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A VALIDATION OF THE BOUNDARY ELEMENT METHOD FOR GROUNDING GRID DESIGN AND COMPUTATION

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SUMMARY

Several widespread intuitive techniques developed during the last two decades for substation grounding analysis, such as the Average Potential Method (APM), have been recently identified as particular cases of a more general Boundary Element formulation [1]. In this approach, problems encountered with the application of these methods [3] can be explained from a mathematically rigorous point of view, and innovative advanced and more efficient techniques can be derived [2].

Numerical results obtained with low and medium levels of discretization (equivalent resistance and leakage current density) seem to be reasonable. However, these solutions still have not been validated. Unrealistic results are obtained when domain discretization is increased, since no one procedure is yet available to eliminate the above mentioned problems. Hence, numerical convergence analyses are precluded. The obtention of highly accurate numerical results by means of standard techniques (FEM, Finite Differences) implies unapproachable computing requirements in practical cases. On the other side, neither practical error estimates have been derived, nor analytical solutions are known for practical cases, nor sufficiently accurate experimental measurements have been reported up to this point.

In this paper, we present a validation of the results obtained by the Boundary Element proposed formulation, including the classical methods. A highly accurate solution to a specially designed test problem is obtained by means of a 2D FEM model, using up to 80,000 degrees of freedom. Results are compared with those carried out by Boundary Elements.

1. INTRODUCTION

The electrokinetic stationary problem, related to fault electric currents dissipation into earth, can be written in the following form [1, 2, 8]

$$\begin{gathered}\boldsymbol{\sigma} = (-\gamma\, \mathbf{grad}\, V), \quad \mathrm{div}\,(\boldsymbol{\sigma}) = 0 \quad \text{in} \quad E, \\ \boldsymbol{\sigma}^T \mathbf{n}_E = 0 \quad \text{in} \quad \Gamma_E, \quad V = 1 \quad \text{in} \quad \Gamma, \quad V \longrightarrow 0 \quad \text{if} \quad |\mathbf{x}| \longrightarrow \infty\end{gathered} \tag{1}$$

where V and $\boldsymbol{\sigma}$ are the potential and current density at an arbitrary point $\mathbf{x}$; domain E is the earth, γ its conductivity (assumed constant), Γ_E its surface (assumed horizontal) and $\mathbf{n}_E$ its normal exterior unit field; and Γ is the grounding electrode surface.

Another important magnitude is the equivalent resistance of the grounding electrode-earth system

$$R_{eq} = \left[\int\!\!\int_{\Gamma} \sigma \, d\Gamma \right]^{-1} \tag{2}$$

being $\sigma = \boldsymbol{\sigma}^T \mathbf{n}$ in Γ, the leakage current density and $\mathbf{n}$ the normal exterior unit field to the grounding electrode surface Γ.

In the last decades some intuitive techniques for grounding grid analysis, such as the Average Potential Method (APM), have been developed. However, significant problems have been encountered in the application of these methods [3].

A new Boundary Element approach has been recently presented [1, 6] that includes the above mentioned intuitive techniques as particular cases. In this kind of formulation the unknown quantity is the leakage current density σ, while the potential at an arbitrary point and the equivalent resistance R_{eq} must be computed subsequently.

Although new more efficient techniques can be derived from this approach, it has not been possible to test the accuracy of the results (that seem reasonable for low and medium levels of discretization) up to this point. Large numerical instabilities appear when discretization is refined, giving unrealistic results [1, 2, 3]. On the other hand, to obtain highly accurate solutions by means of standard techniques (FEM, Finite Differences) should imply unapproachable computing requirements; analytical solutions are not available for real problems; neither accurate experimental measurements nor practical error estimates have been obtained; and, finally, the leakage current density is a difficult magnitude to compare, which real distribution is not obvious.

All these aspects are specially important because the Boundary Element Method could become a powerful numerical computing tool for the systematic analysis of this kind of problems.

A validation of BEM results is presented in this paper. A test has been specifically designed in order to be solved by a 2D Finite Element model and a Boundary Element technique in order to compare the results.

