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Numerical modelling of grounding systems in high-performance parallel computers

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Abstract

The design of safe grounding systems in electrical installations is essential to assure the security of the persons, the protection of the equipment and the continuity of the power supply [1,2]. In order to achieve these goals, it is necessary to compute the equivalent electrical resistance of the system and the potential distribution on the earth surface when a fault condition occurs. While only crude approximations were available before the 60's, some intuitive methods [1] have been proposed in the 70's and the 80's. These non-rigorously established methods are widely used to compute small and medium size installations, in spite of the problems that have been reported [3]. On the other hand, the authors have developed a BEM numerical formulation that has proved to produce highly accurate results in the earthing analysis of large real grounding systems with uniform [4,5] and stratified soil models [6]. At present, single-layer models run in real-time in personal computers, while multiple-layer models break off the design process (since the computing time is not contemptible). In this paper, we present our BEM formulation for the analysis of grounding systems embedded in stratified soils, and we discuss the key points of its implementation in a high-performance parallel computer (HPPC). The feasibility of this approach is demonstrated by its application to the analysis of a real grounding system with a two-layer soil model. As we expected, the speed-up of the algorithm increases when the number of processors does, in accordance with the theoretical predictions. Therefore, the proposed multi-layer BEM formulation could become a real-time design tool in a close future, when high-performance parallel computing becomes a widespread resource in engineering design.

1 Introduction

Fault currents dissipation into the earth can be studied by means of Maxwell's Electromagnetic Theory [5]. Thus, restricting the analysis to the electrokinetic steady-state response and neglecting the inner resistivity of the earthing conductors (then, potential can be assumed constant in every point of the grounding electrode surface), the 3D problem can be written as

$$\begin{array}{c} \mathrm{div}(\boldsymbol{\sigma}) = 0, \qquad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\ \mathbf{grad}(V) \ \text{ in } \ E, \\ \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \ \text{ in } \ \Gamma_E, \quad V = V_\Gamma \ \text{ in } \ \Gamma, \quad V \longrightarrow 0 \ \text{ if } \ |\boldsymbol{x}| \to \infty, \end{array} \tag{1}$$

being E the earth and $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface and $\boldsymbol{n}_E$ its normal exterior unit field, and Γ the electrode surface [5]. Solution to (1) gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$ when the electrode attains a voltage V_Γ (Ground Potential Rise, or GPR) with respect to remote earth. Then, for known values of V on Γ_E and $\boldsymbol{\sigma}$ on Γ, it is straightforward to obtain the design and safety parameters of the grounding system (i.e., the step and touch voltage, and the equivalent resistance of the system) [1,5]. On the other hand, since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, the normalized boundary condition $V_\Gamma = 1$ will be used from here on.

Most of the available methods are based on the assumption that soil can be considered homogeneous and isotropic. Hence, $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ, that must be experimentally obtained [1]. Obviously, this hypothesis does not introduce significant errors if the soil is essentially uniform (both in horizontal and vertical directions) in the surroundings of the grounding grid [1]. Nevertheless, parameters involved in the design of grounding systems can significantly change as soil conductivity varies through the substation site. Therefore, it seems necessary to develop advanced models that could take into account variations of soil conductivity in the surroundings of the grounding site. Obviously, considering the real variation of the soil conductivity in the vicinity of a grounding system would never be affordable, neither from the economical nor from the technical point of view. A more practical approach (and still quite realistic when conductivity is not markedly uniform with depth) consists of considering the soil stratified in a number of horizontal layers. Then, each layer is defined by an appropriate thickness and an apparent scalar conductivity that must be experimentally obtained. In fact, it is widely accepted that two-layer (or even three-layer) soil models should be sufficient to obtain good and safe designs of grounding systems in most practical cases [1].

