

# Utilization of Blind Source Separation Algorithms for MIMO Linear Precoding

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**Abstract.** In this paper we investigate the application of Blind Source Separation (BSS) algorithms for the decoding of linearly precoded MIMO communication systems and for the design of limited feedback channels that send the Channel Status Information (CSI) from the receiver to the transmitter. The advantage of using BSS is that the MIMO channel can be continuously tracked without the need of pilot symbols. CSI is only sent through the feedback channel when the BSS algorithm indicates the presence of a strong channel variation.

*Keywords:* Adaptive blind source separation, MIMO time-varying channels, linear precoding.

## 1 Introduction

The continuous development of the wireless communication industry creates an enormous demand of high bit rate radio interfaces. Recently, it has been demonstrated that it is possible to achieve higher spectral efficiencies when using multiple antennas at both transmission and reception [1]. Transmitting over these Multiple-Input/Multiple-Output (MIMO) channels requires sophisticated signal processing methods in order to compensate the channel impairments. In particular, the receiver has to perform a Space-Time (ST) equalization to separate the streams transmitted through the multiple antennas. ST equalization is a difficult task that is traditionally carried out at the receiving side thus increasing complexity and cost of receivers. The cost of ST equalization at reception can be considerably reduced if an important part of the channel compensation is performed at the transmitter by means of precoding techniques. Besides, jointly optimal ST precoder and decoder designs provide better performance when compared to ST optimization only in the receiver side [2, 3].

Contrary to non linear approaches [4, 5], this paper focuses in linear precoding schemes that end up with the simplest possible receivers. In linearly precoded

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systems, the signals received by the antennas (observations) are instantaneous mixtures of the original signals (sources). The mixing system results from the joint consideration of the precoding matrix and the channel matrix. As a consequence, Blind Source Separation (BSS) algorithms [6] can be employed in order to decode the observations.

An important issue to consider in precoding schemes is that the encoding matrix must be adapted to changes in the channel. Towards this aim, we will apply Adaptive BSS (ABSS) algorithms [7, 8] to update the separating matrix in each data slot in accordance with channel time variations. In order to obtain a good tracking performance, a one-bit flag will be transmitted to the encoder over a limited-rate feedback channel [9]. By means of that bit the transmitter knows if the channel has changed or not and, therefore, if it has to request a training sequence for estimating it. In addition, we will propose a simple way to initialize ABSS algorithms in precoding systems in order to obtain faster convergence.

This paper is organized as follows. Section 2, describes the signal model corresponding to a MIMO communication system with linear precoding. Section 3 presents three novel strategies to decode the received signals using adaptive BSS algorithms. Illustrative computer simulations are presented in Section 4 and some concluding remarks are made in Section 5.

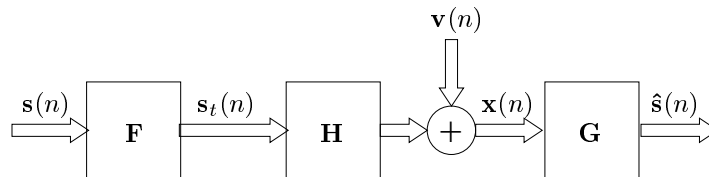


Fig. 1. Precoding communication scheme.

