MODELLING CYCLICAL ASYMMETRIES IN EUROPEAN IMPORTS

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Abstract

This paper applies smooth transition models to capture the nonlinear behavior in the imports data of six major European economies and to assess whether such nonlinearities are related to business cycle asymmetries. Two classes of switch between regimes are considered: endogenously determined transition that assumes nonlinearities are generated by idiosyncratic components specific to foreign trade, and exogenous transition based on GDP growth as a more direct indicator of the cyclical state of the economy. The results support the proposition that the dynamics of imports are nonlinear. In Belgium, France, Spain and the United Kingdom regimes change over the business cycle, while in Germany and Italy the switch between regimes is endogenous. National characteristics play a role in defining the position of extreme regimes, the smoothness of the transition and local dynamics within each state.

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1. INTRODUCTION

Modelling nonlinearities in economic time series related to business cycle asymmetries has long been of interest to applied economists. There has been an explosion of papers on the suitable statistical methods for summarizing and explaining cyclical behavior of macroeconomic data, most of them from an univariate point of view. Three types of models have most commonly been used [Potter, 1999]: the Markov switching model, the Self-Excited Threshold AutoRegression and the Smooth Transition Model (STM).

STMs have been applied to capture asymmetric cyclical behavior in macroeconomic variables like Gross Domestic Product (GDP), industrial production, unemployment, etc, see for instance van Dijk and Franses [1999], Skalin and Teräsvirta [1999, 2002], Öcal and Osborn [2000], and Sensier et al. [2002]. Imports have not received too much attention in the literature, probably because empirical work has concentrated on the United States where the volume of international transactions is low compared to GDP. But international trade represents a significant proportion of economic activity in European countries, and the differences between domestic and imported goods are negligible with regard to that part of imports due to regional trade within the European Union. Therefore imports are expected to be very sensitive to the state of the cycle in Europe.

In such a case the usual approach of modelling imports behavior by estimating aggregate demand functions and focusing on price and income elasticities [Masih and Masih, 2000; Ribeiro, 2001; Sawyer and Sprinkle, 1996 and 1997; Sinha and Sinha, 2000; etc] misses some relevant features. Even though the latest advances in cointegration techniques have been used to separate long-run effects from short-run responses and some tests of the stability of the long-run elasticities have been developed

[Konno and Fukushige, 2002 and 2003], most of the research has been carried out within a linear framework that does not take into account the contribution of business cycle asymmetries.

This paper investigates potential nonlinearities in the imports data of six major European economies: Belgium, France, Germany, Italy, Spain and the United Kingdom. Following standard practice in the literature, firstly Smooth Transition AutoRegressions (STAR) are considered. Next the basic univariate framework is extended to allow for exogenous determination, so that the switch between regimes is a function of the rate of growth of GDP as a more direct indicator of the business cycle. This variant leads to the Smooth Transition AutoRegression with EXogenous Transition (STAR-EXT) model. The two specifications are compared in terms of their adequacy to the data. Finding that STAR-EXTs are preferred would support the proposition that nonlinearities arise from cyclical asymmetries, while opting for pure STARs would be an indication that they are due to idiosyncratic components specific to foreign trade.

The paper is organized as follows. The following section presents the two variants of smooth transition models. Next the estimated models for the quarterly rate of growth of imports are reported. Then the international evidence on the effects of business cycle asymmetries in the imports data is examined. The final section concludes the paper.

2. SMOOTH TRANSITION MODELS

STMs are a special class of state-dependent, nonlinear time series models where the variable is assumed to vary between two extreme regimes and the smoothness of the transition is estimated from the data. The dependent variable is given by a linear combination of predetermined variables plus a random disturbance, where each coefficient is a function of a state variable. This parameterization permits a variety of dynamic behavior and at the same time once the state is given the model is locally linear, which allows an easy interpretation of the local dynamics. Granger and Teräsvirta [1993], Teräsvirta [1994, 1998], and van Dijk et al. [2002] describe STMs with full particulars.

