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Decision Support

A simple model for mixing intuition and analysis

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A B S T R A C T

Firefighters, emergency paramedics, and airplane pilots are able to make correct judgments and choices in challenging situations of scarce information and time pressure. Experts often attribute such successes to intuition and report that they avoid analysis. Similarly, laypeople can effortlessly perform tasks that confuse machine algorithms. OR should ideally respect human intuition while supporting and improving it with analytical modelling. We utilize research on intuitive decision making from psychology to build a model of mixing intuition and analysis over a set of interrelated tasks, where the choice of intuition or analysis in one task affects the choice in other tasks. In this model, people may use any analytical method, such as multi-attribute utility, or a single-cue heuristic, such as availability or recognition. The article makes two contributions. First, we study the model and derive a necessary and sufficient condition for the optimality of using a positive proportion of intuition (i.e., for some tasks): Intuition is more frequently accurate than analysis to a larger extent than analysis is more frequently accurate than guessing. Second, we apply the model to synthetic data and also natural data from a forecasting competition for a Wimbledon tennis tournament and a King's Fund study on how patients choose a London hospital: The optimal proportion of intuition is estimated to range from 25% to 53%. The accuracy benefit of using the optimal mix over analysis alone is estimated between 3% and 27%. Such improvements would be impactful over large numbers of choices as in public health.

1. Introduction

Albert Einstein is quoted as saying that intuitive thinking is a sacred gift and rational analysis a faithful servant, and protesting that society honors the servant and forgets the gift. Firefighters, emergency paramedics, and airplane pilots are often able to make correct judgments and choices in challenging situations with scarce information and under extreme time pressure. In doing so, such experts cite their intuition as the source of success and report that they avoid analysis (Klein, 1998), as illustrated in the case of the "miracle on the Hudson".

On January 15, 2009, US Airways Flight 1549 captain Sullenberger and co-pilot Skiles had to decide whether their plane, with both of its engines disabled, could land safely back at La Guardia airport or they should attempt an emergency landing on the Hudson river. They went for the latter option, based on a thinking process articulated by Skiles (2009): "It's no so much a mathematical calculation as visual in that… a point you can't reach will actually rise in your windshield. A point that you are going to overfly will descend in your windshield". Applying this rule of thumb the plane landed safely and there were no casualties. As simulations showed subsequently, this was the right decision. Making the right decision here is enabled by people's innate capacity of visually tracking moving objects (Gigerenzer, 2022). In general, people can seemingly effortlessly perform tasks which confuse machine algorithms, such as recognizing visual patterns and inferring the hidden meanings of utterances (Gigerenzer, 2022).

Of course, if experts and laypeople always made great decisions based on their intuitions, there would not have been much need for the analyses offered by disciplines such as OR. But this is not the case as work in behavioural operational research (Kunc, Malpass, & White, 2016), and behavioural operations management (Donohue, Katok, & Leider, 2018) has demonstrated. For example, inventory controllers do not place orders that maximize the theoretically expected profit, even when instructed to do so and all variables needed for performing the calculation

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are provided (Schweitzer & Cachon, 2000). In fact, experienced procurement managers do not perform better than university students (Bolton, Ockenfels, & Thonemann, 2012). As another example, Kefalidou (2011) has reviewed experimental evidence which shows that people can solve vehicle routing problems and perform satisfactorily, but do not achieve the theoretical optimal.

Ideally, OR should respect human intuition while also supporting and improving it with analytical modelling. Liebowitz (2015) suggests that "practitioners use informed intuition, founded upon years in business and inspired by trends in big data, to navigate the future of business". This view echoes Aneesh Chopra's (Keiger, 2014)–the first chief technology officer in the US–who proposes that major decisions should be made by *marrying* gut feelings and careful data analysis. As Hämäläinen, Luoma, & Saarinen (2013) put it in their behavioural OR manifesto, OR tools and processes should be augmented by taking into account people's behaviour and cognition. To heed one of the founding fathers of OR, "analysis is necessary but not sufficient" (Koopman, 1977, p. 202).

To the best of our knowledge, there are yet no precise answers on how to integrate analysis and intuition. The present work aims to provide some answers by using mathematical modelling. We take the approach of building a simple model, aiming to derive clear and testable insights (Currie et al., 2020; Katsikopoulos, Durbach, & Stewart, 2018; Robinson, Worthington, Burgess, & Radnor, 2014; Tako, Tsioptsias, & Robinson, 2020). The model is rich, drawing heavily from theories of intuition in psychology (Klein, 1998; Gigerenzer, 2007; Kahneman, 2011), and we employ it to pose and answer new prescriptive questions in the realm of how to integrate intuition with analysis. We do not study the conditions under which intuition outperforms analysis, or vice versa, on a single task (for answers see Hogarth, 2001; Klein, 1998; Gigerenzer, 2007; Kahneman & Klein, 2009). Our model addresses decision making over a *set* of tasks.

To explain, a brief background is needed. Overarching frameworks of human decision making, such as the "adaptive decision maker" (Payne, Bettman, & Johnson, 1993) and the "adaptive toolbox" (Gigerenzer & Selten, 2002), show that, depending on the task, decision makers switch between intuitive and analytical modes of thinking. The intuitive mode is said to be fast and automatic, whereas the controlled mode is postulated to be slow and analytical (Evans (2008); Schneider & Shiffrin (1977)–more precise definitions of intuition are discussed in Section 2). People match intuitive or analytical methods to tasks so that they satisfy criteria such as accuracy, effort, and transparency, sometimes balancing these criteria (Payne et al., 1993), and other times achieving all of them at once (Katsikopoulos, Şimşek, Buckmann, & Gigerenzer, 2020).

Individuals, groups, and organizations often face a set of *interrelated* decision tasks, where the choice of intuition or analysis in one task affects the choice in other tasks. For example, in product development methods Pugh 1990, Katsikopoulos (2009), Saaty, 2014, decision makers would first compare product designs *A* and *B* and then would compare designs *B* and *C*. If the first comparison was made by analyzing both designs *A* and *B*, it is hard to see how this analysis can be forgotten and the assessment of *B* be made intuitively in the second comparison between *B* and *C*. The reverse process is more plausible, meaning that assessing intuitively *A* and *B* might not preclude analyzing both *B* and *C* at a later moment. As in the old saying "you never get a second chance to make a good first impression", the choice of intuition or analysis in one task affects the choice in other tasks. The same issues can be expected to arise when people are asked to make paired comparisons in other operational contexts, as in preference-based multi-objective optimization (Fowler et al., 2010), and more broadly in systems engineering (Clausing & Katsikopoulos, 2008). We investigate how such interrelated choices should be made in order to maximize, or at least improve, accuracy.