2. TEST PROBLEM STATEMENT

Let the grounding electrode be a single cylindrical conductor bar, which radius of the cross section a is small in comparison with its length L ($\simeq 10^{-3}$ times smaller). This seems to be an adecquate geometry for our purposes, since grounding electrodes in practice consist of a number of interconnected bare cylindrical conductors (grounding grids).

Let the electrode be buried so deep that the Neumann boundary condition in (1) can be neglected, domain E can be considered infinite, and the problem becomes symmetric with respect to the axis of the cylinder.

Although an analytical solution for this test problem is available in terms of Bessel expansions [8], we turn our attention to a FEM axial symmetry model, because computing the leakage current density throughout the bar length should be extremely time-consuming by this kind of series.

Taking advantage of the symmetries, we can write our test problems in cylindrical coordinates (Figure 1.a)

$$\begin{aligned} \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} &= 0 \quad \text{in} \quad \Omega \\ V &= 1 \quad \text{in} \quad \Gamma \\ \frac{\partial V}{\partial \mathbf{n}} &= 0 \quad \text{in} \quad \Gamma_q \\ V &\longrightarrow 0 \quad \text{if} \quad |\mathbf{x}| \longrightarrow \infty \end{aligned} \tag{3}$$

being r and z the radial and latitude coordinates.

Problem (3) can now be solved by the Finite Element Method. Results obtained will be potential values at selected nodal points in the discretized domain Ω. In order to validate BEM solutions, the leakage current density σ must be computed from potential values at nodal points close to the cylinder boundary Γ. Because of our high accuracy requirements in computing σ, the finite element mesh arrangement should be carefully designed.

3. NUMERICAL MODEL

A weighted residuals statement for problem (3) can be written as

$$\int_\Omega W \left[r\,\frac{\partial^2 V}{\partial r^2} + \frac{\partial V}{\partial r} + r\,\frac{\partial^2 V}{\partial z^2} \right] d\Omega = 0 \tag{4}$$

Applying Green's identity and taking into account the Neumann boundary conditions we obtain the weaker form

$$\int_\Gamma r\,W\,\frac{\partial V}{\partial \mathbf{n}}\,d\Gamma - \int_\Omega r\left(\frac{\partial W}{\partial r}\frac{\partial V}{\partial r} + \frac{\partial W}{\partial z}\frac{\partial V}{\partial z} \right) d\Omega = 0 \tag{5}$$

for all members W of a suitable class of weight functions on Ω, with boundary conditions

$$V = 1 \quad \text{in} \quad \Gamma, \qquad V \longrightarrow 0 \quad \text{if} \quad (r,z) \longrightarrow \infty \tag{6}$$

Potential V can now be discretized

$$V(r,z) = \sum_{j=1}^{j=N} V_j\,N_j(r,z) \tag{7}$$

being V_j the nodal potential values and N_j the shape functions.

Using Galerkin formulation ($W = N_i$), functional equation (5) results in a linear equations system

$$\begin{aligned} \sum_{j=1}^{j=N} K_{ij}\,V_j &= q_i\,, \quad i = 1, ..., N \\ V_j &= 1\,, \quad \forall\, j \in Q \\ q_i &= 0\,, \quad \forall\, i \notin Q \end{aligned} \tag{8}$$

being Q the set of nodal points on the boundary Γ, and

$$K_{ij} = \int_{\Omega} r \left(\frac{\partial N_i}{\partial r} \frac{\partial N_j}{\partial r} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right) d\Omega \tag{9}$$

Leakage current density σ can be computed subsequently, either by interpolation over potential values on nodal points close to boundary Γ, or by means of the flux quantities q_i in equations system (8). Equivalent resistance R_{eq} can be computed next by means of expression (2).

Since we are analyzing a Dirichlet Exterior Problem, the Finite Element mesh should be large enough (in extension) to adecquately simulate the null potential boundary condition in the infinite, by prescribing null potential values at sufficiently remote nodal points. On the other hand, our purpose is to obtain highly accurate results for leakage current and equivalent resistance. Therefore, we need to design highly dense and regular meshes only in the proximity of the electrode boundary.