The basic univariate version of STMs is the Smooth Transition Autoregression: all predetermined variables are lags of the dependent variable and regimes are endogenously generated by the recent history of the time series itself. The STAR model of order p for a stationary and ergodic process y_t is defined as

$$y_t = \pi_0 + \sum_{i=1}^{p} \pi_i y_{t-i} + F(y_{t-d}) \left[\theta_0 + \sum_{i=1}^{p} \theta_i y_{t-i} \right] + u_t \qquad (2.1)$$

where $F(y_{t-d})$ is a transition function that satisfies $0 \leq F \leq 1$ and d is the transition lag.

The key feature about the transition function $F(y_{t-d})$ is whether it is odd or even; in the first case $F(-\infty)=0$ and $F(\infty)=1$ while in the second $F(\pm\infty)=1$ and $F(c)=0$ for some finite c. In the empirical literature the odd case is usually represented by the logistic function

$$F(y_{t-d}) = \frac{1}{1+\exp[-\gamma(y_{t-d}-c)]} \quad , \qquad \gamma > 0 \qquad (2.2)$$

and the resulting model is known as the logistic STAR or LSTAR. The slope parameter γ determines the smoothness of the transition: the higher it is, the more abrupt the change from one extreme regime to the other. The location parameter c is such that $F(c)=0.5$ and separates the half-intervals of low and high values of the transition function.

The exponential function is used for the even case,

$$F(y_{t-d}) = 1 - \exp\left[-\gamma(y_{t-d}-c)^2\right] \quad , \qquad \gamma > 0 \qquad (2.3)$$

so that (2.1) and (2.3) define the exponential STAR (ESTAR) model. The parameter γ has the same interpretation as in the logistic model and c defines the middle extreme regime so that F(c)=0.

The fact that $F(y_{t-d})$ is even or odd has interesting implications for understanding nonlinearities. At first sight LSTARs seem more adequate, as the two extreme regimes correspond to very high and very low values of the variable. Nevertheless both specifications will be very similar in fitting the data when the estimated location parameter c is either very high or very low, a situation that arises quite frequently in empirical applications [Teräsvirta and Anderson, 1992; Skalin and Teräsvirta, 1999]. Öcal and Osborn [2000] argue that in this case the preference for an exponential transition together with a low value of c may be explained because this combination is best suited than LSTARs for capturing transitions that are sharper near troughs than at peaks, see their paper for the details.

In the basic STAR (2.1) to (2.3) the state variable determining the regime at each t is a lag of the dependent variable. While this may be a valid starting point in a strict univariate framework, for analyzing cyclical asymmetries it seems better to define a more direct indicator of the state of the cycle and to specify the switch as a function of that indicator. This gives rise to a second class of STMs that is midway between STARs and general Smooth Transition Regressions, as it combines univariate dynamic dependence and exogenous regime determination. The model will be referred to as STAR-EXT and is given by

$$y_t = \pi_0 + \sum_{i=1}^{p} \pi_i y_{t-i} + F(x_{t-d}) \left[\theta_0 + \sum_{i=1}^{p} \theta_i y_{t-i} \right] + u_t \qquad (2.4)$$

where x_{t-d} stands for GDP growth, which is a natural candidate for tracking the cycle. In the next sections the relative performance of (2.1) and (2.4) are compared in order to assess the main force driving nonlinear behavior in European imports.

3. EMPIRICAL ANALYSIS

3.1 Data

Quarterly, seasonally adjusted data are considered for imports of goods and services and GDP for Belgium, France, Germany, Italy, Spain and the United Kingdom [Organization for Economic Cooperation and Development Statistical Compendium]. The final sample goes from 1981:Q1 to 2003:Q2. German data have been adjusted because of the unification. Unit root tests indicate that all imports and GDP series are I(1), so from now on all the variables are the first difference of the logarithms.

3.2 Testing linearity

The first step towards building STAR models is to test whether the data display the type of nonlinear behavior generated by smooth transition autoregressions. The usual approach is to compare the nested linear model against all variants of the general nonlinear model. The literature has not considered the case for testing against STAR-EXT in an explicit way, but it is straightforward to adapt the tests derived for general smooth transition regressions to cover this particular situation.

