Preprint of the paper

"A Boundary Element Numerical Approach for Substation Grounding in a Two Layer Earth Structure"

I. Colominas, J. Aneiros, F. Navarrina, M. Casteleiro (1997)

En "Advances in Computational Engineering Science", Sección: "Recent Developments in Boundary Element Method", pp. 756--761; S.N. Atluri, G. Yagawa (Editors); Tech Science Press, Atlanta, USA (ISBN: 0-96570001-0-0)

A BOUNDARY ELEMENT NUMERICAL APPROACH FOR SUBSTATION GROUNDING IN A TWO LAYER EARTH STRUCTURE*

Ignasi Colominas, Juan Aneiros, Fermín Navarrina and Manuel Casteleiro
Departamento de Métodos Matemáticos, Universidad de La Coruña
E.T.S. de Ingenieros de Caminos, Canales y Puertos
Campus de Elviña, 15192— La Coruña, SPAIN

SUMMARY

Analysis and design of substation grounding requires computing the distribution of potential on the earth surface (for reasons of human security) and the equivalent resistance of the earthing system (for reasons of equipment protection) when fault conditions occur (Sverak *et al.*,1981).

A new Boundary Element approach for the numerical computation of substation grounding systems in nonuniform soils is presented in this paper. The formulation is specially derived for two-layer soil models, which are widely considered as adecquate for most practical cases. The feasibility of this BEM approach is demonstrated by solving a real application problem, in which accurate results for the equivalent resistance and the potential distribution on the ground surface are obtained with acceptable computing requirements.

INTRODUCTION

Several methods for grounding design and computation have been proposed in the last three decades. Most of them are founded on semiempirical works or on the basis of intuitive ideas, such as superposition of punctual current sources and error averaging (Heppe,1979). Although these techniques represented a significant improvement in the area of earthing analysis, a number of problems have been reported: applicability limited to very simple grounding arrangements of electrodes in uniform soils, large computational requirements, unrealistic results when discretization of conductors is increased, and uncertainty in the margin of error (Garrett and Pruitt,1985). In the last years a Boundary Element formulation developed by the authors has allowed to identify this family of primitive methods as the result of introducing suitable assumptions in the BEM approach in order to reduce computational cost for specific choices of the test and trial functions. Furthermore, the anomalous asymptotic behaviour of this kind of methods could be mathematically explained, and sources of error have been pointed out (Colominas,1995). On the other hand, this BEM formulation has been succesfully applied (with a very reasonable computational cost) to the analysis of large grounding systems in electrical substations (Colominas *et al.*,1996).

The physical phenomena of fault currents dissipation into the earth can be described by means of Maxwell's Electromagnetic Theory (Durand,1966). Constraining the analysis to the obtention of the electrokinetic steady-state response and neglecting the inner resistivity of the earthing conductors, the 3D problem can be written as

$$\mathbf{div}\boldsymbol{\sigma} = 0,\ \ \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}V \text{ in } E; \quad \boldsymbol{\sigma}^t\boldsymbol{n}_E = 0 \text{ in } \Gamma_E; \quad V = V_\Gamma \text{ in } \Gamma; \quad V \longrightarrow 0, \text{ if } |\boldsymbol{x}| \to \infty; \quad (1)$$

where E is the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the electrode surface (Navarrina *et al.*,1992; Colominas,1995). The solution to this

* Work partially supported by Power Companies "UNIÓN FENOSA" and "FECSA", and research fellowships of the Universidad de La Coruña and the regional government "Xunta de Galicia".

problem gives the potential V and the current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$ when the electrode attains a voltage V_Γ (Ground Potential Rise or GPR) relative to a distant grounding point. Since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, the normalized boundary condition $V_\Gamma = 1$ is not restrictive at all, and will be used from here on.

