ON THE MULTIPLICITY OF SOLUTIONS OF TAYLOR LINEAR SOURCES GENERATING SYMMETRICAL POWER PATTERNS WITH FILLED NULLS

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ABSTRACT

As expressed in earlier works, if filled nulls are required in a power pattern generated by a Taylor line source, their corresponding roots must be complex. This leads to a multiplicity of solutions emerged from the fact that the power pattern keeps unaltered if the signs of the imaginary part of the roots are changed. In view of this attribute, the selection of the most favourable roots set –in terms of variability of amplitude excitation distribution, for example– is allowed. It is shown in this paper that, if the pattern is symmetric, a further consideration, never reported so far, can increase the number of available solutions.

1. INTRODUCTION

It is well known that antenna designers are allowed to control the sidelobe topography of sum patterns generated by line sources through the proper choice of the roots of Taylor distributions [1-2]. That control can be extended to every single side lobe, even for asymmetric topographies [3]. If patterns with filled nulls –as, for example, shaped beams– are needed, they can be generated by making to correspond every filled null with its appropriate complex root in the Taylor's formula [4]. The space factor that summarizes all the mentioned cases is expressed as:

$$F(u) = \frac{\sin \pi u}{\pi u} \left[\frac{\prod_{n=1}^{\bar{n}_L - 1} \left(1 + \frac{u}{z_{n,L}}\right)}{\prod_{n=1}^{\bar{n}_L - 1} \left(1 + \frac{u}{n}\right)} \right] \left[\frac{\prod_{n=1}^{\bar{n}_R - 1} \left(1 - \frac{u}{z_{n,R}}\right)}{\prod_{n=1}^{\bar{n}_R - 1} \left(1 - \frac{u}{n}\right)} \right] \quad \text{with} \begin{cases} z_{n,L} = (u_{n,L}, v_{n,L}) \\ z_{n,R} = (u_{n,R}, v_{n,R}) \end{cases} \tag{1}$$

where u = $(2a/\lambda)\cos\theta$, with θ measured from the endfire of the antenna of length 2a. The subscripts L and R identify the parameters on the left and on the right sides of the pattern whose $\bar{n}_L + \bar{n}_R - 2$ sidelobes are controlled by the proper choice of the real part of the $z_{n,L}$ and $z_{n,R}$ complex roots[1]. The imaginary parts $v_{n,L}$ and $v_{n,R}$ can control the level of every filled null. A remarkable property of eq. (1) is that, once established a set of roots $\{z_{n,L}, z_{n,R}\}$, the power pattern so obtained will keep unchanged if any of the values that belongs to the set are replaced by its conjugate complex. Nevertheless, this change of sign in any of the imaginary parts of the

[1] The $u_{n,L}$ and $u_{n,R}$ indicate the positions of the zeros of the function F(u), but it is perfectly known that these values also determine a specific sidelobe topography.

roots will alter both the phase of the space factor –mostly being of no account in antenna design, which constitutes the main advantage of the method–, and the continuous aperture distribution, whose expression is given by:

$$g(\xi)=\frac{1}{2a}\sum_{m=-(\bar{n}_L-1)}^{(\bar{n}_R-1)} F(m)\, e^{jm\pi\xi/a} \tag{2}$$

Here, $\xi \in [-a,a]$ is the variable along the line source. A subtle examination of such a roots signs property reveals, on the one side, the number of solutions (emerged from the choices of signs combinations) that lead to the same power pattern, and, on the other side, the main characteristics of $g(\xi)$. Moreover, if the power pattern is symmetric, the number of available solutions increases, depending this on how the roots are grouped. This symmetry in addition to further necessary conditions are analysed in next section.

2. ANALYSIS

We initiate our discussion by establishing $\bar{n}_L=\bar{n}_R=\bar{n}$ in (1). Let us call $\mathcal{Z}$ = {z_n} = {(u_n, v_n)}, (with $n\in[1,\bar{n}-1]$), the set of roots obtained by a proper synthesis technique (see, for example, [4]) that, when replaced into both products of (1) –called, from here on, and without any risk of confusion, L and R brackets–, produces a certain symmetrical power pattern. The choice of such symmetry will be justified in paragraph [2.2]. v_n is different from zero only if a filled null is needed at the angular position determined by u_n. Let $M \le \bar{n}-1$ be the number of such filled nulls, counted on one side of the pattern. The possibility of changing the signs of the roots provides 2^{2M} distinct solutions, concerning aperture distributions (including those cases in which one distribution turns out to be a mirror image of any other). The analysis reported up to now in previous papers [4] is revealed in item [2.1], whereas the two remainders describe the unreported ones, but inspired by an analogous work [5]:

