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**ADAPTING RESEARCH METHODS FOR SOLVING THE VRPTW TO REAL LIFE PROBLEMS.  
EVOLUTION OF THE ALADIN METHOD AND IMPLEMENTATION TO A REAL LIFE CASE .**

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## *Presentation outline*

1. Problem Analysis
2. Model development
3. Results
4. Conclusions

- Problem Analysis
- Model Development
- Results
- Conclusions

An aerial photograph of a city grid, likely Orlando, Florida, is shown in a dark blue, semi-transparent overlay. A 3D bar chart is overlaid on the city, with bars of varying heights representing data points across the urban landscape. The chart is centered in the lower half of the image. In the top right corner, there is a white crosshair graphic consisting of a horizontal line and a vertical line.

# 1. Problem Analysis

## *VRPTW characteristics*

### 1. Problem Analysis

2. Model development

3. Results

4. Conclusions

- Central depot and multiple customers geographically dispersed
- Depot with time window
- Customers with variable time windows
- Variable service times for customers
- Limited capacity for all vehicles
- Limited number of vehicles
- Euclidean distances considered
- Goal: Minimize total distance

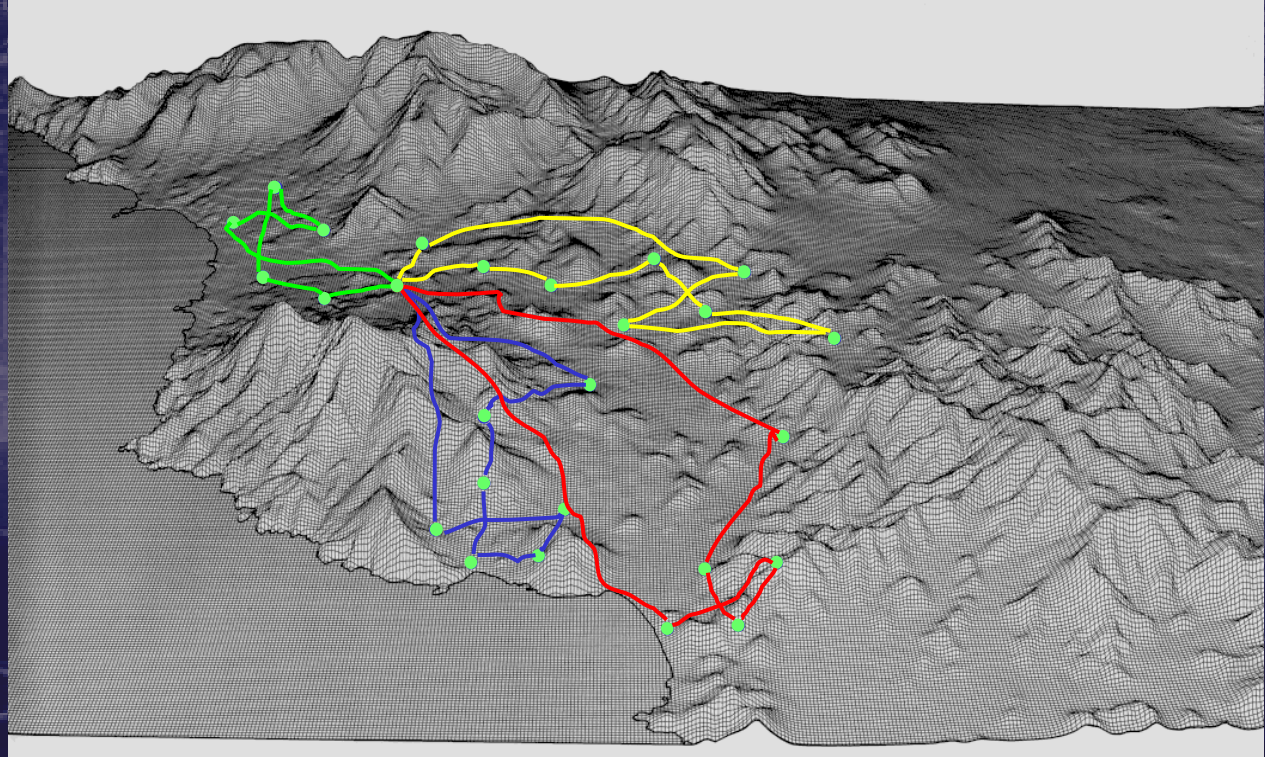
# The Vehicle Routing Problem with Time Windows