In most of the methods proposed up to this moment it is assumed that for practical purposes soil can be considered homogeneous and isotropic. Thus, conductivity tensor $\boldsymbol{\gamma}$ is substituted by a meassured apparent scalar conductivity γ (Sverak *et al.*,1981). In general, it is supposed that this assumption does not introduce significant errors if the soil is essentially uniform (horizontally and vertically) up to a distance of approximately 3 to 5 times the diagonal dimension of the grid, measured from its edge (ANSI/IEEE,1986). However, parameters that are involved in the grounding design can change significantly as soil conductivity varies through the substation site. Therefore, it seems reasonable to seek for more accurate models that could take into account the variation of soil conductivity in the surroundings of the earthing system.

It must be obvious at this point that models describing all variations of soil conductivity in the surroundings of a substation would be unaffordable, from both technical and economical points of view. A more practical (and still quite realistic) approach to situations where conductivity is not markedly uniform with depth consists of considering the earth stratified in a number of horizontal layers, which appropriate thickness and apparent scalar conductivity must be experimentally obtained. In fact, it is widely accepted that two layer earth models should be sufficient to obtain good designs of earthing systems in most practical cases (Sverak *et al.*,1981).

When the grounding electrode is buried in the upper layer, the mathematical problem (1) can be reduced to the Neumann Exterior Problem:

$$\begin{array}{c} \Delta V_1 = 0 \quad \text{in} \quad E_1, \quad \Delta V_2 = 0 \quad \text{in} \quad E_2, \quad \dfrac{dV_1}{dn} = 0 \quad \text{in} \quad \Gamma_E, \quad V_1 = V_2 \quad \text{in} \quad \Gamma_L, \\ \gamma_1 \dfrac{dV_1}{dn} = \gamma_2 \dfrac{dV_2}{dn} \quad \text{in} \quad \Gamma_L, \quad V_1 = V_\Gamma \quad \text{in} \quad \Gamma, \quad V_1 \longrightarrow 0 \quad \text{and} \quad V_2 \longrightarrow 0 \quad \text{if} \quad |\boldsymbol{x}| \to \infty, \end{array} \tag{2}$$

where E_1 and E_2 are the upper and lower layers of the earth, Γ_L is the interface between them, γ_1 and γ_2 are the respective apparent scalar conductivities of both layers, and V_1 and V_2 are the respective expressions of the potential in each one of them (Tagg,1964; Aneiros,1996). Further development in this paper is restricted to the above case. Of course, analogous results can be easily obtained if the grounding system is buried in the lower layer of the earth (Aneiros,1996).

VARIATIONAL STATEMENT OF THE PROBLEM

In most of real electrical installations, the particular geometry of the grounding electrode —a grid of interconnected bare cylindrical conductors, horizontally buried and supplemented by a number of vertical rods, which ratio diameter/lenght uses to be relatively small ($\sim 10^{-3}$)— precludes the obtention of analytical solutions. On the other hand, the use of standard numerical techniques (such as Finite Differences or Finite Elements) requires the discretization of domains E_1 and E_2, and the obtention of sufficiently accurate results would imply an extremely high (out of range) computational effort. At this point, we remark that computation of potential is only required on Γ_E, and the equivalent resistance can be easily obtained in terms of the leakage current density $\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}$ on the grounding electrode surface Γ, being $\boldsymbol{n}$ the normal exterior unit field to Γ (Colominas,1995). Therefore, we turn our attention to a Boundary Integral approach, which will only require the discretization of Γ, and will reduce the 3D problem to a 2D one.

