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A minimum weight FEM formulation for Structural Topological Optimization with local stress constraints

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1. Abstract
Since Bendsøe and Kikuchi proposed the basic concepts in 1988, most of topology structural optimization results have been obtained so far by means of a maximum stiffness (minimum strain energy, minimum compliance) approach. In this kind of approaches, the mass is normally restricted to a given percentage of the total maximum possible mass, while no stress constraints are taken into account. On the other hand, size and shape structural optimization problems are normally stated in terms of a minimum weight with stress constraint approach. These traditional minimum compliance statements for topology optimization problems offer some obvious advantages, since one avoids dealing with a large number of highly non-linear stress constraints.
However, one can argue that this kind of statements has several important drawbacks. Thus, different solutions are obtained for different restrictions on the mass, and the final design could be unfeasible in practice since no constraints are imposed on the maximum allowed stress. On the other hand, the minimum compliance problem is said to be ill-posed, since the solution oscillates as the discretization refinement is increased. This difficulty can be easily overcome by introducing porous materials. However, an optimized material distribution with a large amount of porous material is frequently considered an unwanted result. And, on the other hand, numerical instabilities occur unless additional stabilization techniques (such as the perimeter method, or the filter method) are employed. Thus, the final optimized results normally resemble truss-like structures.
A new FEM formulation for topological optimization of structures is presented in this paper. This new model minimizes the weight of the structure in order to get a more realistic solution, taking into consideration that the materials stresses can not exceed a predetermined maximum value. One gets, therefore, a large number of nonlinear stress constraints which make more difficult the problem from a mathematical point of view but, on the other hand, this technique does not require stabilization schemes because the restrictions are stated in all elements. As an example, several structures optimized with this technique are presented.

2. Keywords: Topological optimization, minimum weight, finite element method, stress constraints.

3. Introduction
Topological optimization problems are solved, generally, by means of a maximum stiffness (minimum compliance) approach. With this formulation the objective function is very complicated, however, there is only one constraint which is, in addition, linear. Consequently, minimum compliance approach has several important advantages. On the other hand, these formulations present several dificulties because they requiere some artificial parameters that do not have an easy physical interpretation. Following the same idea, the objective function does not represent an important physic parameter from an engineering point of view. Mass constraints are not usually employed on structural design. Most common parameters on structural design are displacements and stresses as constraints and the cost as objective.
The most employed formulation applied to solve minimum compliance statements is the so called SIMP (Solid Isotropic Material with Penalty). With this formulation we define a constant relative density of the porous material for each element of the mesh. This relative density oscillates from 0 to 1 (porous-solid). The relative densities are the design variables of the optimization problem. Total amount of material is the linear constraint.
SIMP formulation has several important advantages because the resulting problem is, generally, easy to solve. However, minimum compliance presents numerical instabilities that it is necesary to avoid. Some

techniques, like the perimeter method or the filter method, are usually employed [1]. In addition, a penalization parameter is employed to avoid intermediate densities. Then, the solutions obtained seems to be truss-like structures.
In this paper, we present a minimum weight formulation for structural topological optimization with local stress constraints (MWSC).

4. MWSC Formulation

4.1. The structural problem analysis with relative density
Let the domain Ω^o be occupied by a porous material. Let $\rho(\boldsymbol{r}^o)$ be the relative density of the material (complement of the porosity, which adimensional value must range from 0 to 1) at point $\boldsymbol{P}^o$ of material coordinates $\boldsymbol{r}^o$. Thus, every arbitrary point P^o in Ω^o is mapped into a different position P in Ω. Let $\boldsymbol{r}^o$ and $\boldsymbol{r}$ be the material coordinates vectors of points P^o and P, respectively. Our aim is to compute the displacements

$$\boldsymbol{u}(\boldsymbol{r}^o) = \boldsymbol{r}(\boldsymbol{r}^o) - \boldsymbol{r}^o, \tag{1}$$

which are the key to obtain the strains $\boldsymbol{\varepsilon}(\boldsymbol{r}^o)$ and the stresses $\boldsymbol{\sigma}(\boldsymbol{r}^o)$. In linear elasticity with small displacements and small displacement gradients the corresponding expressions are

$$\boldsymbol{\varepsilon} = \boldsymbol{L}\boldsymbol{u}, \qquad \boldsymbol{\sigma} = \boldsymbol{D}\boldsymbol{\varepsilon}. \tag{2}$$