1. Problem Analysis

2. Model development

3. Results

4. Conclusions



● Open Node

● Closed Node

## Mathematical Model for the VRPTW

$$\min \sum_{k \in V} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}^k$$

$$\sum_{k \in V} \sum_{j \in N} x_{ij}^k = 1 \quad \forall i \in N$$

$$\sum_{j \in N} x_{0j}^k = 1 \quad \forall k \in V$$

$$\sum_{i \in N} x_{i0}^k = 1 \quad \forall k \in V$$

$$\sum_{i \in N} x_{ih}^k - \sum_{j \in N} x_{hj}^k = 0 \quad \forall h \in N, \forall k \in V$$

$$\sum_{i \in N} d_i \sum_{j \in N} x_{ij}^k \leq q \quad \forall k \in V$$

$$x_{ij}^k (S_i^k + t_{ij} - S_j^k) \leq 0 \quad \forall i, j \in N, \forall k \in V$$

$$a_i \leq S_i^k \leq b_i \quad \forall i \in N, \forall k \in V$$

$$x_{ij}^k = (0,1) \quad i, j \in N, \forall k \in V$$

1. Problem Analysis

2. Model development

3. Results

4. Conclusions

## *Solution techniques for the VRPTW*

### 1. Problem Analysis

### 2. Model development

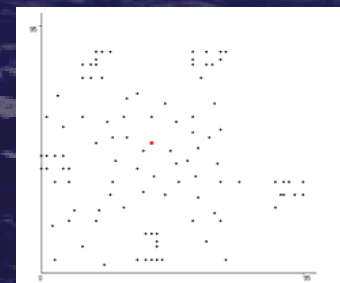
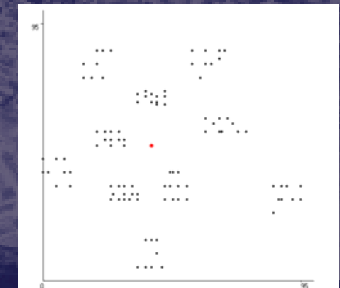
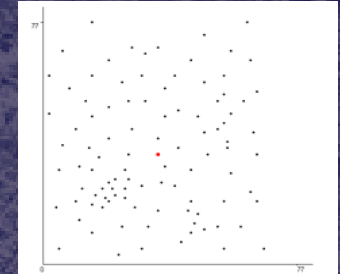
### 3. Results

### 4. Conclusions

- Heuristics
- Construction methods
  - Solomon (1987)
    - Savings methods
    - Gillett and Miller
    - Insertion I1
    - Insertion I2
    - Insertion I3
  - Potvin and Rousseau (1993)
  - Dullaert (2000 and 2001)
  - Ioannou et al (2001)
  - Bramel and Simchi-Levi (1996)

## *Solution techniques for the VRPTW*

- Improvement Methods
  - Intra-route
  - Inter-route
    - Cyclic transfers
  - Antes and Derigs (1995)
  - Russell (1995)
  - Shaw (1997)
  - Cordone and Wolfer Calvo (1998)
  - Bräydis (2001)
- Meta-heuristics
  - Tabu Search
  - Simulated Annealing
  - Genetic Algorithms
  - Others



**VRPTW**  
**Solomon (1987)**

1. Problem Analysis

2. Model development

3. Results

4. Conclusions



The background of the slide is a dark blue aerial photograph of a city grid. Overlaid on this is a 3D bar chart where the height of each bar represents the density or intensity of the city's development at that location. The bars are arranged in a grid pattern, with some taller bars indicating higher density. A white horizontal line is positioned near the top of the slide, and a white crosshair is in the top right corner.