If one considers that the soil is formed by C horizontal layers (each one with a different conductivity) and the grounding electrode is buried in the layer b, mathematical problem (1) can be written in terms of the following Neumann exterior problem [6]

$$\begin{array}{c} \operatorname{div}(\boldsymbol{\sigma}_c) = 0, \qquad \boldsymbol{\sigma}_c = -\boldsymbol{\gamma}_c\, \mathbf{grad}(V_c) \;\text{ in }\; E_c, \qquad (1 \le c \le C) \\ \boldsymbol{\sigma}_1^t \boldsymbol{n}_E = 0 \;\text{ in }\; \Gamma_E, \quad V_b = 1 \;\text{ in }\; \Gamma, \quad V_c \longrightarrow 0 \;\text{ if }\; |\boldsymbol{x}| \to \infty, \\ \boldsymbol{\sigma}_c^t \boldsymbol{n}_c = \boldsymbol{\sigma}_{c+1}^t \boldsymbol{n}_c \;\text{ in }\; \Gamma_c, \qquad (1 \le c \le C-1) \end{array} \qquad (2)$$

where E_c is each one of the soil layers, γ_c is the scalar conductivity of layer E_c, V_c is the potential at an arbitrary point of layer E_c and $\boldsymbol{\sigma}_c$ is the corresponding current density, Γ_c is the interface between layers E_c and E_{c+1} and $\boldsymbol{n}_c$ is the normal field to Γ_c.

2 Variational Statement of the Problem

In most of real electrical substations, grounding systems consist of a grid of interconnected bare cylindrical conductors, horizontally buried and supplemented by rods, which ratio diameter/length uses to be relatively small ($\sim 10^{-3}$). Obviously, it is not possible to obtain analytical solutions to problems with this kind of geometry, and the use of numerical techniques such as FD or FE should involve a completely out of range computing effort, since discretization of the 3D domains E_c is extremely hard. Therefore, taking into account that computation of potential is only required on Γ_E and the equivalent resistance can be easily obtained in terms of the leakage current density σ on Γ ($\sigma = \boldsymbol{\sigma}^t\boldsymbol{n}$, being $\boldsymbol{n}$ the normal exterior unit field to Γ), we turn our attention to a Boundary Element approach, which would only require the discretization of the grounding surface Γ [5].

If one takes into account that the surroundings of the substation site are levelled and regularized during its construction (i.e., the earth surface Γ_E and the interfaces Γ_c can be assumed horizontal), the application of the "method of images" and Green's Identity to problem (2) yields the following integral expression [6] for potential $V_c(\boldsymbol{x}_c)$ at an arbitrary point $\boldsymbol{x}_c \in E_c$, in terms of the unknown leakage current density $\sigma(\boldsymbol{\xi})$ at any point $\boldsymbol{\xi}$ of the electrode surface $\Gamma \subset E_b$:

$$V_c(\boldsymbol{x_c}) = \frac{1}{4\pi\gamma_b} \int\!\!\int_{\boldsymbol{\xi}\in\Gamma} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma \qquad \forall \boldsymbol{x}_c \in E_c, \qquad (3)$$

where integral kernels $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$ are formed by infinite series of terms corresponding to the resultant images obtained when Neumann exterior problem (3) is transformed into a Dirichlet one [6]. This weakly singular kernel depends on the conductivity of the layers, as much as on the inverse of the distances from the point $\boldsymbol{x}_c$ to the point $\boldsymbol{\xi}$ and to all the images of $\boldsymbol{\xi}$ with respect to the earth surface Γ_E and to the interfaces Γ_c between layers [6]. In fact, these integral kernels can be written in the general form:

$$k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi}) = \sum_{l=0}^{\infty} k_{bc}^l(\boldsymbol{x}_c, \boldsymbol{\xi}), \qquad k_{bc}^l(\boldsymbol{x}_c, \boldsymbol{\xi}) = \frac{\psi^l(\kappa)}{r(\boldsymbol{x}_c, \boldsymbol{\xi}^l(\boldsymbol{\xi}))}, \tag{4}$$

where ψ^l is a weighting coefficient that only depends on a certain ratio κ that is defined in terms of the layer conductivities (for the two-layer soil model case $\kappa = (\gamma_1 - \gamma_2)/(\gamma_1 + \gamma_2)$), and $r(\boldsymbol{x}_c, \boldsymbol{\xi}^l(\boldsymbol{\xi}))$ is the Euclidean distance between the points $\boldsymbol{x}_c$ and $\boldsymbol{\xi}^l$, being $\boldsymbol{\xi}^0$ the point $\boldsymbol{\xi}$ on the electrode surface ($\boldsymbol{\xi}^0(\boldsymbol{\xi}) = \boldsymbol{\xi}$), and being $\boldsymbol{\xi}^l$ $(l \neq 0)$ the images of $\boldsymbol{\xi}$ with respect to the earth surface and to the interfaces between layers. Explicit expressions of these kernels for the two-layer soil model can be found in [6].

