Computer software for analysis and design optimization of power transmission structures by Simulated Annealing and sensitivity analysis.

I. Couceiro · J. París · S. Martínez · F. Navarrina · I. Colominas

Abstract This paper presents a computer software for the optimization of power transmission structures. The software employs a modified version of the Simulated Annealing algorithm that has been proven effective in large engineering problems. The target structures are three dimensional steel trusses to be used as supporting towers of electrical lines. A mixed formulation merging continuous and discrete design variables is proposed for optimizing the size and shape of the trusses, including a first order sensitivity analysis that reduces the computational cost. The implementation can be adapted to any kind of transmission tower and allows to quickly create a model to be analyzed and optimized in a few sequential steps. Despite its simplicity of use, the tools provided by the proposed framework allow to perform a full analysis of the design and provide an entire comprehension of its structural behavior. The software also includes a post-process and visualization tool-set in a user-friendly graphical interface.

Keywords structural optimization · optimization software · transmission towers

1 Introduction

Lattice steel transmission towers design is a broad field for the application of mathematical optimization techniques. There is an extensive record of proposals and methodologies for truss structures optimization [1–7]. Transmission towers conception is relatively simple, formed by a finite amount of steel bars with bolted joints. The

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result is a common three dimensional truss structure with pinned connections. Thereby all the theory and practice developed for the optimization of truss structures can be applied to the field.

However, the results of the application of optimization to a certain problem are highly dependent on the form that a particular problem is proposed. Thereby, serious amount of effort is involved in the correct definition of the physical reality and the characteristics that define the structure to be optimized. The mathematical model must represent accurately the particularities of the design and the actual construction of the structure. The conditions or imposed constraints should represent all the limitations the actual structure is subjected to.

A critical aspect is the definition of the design variables. A typical approach in truss structures is sizing optimization [8–12]. In [13] the author proposes a multi-objective optimization of a 272-bar transmission tower using the volume of the structure and the displacement of the top of the tower as objective functions. A very well defined Pareto front is presented where some points are selected as optimum solutions under different defined scenarios. The approach shows a significant efficiency of convergence to Pareto fronts for a wide range of real multi-objective optimization problems. A more complex approach would be to combine sizing and shape optimization. In this case, for the shape optimization part many authors choose to define the coordinates of the joints as design variables [14–16]. Finally, more advanced works are opened to a bigger reconfiguration of the structure by using topology optimization [17,18]. The topology optimization approach for truss structures is to allow the process to decide which elements or bars are needed and which elements can be removed from the structure. The initial design should be a structure with bars connecting every node with

each other, as far as they have physical meaning and then let the algorithm decide the essential bars. However this approach usually results in very large optimization problems that penalize the use of a particular set of optimization algorithms and also increases the cost of the sensitivity analysis needed in many cases. One of the additional problems in topology optimization of truss structures is the fact that elements might be found non-necessary in the structure and therefore be removed from the structure [18]. This introduces severe changes in the design problem and the definition of the optimization problem. Typical examples of truss topology optimization applied to known benchmarks can be found in [19]. Even though topology optimization could probably result in better structural designs, it would destroy the modular conception of the structure and the present methodology is proposed to ensure the construction of the transmission towers with the typical predefined blocks.

However, even though a lot of effort has been put in truss structures optimization, not much of those results were applied to power transmission towers. In [20], the author proposed an optimization algorithm for high-voltage transmission lines with based on fuzzy theory. Shea and Smith [21] proposed a structural topology optimization method for transmission towers through a set of rules that define the allowable design modifications. The paper shows a full-scale example subject to structural and geometrical constraints. In [22], a discrete topology optimization approach is used to optimize a real transmission structure with size, shape and topology variables. Nevertheless, the optimization of a completely real transmission tower used in real situations, considering its full shape, all its elements, the structural constraints actually required by the standards and practical constructional issues is rarely tackled.

One of the most noteworthy and realistic studies is that of [23]. In their work, authors propose an improved Simulated Annealing algorithm to optimize the weight of lattice steel transmission towers using sizing and layout design variables. The optimization software is also integrated in a commercial software called PLS-TOWER. The effectiveness of the methodology is tested in three real and full scale transmission towers. The efficiency of the non-deterministic search technique used is improved by a two phase algorithm to improve the initial design to be used in the complete Simulated Annealing process.

In this paper, a computer software for the optimization of lattice steel transmission towers is presented, considering practical construction issues. The approach of the optimization is particularly detailed in the se-

lection of the design variables and the constraints of the problem so the optimized designs can actually be constructed. A deeper description of the optimization algorithm and its numerical formulation can be found in [24]. In [23] authors used a few panel widths as layout design variables to optimized the shape of the tower while keeping the straightness of the tower body. In this work, the panel widths or block widths are extended to all the height of the transmission structure in search for a better design. The non-straightness of the body can be simply resolved with transition pieces that are already used in current joints of towers where the alignment of the elements changes. Additionally, the proposed work improves the efficiency of the optimization process by including a first order sensitivity analysis. It allows to reduce the number of structural reanalysis needed for the random designs generated and to reduce the global iterations required.

There are several suitable optimization algorithms for this problem. In fact, there are, in general, two main approaches for optimization, Mathematical programming and metaheuristic algorithms [25]. However, for the approach proposed in this article, explained later, metaheuristic or stochastic algorithms are the best possible choice. Also, in [26], authors point out the inefficiency of the common optimization algorithms when facing real towers encounter in practice. Thus, authors considered the application of an algorithm of randomized search, the Simulated Annealing [27], with slight modifications to fit the particularities of the problem. As mentioned in [18], the constantly increasing computational capabilities are in favor of the development of new metaheuristic algorithms. There is a vast field of possibilities to choose as optimizer between metaheuristic algorithms, such us Particle Swarm Optimization (PSO), Dolphin Echolocation Optimization, Colliding Bodies Optimization, Imperialist Competitive Algorithm, an many others as one can consult in [13]. In this work, the selection of the Simulated Annealing algorithm was made based on two main reasons. First, it allows the implementation and optimization of discrete and continuous variables simultaneously. The second reason is computational efficiency. The algorithm also allows the implementation of a first order sensitivity analysis which in this case is used to reduce the computational burden of all the structural reanalysis needed to verify the candidate designs randomly generated by the algorithm. Thus, the Simulated Annealing algorithm combines useful properties from both mathematical programming and metaheuristic algorithms making it a very suitable option for the proposed optimization problem. This properties constitute advantages against other algorithms based on evolutionary processes.

The present software is conceived as a user-friendly interface to ease and speed up the creation of any tower model and the optimization of its design. Pre-process and post-process tools are brought together within a three dimensional CAD framework allowing for the visualization of the model and the results. The whole environment has been developed by the authors in a C++ cross-platform provided by Qt [28].

2 Optimum design of transmission towers

The prime objective of transmission towers is to sustain the power wires of a particular line. Even though this supporting structures can be made of steel, concrete or wood, lattice steel towers are the most common type. These trusses are made by a set of steel bars which, in most of the towers, are typical equal-leg angle sections. The most important fact is that these bars are arranged in blocks of fixed shape and connectivity and symmetric shape. This is also relevant as the applied loads are not symmetric but they can be applied in multiple directions. Therefore, the tower is formed by a vertical series of different blocks. There are different configurations for the shape of the blocks which make them suitable (for example) for the body of the tower, for the bottom positions and transmission of the loads to the foundations, or specific blocks with arms to attach the electrical wires. These practical aspects are some of the particularities that are included in the model and taken into account for the definition of the design problem. This eases the generation of a tower model and reduces the number of variables needed to describe the structure but it also complicates their handling in the optimization process.