# 2. Model Development

1. Problem Analysis

**2. Model development**

3. Results

4. Conclusions

## ***Model characteristics***

*“Paralell construction of the routes in different phases, following different decision rules”*

- Deterministic construction model
- Parameters inclusion
- Temporal and spatial criteria
- Basic rules: assignment, addition, single insertion and double insertion

## Initial data

r103 - Bloc de notas

Archivo Edición Buscar Ayuda

R103

VEHICLE  
NUMBER 25  
CAPACITY 200

CUSTOMER  
CUST NO. XCOORD. YCOORD. DEMAND READY TIME DUE DATE SERVICE TIME

0	35	35	0	0	230	0
1	41	49	10	0	204	10
2	35	17	7	0	202	10
3	55	45	13	0	197	10
4	55	20	19	149	159	10
5	15	30	26	0	199	10
6	25	30	3	99	109	10
7	20	50	5	0	198	10
8	10	43	9	95	105	10
9	55	60	16	97	107	10
10	30	60	16	124	134	10
11	20	65	12	67	77	10
12	50	35	19	0	205	10
13	30	25	23	159	169	10
14	15	10	20	0	187	10
15	30	5	8	61	71	10
16	10	20	19	0	190	10
17	5	30	2	157	167	10
18	20	40	12	0	204	10
19	15	60	17	0	187	10
20	45	65	9	0	188	10
21	45	20	11	0	201	10
22	45	10	18	97	107	10
23	55	5	29	68	78	10
24	65	35	3	0	190	10
25	65	20	6	172	182	10

1. Problem Analysis

2. Model development

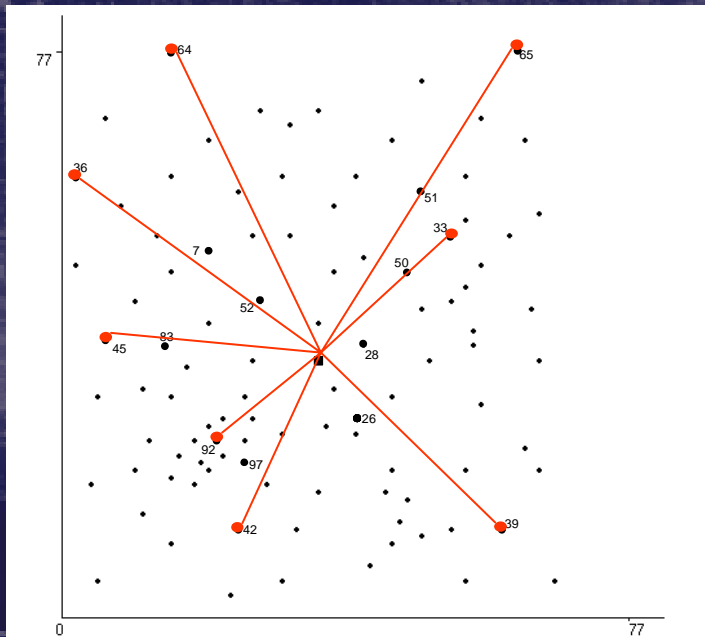
3. Results

4. Conclusions

## Initial assignment rule

R103

$R=8, \beta=1.2 \text{ y } \gamma=2$



### Direct addition

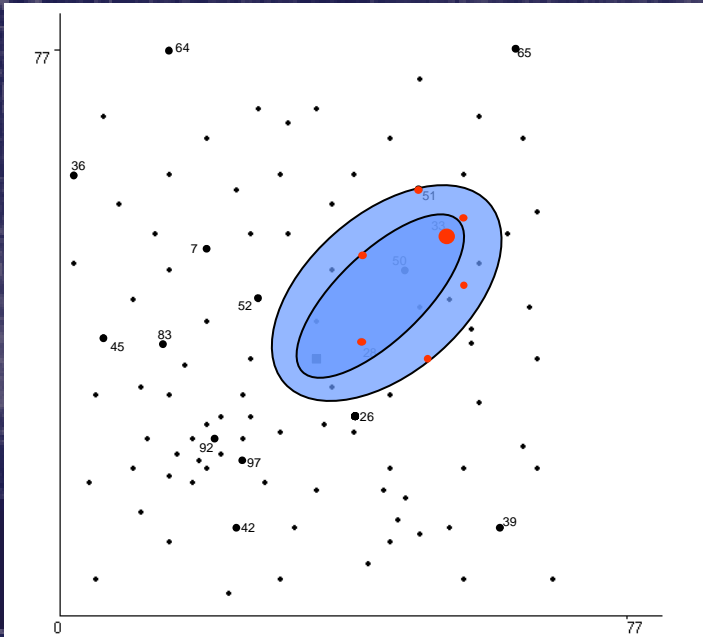
$$\begin{aligned}
 tc_i & \\
 t_{0i} & \\
 h_i &= tc_i - t_{0i} \\
 LR_k &= [0, i] \\
 CR_k &= [i] \\
 c_k &= d_i \\
 tr_k &= t_{0i} + ts_i + te_{ki} \\
 D_k &= t_{0i} \\
 E_k &= te_{ki} \\
 te_{ki} &= [ta_i - (t_{0i})]^+
 \end{aligned}$$