For these reasons, an specific mesh generator for this kind of test problems has been developed. Several layers of small (in comparison with the electrode radius a) regular quadrilateral elements are generated around boundary Γ, in order to capture high precision results in this area. Next, elements size grows exponentially as distance to boundary increases, by alternating layers of triangles and quadrilaterals. Since the number of elements in consecutive layers decreases exponentially, the null potential boundary condition is prescribed in relatively few nodal points. Special care has been taken to preserve regularity of elements through the growing process (see Figures 1.a and 1.b).

In the structured grids that we have generated by means of this strategy, most of elements (around 75 % of total) are placed in the proximity of the electrode surface. Therefore, the greater part of the computing effort is devoted to ensure a sufficiently high level of accuracy in this area.

Different techniques have been tested for solving the linear system of equations (8). Although matrices are symmetric, positive definite and banded, direct methods are out of range due to the size of the problems (up to eighty thousand degrees of freedom). Several iterative methods have been tested for a set of medium size problems. Namely: Jacobi and Gauss-Seidel (with and without overrelaxation and preconditioning) and Conjugate Gradients. The very best results have been obtained by an element-by-element preconditioned conjugate gradients algorithm [7] without assembly of the global matrix. This technique turned out to be extremely efficient, and it was finally used for solving the large scale problems that are presented in this paper.

4. NUMERICAL RESULTS

The test problem characteristics presented are : bar length = 2.0 m; radius of the bar cross section = 0.005 m; and earth resistivity = 1.0 Ohm · m.

Figures 2 and 3 present different leakage current density distributions throughout the bar length obtained by a BEM formulation [2] for several type of elements (constant, linear and parabolic leakage current) with results obtained by FEM with large structured grids (12441 d.o.f./13410 elements in case 3.b, 24987 d.o.f./26914 elements in case 3.d, and 80533 d.o.f./84528 elements in case 3.f). Figure 4 presents a summary of the equivalent resistance values in all these cases.

With respect to the equivalent resistance, it can be shown that BEM results agree significantly with those obtained by FEM. The relative error between the BEM solution for one single parabolic element and the FEM solution for more than eighty thousand degrees of freedom is less than 1.25 %. On the other hand, no significant improvement seems to be achieved by increasing discretization level, since solution obtained by BEM for one single and fifty parabolic elements differs in less than 0.6 %. Actually, the solution obtained by BEM for one single constant element (equivalent to the Average Potential Method with one segmentation [2, 3, 5]) could be considered accurate enough for practical purposes.

However, significant differences are found with respect to the leakage current density distributions throughout the bar length. FEM results (Figures 3.b, 3.d and 3.f) show that real distribution is quite smooth in the center of the bar, but varies sharply near both free ends. All the BEM models give an accurate average (the equivalent resistance differs slightly, as pointed out before), but the leakage current density distribution is substantially different. Obviously, is difficult for the BEM models to adjust the boundary condition $V = 1$ on the electrode surface near the free ends [1, 2]. Nevertheless, this effect is less pronounced in interconnected bars within grounding grids, and has no special importance in practical cases.

Anyhow, it seems important to obtain an accurate enough distribution of the leakage current. In fact, the computed potential level at a certain point on the earth surface can vary significantly if the leakage current distribution in a close electrode is erroneous [3]. After all, a 2.0 % relative error for a $50,000\,V$ fault condition represents a $1,000\,V$ absolute error.

METHOD		Discretization	Equiv. Resist. (Ω)
BEM	–	3 Constant Elements	0.451706
BEM	–	10 Constant Elements	0.449737
BEM	–	100 Constant Elements	0.447752
BEM	–	2 Linear Elements	0.450381
BEM	–	10 Linear Elements	0.448693
BEM	–	100 Linear Elements	0.447327
BEM	–	1 Parabolic Element	0.449764
BEM	–	5 Parabolic Elements	0.448595
BEM	–	50 Parabolic Elements	0.447297
FEM	–	12441 Degrees of Freedom	0.439689
FEM	–	24987 Degrees of Freedom	0.454940
FEM	–	80533 Degrees of Freedom	0.444223

Figure 4.—Equivalent resistances obtained by BEM and FEM for different levels of discretization.