If one further assumes that the earth surface Γ_E and the interface between the two soil layers Γ_L are horizontal, the application of the method of images and some results of Potential

Theory to (2) allow to express the potential $V_1(\boldsymbol{x}_1)$ and $V_2(\boldsymbol{x}_2)$, at arbitrary points $\boldsymbol{x}_1$ in E_1 and $\boldsymbol{x}_2$ in E_2, in terms of the unknown leakage current density $\sigma(\boldsymbol{\xi})$, at any point $\boldsymbol{\xi} \equiv [\xi_x, \xi_y, \xi_z]$ on Γ, in the integral form:

$$V_1(\boldsymbol{x}_1) = \frac{1}{4\pi\gamma_1} \iint_{\boldsymbol{\xi}\in\Gamma} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma,\ \forall \boldsymbol{x}_1 \in E_1; \quad V_2(\boldsymbol{x}_2) = \frac{1}{4\pi\gamma_1} \iint_{\boldsymbol{\xi}\in\Gamma} k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma,\ \forall \boldsymbol{x}_2 \in E_2; \quad (3)$$

being the weakly singular kernels

$$\begin{aligned} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi}) = {} & \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, \xi_z])} + \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -\xi_z])} + \sum_{i=1}^{\infty} \Big[\frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH + \xi_z])} + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH - \xi_z])} \\ & + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH + \xi_z])} + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH - \xi_z])} \Big]; \\ k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi}) = {} & \frac{1+\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, \xi_z])} + \frac{1+\kappa}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, -\xi_z])} + \sum_{i=1}^{\infty} \Big[\frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2iH + \xi_z])} + \frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2, [\xi_x, \xi_y, 2iH - \xi_z])} \Big] \end{aligned} \quad (4)$$

where $r(\boldsymbol{x}, \boldsymbol{\xi})$ indicates the distance between points $\boldsymbol{x}$ and $\boldsymbol{\xi}$, H is the heigth of the upper soil layer and κ is a relation between the conductivities of both layers: $\kappa = (\gamma_1 - \gamma_2)/(\gamma_1 + \gamma_2)$ (Aneiros,1996).

Since the expression for potential $V_1(\boldsymbol{x}_1)$ in (3) holds on the earthing electrode surface Γ, the boundary condition $V_1(\boldsymbol{\chi}) = 1$, $\forall \boldsymbol{\chi} \in \Gamma$ leads to a Fredholm integral equation of the first kind on Γ with quasi-singular kernel $k_{11}(\boldsymbol{\chi}, \boldsymbol{\xi})$, which solution is the unknown leakage current density σ (Colominas,1995). Moreover, for all members $w(\boldsymbol{\chi})$ of a suitable class of test functions defined on Γ, this problem can be written in the weaker variational form:

$$\iint_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi}) \left(\frac{1}{4\pi\gamma_1} \iint_{\boldsymbol{\xi}\in\Gamma} k_{11}(\boldsymbol{\chi}, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi})\ d\Gamma - 1 \right) d\Gamma \ = \ 0. \quad (5)$$

Obviously, a Boundary Element formulation seems to be the right choice to solve variational statement (5). Thus, for given sets of 2D boundary elements and trial functions defined on Γ, both the leakage current density σ and the grounding electrode surface can be discretized (Colominas,1995). Now, for a given set of test functions defined on Γ, variational form (5) is reduced to a system of linear equations, which coefficients matrix is full. However, since computation of each term requires an extremely high number of evaluations of the kernel $k_{11}(\boldsymbol{\chi}, \boldsymbol{\xi})$ and double integration on a 2D domain (Aneiros,1996), it is necessary to introduce some additional simplifications in the BEM approach to decrease the computational cost (Colominas *et al.*,1996).

APPROXIMATED 1D BOUNDARY ELEMENT FORMULATION

Considering the real geometry of grounding systems in most of electrical substations, one can assume that the leakage current density is constant around the cross section of the cylindrical electrode (Navarrina *et al.*,1992). This hypothesis of circumferential uniformity is widely used in most of the practical methods related in the literature (ANSI/IEEE,1986).

Thus, let L be the whole set of axial lines of the buried conductors, $\widehat{\boldsymbol{\xi}}$ the orthogonal projection over the bar axis of a given generic point $\boldsymbol{\xi} \in \Gamma$, $\phi(\widehat{\boldsymbol{\xi}})$ the electrode diameter, and $\widehat{\sigma}(\widehat{\boldsymbol{\xi}})$ the approximated leakage current density at this point (assumed uniform around the cross section).