## 2 Signal Model

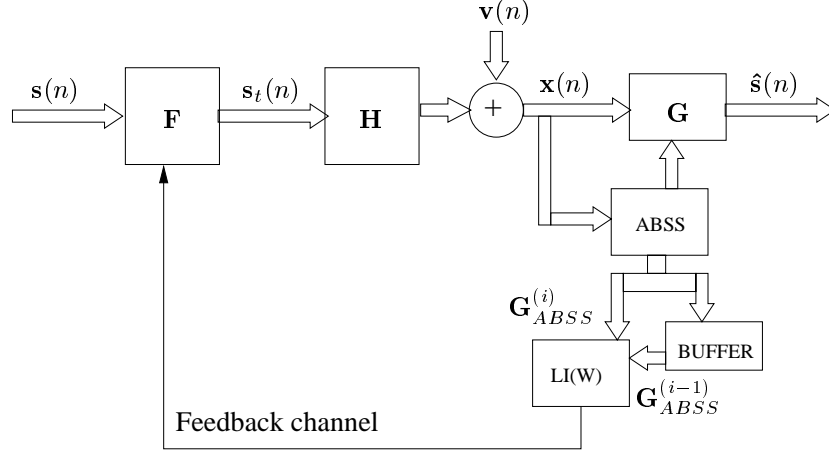
Let us consider the precoded MIMO communication system shown in Figure 1. The input bit-stream is modulated to generate the output stream of (possibly complex) symbols  $\mathbf{s}(n)$ . Let  $\mathbf{s}(n) = [s_1(n), \dots, s_{N_T}(n)]^T$  be the vector formed by  $N_T$  original signals. We assume that they are zero-mean, stationary, temporally-white, non-Gaussian distributed and statistically independent. The signals  $\mathbf{s}(n)$  are then filtered using a linear precoder system represented by an  $N_T \times N_T$  complex-valued matrix  $\mathbf{F}$ . As a consequence, the coded symbols,  $\mathbf{s}_t(n)$ , and the original ones,  $\mathbf{s}(n)$ , are related by the following expression

$$\mathbf{s}_t(n) = \mathbf{F}\mathbf{s}(n) \quad (1)$$

These signals arrive at an array of  $N_R$  antennas whose output at time  $n$ , denoted by  $\mathbf{x}(n) = [x_1(n), \dots, x_{N_R}(n)]^T$ , is given by

$$\mathbf{x}(n) = \mathbf{H}(n)\mathbf{F}\mathbf{s}(n) + \mathbf{v}(n) \quad (2)$$

where  $\mathbf{H}(n)$  is a  $N_R \times N_T$  matrix representing the MIMO channel and  $\mathbf{v}(n)$  is the white Gaussian noise.



**Fig. 2.** Basic Decoding Scheme.

Throughout this paper we will assume that all entries into  $\mathbf{H}(n)$  are complex Gaussian with i.i.d. real and imaginary parts with zero mean and unit variance. Denoting the gain from transmit antenna  $j$  and the receiver antenna  $i$  by  $h_{ij}(n)$ , the magnitudes of the channel gains  $|h_{ij}(n)|$  will have a Rayleigh distribution. We also consider a block fading channel in which the channel matrix response remains constant for blocks of  $L$  symbols and changes according to an autoregressive model of order  $p = 1$  from one block to another in the following form,

$$\mathbf{H}(n) = \mathbf{A}\mathbf{H}(n-1) + (\mathbf{I} - \mathbf{A})\mathbf{w}(n) \quad (3)$$

where  $\mathbf{A}$  is a diagonal matrix whose entries  $a_{kk}$  are given by  $J_0(2\pi f_D T \tau)$ , being  $f_D$  the Doppler frequency,  $T$  the duration of a data frame,  $J_0$  the zero-order Bessel function of the first kind and  $\tau = 1$ . Vector  $\mathbf{w}(n)$  is a zero-mean, i.i.d. and complex Gaussian vector process. The speed in channel changes is decided by means of the parameter  $L$ . For low values of  $L$  we will have fast fading channels whereas higher values of  $L$  lead to more static channels. A flat fading channel is assumed in which the symbol time-period is much larger than the channel delay-spread.

Finally, we assume that a linear decoder is employed to produce estimates of the transmitted symbols. The decoder is represented by a  $N_T \times N_R$  matrix  $\mathbf{G}$

and the symbol estimates are given by

$$\hat{\mathbf{s}}(n) = \mathbf{G}\mathbf{x}(n) \quad (4)$$

## 2.1 Linear transmission/reception optimization

The goal in precoding schemes is to design the matrices  $\mathbf{F}$  and  $\mathbf{G}$  that provide the best performance with respect to some optimization criterion. In this paper, we employ a Zero-Forcing (ZF) approach in order to minimize the symbol estimation errors under a transmit power constraint (see, for instance [3, 10]). The optimization problem can be formulated as