The tests are based on a sequence of auxiliary regressions and a review would demand some technical discussion that goes well beyond the scope of this paper, see Teräsvirta [1994, 1998] for the details. There is, however, a point that deserves some attention for interpreting Table 1. In the standard testing strategy the first step is to determine the lag order p by using the Akaike Information Criterion (AIC) to select the proper number of lags in a linear autoregression, and next linearity tests are carried out conditional on this value. The transition lag d is determined either by varying it and choosing the value minimizing the p-value of his linearity test [Teräsvirta, 1994, p. 211], or by assuming that the transition variable is the linear combination $\sum_{i=1}^{p} \upsilon_i z_{t-i}$, where $\upsilon'=(0 \ldots 1 \ldots 0)'$ is a selection vector with the only unit element corresponding to the unknown transition lag [Teräsvirta, 1998, p. 517] and z_t is the transition variable. The first approach is called the conditional approach, while the second is known as the unconditional procedure.

The lag order p varies from 1 to 8; d goes from 1 to max(p,6) in the endogenous transition model and from 0 to max(p,6) when the switch depends on GDP growth. The maximum lag order is eight because eighth-order dynamics seem to be general enough for quarterly, seasonally adjusted data. Moreover the sample size is 89 observations and an adequate number of degrees of freedom is needed to avoid size distortions in carrying out hypothesis testing.

Table 1 displays a summary of p-values of the linearity tests. To save space it only reports the results against STAR models with exogenous transition for the lag order providing the minimum AIC in the linear autoregression. Both conditional and unconditional approaches are shown. Unconditional tests against STAR-EXT with exponential transition for the United Kingdom were computed for d ranging from 0 to 3, because p is high and there are not enough degrees of freedom for carrying out a joint test against unspecified d in the interval [0,6].

INSERT TABLE 1 HERE

The tests find evidence of nonlinear behavior in Belgium, France, Spain and the United Kingdom, but not for Germany or Italy. The number of rejections is higher against STAR-EXT models than against pure STARs (detailed results are available from the authors), a first indication that nonlinearities seem to be related to the cyclical state of the economy. In fact should the basic testing strategy be taken literally one would not reject linearity against STARs with endogenous transition. Recent literature however advocates attempting to build a valid nonlinear model even in that case, as it is expected that a false rejection of linearity will be discovered at some later stage [van Dijk et al. 2002; Sensier et al. 2002].

3.3 Estimated models

Model building was based on an extensive search. A large number of potential models were specified by considering all the possible combinations of p and d for both logistic and exponential transitions. This amounts to 66 models with endogenous regime determination and 82 models with exogenous transition for each country. All 148 specifications were estimated by nonlinear least squares and the best models were selected for further refinement. Cross-parameter restrictions were evaluated and non-significant coefficients were dropped to conserve degrees of freedom. Standard F-tests and AIC were used to check that the restrictions embedded in the final model were supported by the data. Several misspecification tests were computed to validate the final specifications, see below for further description of the diagnostic tests. All the computations were done in Rats.

Valid STAR and STAR-EXT models were achieved for every country but Germany. Table 1 showed that linearity was not rejected against a STAR-EXT model in Germany, and the modelling process confirmed that it is not possible to derive an adequate STAR-EXT representation for that series. In Table 2 a summary of the estimated models is reported.

INSERT TABLE 2 HERE

The final step is to compare the relative performance of the two approaches in order to determine the origin of the observed nonlinearities. Opting for the model with exogenous transition entails that nonlinear behavior is related to general cyclical conditions, while the preference for the model with endogenous transition may be taken as an indication that nonlinearities are generated by idiosyncratic components specific to foreign trade.

To the authors' knowledge formal tests for comparing non-nested, nonlinear time series models like those reported in Table 2 have not been developed yet, and the only feasible way to select between such competing specifications is to use information criteria. It can be seen in Table 2 that AIC is lower for the exogenous transition model in Belgium, France, Spain and the U.K., while the strict univariate specification is preferred for Italy.

The final selected models are presented in full detail in Table 3, together with some descriptive statistics and diagnostic tests. In regard to the latter LJB is the Lomnicki-Jarque-Bera test of normality; ARCH denotes the statistic of no autoregressive conditional heteroskedasticity with four lags; BCH is Öcal and Osborn's [2000] test of business cycle heteroskedasticity computed by regressing the squared residuals on the values of the transition function. Three tests specially derived for smooth transition models in Eitrheim and Teräsvirta [1996] are also displayed. AUTO tests serial independence against a fourth-order process. NL is a test of no remaining nonlinearity in the residuals. The test is computed for several values of the transition lag under the alternative, and Table 3 reports the value minimizing the p-value of the tests; the p-value in parenthesis is computed from the standard F distribution and understates the actual value that would be obtained by considering the true, unknown distribution of the ordered statistic. PC is a general test of parameter constancy that allows for monotonically and nonmonotonically changing parameters under the alternative.

