A VERY FAST METHOD TO SYNTHESIZE CONFORMAL ARRAYS

J. FONDEVILA-GOMEZ, J. A. RODRIGUEZ-GONZALEZ, J. BRÉGAINS, E. MORENO and F. ARES

I. Abstract

In this paper a very fast technique that allows to synthesize arbitrary footprint patterns by using conformal arrays with many radiating elements is described. This method is based on a combination of Woodward-Lawson and Elliott-Stern techniques, and it was applied to the synthesis of a triangular footprint generated by 657 axial dipoles placed on a cylindrical surface, obtaining acceptable ripple and side lobe levels.

II. Introduction

The term "conformal array antenna" is generally used to refer to a set of radiating elements arranged on a non-planar surface such as an airplane fuselage, a satellite or other structures. The design and analysis of such systems is further complicated by the impossibility of factorizing the analytical expression of the radiation pattern into an element factor and an array factor [1, 2]. In the literature [1], several methods of pattern synthesis using conformal arrays have been proposed: array shape optimization, Fourier methods, aperture projection, alternating projections, adaptive array, iterative least-squares, polarimetric synthesis, genetic algorithm and simulated annealing. Most of these numerical techniques are very dependant on the initial solution and they need several iterations in order to obtain the desired solution, being too slow for application to large arrays. In this paper we describe a new technique that combines the Elliot-Stern [3] with the Woodward-Lawson [4] methods to make the best of both. This quasi-analytical technique is very simple and very fast, being also not dependant on the initial conditions. It can be shown that it allows the synthesis of arrays of thousand of elements, performing, at the same time, control of the ripple in the shaped region, the side lobe level (*SLL*) in the unshaped region, and the drop-off in the intermediate zone. The method has been applied to the synthesis of a triangular pattern by using a large cylindrical array with a computation time of the order of a few seconds.

III. Method

The method of Elliott-Stern [3], hereafter ES, is an extension of Taylor's circular aperture pattern synthesis technique, which permits the generation of a rotationally symmetric flat-topped beam, surrounded by ring side lobes of controllable height, and with the ripple amplitude also controllable. Taylor's sum pattern [2], due to a uni-polarised continuous planar aperture distribution is given by

$$F(t) = 2\frac{J_1(\pi t)}{\pi t}\frac{\prod_{n=1}^{\bar{n}-1}\left(1-\frac{t^2}{u_n^2}\right)}{\prod_{n=1}^{\bar{n}-1}\left(1-\frac{t^2}{\gamma_{1n}^2}\right)} \tag{1}$$

being $t = (2a/\lambda)\sin\theta$ and a the radius of the circular boundary of the aperture.

In eqn. 1, γ_{1n} is the nth root of $J_1(\pi t)$ and $\bar{n}-1$ is the number of controlled sidelobes. The pattern is rotationally symmetric because the aperture distribution has been specified to have the same symmetry. Taylor found a formula for u_n that will cause the side lobes level of $F(t)$ to be quasi-uniform at a prescribed height.

A study of eqn. 1 quickly reveals that $F(u_n) = 0$, so the side lobes are interspersed by deep nulls. But suppose eqn. 1 were replaced by

$$F(t) = 2\frac{J_1(\pi t)}{\pi t}\frac{\prod_{n=1}^{\bar{n}-1}\left(1-\frac{t^2}{(u_n+jv_n)^2}\right)}{\prod_{n=1}^{\bar{n}-1}\left(1-\frac{t^2}{\gamma_{1n}^2}\right)} \tag{2}$$

Now $F(u_n) \neq 0$ unless $v_n = 0$. It is possible to find complex roots $u_n + jv_n$ that will yield a pattern with properly filled nulls in the shaped region, while maintaining controlled side lobe levels in the unshaped region [3]. Therefore, eqn. 2 allows to achieve a flat-topped beam with a half beamwidth HBW_0 (circular footprint) with ripple level at any specified value and side lobes at individually prescribed heights can be achieved.

If we are interested in achieving an arbitrary footprint, it is necessary to have a flat-topped beam in every ϕ-cut, with a half width $HBW(\phi)$ that matches, ϕ-cut by ϕ-cut, the extent of the ground surface to be illuminated. This is performed by stretching or shrinking the continuous circular aperture distribution by using a radius that is inversely proportional to the desired flat-top $HBW(\phi)$ [5], which is equivalent to replace a by $a'(\phi) = \frac{a\,HBW_0}{HBW(\phi)}$ in eqn. (2). With this technique is then possible to obtain a non-ϕ-symmetric pattern $F(\theta,\phi)$ that fits the desired footprint pattern and has the same ripple and SLL than the original circular footprint.

The so obtained irregular aperture distribution is inscribed in a rectangle of size $L_z \times L_y$ along z and y directions, respectively. After that, the rectangular aperture is discretised into a planar array with a rectangular grid of size $d_z \times d_y$, obtaining $N_{ele} = N_z \times N_y$ elements. The following step is to project these elements on a cylinder of curvature radius CR and angular aperture AA, obtaining the position (x_n, y_n, z_n) for the *n-th* element as:

$$
\begin{aligned}
z_n &= (2n-1)\frac{d_z}{2} - \frac{L_z}{2} \\
y_n &= (2n-1)\frac{d_y}{2} - \frac{L_y}{2} \\
x_n &= CR\cos(\phi_n) - x_{\min} \\
\phi_n &= \left(\frac{n}{N_y - 1} - \frac{1}{2}\right) AA
\end{aligned}
\qquad (3)
$$

Where ϕ_n is the angular position of the *n-th* element measured with respect to the *x* axis and the centre of coordinates of the cylindrical array; and $x_{\min}$ is the minimum value of $CR\cos(\phi_n)$. It must be pointed out that the centre of coordinates does not lies, in general, on the main axis of the cylindrical array, but on the center of the plane specified by the minimum *x* coordinates of its elements.