2.1 Problem statement

The general problem of structural optimization can be written in the form:

$$minimize \quad W = F(\boldsymbol{x}) \tag{1}$$

subject to

$$g_j(\boldsymbol{x}) \leq 0, \quad j = 1, ..., m \tag{2}$$

$$(x_i)_{min} \leq x_i \leq (x_i)_{max}, \quad i = 1, ..., n \tag{3}$$

where $\boldsymbol{x}$ represents the vector of n design variables:

$$\boldsymbol{x} = \{x\}_i \quad i = 1, ..., n \tag{4}$$

Eq. (1) is the objective function that represents the total weight of the structure, in this case the optimization objective function. g_j corresponds to the m structural constraints that limit the designs, and inequalities of Eq. (3) indicate the side constraints of the n design variables.

Since the structure is formed by a discrete number of steel bars with a specific cross section and the corresponding area, the total weight of the tower can be expressed as a sum of the individual weights of each bar as:

$$W = \rho \sum_{i=1}^{n_b} l_i \ A_i \tag{5}$$

where ρ is the density of the material, n_b is the number of bars of the structure and l_i and A_i are the length and cross-sectional area of the i-th element, respectively.

Once the aim of the problem is defined, we need to select the design variables for the optimization problem.

2.2 Design variables

Design variables are selected accordingly to the already mentioned construction particularities. Also, the selection of the design variables defines the type of optimization we are facing. In this case, we are dealing with the simultaneous shape and size optimization of the tower. Taking into account how transmission towers are defined (i.e. by a vertical combination of blocks made by steel bars), we chose as design variables the dimensions of the blocks and the cross-section of the groups of bars, since bars are arranged in groups inside each block. The modification of the dimensions of the blocks alters the shape of the tower, while the section of the bars corresponds to the size optimization part.
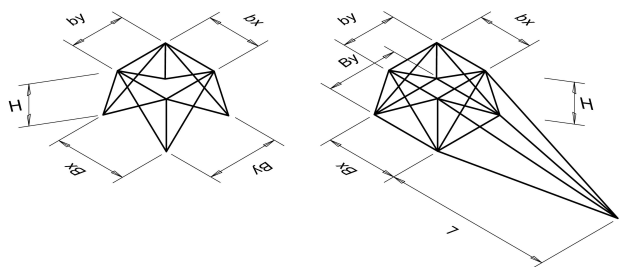
Fig. 1 Dimensions of the blocks.

As shown in Figure 1, blocks are defined by the width of their top and bottom faces, their height and, in special blocks, also by the length of the cross arms wires are attached to. When a project of a transmission line is being developed, both, the height of the tower and

the length of the cross arms are determined by electrical requirements. Therefore, this length and the height of each block are excluded of the design variables and taken as fixed parameters for the optimization process. Thus, the shape optimization of the structure relies on the modification of the top and bottom widths of the blocks. These variables have a continuous nature.
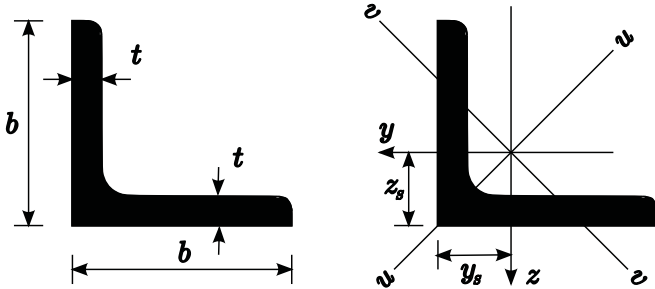

Fig. 2 Equal leg angle section and dimensions.

On the other hand, for the size optimization, the cross sections of the bars forming the structure are also chosen as design variables. Figure 2 details the shape and characteristics of the available profiles. Any equal leg section is defined by its width-thickness pair (b, t). However, in order to be consistent with the particular aspects of the problem, we have to adapt the possible sections of the bars to the actual offer of typical providers of hot rolled-steel bars. This leaves the problem with a discrete inventory of angle sections, each characterized by its (b, t) pair. Moreover, given a range of b and t dimensions, their combinations are limited. For each b value there are only some prescribed values of t from the whole range. Thus, we define a grid (Figure 3) that explicits the cross-sections available to be selected by the algorithm.

It should be noted that the shape and size optimization problem at hand leads to a mixed optimization approach where discrete and continuous variables need to be handled at the same time. Otherwise, a separated treatment of both variables could lead to a stagnation of the design as the optimum might be reachable only by a simultaneous modification of both variables. This aspect has to be considered when selecting the optimization algorithm to be used.

2.3 Structural constraints (ASCE and Eurocode standards)

The general optimization problem stated in Eq. (1) is subject to different structural and side constraints (Eq. (2) and Eq. (3), respectively). These structural constraints are imposed to limit certain characteristics of

Fig. 3 Grid mapping the available equal leg angle sections defined by their (b, t) pair.

the achieved designs or to make them fulfill certain requirements. These requirements imposed to the structural response are stated by current standards.

In the case of lattice steel transmission towers the most relevant standards are the Eurocode 3 [29] and the ASCE 10-97 [30]. Both documents explicit the conditions that need to be imposed to the structural design and the requirements for the structural response given the load cases. Even though both share the same purpose, there are slight differences between their specifications. We considered that the most reasonable approach is to make the designs satisfy both standards. This means that the conditions imposed by the two specifications are compared and then only the more restrictive one is applied so the other is directly satisfied.

Thus, the constraints implemented in the structural model are as follows.

– Cross-section slenderness: The ratio between the flange width b and the thickness t of the steel section is limited to avoid local buckling according to Eurocode 3 [29]. Thus,

$$\frac{b}{t} \leq 15 \, \epsilon \tag{6}$$

where $\epsilon = \sqrt{\frac{235}{f_y}}$, being f_y the yield strength in MPa.
– Element slenderness: The ratio between the length of the elements and the radius of gyration of their

sections is also limited to prevent instabilities under vibrations or deformations. Thus,

$$\frac{l_i}{r_i} \leq g_{0,i} \tag{7}$$

where l_i and r_i are the bar length and radius of gyration of the i-th element, respectively. $g_{0,i}$ is the limit value established by the ASCE 10-97 [30].

− Tensile stress: Tensile stress acting on angle steel sections connected by bolts at both ends is limited to 90% of their tensile strength. This limit is defined by the ASCE 10-97 [30]. The Eurocode 3 [29] does not consider this 10% reduction of the tensile strength, thereby, it is more restrictive the requirement of the American standard. Thus,

$$\frac{N_{Ed,i,j}}{A_i\, 0.90\, f_y} \leq 1.0 \tag{8}$$

where $N_{Ed,i,j}$ is the tension force acting on the i-th element in j-th load case, and A_i the area of the element.

− Compression stress: Steel elements are also checked under buckling when subject to compression forces. Thus,

$$\frac{N_{Ed,i,j}}{\chi_i\, A_i\, f_y} \leq 1.0 \tag{9}$$

where $N_{Ed,i,j}$ is the compression force acting on the i-th element in the j-th load case, and χ_i is the buckling coefficient according to the Eurocode 3 [29].