For a given distribution of (porous) material, defined by the relative density field $\rho(\boldsymbol{r}^o)$, our aim is to compute the displacements Eq.(1) and the associated strains and stresses Eq.(2).
We assume again the linear elasticity hypothesis, implying small displacements and small displacement gradients.
Let $d\Omega$ be the volume of a differential region in the vicinity of point P^o. By definition, the volume occupied by the porous material within the differential region will be $\rho(\boldsymbol{r}^o)d\Omega$. Therefore, the structural analysis problem can be written as [2]

$$\begin{array}{ll}
\textit{Given} & \rho(\Omega^o) \\
\textit{find} & \boldsymbol{u} \in H_u \\
\textit{such that} & a(\boldsymbol{w},\boldsymbol{u}) = (\boldsymbol{w},\boldsymbol{b})_{\Omega^o} + (\boldsymbol{w},\boldsymbol{t})_{\Gamma_\sigma^o} \quad \forall \boldsymbol{w} \in H_w \\
\textit{being} & a(\boldsymbol{w},\boldsymbol{u}) = \displaystyle\iiint_{\Omega^o} (\boldsymbol{L}\boldsymbol{w})^T \boldsymbol{D} (\boldsymbol{L}\boldsymbol{u}) \; \rho \; d\Omega, \\
& (\boldsymbol{w},\boldsymbol{b})_{\Omega^o} = \displaystyle\iiint_{\Omega^o} \boldsymbol{w}^T \boldsymbol{b} \; \rho \; d\Omega, \qquad (\boldsymbol{w},\boldsymbol{t})_{\Gamma_\sigma^o} = \iint_{\Gamma_\sigma^o} \boldsymbol{w}^T \boldsymbol{t} \; d\Gamma.
\end{array} \tag{3}$$

Notice that, in comparison with the original statement of a conventional FEM formulation, the modifications are reduced to take into account the porosity effect in the integration. In fact, once the displacements are known, the strains and stresses fields are computed with the same expressions, independently of the actual material distribution. However, we must exclude the case in which the relative density is locally null, since the concepts of displacement, strain and stress become meaningless. This problem is solved imposing a minimum value of the relative density slightly greater than zero (usually ρ_{min}=0.001) to all the elements of the mesh.

4.2. The Finite Element numerical model with relative density
Let ρ_e be the relative density of element number e, which is assumed constant within the element. Let $\boldsymbol{\rho} = \{\rho_e\}$ $(e = 1, \ldots, nelem)$ be the relative densities vector, which will constitute the design variables of the topology optimization problem. For a given $\boldsymbol{\rho}$, the structural analysis problem to be solved is:

$$\begin{array}{ll}
\textit{Find} & \boldsymbol{\alpha}(\boldsymbol{\rho}) \\
\textit{such that} & \displaystyle\sum_{i=1}^{N} \boldsymbol{K}_{ji}(\boldsymbol{\rho})\boldsymbol{\alpha}_i(\boldsymbol{\rho}) = \boldsymbol{f}_j(\boldsymbol{\rho}), \quad j = 1, \ldots, N.
\end{array} \tag{4}$$

The required terms can be computed on an element by element basis. Thus,

$$\boldsymbol{K}_{ji}(\boldsymbol{\rho}) = \sum_{e=1}^{nelem} \boldsymbol{K}^e_{ji}(\rho_e),$$
$$\boldsymbol{f}_j(\boldsymbol{\rho}) = \iint_{\Gamma^o_\sigma} \boldsymbol{\Phi}_j^T \boldsymbol{t}\ d\Gamma + \sum_{e=1}^{nelem} \boldsymbol{f}^e_j(\rho_e), \tag{5}$$

being the element contributions

$$\boldsymbol{K}^e_{ji}(\rho_e) = \iiint_{E_e} (\boldsymbol{L\Phi}_j)^T \boldsymbol{D}(\boldsymbol{L\Phi}_i)\ \rho_e\ d\Omega,$$
$$\boldsymbol{f}^e_j(\rho_e) = \iiint_{E_e} \Big(\boldsymbol{\Phi}_j^T \boldsymbol{b} - (\boldsymbol{L\Phi}_j)^T \boldsymbol{D}(\boldsymbol{Lu}^p)\Big)\ \rho_e\ d\Omega. \tag{6}$$