The obtained pattern $F(\theta,\phi)$ is then sampled by means of the 3D version of the Woodward-Lawson method [4] (hereafter WL). More specifically, the WL method uses S samples of this pattern $F(\theta_s,\phi_s)$ -whose number is dependant on both the interspacing between elements and the number of elements- located at the angular coordinates (θ_s,ϕ_s) to find the excitations of the radiating elements by using the following equation:

$$
I_n = \sum_{s=1}^{S} F(\theta_s,\phi_s)\exp\{-jk(x_n\ \mathrm{sen}\theta_s\cos\phi_s + y_n\ \mathrm{sen}\theta_s \mathrm{sen}\phi_s + z_n\cos\theta_s)\} \qquad (4)
$$

where *k* is the wavenumber.

The pattern radiated by the cylindrical array is calculated by using the following expression, where the element factor $f_e(\theta,\phi-\phi_n)$ is included:

$$
F_{ES-WL}(\theta,\phi) = \sum_{n}^{N_{ele}} f_e(\theta,\phi-\phi_n)\ I_n \exp\{jk(x_n\sin\theta\cos\phi + y_n\sin\theta\sin\phi + z_n\cos\theta)\} \qquad (5)
$$

IV. Results

We want to synthesize a triangular pattern with its vertexes on the positions (*w*,*v*) in *w*-*v* space: (0.135,0.280), (-0.265,-0.115) and (0.135,-0.115) where $w = \cos\theta$ and $v = \sin\theta \sin\phi$; with a desired ripple of 1 dB and a SLL of -25 dB by using a cylindrical array with its main axis along *z* direction, with an angular aperture of 80º and composed of axial dipoles along z axis. The element factor for these axial dipoles is given by [6]:

$$f_e(\theta,\phi) = \frac{1}{3}\sin(\theta)\left[1 + 2Max\left(\cos(\phi), -1/2\right)\right] \quad (6)$$

Starting from a circular aperture of radius 5λ, we have used eqn. 2 with $\bar{n} = 6$ to synthesize a circular footprint. This pattern has two ripple cycles of -0.1 dB and three inner sidelobes at -30 dB with HBW_0 = 12.7 degrees measured at -0.15 dB.

After applying the above method, the resulting cylindrical array has 35 × 38 elements, and a curvature radius of 14.43 λ.

Finally, in order to reduce the number of elements as well as the dynamic range ratio of excitations ($|I|_{max}/|I|_{min}$), weakly excited elements were removed from the array. The final array has 657 elements with a dynamic range ratio of 40 and it radiates the power pattern shown in Fig. 1. The ripple is about 1.30 dB and the side lobe level about -25 dB.

The process to synthesize this footprint required less than 7 seconds running on a desktop PC with a 3.0 GHz Pentium 4 processor.

V. Conclusions

By combining the Elliott-Stern and Woodward-Lawson footprint synthesis methods in the way describe above, it is possible to perform rapid synthesis of irregular footprints generated by conformal arrays that are too large to allow direct optimization of array excitations by numerical methods. The ripple level can be improved if the complex roots used in eqn. 2 are slightly perturbed by means of an additional optimization process. The described method allows obtaining other radiation diagrams such as sum and difference patterns and can be applied to conformal arrays with arbitrary shape.

Acknowledgement

This work has been supported by the Spanish Ministry of Education and Science under Project TEC2005-07985-C03-03.

References:

[1] Josefsson, L., and Persson, P.: 'Conformal Array Antena Theory and Design', (Wiley-Interscience, 2006).

[2] Mailloux, R. J.: 'Phased Array Antenna Handbook', (2nd ed., Artech House, 2005).

[3] Elliott, R. S., and Stern, G. J.: 'Shaped patterns from a continuous planar aperture distribution', *IEE Proc.*, Pt. H, 1988, 135, (6), pp. 366-370.

[4] Ruze, J.: 'Circular aperture synthesis', IEEE Trans. Antennas Propag., 1964, 12, (6), pp. 691-694.

[5] Ares, F., Elliott, R. S., and Moreno, E.: 'Design of planar arrays to obtain efficient footprint patterns with an arbitrary footprint boundary', IEEE Trans. Antennas Propag., 1994, 42, (11), pp. 1509-1514.

[6] Ares, F., Rengarajan, S.R., Lence, J.A.F., Trastoy, A., and Moreno, E.: 'Synthesis of antenna patterns of circular arc arrays', Electron. Lett., 1996, 32, (20), pp. 1845-1846.

Authors' affiliations:

J. Fondevila-Gomez, J. A. Rodriguez-Gonzalez, J. Bregains, E. Moreno, and F. Ares.
Radiating Systems Group, Department of Applied Physics, Faculty of Physics, Univ. of Santiago de Compostela, 15782 Santiago de Compostela (Spain).
E-mail: faares@usc.es

Figure captions:

Fig. 1 Triangular power pattern (dBs) obtained by a cylindrical array of 657 elements.

Figure 1