INSERT TABLE 3 HERE

4. AN EUROPEAN PERSPECTIVE ON CYCLICAL ASYMMETRIES IN IMPORTS

The previous section pointed out that the dynamics of European imports are regime-dependent; in some cases the relevant regime is related to the cyclical state of the economy, while in the rest it results from more idiosyncratic factors specific to foreign trade. A closer look at Table 2 reveals an interesting stylized fact of European imports, whatever the transition variable is. For every country the variance ratio, defined as the residual variance of the nonlinear model over the residual variance of the best linear autoregression selected with AIC, lies in the interval [0.71, 0.76]: the final nonlinear model explains 25% to 30% of the residual variance of the best linear autoregression in all six countries, although the way that bound is attained is not the same in all cases.

In Belgium and Spain the STAR model already does most of the job and the variance ratios of STAR and STAR-EXT models are very similar. In the two countries the rates of growth of imports and GDP are closely related and lagged imports might be a proxy of GDP in the model with endogenous transition. Thus considering GDP in an explicit way does not lead to a better fit but to a more efficient way of achieving the same explanatory power, both in the sense of requiring less parameters and by shortening the delay of response to changes in the underlying economic conditions.

In France and the United Kingdom a different picture arises: the explanatory power of the univariate model is not much higher compared to the linear autoregression (variance ratios around 0.85), but it increases when an exogenous transition is allowed for. The number of parameters of STAR-EXT models are the same (France) or higher (United Kingdom) than in the STAR specification, and the reduction in AIC is due to a better fit: hence in both countries the regimes are determined by general economic conditions and the lagged growth of imports do not render a good approximation.

The main shortcoming of the Italian model with exogenous transition is that it displays low explanatory power compared to the univariate specification. In Germany the modelling procedure confirmed that the null of linearity is not rejected when a STAR-EXT model is considered under the alternative; it may happen that the unification process has changed the relation between GDP and imports, an issue that is left for further research. Although no valid STAR-EXT model was found for Germany the reduction in its final variance ratio is similar to those reported for the other five countries.

The estimated models show that national characteristics play a role in defining cyclical regimes. In order to describe the situation within each country the shape of the transition function is analyzed together with the local dynamics. A plot of the estimated transition function versus the transition variable shows where the extreme regimes are and how rapid the transition is. Conditional on the regime the models are locally linear and the dynamics can be interpreted in the usual way through the roots of the characteristic polynomial. Unit and explosive roots deserve special attention as the model may be globally stationary but locally unstable, a result that is informative about the way imports evolve over the cycle. In what follows extreme regimes are defined by taking into account both the extreme values of the transition function and the dynamics of the model at their neighbourhood.

Figure 1 depicts the estimated transition functions of the models presented in Table 3. Each dot represents (at least) one observation in the sample. To summarize local dynamics three values of the transition function are considered, the two extreme regimes $F=0$ and $F=1$ and the intermediate situation $F=0.5$, and the roots of the resulting characteristic polynomial are computed. Table 4 reports the main results; to save space only the dominant root is displayed, i.e. the root with the highest modulus that determines the long-run behavior of the series within that regime. For convenience of representation all rates of growth in the following discussion are expressed in annual terms.

INSERT FIGURE 1 HERE

INSERT TABLE 4 HERE

In Belgium the switch between regimes is a function of current GDP growth. The transition is exponential, although it has similar regime implications to the logistic specification as moderate-to-high values of F at the left of the location parameter are seldom observed. Apart from some outliers F is close to 0 when the economy is contracting, and the middle extreme regime can be seen as a recession situation. The other extreme regime is for strong recoveries, GDP growing above 5.3%. The model is always stable and imports are quite volatile as local dynamics are dominated by a pair of complex roots with a period of 2.5 quarters. Volatility increases with GDP growth, as the modulus of the dominant roots goes from 0.70 for F=0 to 0.96 for F=1.