Once the solution $\boldsymbol{\alpha}(\boldsymbol{\rho})$ to problem Eq.(4) is found, we can compute at any arbitrary point $\boldsymbol{r}^o \in \Omega^o$ the approximations

$$\boldsymbol{u}^h(\boldsymbol{r}^o, \boldsymbol{\rho}) = \boldsymbol{u}^p(\boldsymbol{r}^o) + \sum_{i=1}^{N} \boldsymbol{\Phi}_i(\boldsymbol{r}^o)\boldsymbol{\alpha}_i(\boldsymbol{\rho}), \tag{7}$$

$$\boldsymbol{\varepsilon}^h(\boldsymbol{r}^o, \boldsymbol{\rho}) = \boldsymbol{Lu}^h(\boldsymbol{r}^o, \boldsymbol{\rho}), \quad \boldsymbol{\sigma}^h(\boldsymbol{r}^o, \boldsymbol{\rho}) = \boldsymbol{D\varepsilon}^h(\boldsymbol{r}^o, \boldsymbol{\rho}). \tag{8}$$

Notice that acording to Eq.(7) and Eq.(8) displacements, strains and stresses are still computed in the usual way.
Therefore, if we wish to adapt an existing FEM numerical model of structural analysis as a component of a topology optimization system, we only have to modify the element contributions computation. Moreover, the required adjustment is quite simple, since we only need to introduce the relative density in the integration of the corresponding expressions Eq.(6).
Furthermore, computing contributions Eq.(6) is fairly straightforward, since we assume that the relative density is constant within each element. Thus, we just have to multiply the original FEM formulation results by the corresponding relative densities. On the other hand, the original results give the first order derivatives of contributions Eq.(6) with respect to the design variables. Moreover, all the other first and higher order derivatives are obviously null.
We conclude that we do not have to modify the source at the lower level for adapting an existing FEM code into a topology optimization system. In practice, only slight adjustments must be implemented in the data flow between the higher level routines. In fact, any conventional code should contain all the basic tools to perform the required new computations and the associated sensitivity analysis.

4.3. Statement of the Stress Constraints
The values $\boldsymbol{\sigma}^h(\boldsymbol{r}^o, \boldsymbol{\rho})$ computed by means of Eq.(7) and Eq.(8) are numerical approximations to the actual stress tensor components of the material being deformed. Thus, the allowable values of the reference stress $\widehat{\sigma}(\boldsymbol{\sigma})$ at point $\boldsymbol{r}^o_\ell$ can be limited by introducing constraints as

$$G_{\ell,1}(\boldsymbol{\rho}) = \widehat{\sigma}\Big(\boldsymbol{\sigma}^h(\boldsymbol{r}^o_\ell, \boldsymbol{\rho})\Big) - \widehat{\sigma}_{max} \leq 0,$$
$$G_{\ell,2}(\boldsymbol{\rho}) = \widehat{\sigma}_{min} - \widehat{\sigma}\Big(\boldsymbol{\sigma}^h(\boldsymbol{r}^o_\ell, \boldsymbol{\rho})\Big) \leq 0, \tag{9}$$

where $\widehat{\sigma}$ is the stress criterion of comparison employed and $\widehat{\sigma}_{max}$ and $\widehat{\sigma}_{min}$ are the corresponding upper and lower limits.

5. Numerical Application

As we have mentioned before, it is very easy to adapt a conventional FEM formulation for structural topological optimization and only minor changes need to be performed in the integral calculations. Moreover, ρ_e is constant for each element.
Once these changes are made, the problem can be solved in the usual way independently of the value of

the relative densities.

5.1. Sensitivity analysis

Sensitivity analysis is developed by a direct differentiation method over the fundamental equations of the FEM formulation. Sensitivity analysis is developed to calculate the derivatives of the constraints and the objective function over the relative densities. To obtain these derivatives we need to calculate the derivatives of the nodal displacements over the relative densities.
Next, we calculate the derivatives of $\boldsymbol{K}(\boldsymbol{\rho})$ over each relative density as

$$\sum_{i=1}^{N} \boldsymbol{K}_{ji}(\boldsymbol{\rho}) \frac{\partial \boldsymbol{\alpha}_i(\boldsymbol{\rho})}{\partial \rho_e} = \frac{\partial \boldsymbol{f}_j(\boldsymbol{\rho})}{\partial \rho_e} - \sum_{i=1}^{N} \frac{\partial \boldsymbol{K}_{ji}(\boldsymbol{\rho})}{\partial \rho_e} \boldsymbol{\alpha}_i(\boldsymbol{\rho}), \tag{10}$$

