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DIMENSIONING INFLUENCE ON THE FINAL CRITICAL STAGE OF DOUBLE-LAYER SPACE TRUSSES

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SUMMARY

The present paper suggests the application of a non-linear model of analysis to study the final critical stage of double-layer space trusses. A description of the analysis techniques is proposed to take into account the resistant and geometric non-linearity. The model developed has been applied to the study of the repercussion that the dimensioning of the members has on the stress-strain curves of the structure, as well as in the evaluation of its bearing capability when the final limit stage is reached. In order to exemplify this, various solved cases are enclosed, corresponding to different typologies of trusses and dimensions for which different dimensioning criteria have been used. Furthermore, a new method is proposed for the dimensioning of space structures, starting from the described technique of non-linear analysis.

1. INTRODUCTION

It's evident that the application of computer technology in the calculation and dimensioning of space structures of bars has originated a substantial variation in the number and quantity of bars that shape a given structure, thus making less predictable its behaviour when reaching the critical stage of stress. Not only is it less predictable, but also more denotative, when there is a clear tendency to a standardization in types of bars, usually by zones, a design criteria associated with simplified methods of calculation for the assimilation of the structure which advances in computer technology have made obsolete. The latest significant achievements confirm this tendency, especially from the time that systems CAD-CAM have begun to flourish which allow, with guaranteed success, such a number of types of bars within a structure.

In this situation, tending to optimized structures with strict dimensioning it is worthwhile to consider their behaviour upon nearing the final limit stage and what effect the criteria of dimensioning of bars may have.

Clearly, the formulation of analysis for situations at critical stage necessitates abandoning the linear model as a valid instrument for structural analysis of space trusses in situations of fatigue, and requires adopting a non-linear model of analysis.

2. NON-LINEAR MODEL OF ANALYSIS

The non-linear model of analysis must allow us to consider simultaneously:

* **Resistant non-linearity,** which include geometrical, as well as mechanical non-linear behaviour of the bars that make up the space structure.

* **Geometrical non-linearity of the structure,** resulting from the fact that, as we near the critical stages, the displacement of the knots, even in double layer trusses, is not insignificant and, therefore, it is necessary to formulate the balance over the deformed geometry of the structure.

2.1 UNIAXIAL CONSTITUTIVES LAWS

The first step in dealing with the problem of non-linear analysis is to obtain the constitutive equations which control the behaviour of the bars giving shape to structure. The study has been proposed in the case of thin-walled circular hollow sections which, as such, represent the most idoneous typology for these construction and practically dominate the choice of structures built.

The study of buckling in bars with a certain degree of imperfections F_i, assuming its representation in an initial curvature, allow us to obtain **the stability functions** for the **elastic buckling** (1), as well as **anelastic buckling** by applying in this case either the tangential (2) or the reduced modulus theory (3).

$$\varepsilon = \frac{\sigma}{E} + \frac{1}{4} \cdot \left[\frac{\pi}{F_i \cdot (1 - \bar{\sigma} \cdot \bar{\lambda}^2)} \right]^2 \tag{1}$$

$$\varepsilon = \frac{\sigma}{E \cdot \left[1 - \left(\frac{\bar{\sigma} - \bar{\sigma}_p}{1 - \bar{\sigma}_p} \right)^2 \right]} + \frac{1}{4} \cdot \left[\frac{\pi}{F_i \cdot \left[1 - \frac{\bar{\sigma} \cdot \bar{\lambda}^2}{1 - \left(\frac{\bar{\sigma} - \bar{\sigma}_p}{1 - \bar{\sigma}_p} \right)^2} \right]} \right]^2 \tag{2}$$