The transition variable in France is also current GDP growth and the selected specification is a typical ESTAR model, as there are a significant number of data points at the left tail of the exponential function. The middle extreme regime is a low-growth state, below 1.5%. The outer extreme regime is observed during strong recoveries, defined for current GDP growing above 4.8%. Intermediate situations cover both periods of normal growth and classical recessions with declines in the level of GDP. The model is always stable but local dynamics depend on the phase of the cycle. When GDP growth is low (F close to 0) French imports display a cycle with a period of 9.8

quarters and modulus 0.92; as F increases the period of the cycle also increases and when F is close to 1 the rate of growth of imports is dominated by a real, positive root.

Nonlinear behavior of German imports is endogenously determined. The transition is logistic and seems to be very rapid from one extreme regime to the other, but a closer inspection of the model reveals three situations. On the one side there is a lower extreme regime, which is attained for values of the transition lag below 4%; within this regime the model is stable and the dependence of current growth of imports on its past values is moderate, as the modulus of the dominant root is 0.55. This modulus increases with the transition lag and the model becomes explosive for $F>0.5$ or imports growing above 12% in annual terms. Hence the interval (4%, 12%) can be seen as an intermediate situation between the two extreme regimes, the model being locally stable in that case. When the transition lag is above 12% imports enter the third situation, which is dominated by an explosive cycle with a period of 13.5 quarters: the model is locally unstable and imports growth evolves quickly towards more moderate rates.

In the Italian model the switch between regimes is a function of past values of imports growth. The transition function is exponential with a location parameter close to the median of the data. The middle extreme regime extends in practice for F lower than 0.6, as the model becomes unstable when the transition function is below that bound. In terms of lagged imports it corresponds to rates of growth within the interval 1% to 7.3%. The transition function is close to 1 for values of the transition lag that are negative or above 8.3%, and imports are in the outer extreme regime. In the stable part of the model ($F>0.6$) local dynamics vary slightly according to the specific values of the transition function, but the dominant roots tend to generate a cycle with a period of three years and modulus around 0.90.

In Spain the transition is driven by a logistic function of current GDP growth. Imports are in the lower extreme regime when the economy is in recession, and in the upper extreme regime in periods of vigorous expansions (GDP growth above 7.6%). The model is nonstationary in the upper extreme regime because of a bounce-back effect launched by an explosive, negative real root. The dynamics of the model are very simple. The characteristic polynomial includes a negative real root with modulus that is an increasing function of F, and the more rapid the economy grows the more volatile Spanish imports are. This root dominates local dynamics when GDP grows above 6.3%, so the series looks quite erratic in periods of strong recoveries.

The main need for a nonlinear model for the United Kingdom arises from the 1990-1992 recession. The transition function is exponential but the model performs as if it were logistic. Broadly speaking the ESTAR-EXT model is close to a classical two-regime threshold autoregression: one regime for declines in the level of GDP and the other for current rates of growth of GDP above 1%, plus a short interval midway as activity passes on the way up or down. There is an explosive pair of complex roots associated to the recession regime, so the rate of growth of imports is nonstationary when the economy is contracting.

5. CONCLUSIONS

The paper investigated whether European imports display the type of nonlinear behavior related to business cycle asymmetries. Two classes of smooth transition models were considered, strict univariate Smooth Transition AutoRegressions (STAR) and Smooth Transition AutoRegressions with EXogenous Transition (STAR-EXT). STARs assume that the transition between regimes is endogenously determined. STAR-

EXT models parameterize the transition as a function of GDP growth, which acts as an exogenous state variable that captures the cycle of the economy.

The empirical analysis for six major European economies (Belgium, France, Germany, Italy, Spain and the United Kingdom) supported the proposition that the dynamic properties of imports are regime-dependent: modelling the nonlinearity in the data explained 25% to 30% of the residual variance of the best linear autoregression.

In Belgium, France, Spain and the United Kingdom regimes are exogenously determined: in all cases the transition variable is current GDP growth, and the parameters of the model adapt immediately to changes in the economic conditions. In Germany and Italy the switch between regimes is endogenous. As a consequence nonlinear behavior is directly related to cyclical asymmetries in four countries and to idiosyncratic components specific to foreign trade in the other two cases. Together with such common features, national characteristics also play a role in determining the position of extreme regimes, the smoothness of the transition and local behavior within each state.