In these terms, and being $\bar{k}_{11}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ and $\bar{k}_{12}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ the average of kernels (4) around the cross section at $\widehat{\boldsymbol{\xi}}$, we can write expressions (3) as

$$\widehat{V}_1(\boldsymbol{x}_1) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}} \in L} \bar{k}_{11}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL, \forall \boldsymbol{x}_1 \in E_1; \quad \widehat{V}_2(\boldsymbol{x}_2) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}} \in L} \bar{k}_{12}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL, \forall \boldsymbol{x}_2 \in E_2. \qquad (6)$$

Now, since the leakage current is not really uniform around the cross section, variational statement (5) will not hold anymore. However, if we restrict the class of trial functions to those with circumferential uniformity, (5) results in

$$\frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}}) \left[\int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL \right] dL = \int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}})\, dL, \qquad (7)$$

which must hold for all members $\widehat{w}(\widehat{\boldsymbol{\chi}})$ of a suitable class of test functions defined on L, being $\bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ the average of kernel $k_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ in (4) around the cross sections at $\widehat{\boldsymbol{\chi}}$ and $\widehat{\boldsymbol{\xi}}$ (Colominas,1995).

Now, for given sets of 1D boundary elements and trial functions defined on L, the whole set of axial lines of the buried conductors L and the unknown leakage current density $\widehat{\sigma}$ can be discretized. Then, for a given set of test functions defined on L, variational form (7) is reduced to a system of linear equations (Colominas *et al.*,1996; Aneiros,1996). The matrix of coefficients of this approximated 1D problem is still full. However, on a regular basis we can say that the computational cost has been drastically reduced, since the actual discretization (1D) for a given problem will be much simpler than before (2D). Furthermore, suitable unexpensive approximations (Colominas,1995) can be introduced to evaluate the averaged kernels $\bar{k}_{11}(\widehat{x}, \widehat{\boldsymbol{\xi}})$, $\bar{k}_{12}(\widehat{x}, \widehat{\boldsymbol{\xi}})$ and $\bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$. Nevertheless, computation of remaining integrals is not obvious, and the cost of numerical integration is still out of range due to the ill-conditioning of integrands. Suitable arrangements in the final expressions of the matrix coefficients allow to use highly efficient analytical integration techniques that have been derived by the authors in cases of earthing systems in uniform soils (Colominas *et al.*,1996), in order to reduce the computational cost.

This BEM approach has been implemented in the CAD system for grounding grids of electrical installations developed by the authors in recent years (Casteleiro *et al.*,1994). It is important to notice that the total computing effort required in some cases is very high, particularly in those in which conductivities of soil layers are very different ($|\kappa| \approx 1$). The fact is that the rate of convergence of averaged kernels $\bar{k}_{11}(\cdot,\cdot)$, $\bar{k}_{12}(\cdot,\cdot)$ and $\bar{\bar{k}}_{11}(\cdot,\cdot)$ is very low when $|\kappa| \approx 1$, which makes necessary to compute an extremely large number of terms in order to obtain accurate results.

APPLICATION TO A REAL CASE

This formulation has been applied to a real case: the *E.R.Barberá* substation grounding (close to the city of Barcelona in Spain). The characteristics and numerical model are summarized in Table 1. The plan is presented in figure 1. Results (such as the equivalent resistance, the fault current and potential profiles along different lines) obtained with this BEM approach by using a two layer soil model are compared with those obtained by using an uniform soil model (figure 1 and table 2). It can be shown that results are noticeably different, and the design parameters of the earthing system computed from them may significantly vary. Therefore, it will be essential to analyze grounding systems with this new BEM technique, in cases in which the conductivity of the soil changes markedly with depth.