More specifically, in Section 2, we utilize research on intuition from psychology to build a model, which can be analyzed by OR tools, for mixing intuition and analysis over a set of interrelated decision tasks. We study the model and mathematically derive answers to questions such as: How does one theoretically determine in which tasks to use intuition and in which to use analysis, in order to maximize overall decision accuracy? Are there interesting conditions under which it is theoretically best to exclusively use one of analysis or intuition? In Section 3, we apply the model to synthetic and natural data and empirically answer questions such as: What are the estimated optimal proportions of using each of intuition and analysis? What is the estimated accuracy benefit of using the optimal mix over analysis or intuition alone? Section 4 concludes by discussing the contributions of this work, acknowledging its limitations, and considering future research and implementation challenges, as well as sketching responses to such challenges.

2. Theory: optimal mix of intuition and analysis

This section is structured as follows. First, we review research on intuitive decision making in psychology, with an eye towards employing this knowledge to formally model intuition. Then, we build two versions of a mathematical model for mixing intuition and analysis, and study those to derive necessary and sufficient conditions for a positive (conversely zero) proportion of intuition to be maximizing accuracy. We also relate this work to less-is-more effects in psychology and business.

2.1. Psychology knowledge for modelling intuition, and our modelling plan

There are three main views of intuition in cognitive psychology and judgment and decision making research (Klein, 2015). These views emanate from the *naturalistic decision making* paradigm (Klein, Orasanu, Calderwood, & Zsambok, 1993; Zsambok & Klein, 2014), the *heuristics-and-biases* program (Gilovich, Griffin, & Kahneman, 2002; Kahneman, Slovic, & Tversky, 1982), and the *fast-and-frugal-heuristics* program (Gigerenzer, Hertwig, & Pachur, 2011; Gigerenzer & Todd, 1999).

The naturalistic decision making paradigm has studied extensively the decision making of expert practitioners, as it occurs in the field/the wild, that is, outside the scientific laboratory. In this view, intuition is "an expression of experience as people build up patterns that enable them to rapidly size up situations and make rapid decisions without having to compare options" (Klein, 2015, p. 164). For example, experienced firefighters are able to swiftly recognize an effective course of action, mentally simulate its effects, and execute it, all without considering inferior alternatives (Klein & Calderwood, 1991).

The heuristics-and-biases program initially focused on the investigation of the decision making of laypeople in the laboratory, and later tested some of its findings in real-world arenas such as the financial market. Kahneman & Klein (2009, p. 519) jointly contrast the views of intuition in their respective research programs: "Intuitive judgments that arise from experience and manifest skill are the province of naturalistic decision making... In contrast, heuristics-and-biases researchers have been mainly concerned with intuitive judgments that arise from simplifying heuristics, not from specific experience. These intuitive judgments are less likely to be accurate and are prone to systematic biases".

A critique of both of these approaches to intuition, which is relevant to our modelling purposes here, is that they have not generated formal models of people's heuristics (Gigerenzer & Todd, 1999; Katsikopoulos et al., 2020). This issue has been addressed by the fast-and-frugal-heuristics program, which has developed mathematical and computer models of the heuristics of laypeople and experts, for broadly-construed decision tasks such as multi-attribute choice, classification, and forecasting (Katsikopoulos et al., 2018; Todd, 2007). These models include unit-weight linear regression as well as lexicographic heuristics such as deterministic elimination by aspects (Baucells, Carrasco, & Hogarth, 2008; Hogarth & Karelaia, 2005; Katsikopoulos, 2013) and fast-and-frugal decision trees (Luan, Schooler, & Gigerenzer, 2011; Martignon, Katsikopoulos, & Woike, 2008). Such models offer precise explications of how practitioners can make decisions rapidly and accurately as found in the naturalistic decision making paradigm, and also of how people's decisions can be inferior as found in the heuristics-and-biases program. In behavioural OR, these models have been proposed as prescriptive models of decision making, under some conditions (Keller & Katsikopoulos, 2016; Pande, Papamichail, & Kawalek, 2021).

Because intuition is *automatic* (Kahneman, 2011; Kruglanski & Gigerenzer, 2011), as candidates for models of intuition, one should consider those models of fast-and-frugal heuristics that are *especially* fast and frugal, so that their output would come to the human mind quickly and seemingly effortlessly. An obvious option is models that use only a single *cue*–the psychological term for attribute–Şimşek & Buckmann (2015); Hogarth & Karelaia (2005); Kahneman & Frederick (2002); Katsikopoulos, Şimşek, Buckmann, & Gigerenzer (2022); Slovic, Finucane, Peters, & MacGregor (2007). Interestingly, the idea of single-cue decision making is shared by the three views of intuition discussed here. The cue can be binary as the recognition or not of a decision option, see Klein & Calderwood (1991) and Goldstein & Gigerenzer (2002); or continuous as the availability of instances of an event in memory, see Tversky & Kahneman (1974) and Schooler & Hertwig (2005).

Based on the above knowledge, we can now outline our modelling plan. We will use a binary-cue heuristic to model intuition. Note that it makes little sense to model intuition as a single continuous variable. For example, consider the availability heuristic for judging an event's frequency, where availability is defined as the "ease by which instances or occurrences of the event can be brought to mind" (Tversky & Kahneman, 1974, p. 1127). Schooler & Hertwig (2005) proposed measuring "ease" by the speed of retrieval from memory, experimentally or via a cognitive architecture such as ACT-R. The issue is that such a continuous cue would always discriminate between two options (i.e., one option would have an, ever so slightly, smaller retrieval speed than the other), and analysis would never be used for choosing one of two options. This prediction is too extreme. And it would reduce the research problem to a comparison of the accuracy of a single cue with that of an analytical method when the two are not mixed, which has been studied elsewhere (Şimşek & Buckmann, 2015; Hogarth & Karelaia, 2005).

In what follows, we will consider a decision maker who has in their repertoire one single-cue heuristic and one analytical method (possible extensions to multiple heuristics and analytical methods will be outlined in Section 4, together with alternative approaches to modelling intuition). The model applies to any analytical method, and to two main cases of using single-cue heuristics to capture intuition. It will be clear how the model can be formulated for any single-cue heuristic. The decision maker employs a *method selection strategy* wherein s/he first attempts to use the heuristic, and only if this cannot lead to a choice, then s/he employs analysis. This strategy reflects the primacy of intuition over analysis, which is postulated in Kahneman (2011) System 1/System 2 metaphor.

To fix ideas, in the next Section 2.2 we assume that intuition is captured by the *recognition heuristic* (Goldstein & Gigerenzer, 2002). This heuristic is one of the most studied single-cue heuristics that are also formally specified. Other well-known heuristics, such as the affect heuristic (Slovic et al., 2007), can also be viewed as being based on a single cue (e.g., the affective evaluation of an option), but are not mathematically specified. The recognition heuristic holds that a recognized option is chosen over an unrecognized one. If both options are recognized, then presumably some attribute values of the options are known too, and the decision maker applies a method such as multi-attribute utility analysis. If options are not recognized, then presumably no attribute values are known, and an option is chosen by random guessing. This version of the model is applied to synthetic data and a natural dataset from a forecasting competition for a Wimbledon tennis tournament in Section 3.

