MATRIX PSEUDO-INVERSION TECHNIQUE FOR THE DIAGNOSTICS OF PLANAR ARRAYS

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ABSTRACT

In this paper we show, by means of numerical simulation, that the Moore-Penrose pseudoinverse of a matrix taken from an overdetermined system can be applied to retrieve the excitation distribution of a planar array of parallel dipoles with faulty elements, by measuring the complex radiated field in its near zone. Failures on voltage (considering mutual coupling) and currents of several elements, and systematic or random measurement errors are considered in the simulation.

INTRODUCTION

In previous papers, several techniques have been presented in order to determine the location of defective elements in antenna arrays. Some of them try to retrieve the excitation distribution through field measurements (see [1] or [2], for example). One of the most simple and efficient procedures is that one based on the direct solution to a system of equations specified by the measured fields and the geometrical and fed characteristics of the array. If the number of field measurements equals the number of elements of the array, a straight inversion of a transform matrix is required to know the precise excitation distribution that is generating those field values. But, sometimes, the system is overdetermined or ill-conditioned and then some alternatives there must be considered to determine its solution. One of these alternatives, for example, appears in a work due to Gattoufi et al. [3], in which the regularized matrix method is used to retrieve the excitation distribution of a planar array of dipoles from a set of numerically-simulated field values, through near-field measurements taken over a plane parallel to the array. As a complement of that work, in this paper it is shown that a technique –based on the singular matrix decomposition (SVD) procedure (a possibility commented in [3]) and using the Moore-Penrose pseudoinverse ('pinv' function [4])– can be similarly applied to a planar array of parallel dipoles. It is pointed out that mutual coupling between elements is taken into account during the simulation, a behaviour that is mentioned, but not clearly specified in [3]. Another issue that is included in this work is the use of the most usual near-zone measurement surfaces [5]: a plane, a half-cylinder and a hemisphere (though the bi-polar measurement [6] is also available). As a test of the robustness of the technique, some specific simulation of failures and measurement errors are carried out, as described below.

EXPLAINING THE METHOD

We depart from a planar array (rectangular grid on the x-z plane) of $N = N_X \times N_Z$ parallel dipoles of length $2L$, separated δ_x and δ_z apart, and whose axes are taken to be aligned with the z coordinate axis. In the simulation the near field expression of each dipole [7] is used, taking the total field as the superposition of the radiation of every one. The mutual coupling between them [8], if required, is also obtained. In that case, a constant–voltage feeding network is considered. So, if $[Z]_{N\times N} \equiv Z_{nm}$ ($n,m = 1...N,$ where, for simplicity, the elements of the array are numbered from 1 to N instead of having two indexes) represents the matrix of mutual impedances (the diagonal are the self-impedances), the voltage distribution $[V]_{N\times 1} \equiv V_n$ is obtained from:

$$[V]_{Nx1} = [Z]_{NxN}\,[I]_{Nx1} \tag{1}$$

where $[I]_{N\times 1} \equiv I_n$ is the one-column matrix of the excitations that give the desired pattern. In this work, a simple separable Chebyshev distribution [7] at two different sidelobe levels (measured in main planes perpendicular to the array) is selected during the simulation. As it is difficult to know, a priori, what kind of failure will be produced on a specific element (or several ones), as a first approach, we establish that some of the voltages (20% of N) applied to the elements modify randomly their initial values. With this change on the voltages, (1) is used again to obtain, by direct inversion of ***[Z]***, the corresponding ***[I]*** of the failed case. In a second approach, some of the ***[I]*** values are randomly changed.

The scanning surfaces selected to obtain *M* uniformly distributed measurement points are: a plane (parallel to the array), a hemi-cylinder and a hemi-sphere, all of them within the near-zone of the antenna [6]. It is intended that the field measured is the tangential component to the surface at each point. Probe corrections [6] are stipulated as being made. Finally, and due to the difficulty of simulate all of the possible errors that can appear during the measurement [9], a systematic shifting in field amplitude is considered in some cases, and a certain "noise" of random errors in amplitude and phase (certain percentage of their maximum values) is established in other simulations. They are specified in next section.