## Time gap analysis for possible insertions

R103

$R=8, \beta=1.2$  y  $\gamma=2$

$\min h_i = tc_i - t_{0i}$



### L2 creation

$$ta_m \leq t_{0m}$$

$$ta_m + ts_m + t_{mi} \leq tc_i$$

$$t_{0m} + ts_m + t_{mi} \leq tc_i$$

$$t_{0m} + ts_m + t_{mi} + nh_i = tc_i$$

$$nh_i \geq 0$$

$$t_{0m} + t_{mi} \leq \beta \cdot t_{0i}$$

$$t_{0m} + t_{mi} + t_{i0} + te_{ki} \leq tc_0$$

$$te_{ki} = [ta_i - (t_{0m} + ts_m + t_{mi})]^+$$

$$d_m + d_i \leq q$$

### Direct addition

$$LR_k = [0, i]$$

$$CR_k = [i]$$

$$c_k = d_i$$

$$tr_k = t_{0i} + ts_i + te_{ki}$$

$$D_k = t_{0i}$$

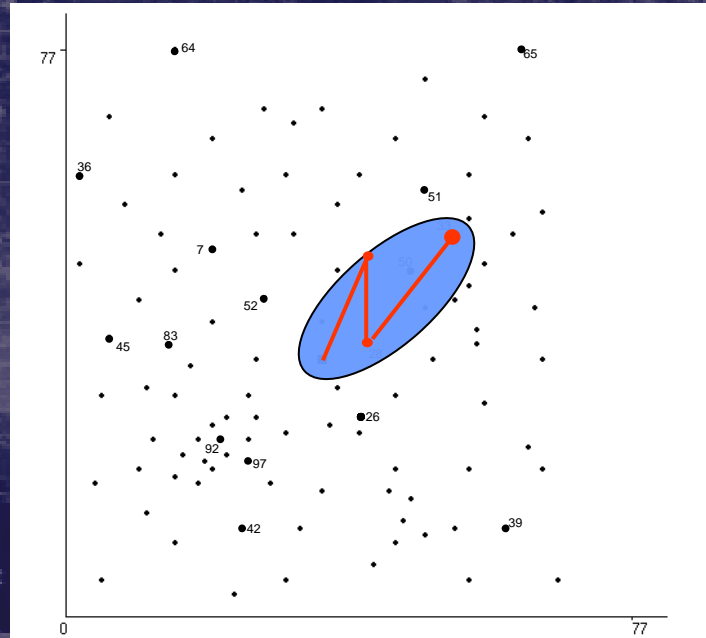
$$E_k = te_{ki}$$

$$te_{ki} = [ta_i - (t_{0i})]^+$$

## Double Insertion Requirements

R103

$R=8, \beta=1.2$  y  $\gamma=2$



### L3 creation

$$ta_m \leq t_{0m}$$

$$ta_l \leq t_{0m} + ts_m + t_{ml}$$

$$t_{0m} + ts_m + t_{ml} + ts_l + t_{li} \leq tc_i$$

$$t_{0m} + ts_m + t_{ml} + ts_l + t_{li} + nh'_i = tc_i$$

$$nh'_i \geq 0$$

$$t_{0m} + t_{ml} + t_{li} \leq \gamma \beta \cdot t_{0i}$$

$$t_{0m} + t_{ml} + t_{li} + t_{i0} + te_{ki} \leq tc_0$$

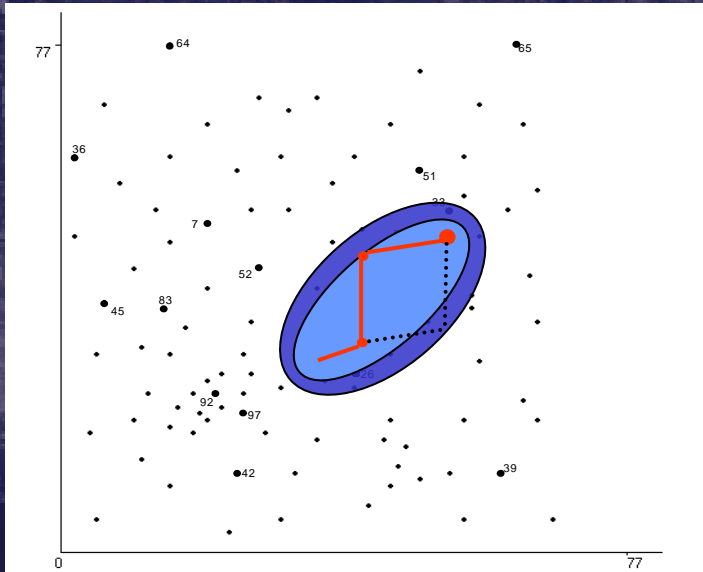
$$te_{ki} = [ta_i - (t_{0m} + ts_m + t_{ml} + ts_l + t_{li})]^+$$

$$d_m + d_l + d_i \leq q$$

