} = \frac{E_N|_{\text{TMA}}}{E_N|_{\text{STA}}}. \tag{14}$$

Taking as a reference channel the one corresponding to a scenario with a conventional STA at the receiver, with gain given by Eq. (9), we can evaluate, over a reference range of E_N denoted by $E_{N\text{ref}} = E_N|_{\text{STA}}$, the BER for the complete family of LDM signals involving a receive-TMA solution with respect to the STA counterpart by substituting $E_N|_{\text{TMA}} = \alpha E_N|_{\text{STA}}$ from Eq. (14) in Eq. (13) writing, finally:

$$\text{BER}_{\text{mod}}|_{\text{TMA}} = f_{\text{mod}}(M) Q\left(\sqrt{g_{\text{mod}}(M)\alpha(I_n, \xi_n) E_{N\text{ref}}}\right). \tag{15}$$

4 Numerical Examples

This section presents the results of numerical examples to quantify the impact on the performance of a QAM digital communication system [7] when using a TMA at the receiver and a STA at the transmitter side. We consider the excitation amplitudes of the static feeding network corresponding to two cases: uniform [10], and Dolph-Tschebyscheff [8, 1] distributions, as explained below.

4.1 Optimized Uniform Distribution

Let us consider a 30-element array with uniform current excitations, i.e. $I_n = I_{U_n} = 1$. By means of Time Modulation (TM) we determine a new pattern with a better maximum SLL whose coefficients are I_{TMA_n}. Such coefficients can be obtained by applying the normalized TM widths ξ_n to the uniform excitations, i.e.

$$I_{\mathrm{TMA}_n} = I_{\mathrm{U}_n}\xi_n = \xi_n. \tag{16}$$

We consider the optimized values for ξ_n obtained in [10][4], which lower the SLL from $-13.24\,\mathrm{dB}$ to $-19.43\,\mathrm{dB}$ (hence giving a $\Delta\mathrm{SLL}_{\mathrm{dB}} = |\mathrm{SLL}_{\mathrm{TMA}} - \mathrm{SLL}_{\mathrm{U}}| = 6.19\,\mathrm{dB}$) and provide a first side-band maximum harmonic level below $|F_{\mathrm{TMA}_1}|_{\max} = -30\,\mathrm{dB}$. We then compute the gain cost (see Eqs. (8), (9) and (16)) with $\eta_s = 1$ (ideal switches [5]), the HPBW percentage variation $\Delta\mathrm{HPBW} = \mathrm{HPBW}_{\mathrm{TMA}} - \mathrm{HPBW}_{\mathrm{U}}$, and the dynamic range ratio of the static normalized amplitude distribution, defined as a percentage, i.e.

$$\Delta|I_n|_{\%} = \max\{100(I_{n+1} - I_n)\} \text{ for } n = 0, \ldots, N-2 \tag{17}$$

Finally, we repeat the comparison by changing the static amplitude distribution to $I_n = I_{\mathrm{TMA}_n}$, which would lead to the case where the STA has the same pattern than the optimized TMA. Table 1 shows the results derived from the mentioned comparisons.

Figure 3 compares the same systems as those given in Table 1 but now in terms of the BER curves when QAM signals are sent. Notice that in the first case a considerable improvement in terms of SLL is achieved with a modest degradation both on the beamwidth and gain, resulting in a slight degradation of the BER curves. Recall that in the second case the power pattern is the same for TMA and STA. This indicates that the STA can be replaced by TMA hence showing that the less complex TMA feeding network fully compensates a practically negligible gain cost reduction.

4.2 Dolph-Tschebyscheff Distribution

We now consider a 16-element array with $I_n = I_{DT_n}$ corresponding to a $-30\,\mathrm{dB}$ DT normalized excitation distribution [8, 1]. We then apply TM to achieve a new pattern with an $\mathrm{SLL} = -40\,\mathrm{dB}$.

Table 2 and Fig. 4 show that, when compared to the $\mathrm{STA}_{\mathrm{DT}} = 30\,\mathrm{dB}$, the TMA provides an SLL improvement (10 dB) that still could compensate for a relatively low impact on gain (0.826) and, consequently in BER. However, when compared to a STA with the same power pattern, the TMA provides a clear solution according to its modest gain cost and the reduction of the dynamic range of the excitations. Note that, according to Eq. (15) and taking

[4] In fact, it corresponds to a simplified version of the distribution given in [10], where the very small (close to 0) and very large (close to 1) ξ_n are replaced by 0 and 1, respectively. Such a simplification, which further reduces the first harmonic level and does not change the SLL significantly, is done also in view of the fact that very small or very large time durations would lead to technical problems (too fast switches).