being

$$\frac{\partial \boldsymbol{K}_{ji}(\boldsymbol{\rho})}{\partial \rho_e} = \boldsymbol{K}^e_{ji}(\boldsymbol{\rho})\Big|_{\rho_e=1} \qquad and \qquad \frac{\partial \boldsymbol{f}_j(\boldsymbol{\rho})}{\partial \rho_e} = \boldsymbol{f}^e_j(\boldsymbol{\rho})\Big|_{\rho_e=1}. \tag{11}$$

The problem above is similar to obtain the nodal displacements of the original FEM problem because the matrix $\boldsymbol{K}(\boldsymbol{\rho})$ is the same. The resulting linear equation system can be solved in a similar way too. We employ a factorization technique because it is possible to store the factorized matrix and use it to solve several equation systems with the same rigidity matrix and different loads. In addition, second derivatives will be obtained by a similar procedure and it will be necessary to solve more linear equation systems.
Now, the derivatives of the stresses over the relative densities can be easily obtained as

$$\begin{gathered} \frac{\partial \boldsymbol{u}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial \rho_e} = \sum_{i=1}^{N} \boldsymbol{\Phi}_i(\boldsymbol{r}^0) \frac{\partial \boldsymbol{\alpha}_i(\boldsymbol{\rho})}{\partial \rho_e}, \\ \frac{\partial \boldsymbol{\varepsilon}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial \rho_e} = \boldsymbol{L} \frac{\partial \boldsymbol{u}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial \rho_e} \qquad \frac{\partial \boldsymbol{\sigma}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial \rho_e} = \boldsymbol{D} \frac{\partial \boldsymbol{\varepsilon}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial \rho_e}. \end{gathered} \tag{12}$$

The derivatives of the nodal displacements over the relative densities are obtained from Eq.(10).
Once we have calculated the first order derivatives, we calculate the second order derivatives. We should obtain them by a directional search diferentiation because the full second order derivatives would require a large amount of data storage. Thus,

$$\sum_{i=1}^{N} \boldsymbol{K}_{ji}(\boldsymbol{\rho}) \frac{\partial^2 \boldsymbol{\alpha}_i(\boldsymbol{\rho})}{\partial s^2} = \frac{\partial^2 \boldsymbol{f}_j(\boldsymbol{\rho})}{\partial s^2} - 2 \sum_{i=1}^{N} \frac{\partial \boldsymbol{K}_{ji}(\boldsymbol{\rho})}{\partial s} \frac{\partial \boldsymbol{\alpha}_i(\boldsymbol{\rho})}{\partial s} - \sum_{i=1}^{N} \frac{\partial^2 \boldsymbol{K}_{ji}(\boldsymbol{\rho})}{\partial s^2} \boldsymbol{\alpha}_i \tag{13}$$

where s is the search direction.
Furthermore, we could simplify this expression because several terms are null

$$\sum_{i=1}^{N} \boldsymbol{K}_{ji}(\boldsymbol{\rho}) \frac{\partial^2 \boldsymbol{\alpha}_i(\boldsymbol{\rho})}{\partial s^2} = -2 \sum_{i=1}^{N} \frac{\partial \boldsymbol{K}_{ji}(\boldsymbol{\rho})}{\partial s} \frac{\partial \boldsymbol{\alpha}_i(\boldsymbol{\rho})}{\partial s}. \tag{14}$$

The second order directional derivatives can now be obtained from Eq.(12) and Eq.(14) as

$$\begin{gathered} \frac{\partial^2 \boldsymbol{u}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial s^2} = \sum_{i=1}^{N} \boldsymbol{\Phi}_i \frac{\partial^2 \boldsymbol{\alpha}_i(\boldsymbol{\rho})}{\partial s^2}, \\ \frac{\partial^2 \boldsymbol{\varepsilon}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial s^2} = \boldsymbol{L} \frac{\partial^2 \boldsymbol{u}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial s^2}, \qquad \frac{\partial^2 \boldsymbol{\sigma}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial s^2} = \boldsymbol{D} \frac{\partial^2 \boldsymbol{\varepsilon}^h(\boldsymbol{r}^0,\boldsymbol{\rho})}{\partial s^2}. \end{gathered} \tag{15}$$

Moreover, the derivatives of the objective function can be obtained directly because the weight of each element is linearly dependent of its relative density. Then, these derivatives can be obtained by calculating the weight of each element without multiplying it by the relative density.