$$\varepsilon = \frac{\sigma}{E \cdot \left[1 - K \cdot \left(\frac{\bar{\sigma} - \bar{\sigma}_p}{1 - \bar{\sigma}_p} \right)^2 \right]} + \frac{1}{4} \cdot \left[\frac{\pi}{F_i \cdot \left[1 - \frac{\bar{\sigma} \cdot \bar{\lambda}^2}{1 - K \cdot \left(\frac{\bar{\sigma} - \bar{\sigma}_p}{1 - \bar{\sigma}_p} \right)^2} \right]} \right]^2 \tag{3}$$

with

$$K = 1 - \frac{1}{\pi} \cdot \left[\beta - \frac{3 \cdot \sin 2\beta}{2 \cdot (1 + 2 \cdot \cos^2\beta)} \right] \tag{4}$$

where ß define the position of the neutral axis and is determined by the intersection of the curves given by the equations (5) and (6).

$$\overline{N} = \overline{\sigma}_p - \frac{(1-\overline{\sigma}_p)^2}{2.K.\overline{\sigma}_{euler}} + \sqrt{\left(\overline{\sigma}_p - \frac{(1-\overline{\sigma}_p)^2}{2.K.\overline{\sigma}_{euler}}\right)^2 - \overline{\sigma}_p + \frac{(1-\overline{\sigma}_p)^2}{K}} \tag{5}$$

$$\overline{N} = \left[\overline{\sigma}_p + (1-\overline{\sigma}_p) . \sqrt{\frac{\pi}{tg\beta - \beta + \pi}} \right] \tag{6}$$

In order to analyze the collapse of the space structure, it is of utmost importance to reproduce **the behaviour of the bars in post-buckling** so as to evaluate its contribution to the load-bearing capacity of the structure.

The integration of the differential equation for the elastic, obtained form the exact value of the curvature, allows us to obtain the expression for the shortening through bending, resulting from buckling , subject to angle α which forms the tangent to the deformation in the end joints with the rectilinear axis of the bar (7).

$$\varepsilon_M = 2 . \left[1 - \frac{\int_0^{\frac{\pi}{2}} \sqrt{1 - p^2 . \sin^2\Omega} . d\Omega}{\int_0^{\frac{\pi}{2}} \frac{d\Omega}{\sqrt{1 - p^2 . \sin^2\Omega}}} \right] \tag{7}$$

The value of angle α is obtained by formulation the balance for the transversal section corresponding to the midpoint of the bar, assuming that the section is totally plasticized, which allow us to determinate the value of the maximum deflection which, due to the balance of the bar, is equal to the value of the maximum deflection of the elastic (8).

$$\frac{p}{\int_0^{\frac{\pi}{2}} \frac{d\Omega}{\sqrt{1 - p^2 . \sin^2\Omega}}} = \frac{2 . \sqrt{2}}{\lambda . \pi . \sigma} . \cos\left(\overline{\sigma} . \frac{\pi}{2} \right) \tag{8}$$

In this way we may determine the shortening due to the bending produced by buckling which, when added to the axial shortening, by applying the tangential modulus (9) or the reduced modulus (10), leads us to the total unit shortening.

$$\varepsilon_A = \frac{\sigma_F}{E} . \left[\overline{N} . \frac{(\overline{N} - \overline{\sigma}_p)^2}{(1 - \overline{\sigma}_p)^2 - (\overline{N} - \overline{\sigma}_p)^2} + \overline{\sigma} \right] \tag{9}$$

$$\varepsilon_A = \frac{\sigma_F}{E} \cdot \left[\overline{N} \cdot \frac{(\overline{N} - \overline{\sigma}_p)^2 \cdot K}{(1 - \overline{\sigma}_p)^2 - (\overline{N} - \overline{\sigma}_p)^2 \cdot K} + \overline{\sigma} \right] \qquad (10)$$

The above equations permit us to represent, depending on the slenderness of the bar, the type of steel use, and the degree of imperfections considered, the constitutive law of the members of space trusses. The results obtained show a substantial concordance with the available experimental ones and with those obtained through other techniques of analysis.