All the constraints are implemented in the methodology in a normalized form. Each constraint is defined by the value of the analyzed quantity and the reference or limit value set by the standard. Thus, the normalized constraints can take values from 0 to 1. Values greater than 0.9 mean an active constraint and values greater than 1 a violated constraint. The general expression for every constraint is then:

$$g_j^*(\boldsymbol{x}) = \frac{\varphi_j(\boldsymbol{x})}{\Psi_j(\boldsymbol{x})} \leq 1, \quad j = 1,...,m \tag{10}$$

where g_j^* is the normalized constraint, φ_j is the actual value, Ψ_j is the limit value and m the number of constraints.

The first two described constraints are applied to each bar of the structure. Thus, for each element there is a cross-section slenderness constraint and a element slenderness constraint. However, for the tensile or compression stress constraints there is one constraint for each element in each load case. Typically, in the design of transmission towers 10 or more load cases are considered. Thereby, the optimization problem ends up

having a large number of structural constraints to handle. This aspect is particularly decisive when selecting the type of optimization algorithm to apply.

2.4 Optimization algorithm

The truss design problem requires an optimization algorithm capable of handling discrete and continuous variables simultaneously and able to deal with a large number of constraints. In this scenario, a stochastic optimization algorithm like the Simulated Annealing appears as an interesting option since it might achieve very good results without exploring the whole search space thoroughly.

The Simulated Annealing algorithm is a stochastic algorithm that involves randomness in the search for new designs by random modifications of the design variables. Based on the Metropolis' criterion [31], Kirkpatrick et al.[27] developed the simulated annealing algorithm inspired by the annealing process in metallurgy. They defined an analogy between the thermodynamic process and the optimization process. Thus, each random design generated by the algorithm corresponds, in the analogy, to a thermodynamic configuration. The equilibrium configuration is therefore the optimum solution. Simulated Annealing was later extended in [32–37].

In the algorithm, the transition between the current state and the randomly generated new design is governed by the probability function:

$$q = min\left\{1, e^{-\Delta E/k_B\, T}\right\} \tag{11}$$

where q is the probability of accepting the new design. ΔE is the energy change between states and the energy, in the analogy, represents the value of the objective function. k_B is the Boltzmann constant and T is the system temperature, a control variable of the process.

The temperature parameter is set to be decreased by the exponential function (12) with $\alpha < 1$. Other rates of variation for this parameter can be found in the bibliography [38,39].

$$T_k = \alpha_B^k\, T_0 \tag{12}$$

For further advanced research on Simulated Annealing one can consult [40–43].

The details of the Simulated Annealing are widely known and may be consulted in the cited references. Nevertheless, the algorithm implemented by the present authors includes certain enhancements in order to improve its performance. One of them is the use of Taylor's expansion, to predict the structural response, based on
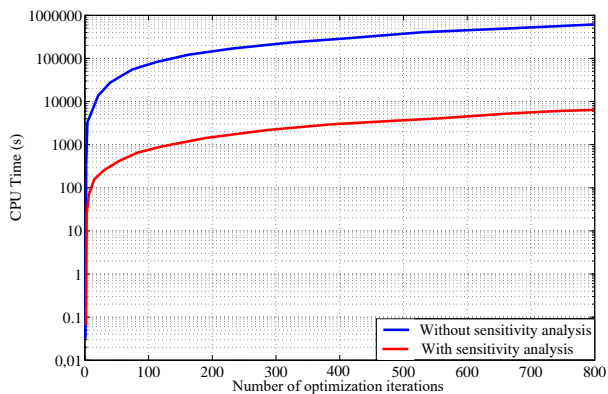
Fig. 4 CPU time reduction with sensitivity analysis.

a first order sensitivity analysis. This technique was proposed and coined by Schmit as *Structural Synthesis* [44] . For a given modification of the design variables, the Simulated Annealing needs to evaluate the values of the objective function and constraints. This action is repeated many times during the optimization process. This may seem obvious and insubstantial but with a great number of constraints, the time consumption per iteration might become quite large.

Thus, instead of computing the objective function and constraints for each iteration, which requires performing a complete new structural analysis, those values are approximated according to the *Structural Synthesis* theory as:

$$\boldsymbol{f}(\boldsymbol{x}^{k+1}) = \boldsymbol{f}(\boldsymbol{x}^k) + \boldsymbol{f'}(\boldsymbol{x}^k) \cdot \Delta\boldsymbol{x}^{k+1} + \Theta(\left\|\Delta\boldsymbol{x}^{k+1}\right\|^2) \quad (13)$$

where $\boldsymbol{f}$ represents the function evaluated, $\boldsymbol{x}$ are the design variables and, k and $k+1$ indicate the current state and the state given for a variation of the design variables. Thus, $\Delta\boldsymbol{x}^{k+1} = \boldsymbol{x}^{k+1} - \boldsymbol{x}^k$ is the variation of the design variables in the new design proposed.

Given the simplicity of the functions that describe the constraints and the objective function, their derivatives can be obtained by Direct Differentiation. The use of the Taylor's expansions achieves considerable CPU time reduction in comparison with traditional Simulated Annealing versions as shown in Figure 4.

As mentioned before, the algorithm needs to deal with the discrete and the continuous variables simultaneously. This aspect needs for a special treatment to prevent some undesired effects. The objective function and the constraints respond differently to the variation of the two variables, they are particularly sensitive to the modification of the discrete variables. Therefore the algorithm may stagnate at certain points since the simultaneous change in both variables leads only to designs out of the feasible region. This issue is solved by

the present authors by restricting the design variables to modify when the algorithm stagnates. In this case, the selection of which variables will be modified is randomized so that it is chosen, with the same probability, if the new design is obtained by modifying both variables again, only the discrete variables or only the continuous variables.

In addition, when the design is close to the optimum, small changes of the discrete variables can lead to big changes of the design and make it leave the feasible design region, as it is schematically shown in figure 5. The design is considered to be close to the optimum if the algorithm does not find a suitable solution for five consecutive iterations. Thus, when the design is close to the optimum, the algorithm limits the number of discrete variables to be modified and adjusts the design changing mostly the continuous design variables.
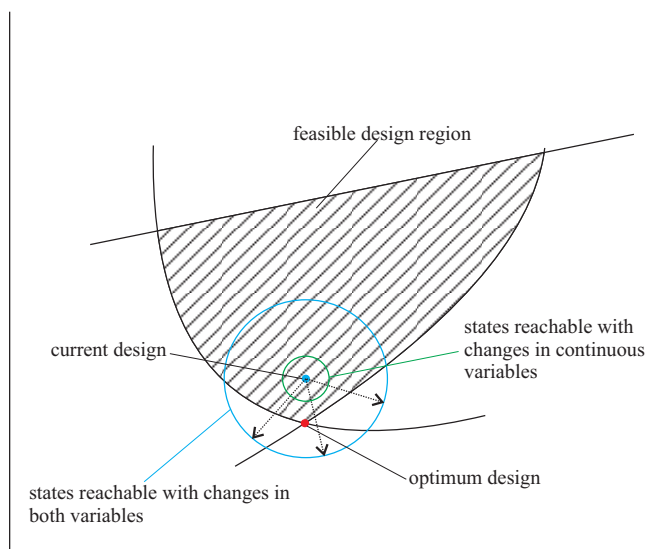
Fig. 5 Designs achievable with modifications of different groups of variables.