5.2. Optimization problem

The optimization problem can be formulated from a generic point of view as

$$\begin{array}{lll} \text{Minimize} & F(\boldsymbol{\rho}) = Cost(\boldsymbol{\rho}) & \\ \text{subject to:} & G_\ell(\boldsymbol{\sigma}_i) \ \leq \ 0 & \ell = 1, \ldots, N_{const} \\ & 0 < \rho_{min} \leq \rho_e \leq 1, & e = 1, \ldots, N_{elem} \\ & \rho_{min} = 0.001 & \text{(usually)} \end{array} \tag{16}$$

The objective function can be defined as

$$F(\boldsymbol{\rho}) = \sum_{i=1}^{Nelem} \int_{\Omega_e} (\rho_e)^{1/q} \; d\Omega \tag{17}$$

where the parameter q is a penalty parameter to avoid intermediate densities in the optimized solution [2]. If no penalization is used ($q = 1$) the objective function to minimize is the total weight of the structure.

However, constraints can be formulated in many different ways. We propose to set a stress constraint in the central point of each element. We have used Von Mises criterion for material failure because we solve steel structures. If one wants to use another material it is necesary to change failure criterion according to material properties and their derivatives. In our case,

$$\widehat{\sigma}_{vm} = \sqrt{\frac{1}{2}\Big[(\sigma^I - \sigma^{II})^2 + (\sigma^{II} - \sigma^{III})^2 + (\sigma^{III} - \sigma^I)^2\Big]}. \tag{18}$$

6. Optimization algorithm

The optimization algorithm is developed in [3]. We use a Sequential Linear Programming (SLP) algorithm to obtain a feasible search direction with linear approximation. If the problem is linear the solution is obtained in only one iteration, but stress constraints are highly non linear and it is necesary to solve a sequence of linearized problems to obtain the optimum. Once we have obtained a valid search direction it is necesary to calculate an advance factor which minimizes the objective function and does not violate any contraints in this direction. We use a second order line search to obtain the advance factor. This new solution can be used as a valid basic value of the vector of design variables to repeat the optimization algorithm. Thus, the iterative problem can be formulated as

$$\begin{array}{ll} given & \boldsymbol{\rho}^k \\ obtain & \boldsymbol{\rho}^{k+1} = \boldsymbol{\rho}^k + \boldsymbol{\Delta\rho}^k. \end{array} \tag{19}$$

The objective function and the contraints can be linearized as:

$$\begin{array}{ll} F(\boldsymbol{\rho}^k + \boldsymbol{\Delta\rho}^k) \ \approx F(\boldsymbol{\rho}^k) + \boldsymbol{\nabla} F(\boldsymbol{\rho}^k)\ \boldsymbol{\Delta\rho}^k & \\ G_\ell(\boldsymbol{\rho}^k + \boldsymbol{\Delta\rho}^k) \approx G_\ell(\boldsymbol{\rho}^k) + \boldsymbol{\nabla} G_\ell(\boldsymbol{\rho}^k)\ \boldsymbol{\Delta\rho}^k & \quad j = 1, \ldots, N_{const} \end{array} \tag{20}$$

The feasible search direction can be obtained with three diferent methods. If there are no violated constraints we use a search direction wich reduces the objective function. This situation is quite infrecuent and then we use as the search direction the negative of the gradient of the objective function.

If there is any violated constraint we use a linear programming method based on the Simplex algorithm to obtain the search direction whitout violating the constraints and reducing the objective function. This direction is usually found but if there is a very high number of constraints or if there are strongly violated constraints the procedure could fail. If this happens we use another algorithm to obtain a direction that forces the design to proceed to the feasible region, although the objective function could be increased.

