Asymptotic behavior of the Stokes system in a thin film flow with a rough boundary

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In this work we consider the Stokes system in an open set of small high ε. Our aim is to study the asymptotic behavior of this fluid for a rugous bottom, where the roughness is periodic of period r_ε and amplitude δ_ε, where $\delta_\varepsilon \ll r_\varepsilon \ll \varepsilon$.

We assume slip boundary conditions on the rough boundary and adherence on the top. We show that the asymptotic behavior depend on the limit $\lambda = \lim_{\varepsilon\to 0} \frac{\delta_\varepsilon}{r_\varepsilon}\sqrt{\frac{\varepsilon}{r_\varepsilon}}$.

In the case of $\lambda = 0$ we prove that the fluid behaves as if the rugous boundary were a plane boundary. If $\lambda = +\infty$, then the slip condition implies adherence condition in the limit. However, in the $\lambda \in (0, +\infty)$ we obtain a Fourier condition for the tangential component of the limit velocity v,

$$-\partial_{y_3} v(y', 0) + \lambda^2 R v(y', 0) = 0,$$

and the following equation for the limit pressure p,

$$-\mathrm{div}_{y'}\left(\left[\frac{1}{12}I + \frac{1}{4}(I+\lambda^2 R)^{-1}\right]\nabla_{y'} p(y')\right) = -\mathrm{div}_{y'}\left(\left[\frac{1}{12}I + \frac{1}{4}(I+\lambda^2 R)^{-1}\right]\nabla_{y'} f(y')\right).$$

References

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