We also included a reheating technique to the Simulated Annealing algorithm, allowing the search process to rise the value of the temperature instead of always decreasing it. This enhancement was introduced by Bonnemoy and Hamma [45] and allows the algorithm to restart itself at certain steps of the process. It is designed to avoid the convergence to local minima.

The improved version of Simulated Annealing is the core of the optimization software application. For further details [24] can be consulted.
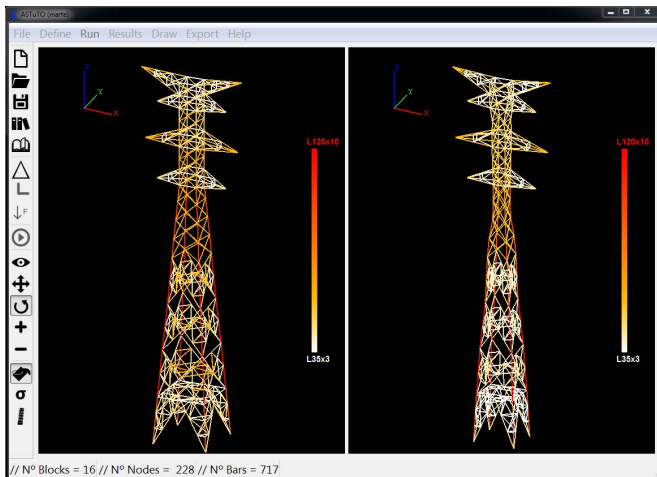
Fig. 6 Main window and CAD environment. Initial design (left window) and optimized design (right window).

3 Advanced software for transmission towers optimization

The application has been developed in C++ with the cross-platform application framework Qt [28]. The result is a native-looking user interface that works in multiple platforms and operating systems. The analysis and optimization core of the software has been developed in Fortran.

3.1 Main features

The computer software developed in this study is an Advanced Software for Transmission Towers Optimization which includes all the necessary tools for the creation, analysis and optimization of a given transmission tower design. The user interface is built around a CAD environment window displaying a three-dimensional model of the structure as shown in Figure 6. Pre-process as well as post-process tools are available in the application menus, allowing for a full description of the structural model and a deep comprehension of the behavior and the response of both initial and optimized designs.

As shown in Figure 6, the main window is formed mainly by two CAD windows with a menu bar on top, a tool bar for quick access icons on the left and an informative status bar at the bottom. The menu bar holds access to all the sub-menus and actions enclosed in the application, from typical file operations (new, save, load, import...) to graphic tools and optimization results. All these actions are arranged in order of utilization (according to the workflow presented in Figure 7) from left to right and, inside each menu, from top to bottom. Thus, by following a guided sequence of steps for the definition, optimization and analysis of any

model, as shown in figure 7, a real application example can be easily solved.

The icons on the left of the window give quick access to the principal features and functionalities of the application. A model of a tower can be generated and studied rapidly only with the actions triggered by these icons.

The main window has been vertically divided into two graphic windows in order to be able to compare the initial design and the optimum design. Thereby, both geometries and results can be viewed simultaneously, allowing an immediate comparison of both designs.

3.2 Working flow and general scheme

The available menus of the application create a sequence of steps for the optimization of a given structure. These steps are pictured in Figure 7 as the general working flow of the application.

However, there are some shortcuts in the general scheme. It gives the possibility of saving models at any step of the process. It also allows to save files for optimization and analysis results. Thus, all these files can be loaded so the process can be resumed from any previous step. Any saved file can also be modified to generate a completely different model through the available menus.

The program needs a valid cross-section inventory file. The software provides a default inventory of equal leg angle cross sections, although, custom catalogs can be used by editing a new inventory file. Within the inventory file, properties for each steel profile need to be described. Dimensions, moment of inertia, area and radius of gyration of the section are the required characteristics to be given in input to the software.

3.3 Tower definition and modeling

The dialogs provided, particularly those enclosed inside the *Define* menu, were conceived to make the data definition as simple and intuitive as possible.

The definition of a tower takes 4 steps, all of them reachable from the quick access icons on the left of the window. The first one is a quick previous dialog where the characteristics of the material, in most cases steel, must be introduced (see Figure 8). The required properties are the Young's modulus E and the strength limit f_y. In addition, the number of blocks forming the tower needs to be specified in this dialog in order to generate the next window for the complete definition of the geometry. Still, the number of blocks can be modified later.
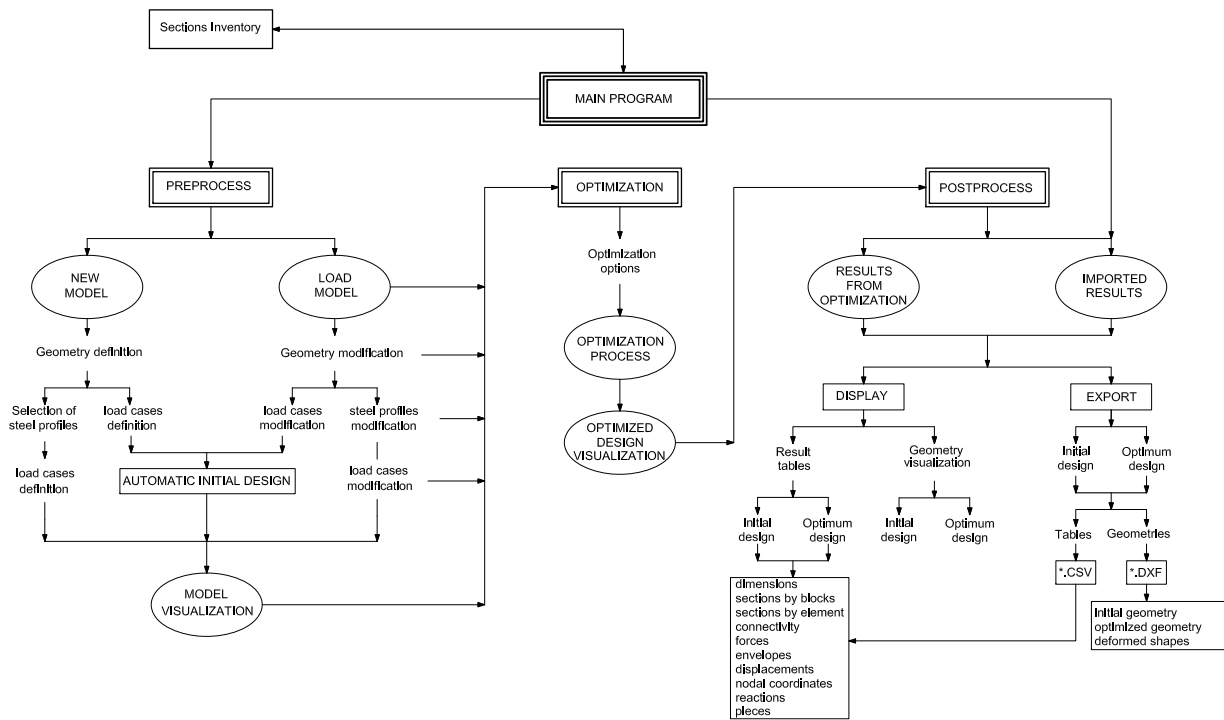
Fig. 7 Flow chart of the proposed design optimization framework for power transmission structures.
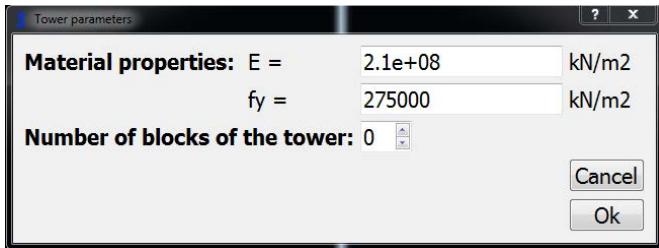

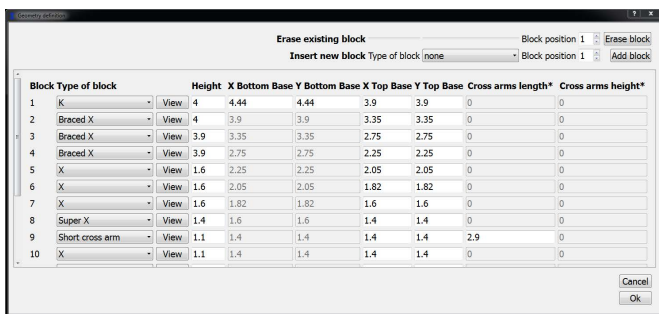**Fig. 8** Material properties input window.