Being $\vec{F}(\vec{r}) = \sum_{n=1}^{N} \vec{f}_n(\vec{r}_n,\vec{r}) I_n$ the field expression of the array, where $\vec{f}_n(\vec{r}_n,\vec{r})$ is the radiated field of every dipole –whose center is located at $\vec{r}_n = (x_n, 0, z_n)$– and $\vec{r} = (r,\theta,\phi)$ the field point, one can obtain a certain tangential component (with respect to the measurement surface) $F_T(\vec{r}) = \sum_{n=1}^{N} f_{T,n}(\vec{r}_n,\vec{r}) I_n = \sum_{n=1}^{N} e_n I_n$. So, if *M* measurement (field) points (r_m,θ_m,ϕ_m) are selected, a matrix equation of the kind $\boldsymbol{[F]_{Mx1} = [e]_{MxN}\,[I]_{Nx1}}$ can be performed. When $M>N$, the system to be solved is overdetermined, and there are several ways that could be taken to achieve a solution. One of them is the use of the Moore-Penrose pseudoinverse, which is the optimal solution in the minimum square error sense, and can be obtained by means of the SVD technique [4]. So, the retrieved excitation distribution can be found as (***[e]'*** is the pseudoinverse of ***[e]***):

$$I_{R,n} \equiv \boldsymbol{[I_{Ret}\,]_{Nx1} = [e]'_{NxM}\,[F]_{Mx1}} \qquad (2)$$

EXAMPLES AND SOME FINAL COMMENTS

Let us make $N_X = N_Y = 10$, $L_d = 0.5\ \lambda$, $r_d = 0.004763\,\lambda$ (length and radius of every dipole, respectively, being λ the wavelength), $\delta_X = \delta_Z = 0.50\ \lambda$. The maximum length (diagonal) of the antenna is found to be 6.727 λ. The excitation distribution I_n is selected so as to obtain two Chebyshev-like levels of –20 and –25 dB with the healthy array. As a general measurement distance we establish a certain constant R_K = 19.845 λ, that situates the points within the Fresnel zone: $\sqrt[3]{D/2\lambda}\,(D/2)+\lambda = 6.04\lambda \le R_K \le 2D^2/\lambda + \lambda = 91.49\lambda$. To specify a number of measurements different from the number of elements of the array, a grid of 30x30 equispaced points are taken for measurement[1], S.1) on a plane, making, y_m = constant = R_K, $-R_K \le x_m \le R_K$, $-R_K \le z_m \le R_K$, ($\Rightarrow \Delta x = \Delta z = 1.37\lambda$), S.2) on a cylinder, making $\rho_m = R_K$, $5^\circ \le \theta_m \le 175^\circ$, $-R_K \le z_m \le R_K$, ($\Rightarrow \Delta\theta = 5.86^\circ$, Δz=1.37λ) and S.3) on a sphere, with $r_m = R_K$, $5^\circ \le \theta_m, \phi_m \le 175^\circ$ ($\Rightarrow \Delta\theta = \Delta\phi = 5.86^\circ$).

Type and number of failures: F.1) Voltage failures: 20 elements randomly selected and with randomly generated values of failure, F.2) Current failures: 20 elements randomly selected and with randomly generated values of failure (ranged within normalized values). Type of measurement errors: E.1) Systematic error in amplitude: the amplitudes of the values of ***[F]*** are shifted a 10% on the maximum, previously calculated; E.2) Random error: the real and imaginary values of ***[F]*** are randomly changed at most ±10% of their maximum values.

Table 1 indicates the maximum percentage error (absolute value) between the "actual" values of the amplitude and phase of the I_n of each generated case and the corresponding $I_{R,n}$. It shows that the errors are near the range of the measurement error of the field values. As an example, figure 1 shows a comparison of $|I_n|$ and $|I_{R,n}|$ in which measurements were made on a sphere, considering F.1 and E.2.

It can be seen here that the technique presented in this work can be used as a very reliable alternative for diagnostics of parallel dipoles planar (or linear) arrays. Several arrays configurations were further simulated by the authors, revealing very similar results. The technique can be straightforwardly extended to conformal arrays.

[1] If the measurements are made to characterize the pattern (this is not, obviously, the case), the density of the grid must be increased, being the minimum density established by the sampling theorem [6].

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LEGENDS FOR FIGURES AND TABLES