Moreover, it is necesary to consider lateral constraints because the design variables can only take a value between ρ_{min} and 1.00. Then, it is necessary to modify the search direction when lateral constraints become active. Thus,

$$\text{if} \quad \left\{ \begin{array}{lll} \rho_e = \rho_{min} & \text{and} & \boldsymbol{s}_e^k < 0, \quad \text{or} \\ \rho_e = 1 & \text{and} & \boldsymbol{s}_e^k > 0 \end{array} \right\} \quad \text{then} \quad \boldsymbol{s}_e^k = 0. \tag{21}$$

Finally, the search direction is normalized to avoid possible scale effects between the different methods employed to obtain the search direction. In addition, if $\boldsymbol{s}$ is normalized the value of θ_k is the magnitude of the design modification.
Now, the advance factor (θ_k) can be obtained by second order approximations of the objective function and the stress constraints. The required second order derivatives can be calculated according to the sensitivity analysis expressions in Eq.(14) and Eq.(15). Then, the objective function and the constraints can be quadratically approximated as

$$\begin{aligned} F(\boldsymbol{\rho}^{k+1}) &\approx F(\boldsymbol{\rho}^k) + \frac{\partial F(\boldsymbol{\rho}^k)}{\partial s_k}\,\theta_k + \frac{1}{2}\,\frac{\partial^2 F(\boldsymbol{\rho}^k)}{\partial {s_k}^2}\,{\theta_k}^2 \\ G_\ell(\boldsymbol{\rho}^{k+1}) &\approx G_\ell(\boldsymbol{\rho}^k) + \frac{\partial G_\ell(\boldsymbol{\rho}^k)}{\partial s_k}\,\theta_k + \frac{1}{2}\,\frac{\partial^2 G_\ell(\boldsymbol{\rho}^k)}{\partial {s_k}^2}{\theta_k}^2 \qquad \ell = 1, \ldots, N_{const}. \end{aligned} \tag{22}$$

Once we have calculated the advance factor and the search direction we can obtain a new solution to the problem. Iterations will stop if none initial solution is found, if convergence is achieved or if the maximum number of allowed iterations is exceeded.

7. Application examples

We present two structures calculated in plane stress. For this kind of examples the design variables are easy to represent graphycally because we can assume that the relative density can be shown as the thickness of each element of the structure. Then, the structure can be shown as a 3D volume although the solution is calculated as a 2D structure.
For simplicity we use a predefined rectangular mesh with homogeneously distributed rectangular elements. The length and the height are previously defined.
The first example is a beam 40 m long and 15 m high. It has vertical and horizontal supports on the left extreme and on the right extreme as it is shown in figure 1. The external load is a punctual load of 6 10^5 kN applied at a distance of 1/3 of the total length from the left support. In addition, self weight is considered. We use steel with an elastic limit of $\sigma_e = 230\ MPa$ and a Young Module of $E_e = 2.1\ 10^5\ MPa$. According to [4], the Poisson value is ($\nu = 0.3$) and the mass density of steel is $\gamma_{mat} = 76.5\ kN/m^3$. We have used a mesh with $36 \times 16 = 576$ rectangular elements. Notice that elements where punctual loads (forces or reactions) are applied are not optimized to avoid the effect of stress accumulation.
As it can be seen, the solution seems to be good from an engineering point of view because it is very similar to an arch (figure 4-right). In this example, the thickness of the elements has ben multiplied by a constant to give a better graphic comprehension of the solution. The final weight of the structure corresponds to 17.16 % of the initial one.