Expression (3) also holds on Γ, where potential is given by the boundary condition ($V_b(\boldsymbol{\chi}) = 1$, $\forall \boldsymbol{\chi} \in \Gamma$). Therefore, the leakage current density σ must satisfy a Fredholm integral equation of the first kind on Γ, that can be written in the weaker variational form:

$$\iint_{\boldsymbol{\chi} \in \Gamma} w(\boldsymbol{\chi}) \left(\frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi} \in \Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\ d\Gamma - 1 \right) d\Gamma = 0, \tag{5}$$

which must hold for all members $w(\boldsymbol{\chi})$ of a suitable class of test functions defined on Γ [5]. Obviously, a Boundary Element approach seems to be the right choice to solve equation (5).

3 Numerical Formulation

3.1 General 2D boundary element general approach

The leakage current density σ that flows from the grounded electrode, and the electrode surface Γ can be discretized as follows:

$$\sigma(\boldsymbol{\xi}) = \sum_{i=1}^{\mathcal{N}} \sigma_i\, N_i(\boldsymbol{\xi}), \qquad \Gamma = \bigcup_{\alpha=1}^{\mathcal{M}} \Gamma^\alpha, \tag{6}$$

for given sets of $\mathcal{N}$ trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on Γ, and $\mathcal{M}$ two dimensional boundary elements $\{\Gamma^\alpha\}$. Then, taking into account that kernels (4) are given by series, integral expression (3) for potential $V_c(\boldsymbol{x}_c)$ can also be discretized as

$$V_c(\boldsymbol{x}_c) = \sum_{i=1}^{\mathcal{N}} \sigma_i V_{c,i}(\boldsymbol{x}_c), \qquad V_{c,i}(\boldsymbol{x}_c) = \sum_{\alpha=1}^{\mathcal{M}} \sum_{l=0}^{l_V} V_{c,i}^{\alpha l}(\boldsymbol{x}_c), \tag{7}$$

$$V_{c,i}^{\alpha l}(\boldsymbol{x}_c) = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\xi} \in \Gamma^\alpha} k_{bc}^l(\boldsymbol{x}_c, \boldsymbol{\xi})\, N_i(\boldsymbol{\xi})\ d\Gamma^\alpha, \tag{8}$$

where l_V represents the number of terms that is necessary to consider until convergence is achieved.

Furthermore, for a given set of $\mathcal{N}$ test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ, variational form (5) is reduced to the following linear system:

$$\sum_{i=1}^{\mathcal{N}} R_{ji}\sigma_i = \nu_j \qquad (j = 1, \dots, \mathcal{N})$$
$$R_{ji} = \sum_{\beta=1}^{\mathcal{M}} \sum_{\alpha=1}^{\mathcal{M}} \sum_{l=0}^{l_R} R_{ji}^{\beta\alpha l}, \;\; \nu_j = \sum_{\beta=1}^{\mathcal{M}} \nu_j^{\beta}, \;\; \nu_j^{\beta} = \iint_{\boldsymbol{\chi}\in\Gamma^{\beta}} w_j(\boldsymbol{\chi}) d\Gamma^{\beta}, \tag{9}$$

$$R_{ji}^{\beta\alpha l} = \frac{1}{4\pi\gamma_b} \iint_{\boldsymbol{\chi}\in\Gamma^{\beta}} w_j(\boldsymbol{\chi}) \iint_{\boldsymbol{\xi}\in\Gamma^{\alpha}} k_{bb}^{l}(\boldsymbol{\chi}, \boldsymbol{\xi}) N_i(\boldsymbol{\xi}) d\Gamma^{\alpha} d\Gamma^{\beta}, \tag{10}$$

where l_R represents the number of terms that is necessary to consider until convergence is achieved.