[2.1] The replacement of $\mathcal{Z}$ into L and R brackets implies $u_{n,L} = u_{n,R} = u_n$, and $\pm v_{n,L} = \pm v_{n,R} = \pm v_n$, and determines 2^M different solutions, provided by the choice of $+v_n$ or $-v_n$ is available whenever a certain null is filled at u_n. The so obtained aperture distributions are complex and symmetric, and the designer is free to select any of them by applying a specific criterion, such as minimal variability of amplitude distribution, reduced edge brightening, maximum absolute value of slope, etc.

[2.2] The substitution of $\mathcal{Z}$ into R and $\mathcal{Z}^*$ into L (the asterisk indicates conjugate complex), determines, again, 2^M solutions, provided the combination of signs is still available, but in this case the v_n on the left and on the right sides are compelled to be opposite. This means $u_{n,L} = u_{n,R} = u_n$, and $\pm v_{n,L} = \mp v_{n,R} = \mp v_n$, giving, as a result, a $g(\xi)$ real-valued (with a phase aperture distribution piecewise constant, oscillating between 0 or 180º) and asymmetric. This possibility, not mentioned in the earlier literature, arises –as in [2.3]– due to the symmetry of the power pattern, and provided by the roots conditions. As in the previous –and next– points, the designer is free to choose the most convenient aperture distribution.

[2.3] The remainder cases offer 2^{2M}-2×2^{M} = 2^{2M}-2^{M+1} new solutions, all of them representing complex, but this time asymmetric, aperture distributions. No further restrictions are imposed to the v_{nL} and v_{nR}, except that, obviously, they must not fulfil the abovementioned requirements.

3. EXAMPLE

By applying Ares-Elliott method [4] we synthesized, as an example given to show the scope of the method, a flat topped beam generated by an antenna of a = 5λ, setting M = $\bar{n}-1$ = 8, establishing four sidelobes at 0 dB (normalized power), and all the remainders under control at –20 dB. To provide the coverage zone, two nulls on either side of the central lobe were raised to –1 dB (which implies ±0.5 dB of ripple), whereas the remainders were kept to –25 dB. The pattern so obtained, shown in figure 1 (in which are listed the roots of the set Z), provides 2^{2x8} = 65536 different cases. From those, we show six solutions, as regards these (arbitrarily chosen) criterions: in the case of complex aperture distributions (symmetric [2.1] and asymmetric [2.2]), we calculated the dynamic range ratio ($\delta = |g_{max}|/|g_{min}|$) and, in the case of real aperture distributions ([2.2]) we computed the maximum slope of the amplitude –being the slope the absolute quantity of the numerical derivative of the normalized[2] g(ξ), and calculated within the range [-a,a]–. Figure 2 shows the better (at the top) and worst solutions of the set. As can be seen, the minimum (1.91) δ correspond to the complex symmetric (CS) solution, whereas the maximum correspond to the complex asymmetric (CA) one (1901.00). The slope of the real asymmetric (RA) cases is bounded between 0.72 and 1.65. Plots of phase distributions were omitted for the sake of brevity.

4. FINAL REMARKS

By exploiting the characteristics of symmetrical power patterns generated by Taylor line sources it is possible to obtain 2^{2M} different solutions if M filled nulls are needed on either side of the main lobe. Previous works reported the 2^{M} cases that correspond to symmetric complex aperture distributions, which are useful, by proper sampling, for corporate-fed arrays. In this work we show that 2^{2M}-2^{M} further asymmetric solutions can be obtained (2^{M} corresponding to real apertures, the remainders leading to complex ones), mainly useful in end-fed arrays. Symmetric real apertures are available only if each of the complex roots of F(u) are accompanied by its complex conjugate, as seen in [6], but those roots sets do not match the ones include here. Besides, the main beam of the power pattern obtained in that manner is considerably broadened, if the same number of filled nulls is required.

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[2] The aperture distribution g was normalized to its maximum within the range [-a,a].

ACKNOWLEDGEMENTS

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CAPTIONS OF FIGURES

FIGURE 1. Symmetric flat topped beam obtained by controlling $\bar{n}-1 = 8$ roots on either side of the main lobe. Note that the last sidelobes were not controlled (as well known, up to eighths lobes the topography will decay as a uniform aperture distribution).