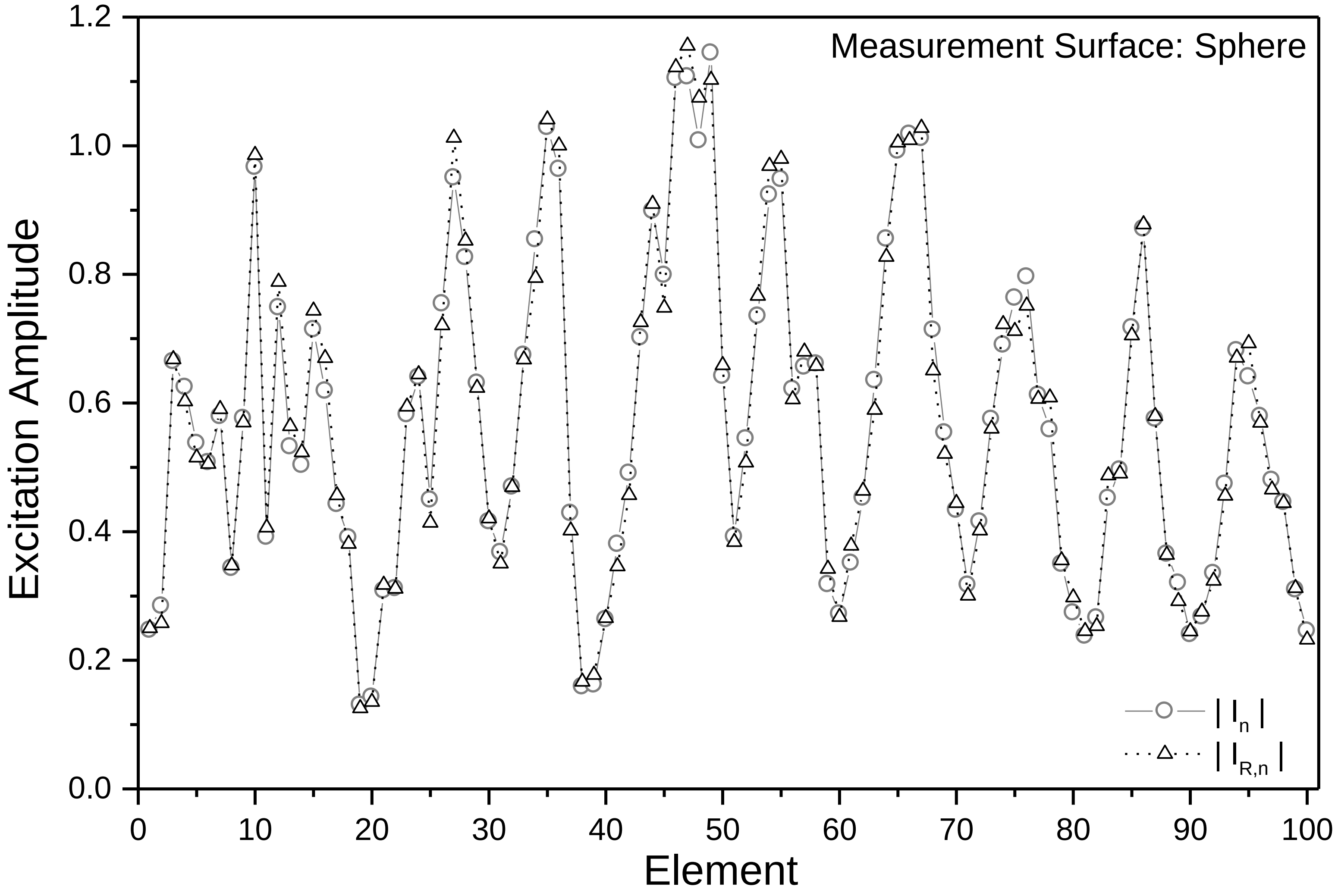
TABLE 1. Percentage errors of amplitude and phase (argument) for each simulated case. "Surface" column indicates the measurement surface. The "Type of error" column specifies the simulated errors in measurements (N=none, SA=Systematic amplitude, R=Random). "Failure" column specifies the type of failure (V=Voltage, C=Current).

FIGURE 1. Excitation distributions obtained after measuring on a near-field sphere. Case E.1, F.2 (see text). 20 (voltage) failed elements were considered.

Table 1

Measurement Surface	**Type of Failure**	**Type of Simulated Error**	**$\{\| \|I_n\|-\|I_{R,n}\| \|\}_{max}$ %**	**$\{\| arg(I_n)-arg(I_{R,n}) \|\}_{max}$ %**
Cylinder	None	N	0.007	0.002
		R	9.184	9.413
		SA	0.661	1.368
	Currents	N	0.008	0.001
		R	8.300	9.671
		SA	1.688	2.873
	Voltages	N	0.008	0.009
		R	8.919	9.043
		SA	0.259	1.677
Plane	None	N	0.006	0.004
		R	9.722	7.667
		SA	1.361	3.002
	Currents	N	0.007	0.026
		R	9.971	8.392
		SA	0.788	3.211
	Voltages	N	0.008	0.009
		R	8.697	9.642
		SA	2.177	4.433
Sphere	None	N	0.008	0.002
		R	6.965	6.664
		SA	0.552	4.826
	Currents	N	0.008	0.015
		R	8.002	9.012
		SA	3.800	5.050
	Voltages	N	0.009	0.013
		R	7.840	8.440
		SA	0.006	7.060

Figure 1.


Measurement Surface: Sphere
Excitation Amplitude
Element
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
10
20
30
40
50
60
70
80
90
100
$|I_n|$
$|I_{R,n}|$