TWO LAYER SOIL MODEL		UNIFORM SOIL MODEL	
Upper Layer Resistivity :	200 Ω m	—	
Lower Layer Resistivity :	60 Ω m	—	
Height of Upper Layer :	1.2 m	Earth Resistivity :	60 Ω m
Fault Current :	25.88 kV	Fault Current :	31.85 kV
Equivalent Resistance :	0.386 Ω	Equivalent Resistance :	0.314 Ω
CPU Time (VAX 4300):	835.5 min.	CPU Time (VAX 4300):	7.5 min.

Table 2.—E. R. Barberá Substation: Characteristics and Results by using different soil models

CONCLUSIONS

A Boundary Element formulation for the analysis of substation grounding systems embedded in layered soils has been derived. This approach has been applied to the practical case of an earthing system in an equivalent two layer soil. Some reasonable assumptions allow to reduce a general 2D BEM approach to an approximated 1D version, according to specific characteristics of these installations in practice. By means of the scheme of analytical integration techniques that has been recently derived by the authors for the case of grounding systems in uniform soils, highly accurate results can be obtained in real problems. At present, the study of larger installations still requires an important computing effort due to the large number of terms of integral kernels that it is necessary to evaluate in order to obtain accurate results. However, the application of new techniques that are being derived by the authors, will allow to accelerate their rate of convergence and reduce the actual computational cost.

REFERENCES

Aneiros J.M. (1996): Una Formulación Numérica para Cálculo y Diseño de Tomas de Tierra de Subestaciones Eléctricas con Modelos de Terreno de Dos Capas. Research Report. E.T.S.I.C.C.P.

ANSI/IEEE Std.80 (1986): Guide for Safety in AC Substation Grounding. IEEE Inc., New York

Casteleiro M. *et al.* (1994): Memoria y Manual de Usuario del Sistema TOTBEM para Cálculo y Diseño Asistido por Ordenador de Tomas de Tierra de Instalaciones Eléctricas. E.T.S.I.C.C.P.

Colominas I. (1995): Cálculo y Diseño por Ordenador de Tomas de Tierra en Instalaciones Eléctricas: Una Formulación Numérica basada en el MEC. Ph.D.Thesis. E.T.S.I.C.C.P.

Colominas I., Navarrina F. and Casteleiro M. (1996): "A Boundary Element Numerical Approach for Earthing Grid Computation", Submitted to the Int. J. Num. Meth. Engng.

Durand E. (1966): Électrostatique. Masson Ed., Paris

Garret D.L. and Pruitt J.G. (1985): "Problems Encountered with the APM of Analyzing Substation Grounding Systems", IEEE Trans. P.A.S., Vol. 104, No. 12, 4006–4023

Heppe R.J. (1979): "Computation of potential at surface above an energized grid or other electrode, allowing for non-uniform current distribution", IEEE Trans. P.A.S., Vol. 98, No. 12, 1978–1988

Navarrina F., Colominas I. and Casteleiro M. (1992): "Analytical Integration Techniques for Earthing Grid Computation by BEM", Int. Co. Num. Meth. Engng. Ap. Sci., CIMNE Pub., 1197–1206

Sunde E.D. (1968): Earth conduction effects in transmission systems. McMillan Ed., New York

Sverak J.G. *et al.* (1981): "Safe Substations Grounding", IEEE Trans. P.A.S., Vol. 100, 4281–4290

Tagg G.F. (1964): Earth Resistances. Pitman Pub. Co., New York

DATA		1D BEM MODEL	
Number of Electrodes:	408	Type of Elements:	Linear
Electrode Diameter:	12.85 mm	Number of Elements:	408
Installation Depth:	0.8 m	Degrees of Freedom:	238
Max. Grid Dimensions:	145 m × 90 m		
Ground Potential Rise:	10 kV		