Fig. 9 Geometry definition input window.

The next step is to define the geometry of the tower. Once the material properties and the previous number of blocks are introduced, the geometry definition dialog box shown in Figure 9 pops up. The blocks of the tower are arranged from 1 to the number specified in the previous step, from bottom to top of the structure.

The geometry of the tower is defined by the type of each block and their respective dimensions (bottom and top widths, height and cross arms length and height when applicable). The user can select the type of block from a given list included by default in the software. The list contains a wide set of different blocks commonly used to build transmission structures. A wide variety of towers can be modeled by a combination of the blocks available. All the blocks can be visualized in a 3D view window.

With all these dimensions the tower geometry is fully determined. Only the width of the bases of the blocks enter the optimization process as design variables, since the other three dimensions are restricted by electrical conditions and only structural considerations are taken into account in the proposed approach.
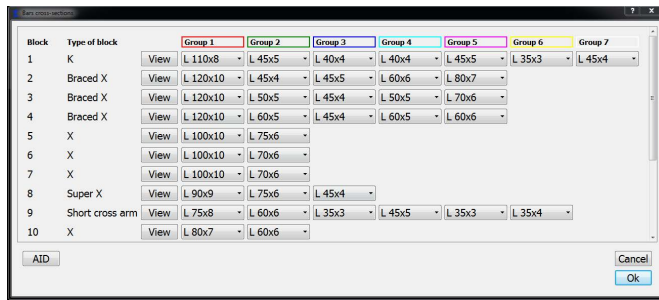
Fig. 10 Cross-sections selection window.

Fig. 11 Load cases definition window.

Once these values are introduced, the geometry of the initial design is displayed on the left CAD window of the application. Any modification will be reflected instantly in the CAD window.

Even though the number of blocks forming the tower was introduced in the preceding dialog, this quantity can also be modified here. The dialog provides some controls in the top right of the window as shown in figure 9. With these tools, any block can be deleted, leaving the rest of the blocks unmodified, or inserted in any desired position. Designs of power transmission structures tend to be quite repetitive along a line with slight variations in some positions or under certain conditions like greater spans, greater forces applied for various reasons, changes of direction, etc. Thereby, the option of loading a file of a previous design and change its properties, or even remove and insert blocks constitutes a very useful tool in the engineering practice.

The process of deleting or adding new blocks to the model does not modifies the dimensions of the rest of the blocks.

The third step is to assign cross-sections to the bars. The dialog box shown in Figure 10 aims to provide users with a simple and intuitive way to specify element cross-sections.

Most of the blocks typically used in these structures have a strong symmetry along two axes. Thus, bars placed at symmetric positions share the same cross-section. Taking this into account, bars with the same cross-section inside each block, either by symmetry or by being intended for the same structural purpose, are grouped together. Thereby, the cross-sections of the bars are introduced by groups for each block.

The cross-sections available are taken from a given list or inventory selected by the user. The application already provides a default inventory with a wide range of equal leg angle profiles.

Nevertheless, assigning sections to every group of bars of the structure is not trivial. Moreover, the optimization algorithm performs better when the initial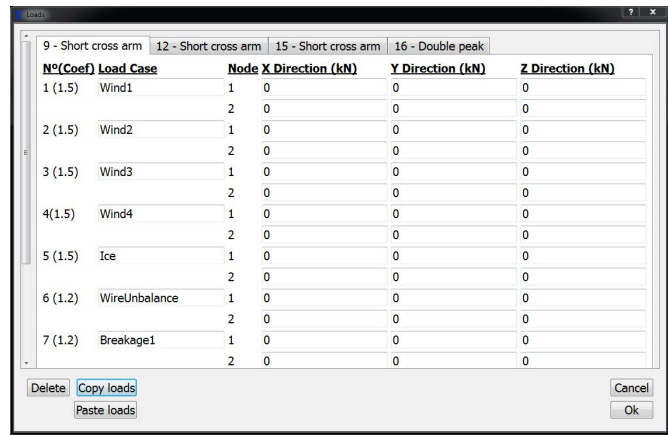 given design is actually inside the feasible region. Selecting cross-sections for the bars ensuring that the design will satisfy the implemented constraints is everything but obvious, and it is a classic structural design problem itself. For this reason authors considered useful the implementation of an algorithm to automatically assign cross-sections to the bars. The initial design provided by the AID algorithm (Assistant for Initial Design) guarantees the fulfillment of the constraints. This algorithm has also been implemented in order to ensure a satisfactory performance of the Simulated Annealing process to run later. Authors have study the robustness of the SA algorithm when starting from different initial configurations and a significant amount of tests failed when the optimization process started from a non-feasible design. The AID is set to ensure the feasibility of the initial design with different safety coefficients. As long as the fulfillment of the constraints in the initial design is guaranteed, the simulated annealing algorithm shows a good performance in general.

Before the AID algorithm starts, the applied load cases need to be defined. The window shown in figure 11 allows the definition of loads for each loaded block up to ten load cases. The loaded blocks are arranged in tabs where the information for each load can be filled.

Once the load cases are specified the AID algorithm starts. It carries out an iterative process that assigns proper cross-sections to the bars in each step until the constraints for the bars are strictly satisfied. When the bars can no longer be adjusted in size without leaving the feasible design region, the process finishes, establishing the initial design. The AID process runs in the following phases:

1. Assign the biggest cross-section of the inventory to all the bars to initiate the process.
2. Run a simple structural analysis to obtain the stresses.
3. Using the stresses envelope, compute the minimum properties of the cross-section needed to satisfy the

equations for element slenderness, tension stress and compression stress.

$$tensile\ stress \rightarrow A_i^{k+1} \geq \frac{N_{Ed,i,max}}{0.90\ f_y} \qquad (14)$$

$$compression\ stress \rightarrow A_i^{k+1} \geq \frac{N_{Ed,i,min}}{\chi_i^k\ f_y} \qquad (15)$$

$$element\ slenderness \rightarrow A_i^{k+1} \geq \frac{4\ t_i^k\ \sqrt{6}\ l_i}{g_{0,i}} \qquad (16)$$

where $N_{Ed,i,max}$ and $N_{Ed,i,min}$ correspond to the maximum absolute value of tension and compression force.

4. Select the limiting A_i^{k+1} for each bar.
5. Assign a cross-section that matches the limiting property.
6. Return to step 2 and check constraints. If any constraint is violated, continue with the selected cross-sections, otherwise, the initial design has been achieved.