Figure 3 shows that actual BEM models (including the classical methods) fail to provide an accurate leakage current distribution when discretization is increased, due to numerical instabilities which origin can now be explained [2]. For higher orders of discretization, instabilities can strecht out the whole bar length, giving unrealistic results in subsequent computation of potential levels on the earth surface [3]. However, results obtained for low levels of discretization (Figure 2) are accurate enough for most practical purposes. In fact, the solutions obtained for one single parabolic element (Figure 2.e), and two linear elements (Figure 2.c) seem to be quite reasonable. On the other hand, results obtained by BEM for higher levels of discretization (Figures 3.a, 3.c and 3.e) could be considered extremely accurate if oscillations around the real solution could be avoided or eliminated by smoothing.

Passing now to other matters, some additional tests have also been carried out with the FEM model. No significant variations have been observed when the internal electrode resistivity is considered. On the other hand, the current density leaving the ends of the conductor has been computed. Obtained values are negligible, in the oreder of magnitude predicted by other authors [3].

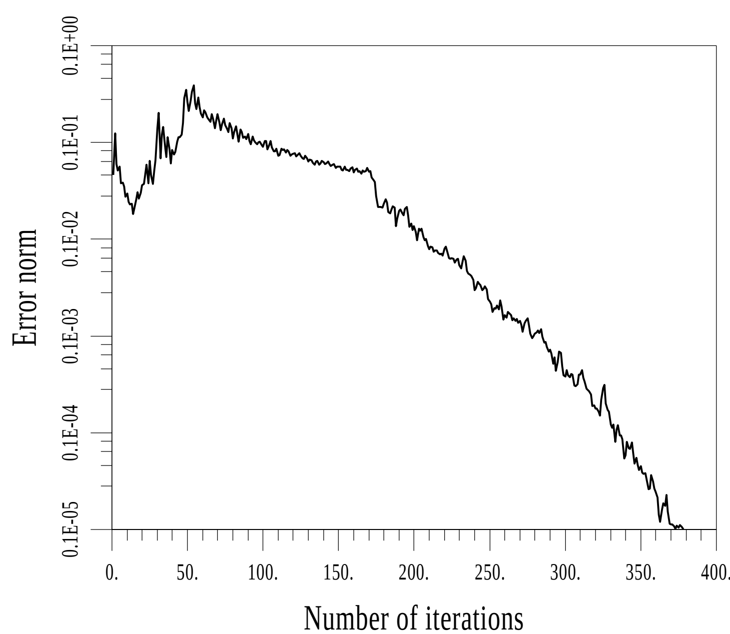

Figure 5.—Evolution of Logarithmic Error Norm versus the Number of Iterations in the algorithm of Preconditioned Conjugate Gradient Method.

5. CONCLUSIONS

In this paper, a numerical validation of a general BEM formulation for substation grounding analysis has been presented. The proposed BEM formulation includes several widespread intuitive techniques (such as the Average Potential Method) as particular cases.

A highly accurate solution to a specially designed test problem has been obtained by means of a 2D FEM model, using up to 80,000 degrees of freedom. Results have been compared with those carried out by the proposed BEM formulation [1, 2].

Numerical results show that, for practical purposes, BEM formulations provide accurate enough results with low/medium levels of discretization, while higher order elements (linear or parabolics) seem to be quite more efficient than traditional constant elements.

However, further research will be necessary to avoid or eliminate numerical instabilities (which origin can now be explained [2]), that produce unrealistic results when discretization level is increased. These powerful numerical tools should probably become widespread techniques for the systematic analysis of substation groundings in a near future.