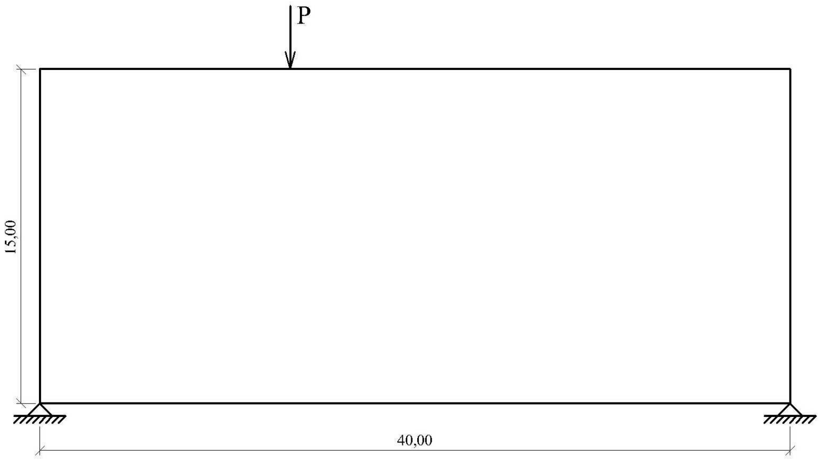

Figure 1: Initial scheme

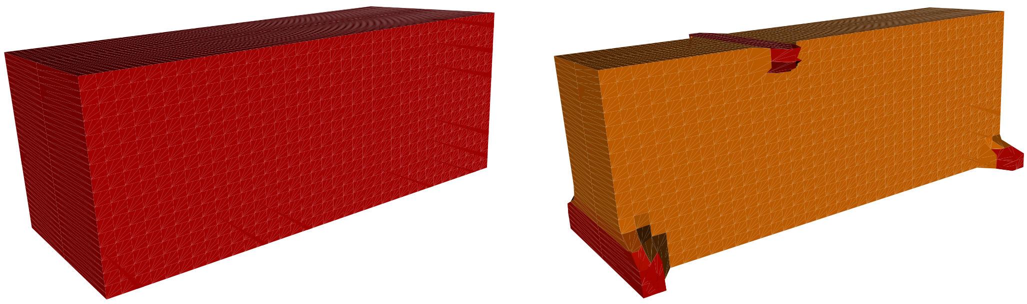

Figure 2: Example 1: Initial solution (left) and iteration 10 (right)

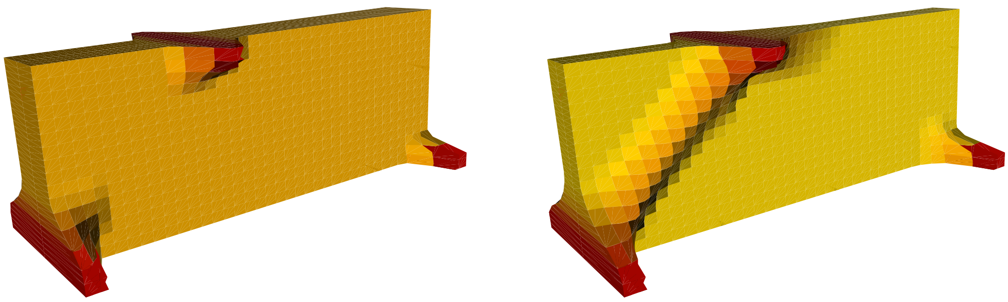

Figure 3: Example 1: iteration 20 (left) and iteration 35 (right)

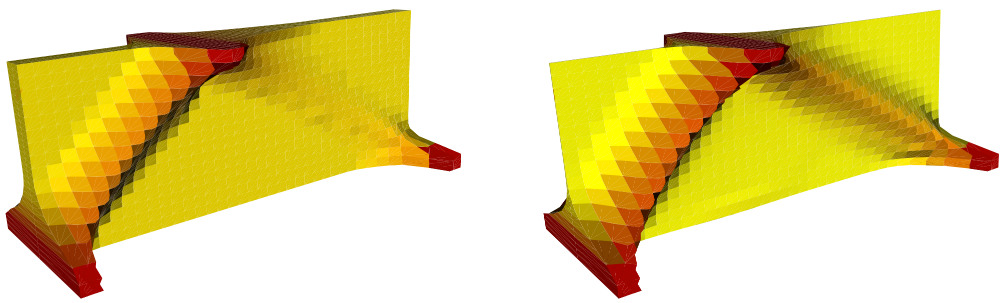

Figure 4: Example 1: iteration 50 (left) and optimized solution (right)

The second example (figure 5) is a beam 40 m long and 1 m high. It is supported on the left edge, on the center and on the right edge. The horizontal deformation is not restricted. Furthermore, a vertical punctual load of 10^4 kN is applied in the middle of the left span. The steel has the same properties as in example 1 but now the mesh is made up of $60 \times 12 = 720$ elements. The total weight of the optimized structure is about 24.49 % of the initial one.