Section 3.3 formulates the model for the more general case where intuition is captured by a single-cue heuristic where the cue is not special in the sense that recognition is. That is, the cue in the heuristic does not constrain the decision maker's other attribute information about the options. For example, if the decision maker is trying to choose between two medical treatments, and intuitively would go for the treatment after which most people reported a health improvement but cannot establish which treatment this is, other attribute values such as the treatment cost and the distance to the hospital can be known and be used. The mathematical treatment of this case is similar to the one for the recognition heuristic case–except that there is no need for guessing–and is presented more quickly. In Section 3, this version of the model is also applied to the same synthetic data, and to another natural dataset from a King's Fund study on choosing a London hospital.

2.2. Model version 1: intuition as recognition heuristic

The decision maker makes a choice for every pair of decision options sampled out of a population of N options. Let $\{1, 2, \ldots, N\}$ denote the options in decreasing order of "utility". That is, choosing i over j is *accurate* if and only if $i < j$. For each option $i = 1, 2, \ldots, N$, X_i is a binary variable denoting whether the decision maker recognizes the option ($X_i = 1$) or not ($X_i = 0$). Note that X_i can be viewed as a variable controlled by the decision maker, as when a consumer chooses to learn about products by attending to adverts or by actively seeking information. Alternatively, X_i can be viewed as exogenously determined. The corresponding choice vector is $X = (X_1, X_2, \ldots, X_N)$. We are interested in determining the value(s) of X that theoretically *optimize* (maximize) *accuracy*, which is the proportion of accurate choices. A decision maker who optimizes accuracy is called *optimal*.

For each pair of sampled options i and j and their associated realizations x_i and x_j, the heuristic states: "If $x_i = 1$ and $x_j = 0$, then choose i". The heuristic defines a partial order over the set of options as it does not apply to pairs with $x_i = x_j$. If $n = \sum_{i=1}^{N} x_i$, then the proportion of choices made intuitively equals $\frac{2n(N-n)}{N(N-1)}$, which can also be written as

$$P_I(\mathbf{x}) = \frac{2\sum_{i=1}^{N} x_i \sum_{i=1}^{N}(1 - x_i)}{N(N-1)},$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_N)$.

The accuracy of intuition is

$$\alpha(\mathbf{x}) = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i(1 - x_j)}{\sum_{i=1}^{N} x_i \sum_{i=1}^{N}(1 - x_i)},$$

because, if $i < j$, among all pairs with $\{x_i = 1 \text{ and } x_j = 0\}$ or $\{x_i = 0 \text{ and } x_j = 1\}$, the heuristic makes an accurate choice only in the former case.

The analytical method $\mathbf{M}$ is defined in a general way, by a complete order over the set of options. $\mathbf{M} = (M_{i,j})$ is an $N \times N$ matrix, and $M_{i,j}$ equals 1 when the analytical method chooses i over j and 0 otherwise. So, $M_{i,j} + M_{j,i} = 1$ for all $i \neq j$.

The analytical method $\mathbf{M}$ applies only to pairs with $x_i = x_j = 1$. Thus, the proportion of choices made analytically equals $\frac{n(n-1)}{N(N-1)}$, which can also be written as

$$P_M(\mathbf{x}) = \frac{\sum_{i=1}^{N} x_i \left(\sum_{i=1}^{N} x_i - 1\right)}{N(N-1)}$$

or as

$$\frac{2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i x_j}{N(N-1)}.$$

Based on a similar logic as in the expression for $\alpha(\mathbf{x})$, the accuracy of analysis equals

$$\beta(\mathbf{x}, \mathbf{M}) = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i x_j M_{i,j}}{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i x_j}.$$

If $\mathbf{x} = (1, 1, \dots, 1) = \mathbf{1}$, then $P_M(\mathbf{1}) = 1$ and the decision maker exclusively uses analysis. In contrast, it is not possible in our binary-cue model for the decision maker to exclusively use intuition because there does not exist an $\mathbf{x}$ such that $P_I(\mathbf{x}) = 1$; in fact, $P_I(\mathbf{x})$ is *capped* a bit under or over 0.5 (depending on whether N is even or odd; for very small N, the cap can be up to 0.67).

Finally, the decision maker has to guess in a proportion of choices $\frac{(N-n)(N-n-1)}{N(N-1)}$, which can be written as

$$P_G(\mathbf{x}) = \frac{\sum_{i=1}^{N}(1-x_i)\left(\sum_{i=1}^{N}(1-x_i) - 1\right)}{N(N-1)}$$

or as

$$\frac{2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)(1-x_j)}{N(N-1)}.$$

Both $P_M(\mathbf{x})$ and $P_G(\mathbf{x})$ can range from 0 to 1, subject to the constraint that $P_I(\mathbf{x}) + P_M(\mathbf{x}) + P_G(\mathbf{x}) = 1$. If the decision maker is viewed as deciding how to set their n, then s/he is effectively setting the mix of intuition, analysis, and guessing. The accuracy of guessing equals 1/2.

The decision maker's accuracy equals

$$f_1(\mathbf{x}, \mathbf{M}) = \alpha(\mathbf{x})P_I(\mathbf{x}) + \beta(\mathbf{x}, \mathbf{M})P_M(\mathbf{x}) + (1/2)P_G(\mathbf{x}). \tag{1}$$

which, based on the above, reduces to

$$f_1(\mathbf{x}, \mathbf{M}) = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left[2x_i(1-x_j) + 2x_i x_j M_{i,j} + (1-x_i)(1-x_j)\right]}{N(N-1)}. \tag{2}$$

Eq. (2) can be used to derive a necessary and sufficient condition for a positive proportion of intuition (i.e., for *some* choices between two options) to lead to optimal accuracy, as shown below.

A positive proportion of intuition is optimal if and only if there exists an $\mathbf{x} \neq \mathbf{1}$ such that, for an arbitrary but fixed $\mathbf{M}$,

$$f_1(\mathbf{x}, \mathbf{M}) > f_1(\mathbf{1}, \mathbf{M}). \tag{3}$$

Note that

$$f_1(\mathbf{1}, \mathbf{M}) = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} 2M_{i,j}}{N(N-1)}$$

and hence from Eq. (2), the condition (3) can be written as follows:

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left[2x_i(1-x_j) + 2x_i x_j M_{i,j} + (1-x_i)(1-x_j)\right] > \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} M_{i,j}.$$

By expanding this inequality using

$$x_i(1-x_j) = x_i(1-x_j)(M_{i,j} + M_{j,i})$$

in the left-hand side and

$$M_{i,j} = M_{i,j}[x_i x_j + x_i(1-x_j) + (1-x_i)x_j + (1-x_i)(1-x_j)]$$

in the right-hand side, and by rearranging terms, the following result is obtained.

