High-order discretizations for the wave equation based on the modified equation technique

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Highly accurate solution of the full wave equation implies very high computational burdens. Indeed, to improve the accuracy of the numerical solution, one must considerably reduce the space step. Obviously this will result in increasing the number of unknowns of the discrete problem. Besides, the time step is linked to the space step through the CFL (Courant-Friedrichs-Lewy) condition. In the three-dimensional case the problem can have more than ten million unknowns, which must be evaluated at each time-iteration. However, high-order numerical methods can be used for computing accurate solutions with larger space and time steps. Recently, Joly and Gilbert (cf. [2]) have optimized the Modified Equation Technique (MET), which was proposed in [1] for solving the wave equation, and it seems to be very promising given some improvements. In this work, we apply this technique in a new way. Normally, most of the study devoted to the solution of the wave equation consider first the space discretization of the system before addressing the question of the time discretization. We intend here to invert the discretization process by applying first the time discretization using the MET and then to consider the space discretization. The time discretization causes high-order operators to appear (such as p-Laplacian) and we have therefore to consider appropriate methods to discretize them. The Discontinuous Galerkin Methods are well adapted to this discretization, since they allow to consider piecewise discontinuous functions. In particular, using the Interior Penalty Discontinuous

Galerkin (IPDG) method (cf. [3, 4]), one can enforce through the elements high-order transmission conditions, which are adapted to the high order operators to be discretized.

References

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