$E_N = E_N|_{\rm STA}$ in Eq. (13), the difference between the absolute values of the slopes of the BER curves for TMA and its STA counterpart can be quantified through

$$m_{\rm TMA} - m_{\rm STA} = C_1 \sqrt{\frac{\alpha}{E_N}} \left(\frac{1}{\mathrm{e}^{C_2 E_N}} - \frac{1}{\mathrm{e}^{\alpha C_2 E_N}} \right), \qquad (18)$$

with $C_1 = f_{\rm mod}(M)\sqrt{g_{\rm mod}(M)/(8\pi)}$ and $C_2 = g_{\rm mod}(M)/2$ observing that, as $0 < \alpha < 1$, then $m_{\rm TMA} < m_{\rm STA}$.

5 Conclusion

We have characterized the BER of a linearly modulated digital communication system that incorporates a receive-TMA to synthesize pencil beam radiation patterns. The system BER is properly connected to the classical trade-off antenna variables for TMA synthesis purposes. The analysis shows the benefits provided by the TMA technique, namely the reconfigurability of the power pattern while achieving ultra-low levels of SLL with a less complex feeding network, are obtained with minimal impact on the BER.

Acknowledgment

This work has been funded by Xunta de Galicia, MINECO of Spain, and FEDER funds of the E.U. under Grant 2012/287, and Grant TEC2013-47141-C4-1-R.

References

[1] Kummer W, Villeneuve A, Fong T, Terrio F. Ultra-low sidelobes from time-modulated arrays. IEEE Transactions on Antennas and Propagation. 1963; 11(6):633–639.

[2] Poli L, Rocca P, Manica L, Massa A. Handling sideband radiations in time-modulated arrays through particle swarm optimization. Antennas and Propagation, IEEE Transactions on. 2010 April;58(4):1408–1411.

[3] Rocca P, Zhu Q, Bekele E, Yang S, Massa A. 4-d arrays as enabling technology for cognitive radio systems. Antennas and Propagation, IEEE Transactions on. 2014 March;62(3):1102–1116.

[4] Bregains J, Fondevila-Gomez J, Franceschetti G, Ares F. Signal radiation and power losses of time-modulated arrays. IEEE Transactions on Antennas and Propagation. 2008;56(6):1799–1804.

[5] Zhu Q, Yang S, Yao R, Huang M, Nie Z. Unified time- and frequency-domain study on time-modulated arrays. Antennas and Propagation, IEEE Transactions on. 2013 June;61(6):3069–3076.

[6] Maneiro-Catoira R, Bregains J, Garcia-Naya JA, Castedo L. On the feasibility of time-modulated arrays for digital linear modulations: A theoretical analysis. IEEE Transactions on Antennas and Propagation. 2014 Dec; 62(12):6114–6122.

[7] Proakis J. Digital communications. 5th ed. McGraw-Hill; 2008.

[8] Balanis C. Antenna theory: Analysis and design. 3rd ed. Wiley-Interscience; 2005.

[9] Kay S. Fundamentals of statistical signal processing, detection theory. vol ii. 1st ed. Prentice Hall; 1998.

[10] Fondevila J, Bregains J, Ares F, Moreno E. Optimizing uniformly excited linear arrays through time modulation. IEEE Antennas and Wireless Propagation Letters. 2004;3(1):298–301.

Table 1: Comparison between a TMA uniform optimized distribution –built from a static uniform distribution– and the STA counterpart.

	TMA SLL $= -19.43\,\text{dB}; N = 30$ HPBW $= 3.6°; \|F_1\|_{\max} = -32.24\,\text{dB}$			
	α	ΔSLL	ΔHPBW	$\Delta\{\Delta\|I_n\|\}$
STA uniform	0.82	6.19 dB	0.36°	0.00%
STA uniform optimized	0.98	0.00 dB	0.00°	93.65%

Table 2: Comparison between a TMA_{DT} with SLL = 40 dB –built from a STA_{DT} with SLL = 30 dB– and the STA counterparts.

	TMA SLL $= -40\,\text{dB}; N = 16$ HPBW $= 8.98°; \|F_1\|_{\max} = -19.94\,\text{dB}$			
	α	ΔSLL	ΔHPBW	$\Delta\{\Delta\|I_n\|\}$
STA_{DT} SLL=30 dB	0.83	10.00 dB	0.72°	0.00%
STA_{DT} SLL=40 dB	0.93	0.00 dB	0.00°	148.30%

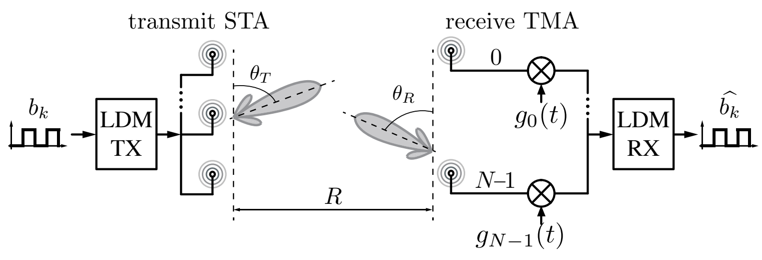

Figure 1: Digital communication system based on an M-ary linear modulation scheme with a conventional transmit-STA and a receive-TMA synthesizing pencil beam radiation patterns.