Proposition 2.1. *A positive proportion of intuition is optimal if and only if there exists* $\mathbf{x} \neq \mathbf{1}$ *such that, for an arbitrary but fixed* $\mathbf{M}$,

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i(1-x_j)M_{j,i} - \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)x_j M_{i,j} > \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)(1-x_j)M_{i,j} - \frac{1}{2}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)(1-x_j).$$

As a numerical example, take $\mathbf{x} = (1, 1, \dots 1, 0)$ and $\mathbf{M}$ such that $M_{N,i} = 1$ for all $i < N$. The condition of Proposition 2.1 holds because $(N-1) - 0 > 0 - 0$. Additionally, from Eq. (2),

$$f_1(\mathbf{x}, \mathbf{M}) = \frac{2(N-1) + \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} 2M_{i,j}}{N(N-1)},$$

which is indeed larger than

$$\frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} 2M_{i,j}}{N(N-1)} = f_1(\mathbf{1}, \mathbf{M}).$$

It is important to note that the terms in Proposition 2.1 have clear interpretations:

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i(1-x_j)M_{j,i}$$

counts the number of pairs in which intuition is correct when intuition and analysis disagree;

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)x_j M_{i,j}$$

counts the number of pairs in which analysis is correct when intuition and analysis disagree;

$$\frac{1}{2}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)(1-x_j)$$

counts the number of pairs in which guessing applies and it is correct and

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)(1-x_j)M_{i,j}$$

counts the number of pairs in which guessing applies and analysis would have been correct (if analysis, rather than guessing, were applied, which is a counterfactual quantity given the method selection strategy assumed, but may be computed).

Putting all this together shows that, the reason for an optimal decision maker to switch from analysis to intuition is not because intuition is more accurate than analysis as one might expect at a first glance and has also been claimed (see the comments in the next paragraph on less-is-more effects). Rather, Proposition 2.1 says that the reason for it being optimal to use some intuition is that *intuition is more frequently accurate than analysis to a larger extent than analysis is more frequently accurate than guessing* (conditional on the pairs where the two methods, in each comparison, are applied). In this sense, for an optimal decision maker to use some intuition, it has to be the case that intuition picks up the *slack* created by guessing compared to analysis.

This result connects with the theory of *less-is-more effects* (Goldstein & Gigerenzer, 2002), which has been applied to forecasting (Pachur & Biele, 2007; Scheibehenne & Bröder, 2007), financial investment (Ortmann, Gigerenzer, Borges, & Goldstein, 2008), consumer choice (Hilbig, 2014), and marketing (Hauser, 2014). The differences are that (Goldstein & Gigerenzer, 2002) (i) modelled recognition by n and (ii) the accuracy of the heuristic and analysis, respectively α and β, were assumed constant. Our model for the recognition-heuristic case moves from the summary statistic n to the level of individual observations x_i and does not assume constant α and β, an assumption challenged theoretically (Egozcue, García, Katsikopoulos, & Smithson, 2017; Smithson, 2010) and empirically (Lee, 2015). The variable n is the amount of information the decision maker has. If $n = N$, the decision maker has maximum information. When maximum accuracy is obtained for $n^* < N$, a less-is-more effect occurs. A less-is-more effect maps to the optimality of a positive proportion of intuition. The necessary and sufficient condition for a less-is-more effect is $\alpha > \beta$ (Goldstein & Gigerenzer, 2002). This condition says that the reason for the less-is-more effect is that intuition is more accurate than analysis. Katsikopoulos (2010) labelled this the "accurate-heuristics explanation" and showed that it does not work when memory is imperfect (Pleskac, 2007); additionally, Smithson (2010) showed that the explanation fails if α and β are not constant, even if memory were perfect. A subtler explanation is needed, and this is provided by Proposition 2.1. The proposition implies Smithson (2010) result that the accurate-heuristics explanation is a necessary condition for the less-is-more effect if it is assumed that analysis is more accurate than guessing.

2.3. Model version 2: intuition as other (non-recognition) single-cue heuristics

Here we also use the set-up and notation of Section 2.2. But in this more general case of single-cue heuristics, if the single cue X used by intuition is such that $x_i = x_j = 0$ for two decision options i and j, other attribute values can be known, and thus analysis can be used. There is no other difference with version 1 of the model. Thus, Eq. (1) is now substituted by

$$f_2(\mathbf{x}, \mathbf{M}) = \alpha(\mathbf{x})P_I(\mathbf{x}) + \beta(\mathbf{x}, \mathbf{M})[P_M(\mathbf{x}) + P_G(\mathbf{x})], \tag{4}$$

which reduces to

$$f_2(\mathbf{x}, \mathbf{M}) = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} 2[x_i(1-x_j) + [x_i x_j + (1-x_i)(1-x_j)]M_{i,j}]}{N(N-1)}. \tag{5}$$

As in the recognition-heuristic case, the decision maker cannot exclusively use intuition. Unlike the recognition-heuristic case, now the decision maker exclusively uses analysis if $P_M(\mathbf{x}) + P_G(\mathbf{x}) = 1$, which occurs for $\mathbf{x} = (1, 1, \ldots, 1) = \mathbf{1}$ or $\mathbf{x} = (0, 0, \ldots, 0) = \mathbf{0}$. Accuracy is equal for these two vectors, that is

$$f_2(\mathbf{1}, \mathbf{M}) = f_2(\mathbf{0}, \mathbf{M}) = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} 2M_{i,j}}{N(N-1)}.$$

The necessary and sufficient condition for a positive proportion of intuition to be optimal is now that there exists an $\mathbf{x} \neq \mathbf{1}, \mathbf{0}$ such that, for an arbitrary but fixed $\mathbf{M}$,

$$f_2(\mathbf{x}, \mathbf{M}) > f_2(\mathbf{1}, \mathbf{M}) = f_2(\mathbf{0}, \mathbf{M}). \tag{6}$$

Using Eq. (5), and performing essentially the same (as in Section 2.2) algebra on the condition (6), the following result is derived.

Proposition 2.2. *A positive proportion of intuition is optimal if and only if there exists an* $\mathbf{x} \neq \mathbf{1}, \mathbf{0}$ *such that, for an arbitrary but fixed* $\mathbf{M}$,

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} x_i(1-x_j)M_{j,i} > \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(1-x_i)x_j M_{i,j}.$$

This result is very similar to the result in version 1 of the model for the recognition heuristic. The same numerical example works, and the terms in Proposition 2.2 can be interpreted in the same way as in Proposition 2.1. The only difference is that the condition for the optimality of a positive proportion of intuition is simpler for the more general case of other (non-recognition) single-cue heuristics capturing intuition. More specifically, Proposition 2.2 says that the reason for it being optimal to use some intuition is that *intuition is more frequently accurate than analysis* (conditional on the pairs where the two methods are applied).