FIGURE 2. Top: Amplitude distributions that correspond to the most favourable solutions that generate the power pattern given in figure 1. Bottom: As mentioned for top, but showing the worst cases. It is understood the curves that correspond to RA distributions going below zero represent a constant phase of 180º, whereas the part going up to zero correspond to 0º, or vice–versa.

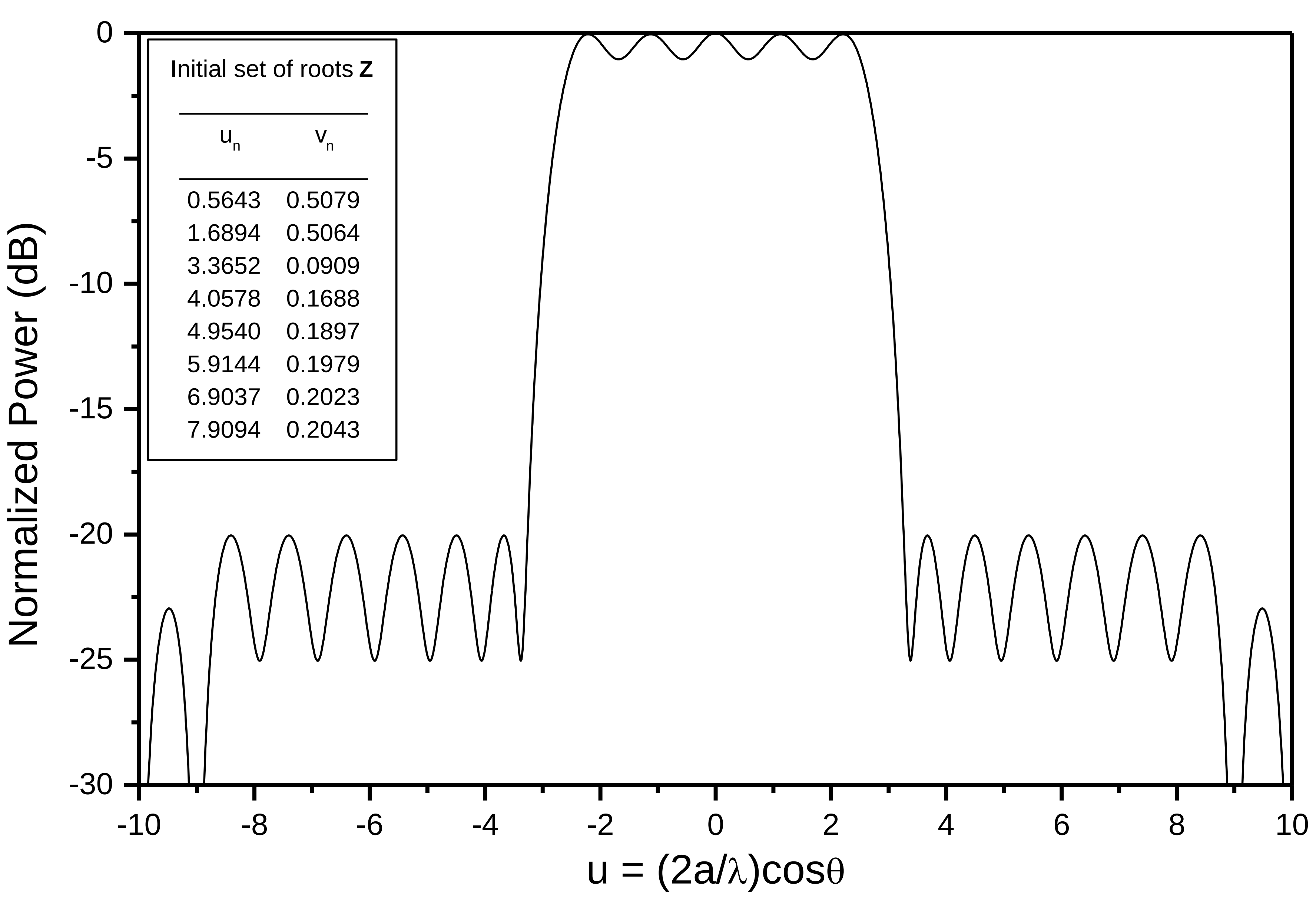
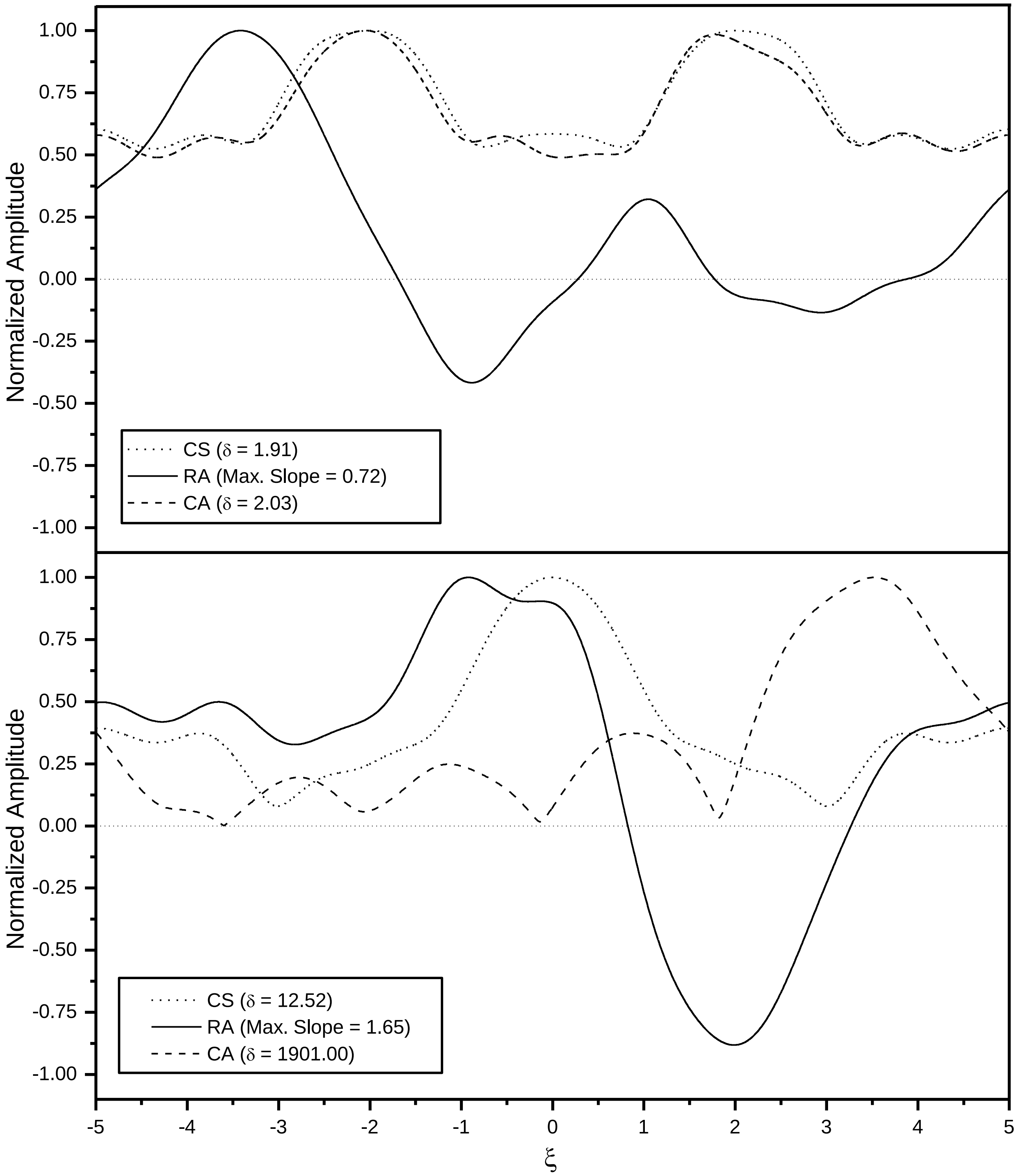

Initial set of roots **z**
u_n v_n
0.5643 0.5079
1.6894 0.5064
3.3652 0.0909
4.0578 0.1688
4.9540 0.1897
5.9144 0.1979
6.9037 0.2023
7.9094 0.2043
Normalized Power (dB)
0
-5
-10
-15
-20
-25
-30
-10 -8 -6 -4 -2 0 2 4 6 8 10
u = (2a/λ)cosθ

Figure 1



Normalized Amplitude
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
CS (δ = 1.91)
RA (Max. Slope = 0.72)
CA (δ = 2.03)
Normalized Amplitude
CS (δ = 12.52)
RA (Max. Slope = 1.65)
CA (δ = 1901.00)
-5
-4
-3
-2
-1
0
1
2
3
4
5
ξ

Figure 2