At this point, it is important to remark that system (9) is the key to solve the problem, since its solution provides the values of the unknowns σ_i $(i = 1, \dots, \mathcal{N})$, that is necessary to compute the potential at any point on the earth surface (7) and the leakage current density (6), and all the design and safety parameters of the grounding system [1,5]. However, the statement of linear system (9) requires the discretization of a 2D domain (the whole surface Γ of the grounding electrodes), which involves a large number of degrees of freedom in practical cases. Besides, the matrix is full and the computation of its coefficients requires to perform double integration on 2D domains. For all these reasons, it is necessary to introduce some additional hypotheses in order to decrease the computational cost.

3.2 Approximated 1D boundary element approach

Taking into account the real geometry of grounding systems in practical cases, we introduce an assumption that is widely used in most of the theoretical developments in grounding analysis: the hypothesis of circumferential uniformity. Thus, the leakage current density σ is assumed constant around the cross section of the cylindrical conductors of the grid [1]. Therefore, discretizations (6) and (9) become much simpler, since the classes of test and trial functions are restricted to those with circumferential uniformity, while only the axial lines of the grounding electrodes have to be discretized [5].

Thus, for a given level of mesh refinement, the number of element contributions $R_{ji}^{\beta\alpha}$ and ν_j^{β} that we need to compute in order to state linear system (9), as much as the number of unknowns σ_i, are much lower. Hence, the computational work required to solve a real problem by means of this approximated 1D BEM version is drastically reduced with respect to the general 2D BEM formulation. However, extensive computing is still required, mainly because of the double integration on 2D domains that is necessary to obtain element contributions (8) and (10). By means of suitable simplifications [5], circumferential integration can be easily avoided. However, the computation of the remaining integrals is not obvious. In fact, the use of numerical quadratures is precluded, due to the undesirable behaviour of the integrands. The expressions of terms $V^{\alpha l}_{c,i}$ and $R^{\beta\alpha l}_{ji}$ in (8) and (10) are formally equivalent to those obtained in the case of uniform soil models. The authors have derived highly efficient analytical integration techniques to compute this kind of terms in the case of constant, linear and parabolic leakage current elements [5]. Therefore, terms (8) and (10) can be computed by means of explicit formulae [6].

Further discussion is restricted to the case of a Galerkin type formulation, in which the matrix of coefficients in (9) is symmetric and positive definite [5,6]. The example presented in this paper correspond to a two-layer soil model. Obviously, this BEM formulation can be applied to any other case with a higher number of layers. However, CPU time may increase exponentially, mainly because of the poor rate of convergence of the underlying series expansions.

3.3 Overall efficiency and Parallelization of the algorithm

With regard to the overall computational cost, for a given discretization ($\mathcal{M}$ elements of p nodes each, and a total number of $\mathcal{N}$ degrees of freedom) a linear system (9) of order $\mathcal{N}$ must be generated and solved. Since the matrix is symmetric, but not sparse, its resolution by means of a direct method requires $O(\mathcal{N}^3/3)$ operations. Matrix generation requires $O(\mathcal{M}^2 p^2/2)$ operations, since p^2 series of contributions of type (10) have to be computed for every pair of elements, and approximately half of them are discarded because of symmetry.

Hence, most of computing effort is devoted to matrix generation in small/medium problems, while linear system resolution prevails in medium/large ones. In these cases, the use of direct methods for the linear system resolution is out of range. Therefore iterative or semiiterative techniques will be preferable. The best results have been obtained by a diagonal preconditioned conjugate gradient algorithm with assembly of the global matrix [5]. This technique has turned out to be extremely efficient for solving large scale problems, with a very low computational cost in comparison with matrix generation.

On the other hand, once the leakage current has been obtained, the cost of computing the equivalent resistance is negligible. The additional cost of computing potential at any given point (normally at the earth surface) by means of (7) requires only $O(\mathcal{M}p)$ operations, since p series of contributions of type (8) have to be computed for every element. However, if it is necessary to compute potentials at a large number of points (i.e. to draw contours), computing time may also be important.