In 2009 Daniel Kahneman was awarded an honorary doctorate from Erasmus University Rotterdam. In his introduction, decision theorist Peter Wakker referred to Kahneman's work as a *rational theory of irrationality*. Prospect theory (Kahneman & Tversky, 1979) is a mathematically simple revision of expected utility theory aiming to fit human decision making, and which has served as an inspiration for modelling behavioural operations (Donohue et al., 2018). The work presented in Section 2 is inspired by Gerd Gigerenzer and colleagues' work on modelling bounded rationality (Gigerenzer & Todd, 1999; Katsikopoulos et al., 2020). Paraphrasing Peter Wakker, our mathematically simple model can be said to be a first step towards an *analytical theory of intuition*. How might the model be utilized to improve human decision making? Section 3 provides answers.

3. Data: estimation of optimal proportion of intuition and its accuracy benefit

Propositions 2.1 and 2.2 establish the optimality of a positive proportion of intuition under some conditions but do not specify what the value of this optimal proportion actually is. What is the optimal $\mathbf{x}^*$ which determines the optimal value of $n^* = \sum_{i=1}^{N} x_i^*$, and in turn the optimal proportion of tasks (choices between pairs of options) $\frac{2n^*(N-n^*)}{N(N-1)}$ that intuition should be used on? And what is the accuracy benefit of using the optimal mix of intuition and analysis over analysis alone? We apply our model to different types of data and empirically estimate these quantities.

3.1. Synthetic data: intuition as recognition and as other single-cue heuristics

Synthetic data can provide exact answers because $\mathbf{x}$ and $\mathbf{M}$ are discrete parameters and their spaces can in principle be fully enumerated—there are 2^N possible $\mathbf{x}$ and $N!$ possible $\mathbf{M}$. For N from 3 to 9 we exhaustively enumerated all combinations of $\mathbf{x}$ and $\mathbf{M}$. For N from 10 to 15 we randomly sampled 10,000 combinations of $\mathbf{x}$ and $\mathbf{M}$. In all cases we considered only those $\mathbf{M}$ with accuracy greater or equal to 1/2. We repeated this analysis for both versions of the model.

For version 1 of the model, Table 1 displays, for all N from 3 to 9, the *prevalence* of an optimal positive proportion of intuition, which is the proportion of all $\mathbf{M}$ for each N such that $\mathbf{x}^* \neq \mathbf{1}$ (note that guessing is never optimal because it always holds that $\mathbf{x}^* \neq \mathbf{0}$). The table also includes the mean *optimal proportion* of intuition, measured by the mean proportion of tasks $\frac{2n^*(N-n^*)}{N(N-1)}$ that intuition should be used on as well as the range of this proportion, both calculated across all $\mathbf{M}$ for each N. The calculation of the mean includes cases where a zero proportion of intuition is optimal whereas the calculation of the range does not include such cases because if it did the range would have started from zero for all $N \neq 9$. Table 1 also provides the mean *accuracy benefit* of using the optimal mix of intuition and analysis over analysis alone, which is the value of $f_1(\mathbf{x}^*, \mathbf{M}) - f_1(\mathbf{1}, \mathbf{M})$ averaged across all $\mathbf{M}$ for each N. In all cases, ties in $\mathbf{x}^*$ and n^* were broken randomly. The results are given in percentage points.

Table 1
Synthetic data, with N from 3 to 9 and exhaustive enumeration of all combinations of $\mathbf{x}$ and $\mathbf{M}$ (with accuracy greater or equal to 1/2). For model version 1 (Section 2.2), we estimated statistical measures characterizing the proportion of intuition in the optimal mix of intuition and analysis, and the accuracy benefit of the mix over analysis alone.

Number of decision options (N)	Prevalence of an optimal positive proportion of intuition (%)	Range of optimal positive proportion of intuition (%)	Mean optimal proportion of intuition (%)	Mean accuracy benefit of optimal mix over analysis alone (%)
3	67	67–67	67	25
4	78	50–67	58	19
5	92	40–60	55	19
6	97	33–60	53	21
7	99	28–57	52	21
8	99	25–57	51	21
9	100	22–55	51	21

Table 2
Same as in Table 1, but for model version 2 (Section 2.3).

Number of decision options (N)	Prevalence of an optimal positive proportion of intuition (%)	Range of optimal positive proportion of intuition (%)	Mean optimal proportion of intuition (%)	Mean accuracy benefit of optimal mix over analysis alone (%)
3	67	67–67	67	33
4	89	50–67	57	25
5	98	40–60	52	25
6	100	33–60	51	27
7	100	28–57	51	27
8	100	25–57	50	26
9	100	22–55	50	26

Table 3
Same as in Table 1, but with N from 10 to 15, and estimates being calculated by using 10,000 randomly sampled combinations of $\mathbf{x}$ and $\mathbf{M}$.

Number of decision options (N)	Prevalence of an optimal positive proportion of intuition (%)	Range of optimal positive proportion of intuition (%)	Mean optimal proportion of intuition (%)	Mean accuracy benefit of optimal mix over analysis alone (%)
10	100	20–55	52	22
11	100	18–54	51	22
12	100	16–54	51	22
13	100	15–53	51	22
14	100	14–53	51	22
15	100	13–53	51	22

Table 1 shows that it is from very likely to certain that the optimal mix of intuition and analysis includes some intuition, especially for N from 5 to 9. The optimal proportion of intuition can have a considerable range but it includes its maximum possible value (e.g., 67% for $N = 3$). Consistently, the mean optimal proportion of tasks where intuition should be used is more than 50% for all N. The mean accuracy benefit of including intuition optimally over using analysis alone is estimated as typically more than 20%.

The same analysis was repeated for version 2 of the model, and Table 2 displays the results. The range of the optimal proportion of intuition and its mean are almost the same as those in Table 1. Here analysis has taken the place of random guessing and the mean accuracy benefit of including intuition optimally over using analysis alone is, for all N, at least 5% higher than in Table 1, rising as high as 33%.

Tables 3 and 4 display, for all N from 10 to 15, the estimates for the same measures as in Tables 1 and 2, respectively for the two model versions. The only difference is that the estimates in Tables 3 and 4 were calculated by using 10,000 randomly sampled combinations of $\mathbf{x}$ and $\mathbf{M}$ (with accuracy greater or equal to 1/2) for each N. The results are similar to those in Tables 1 and 2. In Table 3, the optimal mix should always include some intuition. The mean optimal proportion of intuition is close to the maximum possible value for each N, which is a bit over 50%. The mean accuracy benefit of including intuition optimally over using analysis alone is 22% for all N. In Table 4, the optimal mix should also always include some intuition. The mean optimal proportion of intuition is slightly smaller than in Table 3, at 50% for all N. The mean accuracy benefit of including intuition optimally over using analysis alone is higher by 5% from the benefit in Table 3.

Our synthetic data simulations implicitly assume that all $\mathbf{x}$ and $\mathbf{M}$ are equally likely, which cannot be expected to hold in all real decision problems. To address this issue, we also run simulations using natural data. We used two different datasets, one in which intuition is captured by the recognition heuristic (Section 3.2), and another in which intuition is captured by more general single-cue heuristics (Section 3.3).