6. ACKNOWLEDGMENTS

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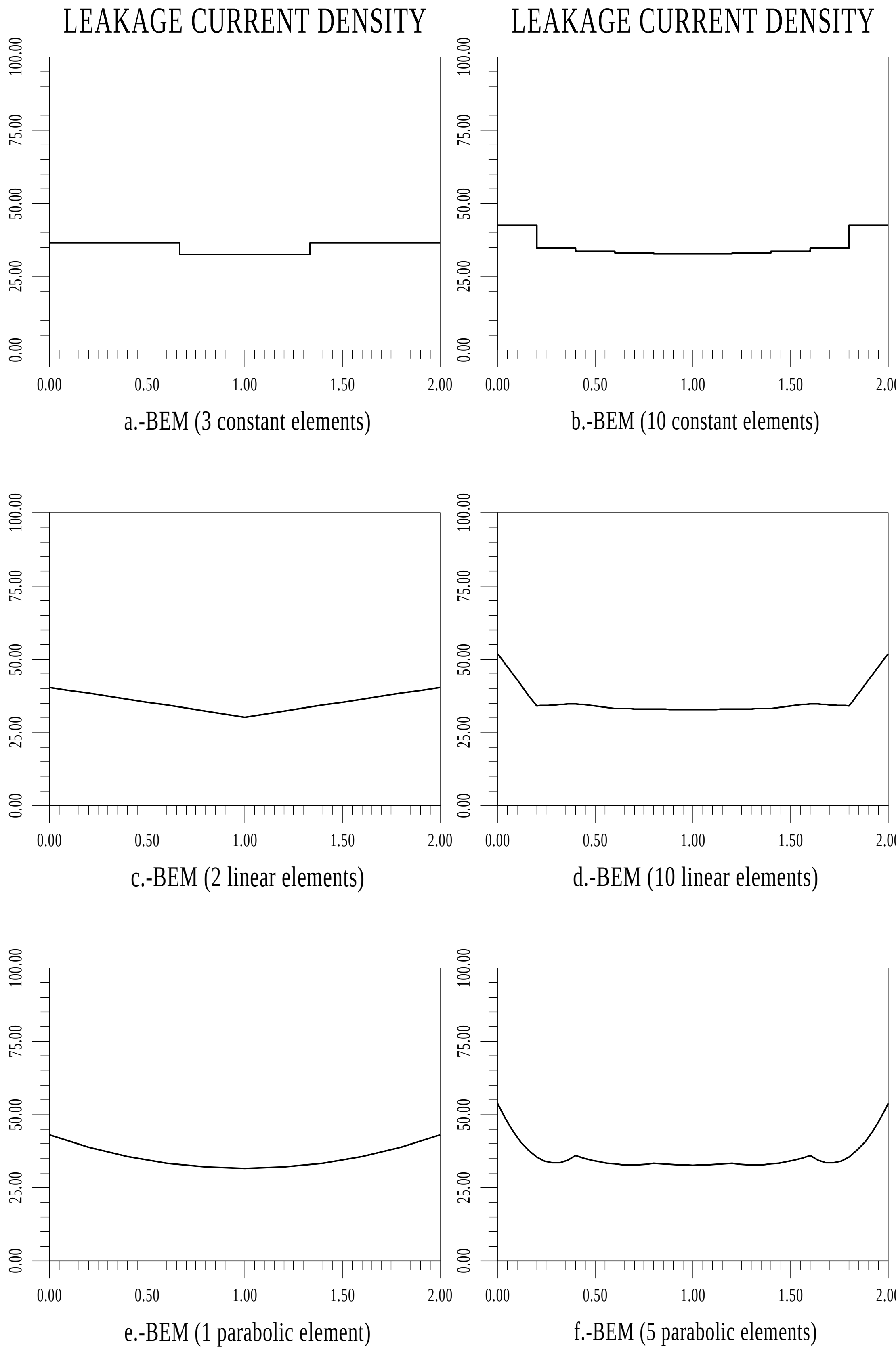

Figure 2.—Comparison between results obtained by BEM with different type and number of elements. Leakage Current Density throughout the bar length.