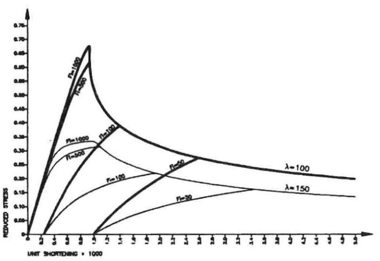

Fig.1.- CONSTITUTIVE CURVES COMPRESSED BARS INFLUENCE OF IMPERFECTION PARAMETER
Slendernesses 100 and 150. Steel A42
Imperfection L/1000,L/500,L/100,L/50

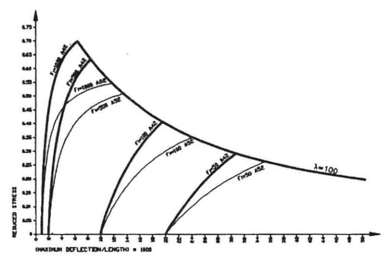

Fig.2.- INFLUENCE OF THE STEEL USED AND IMPERFECTION PARAMETER
Slenderness 100 Steels A42 A52
Imperfections L/1000,L/500,L/100,L/50

The usual non-linear analysis techniques, from the standpoint of idealizing the behaviour of the bars, have focused mainly on the elimination of the bars which reach a critical stage, either of tension (elastic limit stress) or compression (critical buckling stress) and in the balance, in a new cycle of calculation, of forces that result in eliminating the critically compressed bars. While this analysis technique overly simplifies the problem, since it views buckling as a sudden and rough phenomenon with a rapid and total loss of the load-bearing capacity of the affected bars, which neither corresponds to the experimental results nor, as we have seen, to the constitutive laws developed and, evidently, does not consider the collaboration of such bars in the behaviour of the truss. For this reason, idealizing the bars at critical stage has been resorted to as an analysis technique, modifying the modulus of elasticity in accordance with the different sections in which the constitutive law of each bar has been idealized. This technique, although somewhat recent, is by no means new, and has already shown its possibilities in earlier works by other authors, albeit in very simple structures.

The idealization of the constitutive law has been resolved with a total of seven sections which attempt to reproduce as precise as possible the bahavioural characteristics (Fig.3):

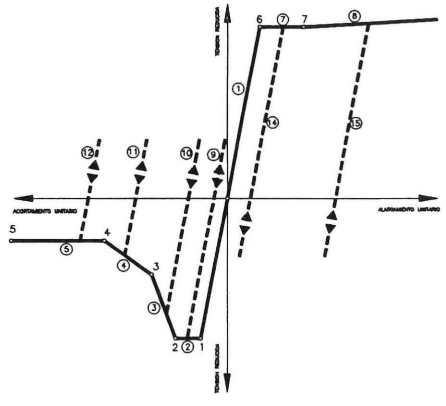

Fig.3.-IDEALIZATION OF THE UNIAXIAL CONSTITUTIVE LAWS

* linear section (1).

* plastic plateaus in areas of tension and compression (7 y 2).

* hardening section through deformation in tension (8).

* sections of unloading in post-buckling (3 y 4).

* remaining or residual load in post-buckling (5).

Likewise, within each section, the corresponding segments of elastic recovery have been predicted (9 a 12,14 y 15). These segments are essential when what is attempted is to carry the study of structural behaviour to stages beyond the final phase of fatigue, since the necessary unloading of the structure causes frequent force inversions and, therefore, elastic recoveries of bars that were already in non-elastic areas of behaviour within their corresponding constitutive laws.

Non-linear analysis war applied to the study of diverse space structures with differing dimensioning criteria. The process was rapid in trusses in which only a few types of bars were used since, generally, the critical area of the truss was situated in very specific, predictable areas and affected very few bars. In those cases in which the dimensioning of the bars was strict, the pattern of behaviour was considerably different so that the number of bars affected was much greater, thus producing a very considerable lengthening of calculation cycles and often entered infinite loops where the critical stage of certain bars and the consequent modification of the rigidity matrix of the structure produced the elastic recovery of bars proceeding from former critical stages and the new assembling of the matrix led us back to the initial problem.