Whatever the source of the nonlinearity may be, either related to cyclical asymmetries or due to factors specific to foreign trade, the paper showed empirical evidence of nonlinear behavior in the imports data. One may argue to what extent linear aggregate import demand functions are misspecified, and whether current estimates of price and income elasticities should be revised to take into account the nonlinear dynamics. These issues will be considered in future research.

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TABLE 1

Linearity tests against exogenous transition models (p-values)

Transition lag	Country											
	Belgium		France		Germany		Italy		Spain		United Kingdom	
	p=5		p=4		p=2		p=5		p=2		p=8	
d	LSTAR	ESTAR	LSTAR	ESTAR	LSTAR	ESTAR	LSTAR	ESTAR	LSTAR	ESTAR	LSTAR	ESTAR
Cond. 0	0.0080	0.0147	0.0004	0.0036	0.4243	0.6778	0.5939	0.4820	0.0001	0.0006	0.0218	0.0247
1	0.3895	0.4350	0.6386	0.5492	0.9059	0.7461	0.1304	0.4987	0.1953	0.1493	0.5268	0.3598
2	0.0463	0.1383	0.3304	0.2204	0.2920	0.5670	0.0809	0.0643	0.2104	0.3170	0.2628	0.5727
3	0.0710	0.1569	0.0696	0.0451	-	-	0.4041	0.2997	-	-	0.5598	0.6099
4	0.3778	0.7737	0.5911	0.8879	-	-	0.8661	0.6021	-	-	0.0299	0.0637
5	0.4256	0.8865	-	-	-	-	0.7980	0.4609	-	-	0.0436	0.2280
6	-	-	-	-	-	-	-	-	-	-	0.6123	0.0497
Uncond. 0-6	0.0172	0.0394	0.0002	0.0014	0.6430	0.8242	0.5239	0.3056	0.0023	0.0329	0.0154	0.3149

TABLE 2

Model selection: summary of the best specifications with endogenous (STAR) and exogenous (STAR-EXT) transition

	Country											
	Belgium		France		Germany		Italy		Spain		United Kingdom	
	STAR	STAR-EXT	STAR	STAR-EXT	STAR	STAR-EXT	STAR	STAR-EXT	STAR	STAR-EXT	STAR	STAR-EXT
Transition	L	E	L	E	L	-	E	L	L	L	L	E
p	8	5	6	7	7	-	8	4	6	2	7	7
d	5	0	3	0	5	-	3	2	4	0	2	0
k	14	9	10	10	11	-	13	10	11	7	10	14
s	0.0207	0.0198	0.0139	0.0126	0.0195	-	0.0283	0.0292	0.0268	0.0258	0.0184	0.0175
R^2	0.38	0.39	0.41	0.51	0.24	-	0.30	0.26	0.34	0.33	0.25	0.35
AIC	-7.5993	-7.7418	-8.4420	-8.6270	-7.7494	-	-6.9820	-6.9566	-7.1120	-7.2364	-7.8795	-7.9316
s^2/s^2_L	0.73	0.72	0.86	0.71	0.76	-	0.75	0.84	0.73	0.71	0.85	0.73

Notes: L (E) stands for a logistic (exponential) transition function; p is the lag order of the model; d the transition lag; k the number of estimated parameters; s the residual standard error; R^2 the determination coefficient; AIC the Akaike Information Criterion; s^2/s^2_L the variance ratio of the residuals from the nonlinear model and the best linear AR selected with AIC.