$$\{\mathbf{F}_{opt}^{ZF}, \mathbf{G}_{opt}^{ZF}\} = \arg \min_{\mathbf{F}, \mathbf{G}} E\{\|\mathbf{s}(n) - \hat{\mathbf{s}}(n)\|_2^2\} \quad (5)$$

subject to  $\mathbf{G}\mathbf{H}\mathbf{F} = \mathbf{I}$  and  $\text{tr}(\mathbf{F}\mathbf{R}_s\mathbf{F}^H) = P_t$ . In order to obtain the optimal linear precoding/decoding matrices  $\mathbf{F}$  and  $\mathbf{G}$  we define the following channel eigenvalue decomposition

$$\mathbf{H}^H(n)\mathbf{R}_n^{-1}(n)\mathbf{H}(n) = \mathbf{V}(n)\mathbf{\Delta}(n)\mathbf{V}^H(n) \quad (6)$$

Applying the Lagrangian method, it is possible to demonstrate that the joint ZF solution for the design of the linear precoder/decoder is given by [10]

$$\mathbf{F}_{opt}^{ZF} = \sqrt{\frac{P_t}{\text{tr}(\mathbf{\Delta}^{-1/2})}} \mathbf{V} \mathbf{\Delta}^{-1/4} \quad (7)$$

$$\mathbf{G}_{opt}^{ZF} = \sqrt{\frac{\text{tr}(\mathbf{\Delta}^{-1/2})}{P_t}} \mathbf{\Delta}^{-3/4} \mathbf{V}^H \mathbf{H}^H \mathbf{R}_n^{-1} \quad (8)$$

## 3 Proposed Decoding Schemes

It is interesting to note that the received signals (observations)  $\mathbf{x}(n)$  given by equation (2) are instantaneous mixtures of the original signals  $\mathbf{s}(n)$ , where  $\mathbf{H}(n)\mathbf{F}$  represents the mixing system. Consequently the decoding matrix  $\mathbf{G}$  can be interpreted as the separating system needed to recover the original signals from the observations and it can be estimated using many BSS algorithms. Based on this idea, we propose in the sequel several decoding strategies for time-varying MIMO channels.

### 3.1 Approach I: Basic decoding scheme

Figure 2 shows a block diagram of a general linearly precoded MIMO system with limited feedback channel. We propose to use an adaptive BSS algorithm to find the separating (decoding) matrix  $\mathbf{G}_{ABSS}$  from the received symbols. The decoded (separated) signals are thus obtained according to

$$\hat{\mathbf{s}}(n) = \mathbf{G}_{ABSS}\mathbf{x}(n) \quad (9)$$

In order to detect variations in the channel we make use the following performance index  $LI(\mathbf{W}^{(l)})$  that measures the likeliness between the decoding matrices obtained for consecutive symbols

$$LI(\mathbf{W}^{(i)}) = \sum_{i=1}^N \left( \sum_{j=1}^N \frac{|w_{ij}|^2}{\max_l(|w_{il}|^2)} - 1 \right) + \sum_{j=1}^N \left( \sum_{i=1}^N \frac{|w_{ij}|^2}{\max_l(|w_{lj}[k]|^2)} - 1 \right) \quad (10)$$

where  $\mathbf{W}^{(i)} = \mathbf{G}_{ABSS}^{(i-1)} (\mathbf{G}_{ABSS}^{(i)})^{-1}$ . The superscript  $(i)$  denotes the symbol for which the decoding matrix has been calculated<sup>1</sup>. We consider that a channel change has occurred when this performance criterion exceeds a threshold value  $\epsilon$ , i.e., when

$$LI(\mathbf{W}^{(i)}) - \frac{1}{i - i_c} \sum_{l=i_c}^{i-1} LI(\mathbf{W}^{(l)}) > \epsilon \quad (11)$$

where  $i_c$  denotes the last symbol for which a channel change has been detected.

When a change in the channel is detected, a bit is transmitted through the feedback channel to indicate that a training sequence must be transmitted. From this training sequence, the receiver estimates the channel matrix  $\mathbf{H}(n)$  using a supervised algorithm. Finally, the feedback channel is also used to send from the receiver to the transmitter the channel estimate needed to adapt  $\mathbf{F}$  in the transmit side by evaluating (7).