The results obtained with the AID algorithm are then loaded in the cross-section selection window and can be modified if required.

These four main steps define the whole model of the transmission structure completely. Once the model of the tower is completely defined the software allows to perform a simple structural analysis or the full optimization process.

3.4 Analysis and optimization

The software allows the user to perform a simple structural analysis of the transmission tower collecting results of stresses, reactions at joints, nodal displacements and all the information needed for a complete comprehension of the structural behavior. However, the strength point of the proposed software is its optimization capability based on the algorithm briefly described in Section 2.4.

Figure 12 shows a window where general and specific options for the structural analysis and optimization can be modified. General properties as the material density for the self weight load consideration or the tower category are displayed. The tower category includes a wind distributed load on each bar on the first four load cases. In first, second and third categories, forces according to a nominal wind speed of 120 km/h are applied whereas, in special category, forces are generated by a speed of 140 km/h.

Some parameters of the Simulated Annealing are open for change too. They are preselected by default with values that ensure a good performance of the algorithm but they can be modified.
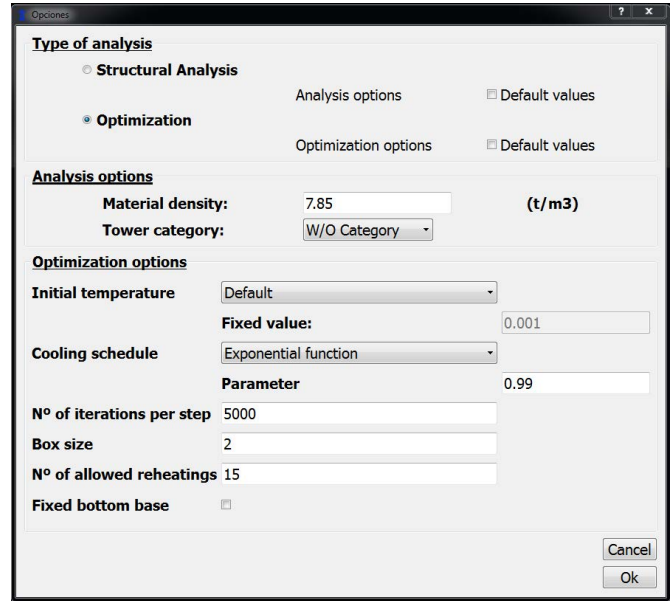

Fig. 12 Window of options for the analysis and optimization.

– Initial temperature: Temperature is a control value inherent to the Simulated Annealing algorithm [27]. Its initial value is defined depending on how close to the optimum the initial design is.
– Cooling schedule: The rate of reduction of the temperature is called cooling schedule. There are several functions used as cooling schedules in the bibliography. The default scheme is an exponential function that ensures an affordable convergence speed [42, 43].
– Number of iterations per step: The Simulated Annealing algorithm tries to find a feasible design many times at each global iteration of the optimization process. This parameter limits the maximum number of trials that the present algorithm can perform for searching new designs on each global iteration.
– Box size: It represents the percentage of variation allowed for the continuous variables per iteration. It is established as a $\pm 2\%$ per global iteration. In the numerical tests carried out by the authors, the Simulated Annealing algorithm showed the best performance for this value.
– Number of allowed reheating cycles: The maximum number of reheating cycles allowed to avoid local optima.

Once these options are settled, the analysis and the optimization are ready to start. A process evolution window will appear displaying relevant information about the ongoing process. In the case of the optimization this information allows to supervise the correct performance of the algorithm. In the case of a simple structural anal-

ysis the process finishes immediately so no information is required to supervise the progress.

As soon as the algorithm finishes the post-processing, options of the application will be available.

3.5 Results visualization, output and post-process

The results of the structural analysis and optimization process found by the proposed software should give a full understanding of the structural behavior of both models. The post-processing graphic tools also allow to represent some of the main results in the visualization window. Moreover, all the analysis data are organized in the *Results* menu in data table form.

The results of analysis provided for both the initial and the optimum designs are:

- Geometry: Dimensions specifying the size of the blocks forming the tower.
- Sections: Cross-sections of the bars arranged by groups inside each block.
- Forces: Axial force per load case in every bar in kN. The post-processing options also allow to view the stress in the elements, tension stress is represented in red and compressive stress in blue. Stresses are given in kN/m^2 (Fig. 13).
- Envelopes: Envelope of axial forces for every element considering all the load cases, in kN.
- Bars: Cross-section and length of each bar, not arranged by groups or blocks.
- Reactions: Reactions in kN at the restrained joints.
- Nodal coordinates: Coordinates of the nodes of the structure.
- Nodal displacements: Displacements of the nodes for each load case. The deformed shape consequence of each load case can be represented in the CAD window.
- Connectivity: The connectivity data link every bar of the structure with the two nodes it is connected to. This information is quite useful and simplifies the process of exporting the model or replicating it with another software.
- Pieces: A very useful information for any project of a transmission tower is the number of steel bars that share the same cross section and the same length. This is needed when placing an order to the steel profiles provider. Instead of counting every element of the structure manually, the software provides this list automatically using a standard sorting algorithm with slight modifications to count and sort the bars twice. The list is then organized by sections from biggest to smallest and at the same time by length from longest to shortest.

Table 1 General characteristics of the real application example

Height	31 m
Initial Weight	5.33 t
Elements	717
Blocks	16
Continuous variables	33
Discrete variables	130
Constraints	7952

All these results are available for the initial design and the optimum design in table form. Several tables can be displayed simultaneously to compare results. In addition, all the tables are exportable to csv files compatible with usual spreadsheet or database management software.

Once again, to help in the process of elaborating a project, the geometry of the tower can be saved in dxf format containing the 3D model. Initial and optimum designs can be saved in dxf files as well as the deformed shapes for any load case.

4 Numerical examples

4.1 Real application example

In order to enlighten the capabilities of the software package described in this article and its optimization algorithm based on Simulated Annealing, the optimization of a real transmission tower is presented.

The selected structure is a 31 m high tower for double electric circuit with double peak for two ground wires. General characteristics of the model are described in Table 1. Figure 14 shows the initial and optimized geometries.

For this structure, the algorithm achieves a reduction of 25% of the weight of the initial design. The final weight of the structure is 3.98 t. The optimization process was completed within about 1.5 hrs. The evolution of the structural weight during the optimization process is shown in Figure 15.