Hence, the first critical time-consuming process is matrix generation, followed by computation of potential at a large number of points. Obviously, both accept massive parallelization. Therefore, it is clear that computing time could be reduced under acceptable levels, even for extremely large models, provided that the number of available processors is high enough, in spite of the efficiency loses due to the data transfer overhead and the system administration workload.

4 Application to a Practical Case

4.1 Description of the grounding system and results

This BEM numerical approach has been applied to the grounding analysis of a real electrical installation: the Santiago II substation, close to the city of *Santiago de Compostela* in Spain. This earthing system is formed by a grid of 534 cylindrical conductors of the same diameter (11.28 mm) buried to a depth of 75 cm, supplemented with 24 ground rods of the same length (4 m) and diameter (15 mm). The grounding system protects a total area of 38,000 m^2. The studied area is a wider superimposed rectangular zone of 300×260 m^2 (i.e., 78,000 m^2). The Ground Potential Rise (GPR) considered in this study is 10 kV. The plan of the earthing grid and the general data were obtained from the grounding plans and specifications of the substation provided by the power company (*Unión Fenosa*). The grid has been discretized in $\mathcal{M}$=582 linear leakage current elements, what leads to a problem with $\mathcal{N}$=386 degrees of freedom. The analysis of this grounding system is particularly difficult because the grid is embedded in both layers. In cases like this, the different expressions must be applied properly, considering the situation of each electrode [6].

Figure 1 compares the potential distributions on the earth surface obtained by means of the homogeneous and isotropic soil model (being the soil conductivity γ=60Ωm) and the proposed two-layer soil model (being the layer conductivities γ_1=200Ωm and γ_2=60Ωm,