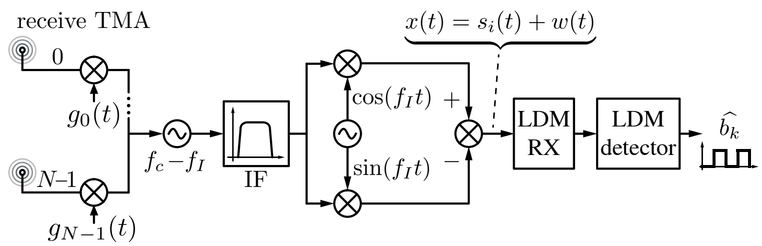

Figure 2: Generalized receiver based in a linear digital modulation scheme incorporating a TMA.

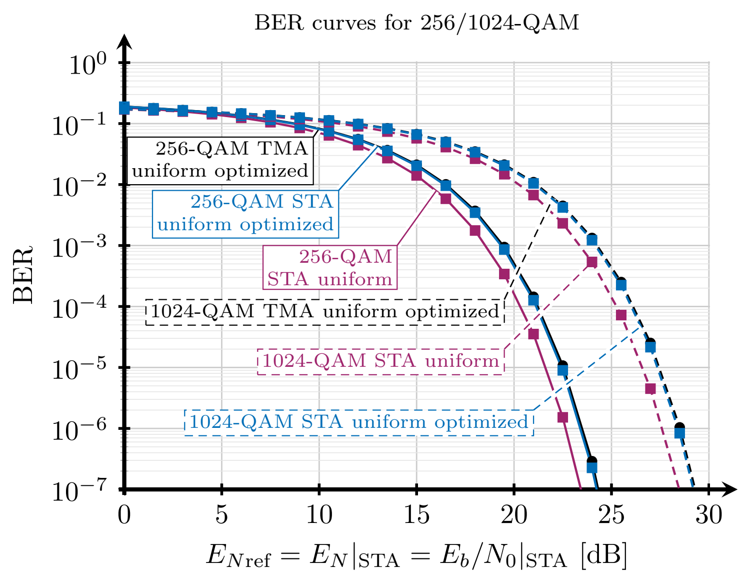

Figure 3: BER curves for 256- and 1024-QAM signals received through the arrays outlined in Table 1. For a reference BER = 10^{-6} the TMA SNR loss with respect to the STA uniform optimized is 0.1 dB for both 256-QAM and 1024-QAM; and with respect to the STA uniform is 0.9 dB for both 256-QAM and 1024-QAM.

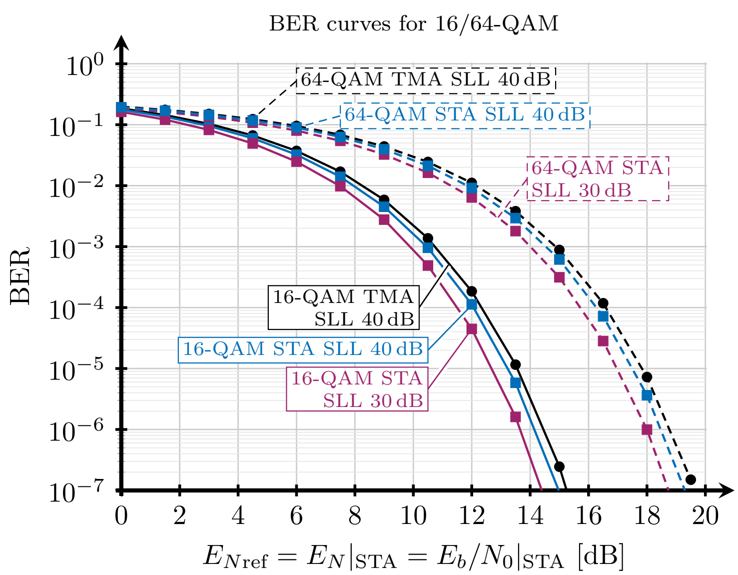

Figure 4: BER curves for 16- and 64-QAM signals received through the arrays outlined in Table 2. For a reference BER = 10^{-6} the TMA SNR loss with respect to the STA with SLL = 40 dB is 0.3 dB for both 16-QAM and 64-QAM; and with respect to the STA with SLL = 30 dB is 0.8 dB for both 16-QAM and 64-QAM.