In the proposed formulation, constraints can take two states, active or inactive. Here, we consider that a constraint is active when its normalized value is greater than 0.9. It can be seen from Figure 16 that the initial design for the example (obtained in 8 iterations of the AID algorithm) presents 99 active constraints corresponding by most part to element slenderness limitations described by Eq. (7). The optimum design ends up with 200 active constraints where the compressive stress constraints become more relevant, as shown in Figure 17. Colored bars represent elements with an ac-
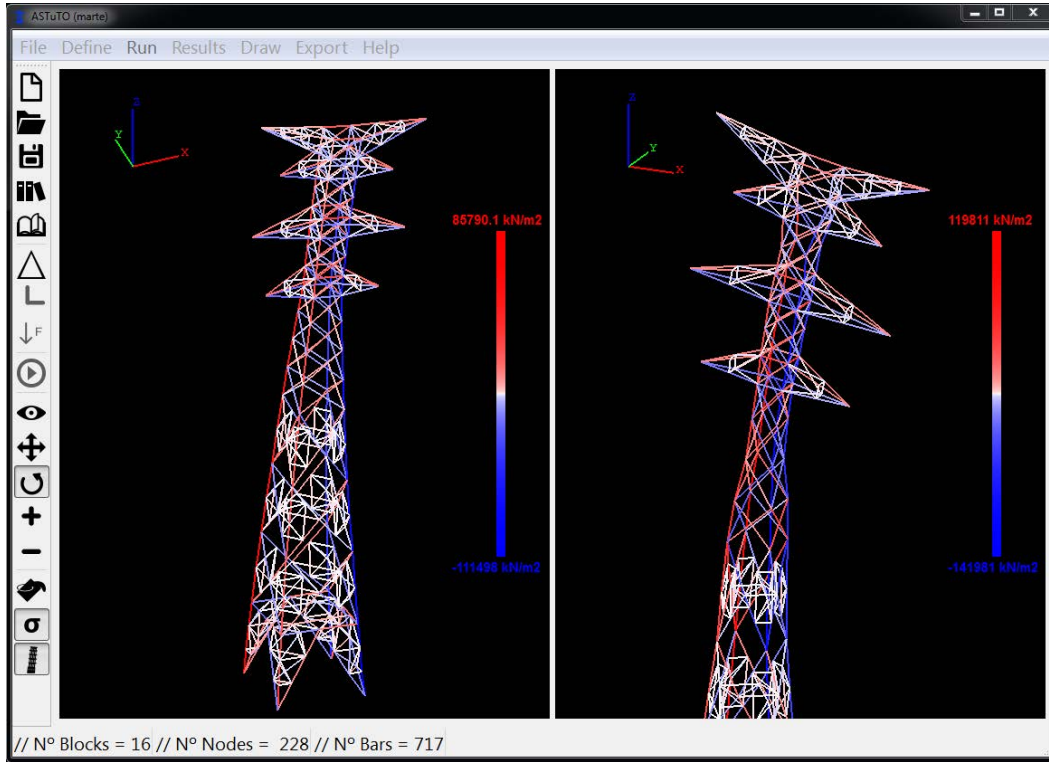
Fig. 13 Stresses in bars for a given load case (zoom in view of the structure).

Table 2 Dimensions of the blocks for the initial design and variation in the optimum geometry. H is the height of the block, W the width of the base (b for bottom, t for top and x and y for the axis)

| Block	H	$W_{b,x}$	$W_{b,y}$	$W_{t,x}$	$W_{t,y}$	$\Delta W_{b,x}$	$\Delta W_{b,y}$	$\Delta W_{t,x}$	$\Delta W_{t,y}$
1	4.0	4.44	4.44	3.90	3.90	-1.0733	-1.5907	-0.8967	-1.3423
2	4.0	3.90	3.90	3.35	3.35	-0.8967	-1.3423	-0.7153	-1.2364
3	3.9	3.35	3.35	2.75	2.75	-0.7153	-1.2364	-0.8051	-1.0725
4	3.9	2.75	2.75	2.25	2.25	-0.8051	-1.0725	-0.6127	-0.6694
5	1.6	2.25	2.25	2.05	2.05	-0.6127	-0.6694	-0.6171	-0.6550
6	1.6	2.05	2.05	1.82	1.82	-0.6171	-0.6550	-0.7876	-0.8269
7	1.6	1.82	1.82	1.60	1.60	-0.7876	-0.8269	-0.6372	-0.6571
8	1.4	1.60	1.60	1.40	1.40	-0.6372	-0.6571	-0.4453	-0.4507
9	1.1	1.40	1.40	1.40	1.40	-0.4453	-0.4507	-0.3356	-0.4336
10	1.1	1.40	1.40	1.40	1.40	-0.3356	-0.4336	-0.3296	-0.4037
11	1.1	1.40	1.40	1.40	1.40	-0.3296	-0.4037	-0.3344	-0.1738
12	1.1	1.40	1.40	1.40	1.40	-0.3344	-0.1738	-0.2919	-0.4051
13	1.1	1.40	1.40	1.40	1.40	-0.2919	-0.4051	-0.4514	-0.1036
14	1.1	1.40	1.40	1.40	1.40	-0.4514	-0.1036	-0.0891	-0.1201
15	1.1	1.40	1.40	1.40	1.40	-0.0891	-0.1201	0.2167	-0.4771
16	1.1	1.40	1.40	1.40	1.40	0.2167	-0.4771	0.0778	-0.4377

tive constraint. Red color states for active compressive stress constraint. Orange represents active element slenderness constraint.

The geometry of the tower is also described in Table 2 by means of the continuous variables that define the dimensions of the blocks. Table 2 shows the general reduction in all the widths along both axes, which drives to a much more slender design. This dimensions reduction, together with the decrement in the size of

the discrete variables (i.e. cross-sections of bars) is the reason for the increment in the stress state of the bars, and consequently the rise in the number of active stress constraints.
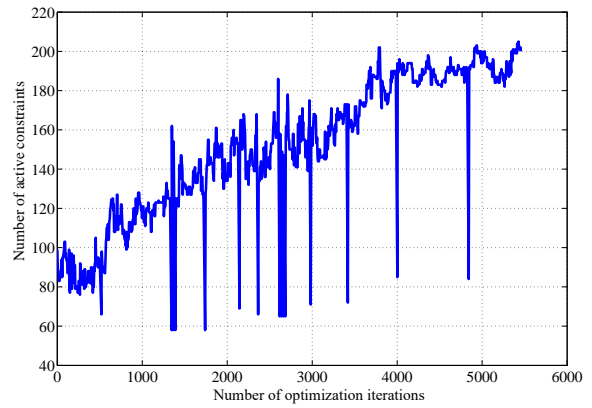
4.2 Comparative example