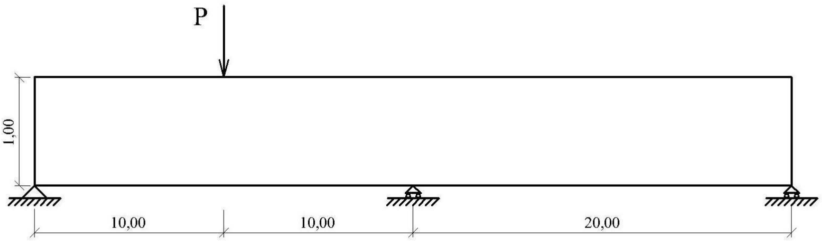

Figure 5: Example 2: initial scheme

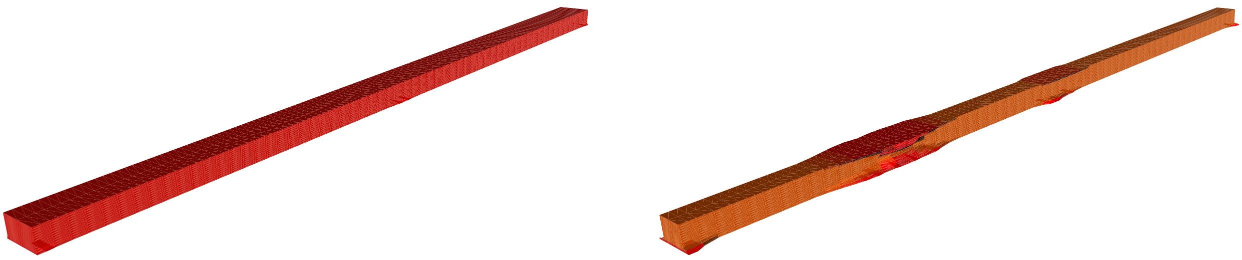

Figure 6: Example 2: initial solution (left) and iteration 5 (right)

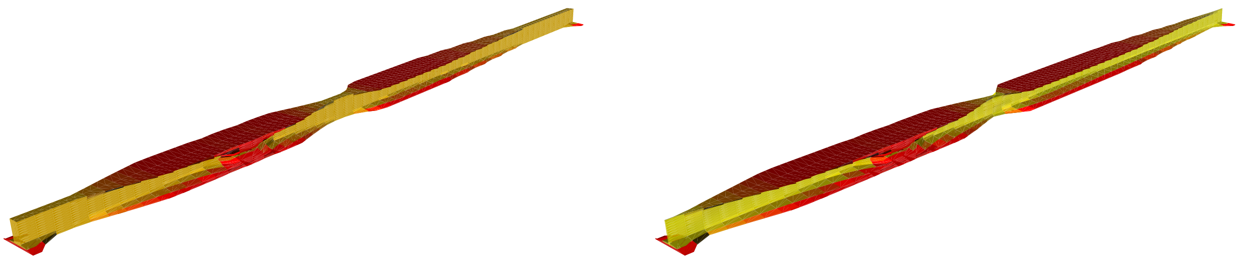

Figure 7: Example 2: iteration 10 (left) and iteration 20 (right)

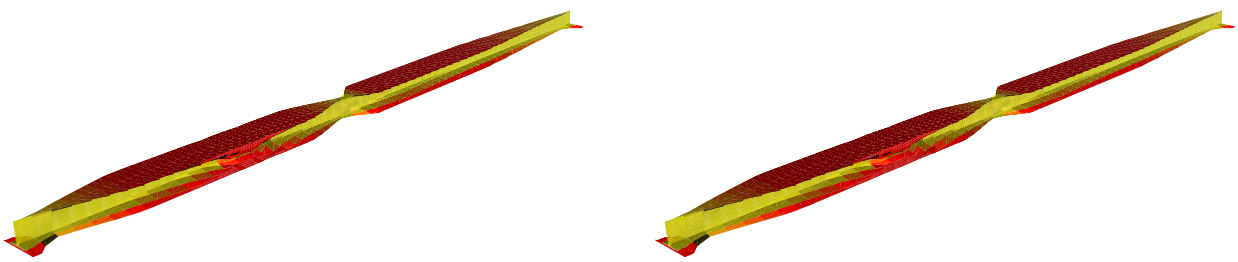

Figure 8: Example 2: iteration 30 (left) and optimized solution (right)

8. Conclusions

We present a minimum weight formulation for structural topological optimization with local stress constraints.

The formulation is based on an optimization method wich includes a conventional FEM approach with simple modifications.

The presented optimization approach does not require neither artificial parameters nor stabilization techniques. Intermediate densities are not penalized neither.

The objective function and the constraints have a clear physical interpretation from an engineering point of view. In addition, another kind of constraints could be used (displacementes, vibration frequencies) and several load cases could be analysed simultaneously.

From a mathematical point of view the formulation is very robust. However, this approach implies a high number of non linear constraints wich implies a large amount of data storage.

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