After successive verifications in several structures what we agreed to call **"CRITICAL BANDS"** was finally decided upon. These will include all those bars which, in a calculation iteration, reach critical stage with variations in the total load value of the structure equal to less than a prefixed value (in the accompanying examples 1Kg./m^2. was considered). This allows, instead of only one bar in each iteration reaching a critical stage (the first reaches a significant point within its idealized constitutive law), all the bars which form the "critical band" corresponding to that iteration reach said critical stage simultaneously. This technique brought with it the advantages of elimination infinite loops and reducing the number of calculation iterations.

2.2 CONSIDERATION OF THE DEFORMED GEOMETRY IN FORMULATING THE BALANCE

Frequently, in double layer structures, a strict formulation is not chosen of the non-linear geometry based on an iterative incremental algorithm through small grades of load within which we would have to formulate the balance over the deformed geometry of the structure and balance the forces that would be unbalances in that iteration. This simplification is reasonable when one considers that, given the great rigidity of the double-layer truss, the complexity of the analysis is not justified by the repercussion that this phenomenon has on the results.

Nonetheless, when we analyze the structure up to a critical situation, the arrival at critical stage of successive critical bands causes a not insignificant lessening of rigidity of the unit and a greater repercussion each time of the importance of considering this non-linear geometrical phenomenon. Therefore, after continued analysis, a mixed solution has been chosen, beginning with a non-linear analysis, keeping in mind the deformed geometry in the formulation of the balance, not by loading grades, but rather by starting from the first critical situation of the structure (first critical band), in which the reduction of unit rigidity is produced. This mixed technique allows us to lessen considerably the number of calculation cycles. This, together with the reduction already obtained upon applying the model of critical bands leads, in the end, to considerable time saving in calculation, without any loss of precision in the structural analysis.

The application of the model, both in flat structures, as well as those of simple or double curvature, allows a complete description of the maximum deflection-critical load curves. Three example of these curves follow.

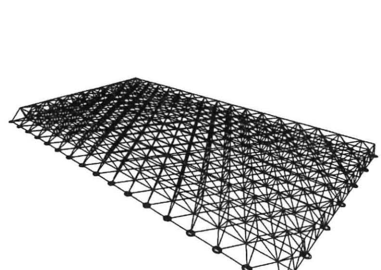

Fig.4.- FLAT TETRAEDRIC SPACE TRUSS
$S=1039$ m^2. $Q=150$ Kg./m^2. $P=12.59$ Kg./m^2.

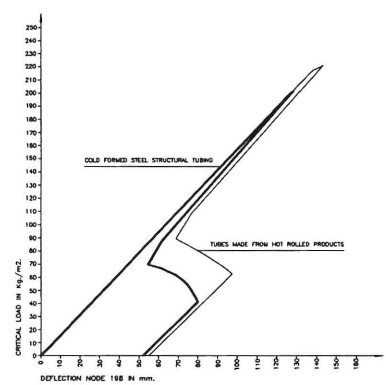

Fig.5.- MAXIMUM DEFLECTION-CRITICAL LOAD
FLAT TETRAEDRIC SPACE TRUSS

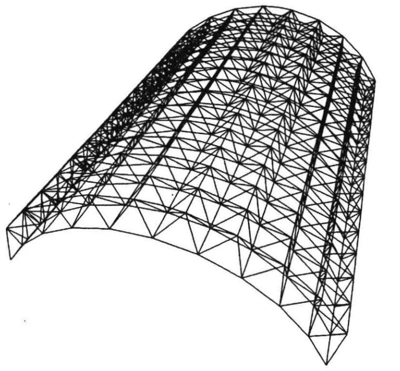

Fig.6.- CYLINDRICAL SPACE TRUSS OF SEMIOCTAHEDRONS
S=1000 m². Q=150 Kg./m². P=13.51 Kg./m².