3.2. Natural data: intuition as recognition heuristic

We used data from a forecasting competition for a tennis tournament. Scheibehenne & Bröder (2007) polled 93 tennis amateurs—enthusiasts who, for example, played regularly in local clubs—and 117 laypeople about whether or not they recognized each one of the $N = 128$ male players who took part in the 2005 Wimbledon singles competition. The probability that each player is recognized is estimated by the proportion of Scheibehenne & Bröder (2007) participants who recognized the player (see supplement S1). We estimated three recognition probability distributions, one from the 93 tennis amateurs, one from the 117 laypeople, and

Table 4
Same as in Table 2, but with N from 10 to 15, and estimates being calculated by using 10,000 randomly sampled combinations of **x** and **M**.

Number of decision options (N)	Prevalence of an optimal positive proportion of intuition (%)	Range of optimal positive proportion of intuition (%)	Mean optimal proportion of intuition (%)	Mean accuracy benefit of optimal mix over analysis alone (%)
10	100	20–55	50	27
11	100	18–54	50	27
12	100	16–54	50	27
13	100	15–55	50	27
14	100	14–55	50	27
15	100	13–55	50	27

one from the combined group of 210 participants. In each of three simulations, 1000 different **x** were realized by randomly sampling the corresponding recognition distribution 1000 times (values of 50 to 500 samples were also tested and results differed by 1–2%) and this process was repeated for 500 runs. In all simulations, there was one **M**, the Champions-Race ranking produced by the Association for Tennis Professionals ahead of the 2005 competition (see supplement S1). This was the analytical method in Scheibehenne & Bröder (2007) that achieved the highest forecasting accuracy of 70%.

As it is often the case when "hard" OR models are applied to the messy real world (Mingers, 2011), applying our model required some extra resourcefulness here (the application of the model in the King's Fund data, as we shall see in Section 3.3, was more straightforward). More specifically, the model needs a complete "true" order of all options, but Wimbledon tournaments do not produce an official ranking of all competing players. The finalists are officially ranked—in 2005, Roger Federer was first and Andy Roddick second—but not the other players. We produced an order (see supplement S1) based on the results of the matches played (see https://www.wimbledon.com/en_GB/draws_archive/index.html), by employing a *lexicographic rule*, as detailed below. Lexicographic rules are routinely used to rank competitors in sports events such as the World Cups in soccer or basketball, using attributes such as the number of rounds passed, matches won, and goals scored (Bennis, Katsikopoulos, Goldstein, Dieckmann, & Berg, 2012). We adapted this approach here and used the following rule:

(i) Players who reached a round closer to the final are ranked higher than the players who reached rounds further away from the final (e.g., all players who reached a semi-final are ranked higher than all players who reached a quarter-final);
(ii) Players who reached the same round are ranked according to the number of sets they won in this round;
(iii) Ties are broken according to the number of games won in this round;
(iv) Remaining ties are broken according to the number of sets won by the opponent in the previous round;
(v) Remaining ties are broken according to the number of games won by the opponent in the previous round;
(vi) Remaining ties are broken according to the following rankings: first Champions-Race, then Wimbledon official seeds, and finally the Entry ranking of the Association for Tennis Professionals (for details see Scheibehenne & Bröder, 2007).

For all three groups (tennis amateurs, laypeople, and combined), the prevalence of an optimal positive proportion of intuition is 100%. Table 5 displays the mean optimal proportion of tasks $\frac{2n^*(N-n^*)}{N(N-1)}$ that intuition should be used on as well as the range of this proportion across the 500 runs for each group. The table also includes, for each group, the mean accuracy of the optimal mix of intuition and analysis, and of analysis alone. We additionally computed the accuracy of a wisdom-of-crowds (Surowiecki, 2005) heuristic for each group, proposed by Scheibehenne & Bröder (2007) which ranks players by the probability that the player is recognized in the group (see supplement S1). We do not replicate the accuracies computed by Scheibehenne & Bröder (2007) because those referred only to the matches played, not all possible matches as is the case here. Note that the wisdom-of-crowd heuristic is distinct from the recognition heuristic that enters the optimal mix of analysis and intuition because the former uses probabilities and the latter uses samples.

Table 5 shows that that the optimal mix of intuition and analysis includes a very high amount of intuition for the tennis amateurs, with a mean of 49% (maximum possible value for $N = 128$ is 50%), as was the case for synthetic data in the recognition-heuristic case (Tables 1 and 3). For the laypeople, the mean optimal proportion of intuition is about half of that of the amateurs, 22%. The optimal mix's accuracy is also highest for the tennis amateurs, at 64%. This yields a benefit of 5% over the 59% accuracy of using analysis alone. The wisdom-of-crowds heuristics performed competitively with the theoretically optimal mix, lagging by 2% on the average, and the heuristics also performed slightly better than analysis, 60% versus 59% on the average.

These results are consistent with those obtained in the study by Scheibehenne & Bröder (2007), where there was an accuracy of 70% for analysis and 68% for heuristics in the amateurs' group, 67% in the laypeople group, and 70% in the combined group. Note that our values are lower than Scheibehenne & Bröder (2007). This might be so because we used the complete lexicographic order of all players, not the partial order produced by the matches played. The lexicographic order of all players is influenced by early upsets such as No. 4 seed Rafael Nadal's loss in the second round to unseeded Gilles Müller.

3.3. Natural data: intuition as other (non-recognition) single-cue heuristics

For the application of version 2 of the model where intuition is captured by single-cue heuristics other than the recognition heuristic (Section 2.3), we used data from a King's Fund study on how patients choose a high-quality hospital (Boyce, Dixon, Fasolo, & Reutskaja, 2010; Fasolo, Reutskaja, Dixon, & Boyce, 2010). This study aimed at informing the UK government's efforts to support patients make better use of information for choosing a hospital for a serious, non-urgent operation, such as a knee or cataract operation. To do so, the researchers investigated hospital choices based on an easier-to-understand version of the information available at the *NHS Choices* website (Boyce et al., 2010). Briefly, 5 real, anonymized London hospitals, and their actual values on 9 attributes, were presented to 744 participants in an online experiment, in order to study hospital choice processes under different behavioural interventions (Boyce et al., 2010). The "true" order of the 5 hospitals was provided by a group of NHS experts (Boyce et al., 2010). The 9 attributes were those that were suggested as most important for hospital choice by 44 participants in a series

Table 5
Natural data, with $N = 128$, one analysis method **M** (Champions-Race ranking), and 1000 **x** sampled in each of 500 runs, utilizing data from a forecasting competition (Scheibehenne & Bröder, 2007) for the 2005 Wimbledon men's singles tournament. For model version 1 (Section 2.2), we estimated statistical measures characterizing the amount of intuition in the optimal mix of intuition and analysis and its accuracy benefit over using analysis alone and wisdom-of-crowds heuristics.