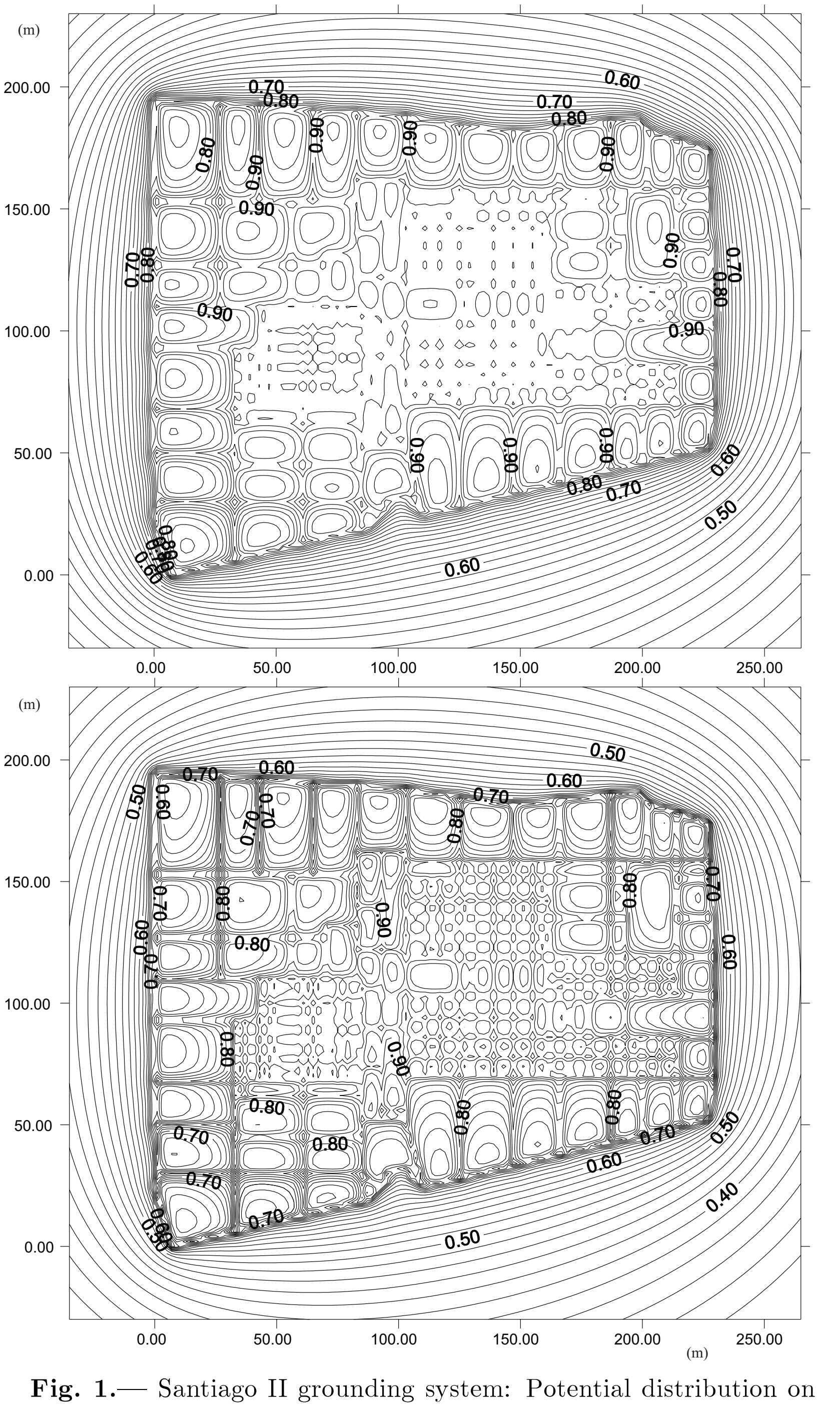

Fig. 1.— Santiago II grounding system: Potential distribution on the ground surface ($\times 10$ kV) computed by means of the isotropic soil model (up) and the two layer soil model (down).

and being the thickness of the upper layer h=1.2m). The computed values of the equivalent resistance and the total fault current of the grounding system were R_{eq}=0.1782Ω and I=5.61kA in the case of the uniform soil model, and R_{eq}=0.1486Ω and I=6.73kA in the case of the two-layer soil model. In the presented example, we show that the results obtained by using a multiple-layer soil model can be noticeably different from those obtained by using a single layer soil model. Therefore, it could be advisable to use multi-layer soil formulations to analyze grounding systems as a general rule, in spite of the increase in the computational effort. In fact, the use of this kind of advanced models should be mandatory in cases where the conductivity of the soil changes markedly with depth.

The uniform soil model results were obtained in real time in a personal computer. The two-layer model results where obtained in a Fujitsu AP3000 supercomputer configured with 16 nodes at the *Centro de Supercomputación de Galicia* (CESGA). The AP3000 system is a distributed-memory parallel server that employs 64-bit UltraSPARC workstations as node processors connected via ultra-high-speed network. Actually, 12 of the 16 nodes are single UltraSPARC II processors at 300 MHz with 128 Mb each configured for parallel processing, while the other 4 nodes are double UltraSparc II processors at 300 MHz with 256 Mb each.

Table 1 shows how the computing time and the speed-up factor change as the number of processors increases. Computing time was not ever contemptible, in spite of the impressive performance of the parallel server. Furthermore, this kind of supercomputing resources are not widespread available for engineers at the present moment (the base AP3000 system started at US$ 267,000 in 1996). However, the speed-up of the algorithm increased with the number of processors in accordance with the theoretical predictions, as we expected, since the structure of the algorithm accepts massive parallelization.

TABLE I

COMPUTING TIME AND SPEED-UP VS. NUMBER OF PROCESSORS

Processors	**Time (seg.)**	**Speed-up**
1	645	1.00
2	329	1.96
4	180	3.58
8	100	6.45

5 Conclusions

At present, uniform soil models for grounding analysis run in real-time in single processor conventional computers. On the other hand, the use of models with a small number of soil layers breaks off the design process (since the computing time is not contemptible), while the use of models with a higher number of layers is precluded. However, it could be advisable, or even mandatory, to use a multi-layer soil model as a general rule.

The authors have developed a BEM approach for the analysis of grounding systems embedded in stratified soils that accepts massive parallelization. The proposed formulation has been implemented in a high-performance parallel computer (HPPC), and the code has been applied to the analysis of a real grounding system. The results prove that the proposed multi-layer BEM formulation will become a real-time design tool in a close future, as high-performance parallel computing becomes a widespread available resource in engineering.

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