Another example is analyzed for comparison by the proposed methodology and using the developed software.
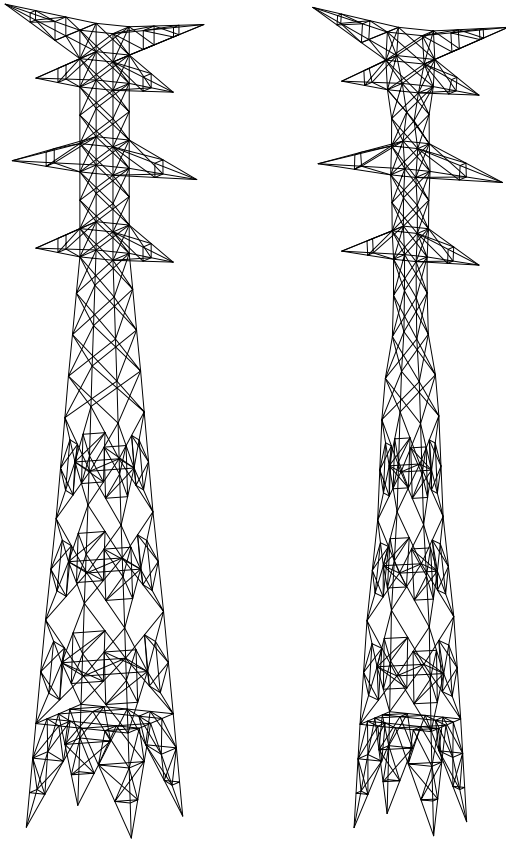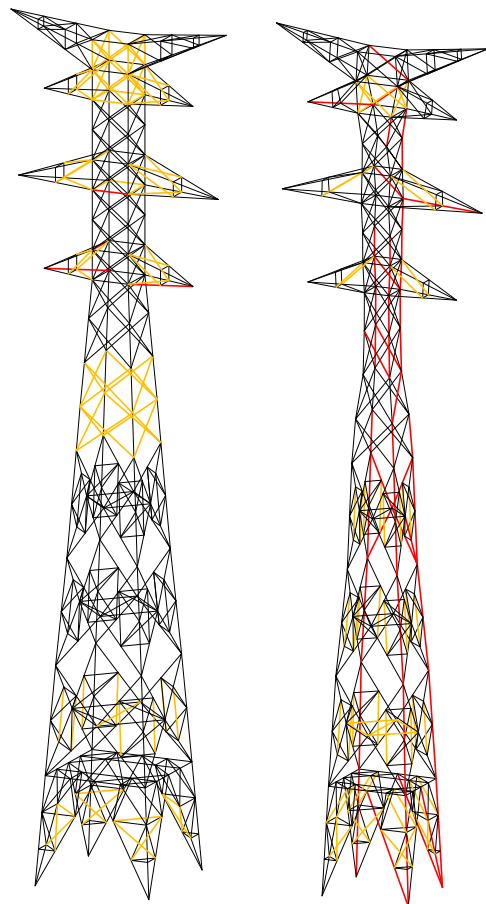
Fig. 14 Initial geometry (left) and optimized geometry (right) of the real tower example.
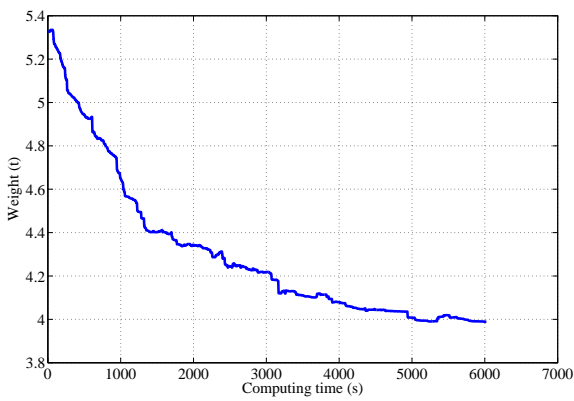
**Fig. 15** Convergence curve recorded for the real tower example.

Fig. 16 Evolution of the number of active constraints in the optimization process of the real tower.

Fig. 17 Active constraints evaluated for the initial design (left) and optimum design (right) of the real tower.

In this case, the structure is a 160-bar power transmission tower that is optimized in [46]. The complete description of the optimization problem, nodal coordinates, connectivity, structural constraints and design variables can also be found in [47–49]. The optimization
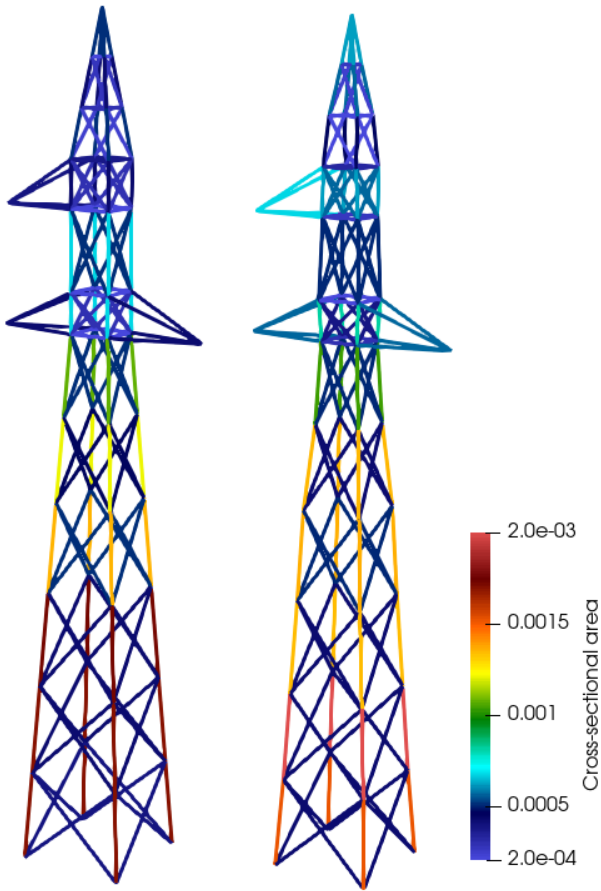
Fig. 18 Initial design (left) and optimum design (right) for the 160-bar power transmission tower.

of the transmission tower is performed using 38 size design variables for the cross-sectional areas of the bars chosen from a discrete set. In this case, the material has a modulus of elasticity of 204.7 MPa, a density of 7.85 t/m³ and a yield strength of 150 MPa. The initial configuration of the structure can be seen in Figure 18.

The optimization is performed introducing the geometrical design variables proposed in this work. Thus, 12 additional design variables are included in the optimization. The resulting geometry and distribution of the cross-sectional areas of the elements can be seen in Figure 18. The changes in geometry are small but allow the design of the transmission tower to reduce its weight further than using only size optimization. Figure 19 shows the convergence curves recorded for the optimization process with and without the geometric design variables. The full curve for the size optimization is not shown completely as it needed 6370 structural reanalysis and reached an optimum weight of 1330.05 kg. As it can be seen, the optimum design including the 12 additional design variables that control the overall shape
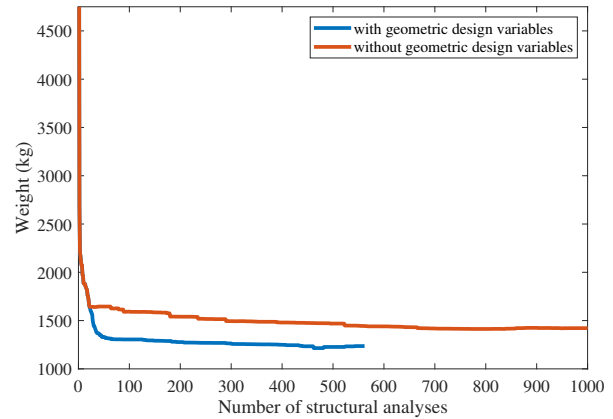
Fig. 19 Convergence curves recorded for the 160-bar power transmission tower using and not using the geometric design variables.

of the tower reaches a minimum weight of 1235.90 kg in only 562 structural reanalysis.

Conclusions

This paper presented an engineering software for the structural optimization of power transmission towers based on the Simulated Annealing algorithm. The proposed algorithm has been enhanced with Taylor's expansions approximation of the objective function and constraints based on a first order sensitivity analysis to reduce the computational cost. The constructional particularities of these structures have also been considered. Discrete and continuous variables are treated simultaneously in the design process to optimize both geometry and size of the towers. The developed software combines pre-processing tools, which allow to create a model of any tower, with post-processing tools which allow to analyze all the results of the optimization. The software includes a CAD environment to visualize a three dimensional model of the tower and present the most important results.

The optimization algorithm and the truss design software been proven effective when dealing with real problems, making easy the design and analysis of real structures. In the numerical example presented in this paper, the proposed software was able to reduce structural weight by 25% with respect to the initial design. The algorithm was used for designing several other structures and achieved weight reductions ranging from 20 to 30% with respect to the initial designs in some cases. The presented software was also used in a comparison example showing that the formulation including the widths of the blocks of the towers allows a reduction of the weight of the transmission towers.

Acknowledgments

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