Group	Range of optimal positive proportion of intuition (%)	Mean optimal proportion of intuition (%)	Mean accuracy of optimal mix (%)	Mean accuracy of analysis (%)	Mean accuracy of wisdom- of-crowds heuristic (%)
Tennis amateurs	44–50	49	64	59	60
Laypeople	13–29	22	59	59	58
Combined	29–45	38	61	59	60

Table 6
Natural data, with $N = 5$, all analysis methods **M** with accuracy greater or equal to $1/2$, and 7 intuition vectors **x** based on data from a King's Fund study on hospital choice (Boyce et al., 2010; Fasolo et al., 2010). For model version 2 (Section 2.3), we estimated, across **M**, statistical measures characterizing the amount of intuition in the optimal mix of intuition and analysis and its accuracy benefit over using analysis alone. The bottom row provides the grand means of prevalence, optimal proportion of intuition, and accuracy benefit across **x**, **M**, and the selection probability distribution in the right-most column.

Intuition vector **x**	Prevalence of an optimal positive proportion of intuition (%)	Mean optimal proportion of intuition (%)	Mean accuracy benefit of optimal mix over analysis alone(%)	Probability of selecting intuition vector **x**
(1,1,0,0,0)	89	53	20	0.18
(1,0,0,0,0)	80	53	13	0.12
(1,0,1,0,0)	73	44	12	0.16
(0,1,0,0,0)	59	24	6	0.10
(1,0,0,0,1)	15	9	1	0.18
(0,1,0,1,0)	15	9	1	0.10
(0,0,0,0,1)	0	0	0	0.16
	48	25	8	

of focus group meetings (Fasolo et al., 2010), and they were the following (in decreasing order of the number of times that each attribute was mentioned, in the focus group, as an important consideration for choosing a hospital):

(i) Number of MRSA blood infections for patients (MRSA is a group of bacteria responsible for difficult-to-treat infections);
(ii) Standardized mortality rate at the hospital;
(iii) Score of cleanliness of treatment areas;
(iv) Number of people developing a wound infection after an operation;
(v) Risk of having to return to the hospital urgently within one month after an operation;
(vi) Number of people who reported an improvement in their health after treated at the hospital;
(vii) Score of dignity and respect shown to patients;
(viii) Score of patient involvement in decision making about treatment;
(ix) Distance of hospital from patient's home.

These 9 attributes were numerically valued in the King's Fund study. To apply the model, we binarized the attributes (see supplement S2) by using a median split over the values of each attribute on the 5 hospitals. A value of 1 is preferable to a value of 0. Each attribute can be employed as a single-cue heuristic, and leads directly to the intuitive choice of a hospital, or necessitates the use of analysis as per the model (Section 2.3). For example, if a patient employs the would-infection attribute (iv), then $\mathbf{x} = (1, 1, 0, 0, 0)$, and intuition is used for 6 pairs ($i = 1, 2$ crossed with $j = 3, 4, 5$) and analysis for 4 pairs ($i = 1, j = 2; i = 3, j = 4; i = 3, j = 5; i = 4, j = 5$). We estimated the *probability* that a patient would select a particular attribute (see supplement S2) as the proportion of times that this attribute was mentioned in the focus group as an important consideration for choosing a hospital (in the initial stage of the focus group meetings, see Fasolo et al., 2010).

Selecting each one of the 9 possible attributes leads to 9 possible **x**, of which 7 are distinct (see supplement S2 and Table 6). Each **x** corresponds to a positive proportion of intuition since all **x** are different from **1** and **0**. We refer to these 7 distinct **x** as the *intuition vectors*. For all possible analysis methods **M** (with accuracy greater or equal to $1/2$), we compared the accuracy of using the mix of intuition and analysis associated with each of the intuition vectors with the accuracy based on **M** alone. If an intuition vector led to greater or equal accuracy than an **M**, this comparison was counted as an instance of an optimal positive proportion of intuition; otherwise it was counted as an instance of optimal zero proportion of intuition.

Table 6 displays, for all intuition vectors **x**, the prevalence of an optimal positive proportion of intuition, which, for each **x** is the proportion of all **M** such that $f_2(\mathbf{x}, \mathbf{M}) > f_2(\mathbf{1}, \mathbf{M})$; if so, then $\mathbf{x}^* = \mathbf{x}$, and otherwise $\mathbf{x}^* = \mathbf{1}$. The table also includes, again for each intuition vector **x**, the mean optimal proportion of intuition as measured by the mean proportion of tasks $\frac{2n^*(N-n^*)}{N(N-1)}$, where $n^* = \sum_{i=1}^{N} x_i^*$, that intuition should be used on. Table 6 also provides the mean absolute accuracy benefit of using the optimal mix of intuition and analysis over analysis alone, which is the value of $f_2(\mathbf{x}^*, \mathbf{M}) - f_2(\mathbf{1}, \mathbf{M})$, averaged across all **M** for each intuition vector **x**. All of these values are given in percentage points. The table also includes, for each intuition vector, the probability that the vector would be selected by a patient. The bottom row of Table 6 provides the *grand means* with respect to this probability distribution, for the prevalence of optimal positive proportion of intuition, the optimal proportion of intuition, and the accuracy benefit of the optimal mix over analysis alone.

Table 6 shows that the performance of intuition varies greatly across the intuition vectors. The prevalence of an optimal positive proportion of intuition ranges from 0% to 89% and the mean optimal proportion of intuition from 0% to 53%. That is, some of the attributes suggested by patients in the King's Fund study are not very accurate single-cue heuristics, as the MRSA attribute which has accuracy 50% and the cleanliness attribute which has accuracy 0% (see supplement S2 for all cue accuracies). Consistently, the grand mean of prevalence is about half of the values in the other simulations (Tables 1–5). On the other hand, because some of the suggested attributes are perfectly accurate single-cue heuristics, as the wound-infection and the return-risk attributes which both have accuracy 100%, the grand mean of the optimal propor-

Table 7
Summary of results in the four empirical studies (see text for details).

Simulation study and its key assumptions	Prevalence of optimal positive proportion of intuition (%)	Optimal proportion of intuition (%)	Accuracy benefit of optimal mix over analysis alone (%)
Synthetic, uniformly distr. data; intuition is recognition heuristic	95	53	21
Synthetic, uniformly distr. data; intuition is not recognition heuristic	95	52	27
Wimbledon (natural) data; true order had to be estimated	100	36	3
King's Fund (natural) data; few intuition vectors tested	48	25	8

tion of intuition equals 25%, which is comparable to the values in other simulations (Tables 4 and 5). In fact, the grand mean accuracy benefit of the optimal mix over analysis alone, 8%, is higher than in the Wimbledon study (3%).