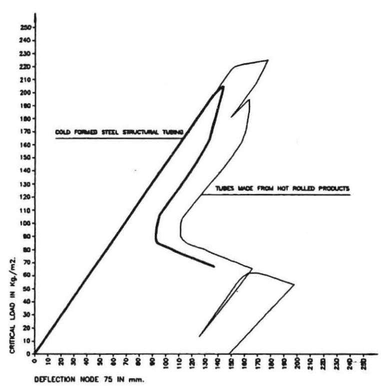

Fig.7.- MAXIMUM DEFLECTION-CRITICAL LOAD CYLINDRICAL SPACE TRUSS

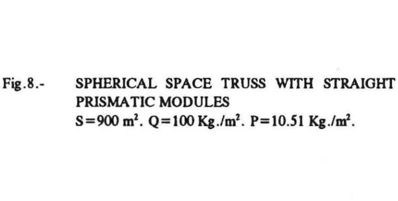

Fig.8.- SPHERICAL SPACE TRUSS WITH STRAIGHT PRISMATIC MODULES
S=900 m². Q=100 Kg./m². P=10.51 Kg./m².

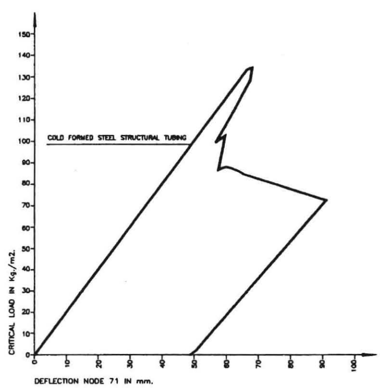

Fig.9.- MAXIMUM DEFLECTION-CRITICAL LOAD SPHERICAL SPACE TRUSS

The formulation of non-linear analysis described allow us, moreover, to evaluate the extraordinary influence which the dimensioning criteria has, both on the deformation-load curves, as well as on the bearing capacity of the structure upon reaching the last critical stage and since the use of uniform dimensioning, besides leading us to significantly lower performance, does not provide a single advantage from the structural point of view. Likewise, it cannot be said to provide a safety reserve in case of an increase in the load initially predicted in design.

To support this argument, the results corresponding to two trusses analyzed are included. En Fig.10 the curves corresponding to a **flat truss with straight prismatic modules** of the following characteristics:

***Truss 30*30m². hold up by 8 supports. Modules 2.5*2.5m. Thickness 2m. Load=150Kg./m².**

*Curve 1.-	Strict dimensioning			Weight = 10.55 Kg/m².
*Curve 2.-	Upper chord 65.3	Lower chord 65.2	Diagonals 50.2	Weight = 10.71 Kg/m².
*Curve 3.-	Upper chord 100.3	Lower chord 125.5	Diagonals 100.4	Weight = 37.72 Kg/m².
*Curve 4.-	Upper chord 80.3	Lower chord 100.3	Diagonals 100.3	Weight = 25.41 Kg/m².

Fig.11 represents the curves in the case of a **flat tetraedrical truss**:

***Truss 30*30m². hold up by 8 supports. Modules 2.5*2.5m. Thickness 2m. Load=150Kg./m².**

*Curve 1.-	Strict dimensioning			Weight = 12.23 Kg/m².
*Curve 2.-	Upper chord 125.5	Lower chord 125.6	Diagonals 200.5	Weight = 77.81 Kg/m².
*Curve 3.-	Idem curve 2 with the first critical memeber 200.5			Weight = 77.85 Kg/m².
*Curve 4.-	Upper chord 125.4	Lower chord 100.6	Diagonals 175.5	Weight = 64.55 Kg/m².
	First critical member 200.5			
*Curve 5.-	Idem curve 4 with the second and the third critical members 200.8			Weight = 64.65 Kg/m².
	Diagonals in supports zones 125.5			Weight = 22.93 Kg/m².