TABLE 3

Estimated models for European imports

BELGIUM (ESTAR-EXT)

$$y_t = \underset{(0.0067)}{-0.0085} - \underset{(0.07)}{0.16}\, y_{t-1} + \underset{(0.22)}{0.27}\, y_{t-2} - \underset{(0.14)}{0.21}\, y_{t-4}$$
$$+\left(\underset{(0.0105)}{0.0382} - \underset{(0.07)}{0.16}\, y_{t-1} - \underset{(0.35)}{0.54}\, y_{t-2} + \underset{(0.14)}{0.21}\, y_{t-4} + \underset{(0.23)}{0.67}\, y_{t-5} \right)$$
$$\times \left[1 - \exp\left\{ - \underset{(0.23)}{0.41} \times 21724.82 \left(x_t + \underset{(0.0012)}{0.0030} \right)^2 \right\} \right] + u_t$$

s=0.0198, R^2=0.39, AIC=-7.7418, s^2/s^2_L=0.72, LJB=1.40 (0.50), ARCH=0.26 (0.90), BCH=0.17 (0.68), AUTO=1.94 (0.11), NL=1.46 (0.14), PC=1.15 (0.34).

FRANCE (ESTAR-EXT)

$$y_t = \underset{(0.0036)}{0.0022} + \underset{(0.15)}{0.23}\, y_{t-2} + \underset{(0.17)}{0.51}\, y_{t-3} - \underset{(0.18)}{0.59}\, y_{t-4} - \underset{(0.15)}{0.36}\, y_{t-5} + \underset{(0.14)}{0.25}\, y_{t-7}$$
$$+\left(\underset{(0.0068)}{0.0142} + \underset{(0.28)}{0.39}\, y_{t-2} - \underset{(0.17)}{0.51}\, y_{t-3} + \underset{(0.18)}{0.59}\, y_{t-4} + \underset{(0.15)}{0.36}\, y_{t-5} - \underset{(0.14)}{0.25}\, y_{t-7} \right)$$
$$\times \left[1 - \exp\left\{ - \underset{(0.25)}{0.58} \times 39810.18 \left(x_t - \underset{(0.0006)}{0.0017} \right)^2 \right\} \right] + u_t$$

s=0.0126, R^2=0.51, AIC=-8.6270, s^2/s^2_L=0.71, LJB=1.71 (0.42), ARCH=0.41 (0.80), BCH=0.03 (0.86), AUTO=0.91 (0.47), NL=1.32 (0.20), PC=2.09 (0.03).

GERMANY (LSTAR)

$$y_t = \underset{(0.0035)}{0.0116} - \underset{(0.15)}{0.18}\, y_{t-1} + \underset{(0.11)}{0.15}\, y_{t-2} + \underset{(0.13)}{0.13}\, y_{t-3}$$
$$+\left(\underset{(0.0127)}{0.0139} + \underset{(0.60)}{1.07}\, y_{t-1} - \underset{(0.62)}{0.88}\, y_{t-3} - \underset{(0.27)}{0.24}\, y_{t-6} - \underset{(0.49)}{0.95}\, y_{t-7} \right) \times \left[1 + \exp\left\{ - \underset{(3.70)}{4.48} \times 46.49 \left(y_{t-5} - \underset{(0.0060)}{0.0287} \right) \right\} \right]^{-1} + u_t$$

s=0.0195, R^2=0.24, AIC=-7.7494, s^2/s^2_L=0.76, LJB=11.72 (0.003), ARCH=0.80 (0.53), BCH=2.35 (0.13), AUTO=0.10 (0.98), NL=1.07 (0.41), PC=0.81 (0.73).

(Table 3, continued)

ITALY (ESTAR)

$$y_t = \underset{(0.0266)}{-0.0267} + \underset{(0.51)}{0.67}\,y_{t-1} + \underset{(0.61)}{0.69}\,y_{t-4} - \underset{(0.10)}{0.17}\,y_{t-6} + \underset{(1.00)}{1.25}\,y_{t-7} + \underset{(0.64)}{1.60}\,y_{t-8}$$
$$+\left(\underset{(0.0273)}{0.0525} - \underset{(0.54)}{0.94}\,y_{t-1} - \underset{(0.65)}{0.93}\,y_{t-4} - \underset{(1.03)}{1.47}\,y_{t-7} - \underset{(0.64)}{1.81}\,y_{t-8}\right) \times \left[1 - \exp\left\{-\underset{(7.28)}{17.36} \times 943.40\left(y_{t-3} - \underset{(0.0010)}{0.0102}\right)^2\right\}\right] + u_t$$

s=0.0283, R^2=0.30, AIC=-6.9820, s^2/s^2_L=0.75, LJB=2.79 (0.25), ARCH=0.28 (0.89), BCH=2.69 (0.10), AUTO=0.32 (0.87), NL=1.17 (0.32), PC=0.76 (0.77).