Note also that since  $\mathbf{G}_{ABSS}$  is an estimation of the mixing system inverse, the channel matrix  $\mathbf{H}(n)$  can be easily estimated without using training sequences by using a totally blind method such that  $\hat{\mathbf{H}}(n) = \mathbf{F}\mathbf{G}_{ABSS}$ . However, we have observed that this approach produces a poor performance in terms of bit rate error.

### 3.2 Approach II: Decoding using a stored matrix $\mathbf{G}$

The performance of the previous approach can be substantially improved for slow fading channels (see Section 4) by evaluating equation (8) at the receiver to obtain matrix  $\mathbf{G}$  each time a change in the channel is detected. Thus, this matrix will be used in the receiver, instead of  $\mathbf{G}_{ABSS}^{(i)}$ , to decode the received signals.

### 3.3 Adaptive algorithms: initial conditions

The initial conditions of the separating system is a crucial issue to consider when adaptive algorithms are used to obtain the separating coefficients. In the previously proposed approaches, the algorithm is used to obtain an estimation of

<sup>1</sup> The index  $LI(\mathbf{W})$  has been used in previous BSS work to measure the performance of BSS algorithms [11].

the decoding matrix  $\mathbf{G}$  given in equation (8). For this reason it is sensitive to think that equation (8) is a good starting point. Recall that this matrix can be obtained for the channel matrix  $\mathbf{H}(n)$  each time a change in the channel is detected. This initialization has two important advantages: the convergence speed is increased and the permutation indeterminacy inherent to BSS algorithms is avoided.

## 4 Computer Simulations

In this section we present the results of several computer simulations that we carried out to validate the proposed systems. It is assumed that the sources have been passed through a mixing system with  $N_T = 3$  and  $N_R = 4$  antennas. We considered 10,000 QPSK symbols transmitted over normalized Rayleigh channels such that  $E[\|\mathbf{H}\|_F^2] = 1$ , where  $\|\cdot\|_F$  denotes the Frobenius norm. The SNR plotted is given by  $input\ SNR = 10 \log P_T / \sigma_v^2$ , where  $P_T$  is the transmit power. Note that possible channel attenuation/gain is not considered. We set the transmit power to  $P_t = 20$ . In order to reduce the computational cost associated to track the channel variations, we have evaluated the decision criterion (11) after processing 10 received symbols instead of symbol by symbol.

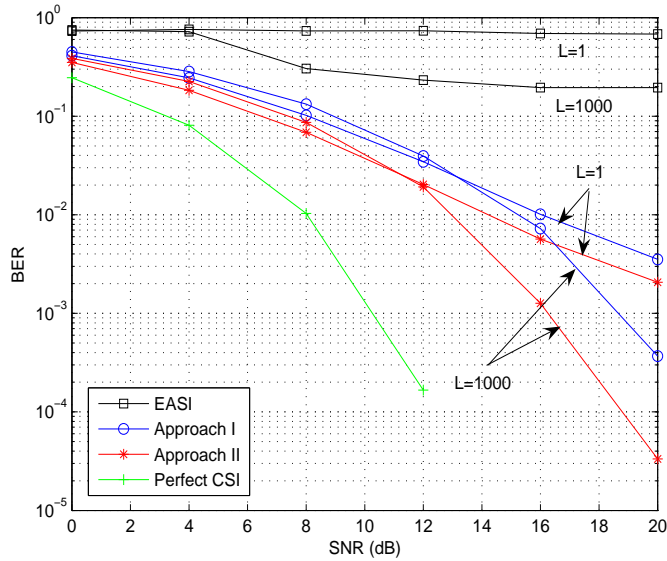
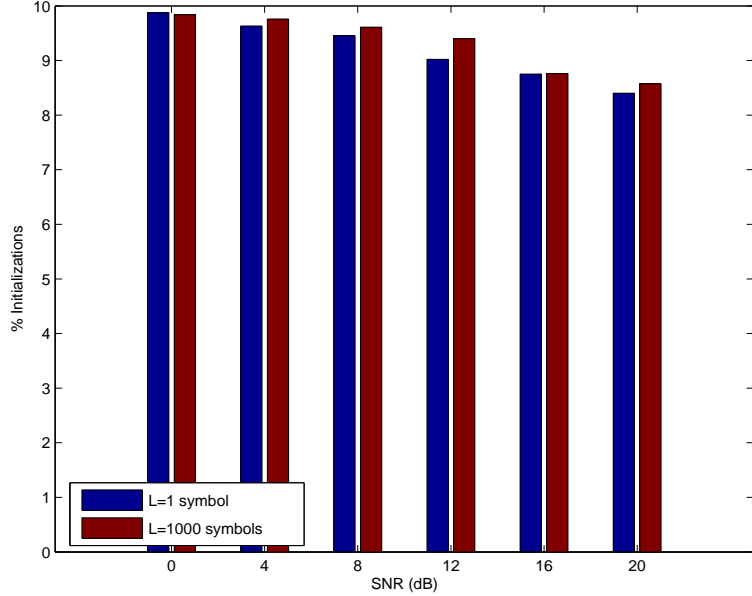