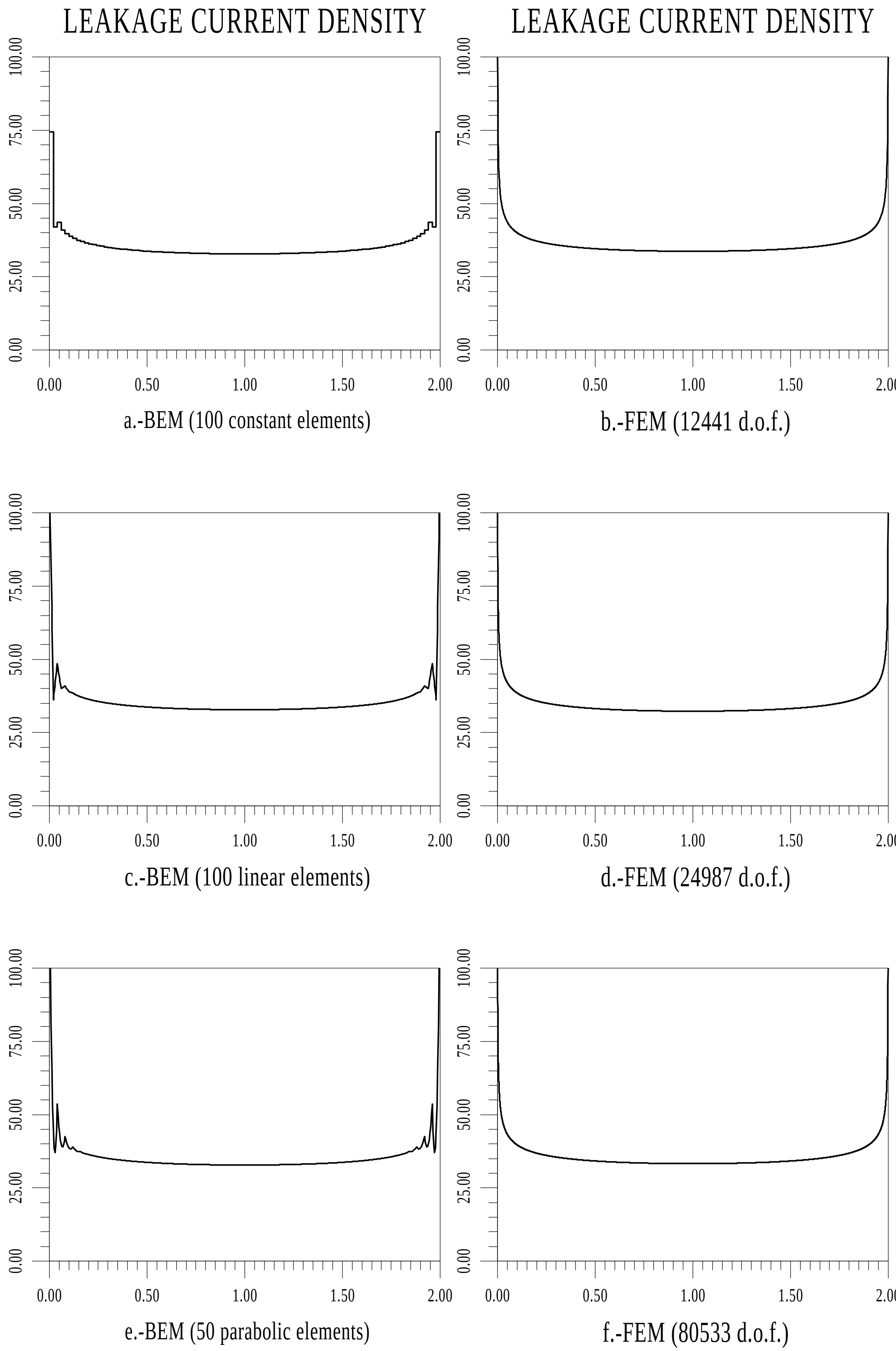

Figure 3.—Comparison between results obtained by BEM with different type and number of elements and results obtained by FEM with three meshes. Leakage Current Density throughout the bar length.