This relatively high benefit of the optimal mix might decrease if the analysis methods were more accurate. To illustrate this, we computed the benefit over two analysis methods suggested in the multi-attribute choice literature when attribute weights have not been elicited (Baucells et al., 2008; Hogarth & Karelaia, 2005; Katsikopoulos, 2013), as in the King's Fund study. The methods are the lexicographic rule (see also Section 3.2) where attributes are ordered by their accuracy (ties are broken by selection probability), and a linear utility model with unit attribute weights (ties are broken randomly). Their respective accuracies are 90% and 70%, and the mean accuracy benefit of the optimal mix is 3%. Table 7 summarizes the main results of the two synthetic and the two natural empirical studies. All values are grand means computed across the variables $N, \mathbf{M}, \mathbf{x}$ (in the Wimbledon study, results are averaged across the amateur and laypeople groups). We also include key, and in some cases limiting, assumptions of the studies.

4. Discussion

4.1. Contributions

Throughout the life of OR, it has been pointed out that formal models of optimization do not necessarily mesh with human decision making (Ackoff, 1979; Kimball, 1958; Mingers, 2011). As mentioned earlier, management students and procurement professionals place inventory orders that deviate from analytically derived orders; as another example, practitioners adjust the output of forecasting software (Goodwin, Moritz, & Siemsen, 2018), and so on as the fields of behavioural operations demonstrate (Donohue et al., 2018; Kunc et al., 2016). Why do such discrepancies occur? First, people's intuitions might just be systematically inferior to analysis. A second possibility is that people rightly distrust that a model can capture well a particular decision making situation, as when the probability distribution of demand is assumed to be known in inventory control models (Ward, Chapman, & Klein, 1991).

In some situations, both factors can play a role, and then it would be reasonable to rely on both intuition and analysis (Syntetos, Kholidasari, & Naim, 2016). Ward et al. (1991) strongly argue that inventory control is such a situation. These authors try to make the standard OR model of the newsvendor problem more intuitive to decision makers by replacing the demand distribution with a small number of concrete scenarios, and by employing a process for working through the computations with the decision makers. Similar approaches have been followed in other contexts where there are both harms and benefits to using human intuition, as when solving traveling salesperson problems (Kefalidou, 2011). For instance, Kefalidou (2017) explores how human interaction with a visual computer interface can help improve upon intuitive solutions to capacitated vehicle routing tasks.

Approaches such as the above aim at integrating intuition and analysis by making the modelling more palatable to decision makers, and in some sense also less formal. The present article takes a complementary approach. It presents the first, to our knowledge, mathematical model of mixing intuition and analysis. A novelty of the present approach is that we considered in depth how knowledge from the psychology of intuitive decision making can be modeled. The models were analyzed and led to closed-form expressions, and then they were applied computationally to laboratory and field data. The results suggest that including intuition optimally in a mix can provide substantial benefits over using analysis alone.

In sum, the present approach combines concepts from behavioural science with standard OR methodologies. Of course, as the first stage in an innovative program, there are limitations to our approach, which in turn introduce challenges for future work.

4.2. Limitations and challenges

We opted for developing a simple model, in order to get clear and testable first insights (Katsikopoulos et al., 2018; Tako et al., 2020). Now, to the extent that human intuition might be a complex phenomenon (Sinclair, 2011), playing a complex part in decision making, more complex models of integrating intuition and analysis might be needed down the line. This hypothesis seems to resonate with Koopman's views (1977), who, unlike Klein (2015) and Gigerenzer (2007), sees intuition as transcending pattern recognition and gut feeling, even though it has to be noted that Koopman primarily considered mathematical intuition. In this subsection, we discuss building further, possibly more complex, models for meeting research and implementation challenges yet unmet by our approach. Research challenges refer to developing rich theoretical models, and implementation challenges refer to applying these models usefully in processes of decision analysis and support in the field.

More complex models could employ rich formal structures for capturing intuitive decision making, including neural networks (Glöckner & Betsch, 2008) and cognitive architectures (Marewski & Mehlhorn, 2011), also taking into account group and organizational contexts (Hoffrage & Marewski, 2015). A possibility is to train neural networks by generating big (synthetic) data from psychological models of human decision making, and building ensembles of such networks and other machine learning models (Bourgin, Peterson, Reichman, Russell, & Griffiths, 2019). This route is worth exploring, but care should be taken to ensure transparency–for example, many psychological models are in fact black boxes and thus may hinder discussion in sensitive contexts such as law, health, and wealth (Katsikopoulos, 2022), and additionally, ensemble models can confuse practitioners (Lessmann, Baesens, Seow, & Thomas, 2015).

More specific research challenges relate to the assumptions that the toolbox of the decision maker consists of one heuristic, one analytical method, and one method-selection strategy. These assumptions often do not hold, neither for relatively straightforward decisions (Gigerenzer & Selten, 2002; Katsikopoulos et al., 2020; Payne et al., 1993), nor for more involved ones, with multiple attributes, criteria, or objectives (Durbach & Stewart, 2012; Fowler et al., 2010;

French, Maule, & Papamichail, 2009). The set-up of the interrelated decision tasks might also be made richer, including sequential or strategic aspects. In general, there is a need of scaling up the mathematical model of mixing intuition and analysis. This would also allow testing the robustness of the empirical findings (Table 7).

Consider next challenges of implementation. Even though the present model may be considered simple and transparent by academic standards, this definitely does not need to be the case from the point of view of practitioners and the decision analysts or facilitators who work with them (Franco & Montibeller, 2010). This is crucial because if people do not understand a model, even if it is meant to celebrate and support their intuition, they are likely to resist or ignore it. Furthermore, experts might feel threatened by attempts to model their intuition formally; for some professionals, a mystique surrounding their processes and tools is a valuable asset that they could choose to keep inaccessible, at least to a good extent (Goodwin, 2022). On the other hand, there have been projects where researchers were able to work together with practitioners and develop successful and intuitive, and at the same time formal, models of fast-and-frugal heuristics, in areas such as predicting impending heart attacks, decreasing civilian casualties in peace keeping operations, forecasting the incidence of influenza, or financially regulating investment banks (Katsikopoulos et al., 2020).

In other words, considering implementation, making models more complex might make the interaction with practitioners more challenging. Thus, a trade-off might exist between meeting both research and implementation challenges with regards to choosing the right level of model complexity.

Ultimately, integrative models such as the one presented here might be more acceptable and also effective in situations where analysts and practitioners agree that conditions for intuitive expertise are met partially, but not fully. Such conditions include opportunities to learn the statistical regularities of the decision environment (Hogarth, 2001; Todd & Gigerenzer, 2011), and metacognitive abilities of people to know what they know and what they do not know (Kahneman & Klein, 2009).

In conclusion, we can return to Albert Einstein and Aneesh Chopra. The marriage of intuition and analysis might at times be trying, but it can be a great one, where partners help each other improve and are mutually valued. In pursuing such aims, interdisciplinary formal modelling can help and provide good counsel.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.03.005.

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