SPAIN (LSTAR-EXT)

$$y_t = \underset{(0.0126)}{-0.0053} + \underset{(0.22)}{0.54}\,y_{t-1} + \left(\underset{(0.0298)}{0.0586} - \underset{(0.53)}{1.32}\,y_{t-1} + \underset{(0.29)}{0.47}\,y_{t-2}\right) \times \left[1 + \exp\left\{-\underset{(0.94)}{1.37} \times 133.61\left(x_t - \underset{(0.0041)}{0.0108}\right)\right\}\right]^{-1} + u_t$$

s=0.0258, R^2=0.33, AIC=-7.2364, s^2/s^2_L=0.71, LJB=0.10 (0.95), ARCH=0.52 (0.72), BCH=0.05 (0.83), AUTO=2.03 (0.10), NL=1.67 (0.10), PC=0.81 (0.70).

UNITED KINGDOM (ESTAR-EXT)

$$y_t = \underset{(0.0138)}{-0.0007} + \underset{(0.82)}{1.19}\,y_{t-1} - \underset{(0.67)}{0.86}\,y_{t-3} - \underset{(0.81)}{0.75}\,y_{t-4} + \underset{(0.10)}{0.19}\,y_{t-5} - \underset{(0.90)}{1.58}\,y_{t-6} - \underset{(0.91)}{0.86}\,y_{t-7}$$
$$+\left(\underset{(0.0146)}{0.0117} - \underset{(0.88)}{1.39}\,y_{t-1} + \underset{(0.69)}{0.98}\,y_{t-3} + \underset{(0.81)}{0.75}\,y_{t-4} + \underset{(0.90)}{1.75}\,y_{t-6} + \underset{(0.91)}{1.08}\,y_{t-7}\right)$$
$$\times\left[1 - \exp\left\{-\underset{(0.50)}{0.82} \times 32209.13\left(x_t + \underset{(0.0011)}{0.0061}\right)^2\right\}\right] + u_t$$

s=0.0175, R^2=0.35, AIC=-7.9316, s^2/s^2_L=0.73, LJB=3.33 (0.19), ARCH=1.16 (0.34), BCH=2.68 (0.11), AUTO=0.17 (0.95), NL=1.13 (0.35), PC=0.60 (0.92).

Notes: y_t (x_t) denotes the quarterly rate of growth of imports (GDP). Values under regression coefficients are standard errors of the estimates; s is the residual standard error; R^2 the determination coefficient; AIC the Akaike Information Criterion; s^2/s^2_L is the variance ratio of the residuals from the nonlinear model and the best linear AR selected with AIC; LJB is the Lomnicki-Jarque-Bera normality test; ARCH is the statistic of no ARCH based on four lags; BCH is a business cycle heteroscedasticity test; AUTO is the test for residual autocorrelation of order 4; NL is the test for no remaining nonlinearity; PC is the general parameter constancy test. Numbers in parentheses after values of LJB, ARCH, BCH, AUTO, NL and PC are p-values.

TABLE 4

Local dynamics: dominant roots in each regime

Country	Regime (value of F)	Root	Modulus	Period
Belgium	0	-0.5846 ± 0.3829i	0.70	2.5
	0.5	-0.7161 ± 0.5050i	0.88	2.5
	1	-0.7640 ± 0.5796i	0.96	2.5
France	0	0.7360 ± 0.5467i	0.92	9.8
	0.5	0.6453 ± 0.4528i	0.79	10.3
	1	0.7897	0.79	-
Germany	0	0.5479	0.55	-
	0.5	0.8820 ± 0.4449i	0.99	13.5
	1	1.0700 ± 0.5387i	1.20	13.5
Italy	0	1.3419	1.34	-
	0.5	1.0664	1.07	-
	1	0.7889 ± 0.4266i	0.90	12.7
Spain	0	0.5393	0.54	-
	0.5	-0.5503	0.55	-
	1	-1.1809	1.18	-
UK	0	1.2547 ± 0.7099i	1.44	12.2
	0.5	0.9223 ± 0.5781i	1.09	11.2
	1	0.9049	0.90	-

FIGURE 1

Transition functions for European imports