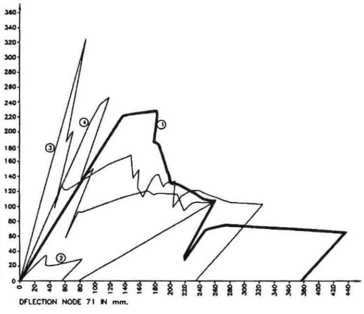

Fig.10.- MAXIMUM DEFLECTION-CRITICAL LOAD
INFLUENCE OF DIMENSIONING OF BARS
FLAT TRUSS OF PRISMATIC MODULES

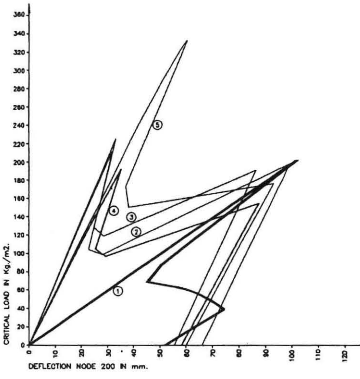

Fig.11.- MAXIMUM DEFLECTION-CRITICAL LOAD
INFLUENCE OF THE DIMENSIONING OF BARS
FLAT TETRAEDRIC TRUSS

Finally, Fig. 12 shows the comparison between the curve corresponding to a strict dimensioning (1) and that obtained when starting with said dimensioning and subsequently adding on the section of the successive bars which enter critical situation (2). In spite of having considered only the first six critical bands, a noticeable improvement in performance is accomplished, which creates interesting future expectations related do the use of non-linear analysis as a procedure in the optimization of the dimensioning of the structure, in order to achieve minimum weight. Likewise, the curve (3) has been graphed when the section of the first 4 critical bars has been reduced. This causes a remarkably lower performance, which supports the validity of the proposed model in the study of structural performance with regards to dimensioning of the bars in a truss.

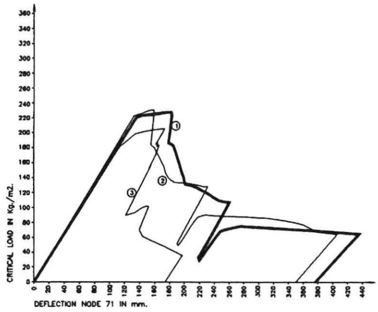

Fig.12.- OPTIMIZATION OF DIMENSIONING FLAT TRUSS OF PRISMATIC MODULES

3.- CONCLUSIONS

The non-linear model of analysis described here has been shown to be an extremely useful instrument for discovering the entire background of the behaviour of a space structure with bars up to the moment it reaches its critical state of fatigue, allowing us to continue the analysis beyond this, to later stages. In addition, the technique described has demonstrated its possibilities for the study of the effects that the dimensioning of bars has on the loading capacity of the structures and constitutes the basis for the definition of an algorithm of optimization of the dimensioning of this type of structure.

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APPENDIX. NOTATION

a_0 . . . . maximum deflection of a bar with an initial curvature.
E . . . . elasticity modulus of a material.
$F_i=L/a_0$ . . . imperfection factor.
K . . . . nondimensional coefficient.
L . . . . length of a bar.
N . . . . coefficient of reduction to buckling.
α . . . . angle which forms the tangent of the elastic on the end joints with the rectilinear axis of the bar.
ß . . . . angle defining the position of the neutral axis of a section.
ϵ_A . . . . unitarian shortening produced by the axial.
ϵ_M . . . . unitarian shortening produced by bending through buckling.
\ . . . . geometrical slenderness.
σ . . . . reduced stress = σ/σ_f.
σ_P . . . . critical stress of proportionality.
σ_{euler} . . . . . Euler's critical stress.