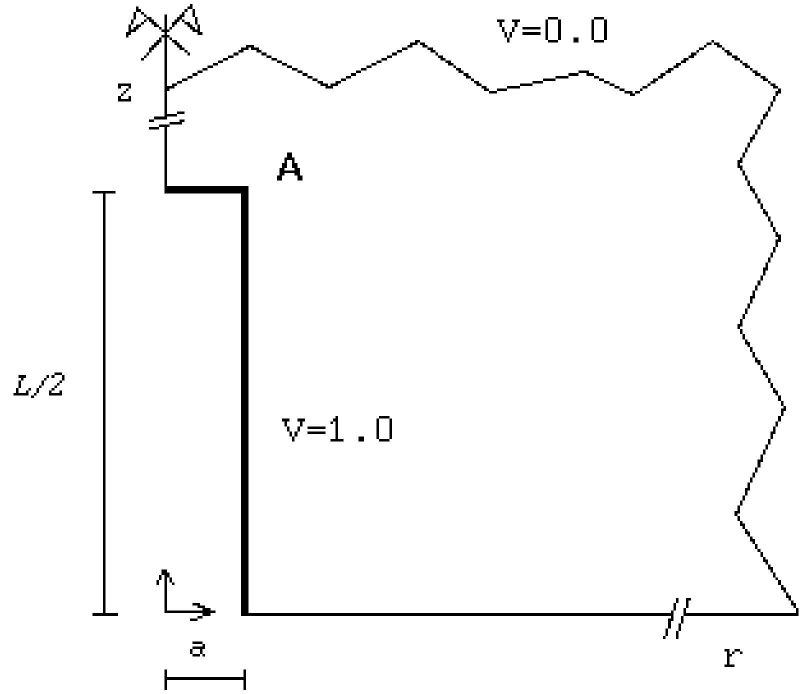

Figure 1.a.—Test Problem Schematic Representation. Bar Boundary marked with thick line.

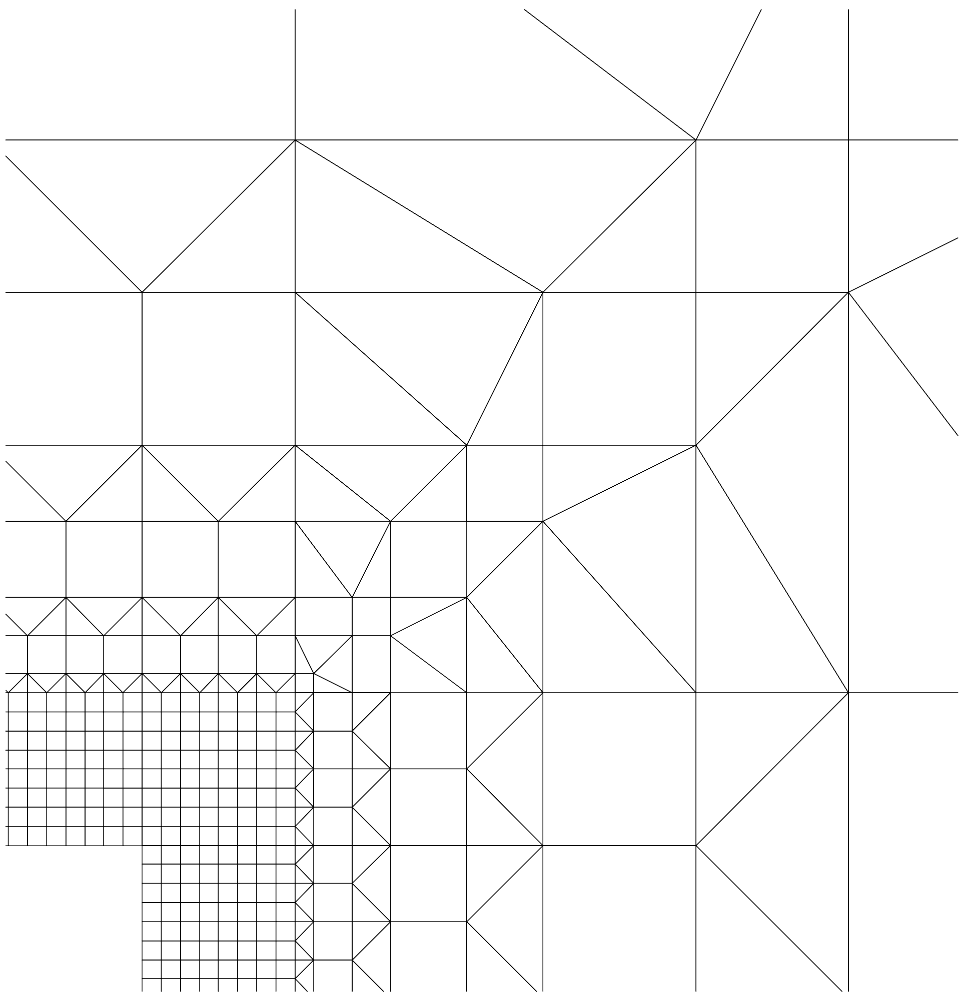

Figure 1.b.—Structured Grid used in FEM. Detail of zone close to point A.