Table 1.—E. R. Barberá Substation: Characteristics and Numerical Model

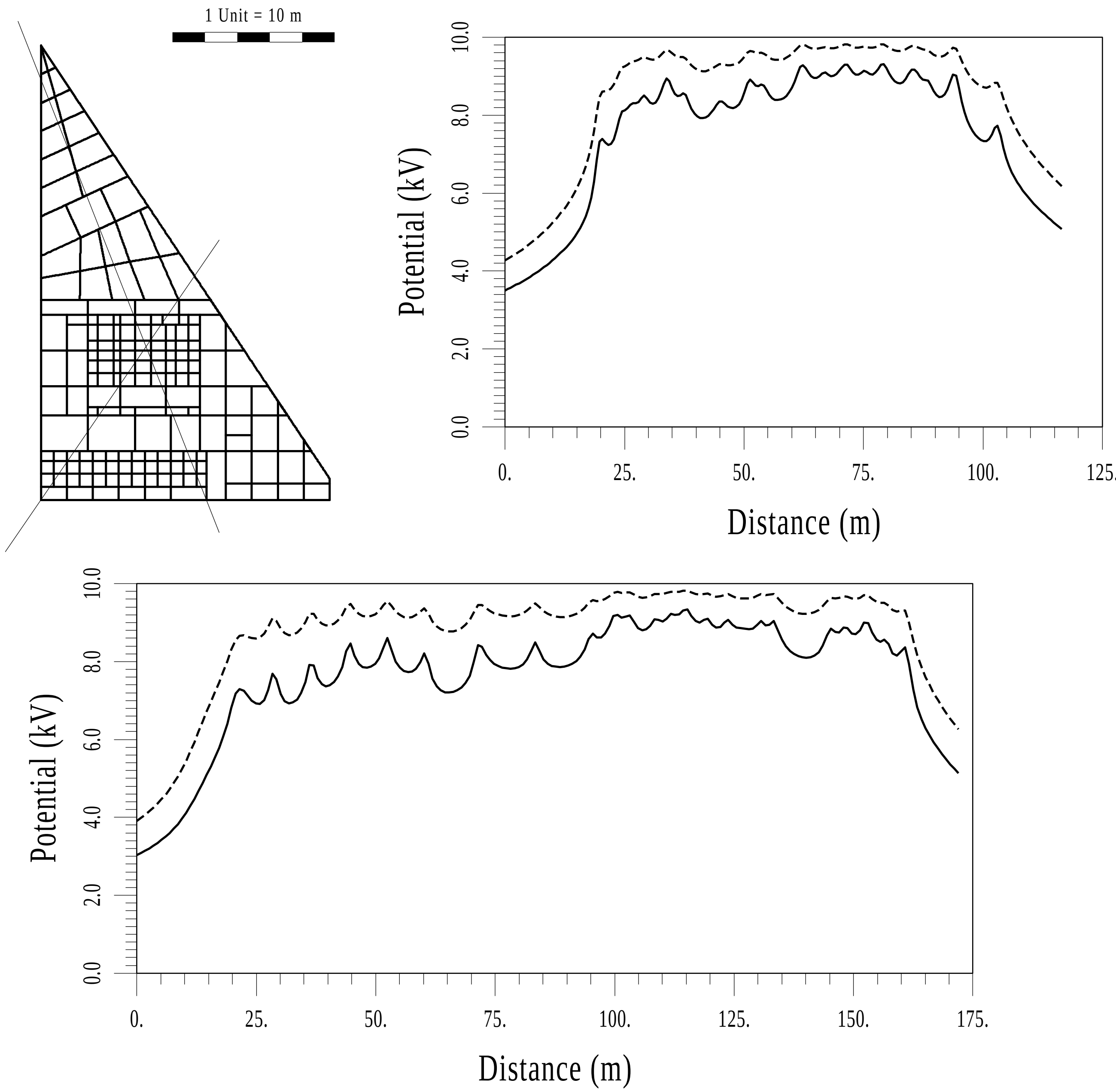

Fig. 1.—E. R. Barberá Substation: Plan of the Grounding Grid and Potential profiles along two different lines (results obtained by using an uniform soil model are indicated with discontinuous line, and those obtained by using a two layer model are given with continuous line).