Fig. 3. BER performance vs. SNR

Many conventional BSS algorithms can be used in the proposed schemes to estimate the mixing system. Among of all of them, we have selected the



**Fig. 4.** Percentage of number of initializations vs SNR

adaptive EASI algorithm proposed by [7, 8]. As regards EASI parameters, we have considered  $\epsilon = 0.05$  and a constant adaptation step equal to  $\lambda = 0.1$ , with non-linear functions given by  $g_i = (\text{diag}(\hat{\mathbf{s}}\hat{\mathbf{s}}^H))_i \hat{\mathbf{s}}_i$ , for  $1 \leq i \leq N_T$  (see [7, 8]). We will track the MIMO channel with the matrix given by the adaptive BSS algorithm according to the system described in Section 3. The channel is constant along  $L = 1$  or  $L = 1000$  data symbols. Obviously, the size of  $L$  determine of speed in channel time variations. You can see in the Figure 3 how for larger blocks (i.e., more slow fading channels) a better performance could be obtained. Finally, you can see in this figure how employing the optimum decoding matrix  $\mathbf{G}_{opt}^{ZF}$  according to the approach II described in Section 3 produces too better results. This is because the ABSS algorithm provides a suboptimum decoding matrix. In the figure can be also seen the curve when perfect channel information is available, i.e., when  $\epsilon = 0$ , at the transmitter side. Smaller values of  $\epsilon$  will yield to better BER performances due to the larger number of channel re-estimations and precoder updates at cost of greater number of initializations and therefore, greater overhead in the feedback channel. This means that the optimal value of  $\epsilon$  must be modified depending on the fading speed to ensure a good performance.

Figure 4 plots the number of initializations of the matrix  $\mathbf{G}_{ABSS}$  (and updates of  $\mathbf{F}$  and  $\mathbf{G}$ ) for different values of SNR as a function of  $L$  when approach II is employed. This value will depend weakly on the block size  $L$  and on the signal to noise ratio employing the proposed approach. Obviously, for lower SNR the precoder matrix  $\mathbf{F}$  will be adapted more times in order to get a correct channel tracking. Note that if we evaluate the proposed decision criterion each

received symbol the algorithm complexity is not only increased but the number of initializations would be extend greatly, specially for low SNR.

## 5 Conclusions

In this work we have studied the utilization of Blind Source Separation algorithms for decoding linearly precoded MIMO communication systems. The basic idea is to consider that the received precoded signals are instantaneous mixtures of the sources and that they can be decoded using adaptive BSS algorithms. This simple strategy has been combined together with the low rate characteristic of limited feedback channels available in wireless communications to track channel variations. Simulation results show that the performance of this scheme can be improved for slow varying channels by including a buffer in the receiver that contains previous decoding matrices if an adequate start matrix for the BSS algorithm is selected. In this approach, we examine the likeness between the separating matrix and the matrices stored in the receiver side in order to track the channel variations.

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