## Double or Single insertion procedures

R103

$R=8, \beta=1.2$  y  $\gamma=2$



### L3 Creation

$$ta_m \leq t_{0m}$$

$$ta_l \leq t_{0m} + ts_m + t_{ml}$$

$$t_{0m} + ts_m + t_{ml} + ts_l + t_{li} \leq tc_i$$

$$t_{0m} + ts_m + t_{ml} + ts_l + t_{li} + nh'_i = tc_i$$

$$nh'_i \geq 0$$

$$t_{0m} + t_{ml} + t_{li} \leq \gamma \beta \cdot t_{0i}$$

$$t_{0m} + t_{ml} + t_{li} + t_{i0} + te_{ki} \leq tc_0$$

$$te_{ki} = [ta_i - (t_{0m} + ts_m + t_{ml} + ts_l + t_{li})]^+$$

$$d_m + d_l + d_i \leq q$$

### Double Insertion

$$LR_k = [0, m, l, i]$$

$$CR_k = [i]$$

$$c_k = d_m + d_l + d_i$$

$$tr_k = t_{0m} + ts_m + t_{ml} + ts_l + t_{li} + ts_i + te_{ki}$$

$$D_k = t_{0m} + t_{ml} + t_{li}$$

$$E_k = te_{ki}$$

$$te_{ki} = [ta_i - (t_{0m} + ts_m + t_{ml} + ts_l + t_{li})]^+$$

### Single Insertion

$$LR_k = [0, m, i]$$

$$CR_k = [i]$$

$$c_k = d_m + d_i$$

$$tr_k = t_{0m} + ts_m + t_{mi} + ts_i + te_{ki}$$

$$D_k = t_{0m} + t_{mi}$$

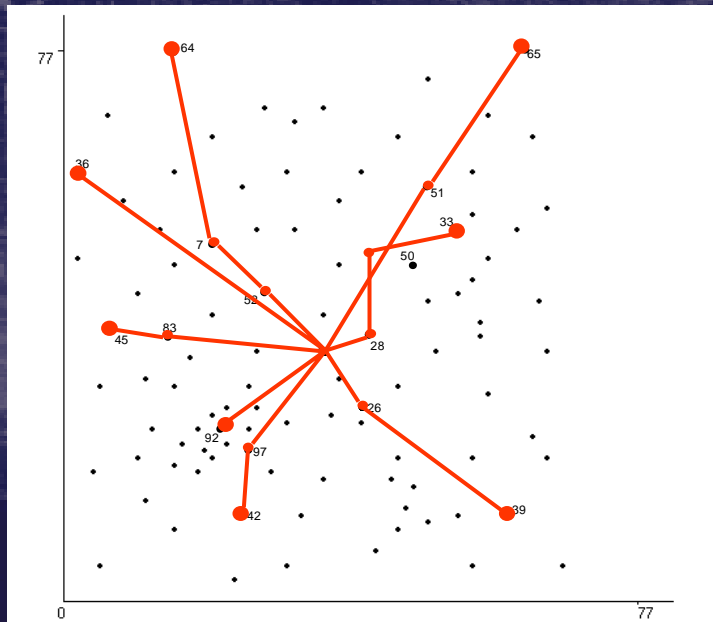
$$E_k = te_{ki}$$

$$te_{ki} = [ta_i - [t_{0m} + ts_m + t_{mi}]]^+$$

## End of first phase

R103

$R=8, \beta=1.2$  y  $\gamma=2$



End of first phase

Routes update

New selection of  $R$  nodes with  
 $\min tc_i$

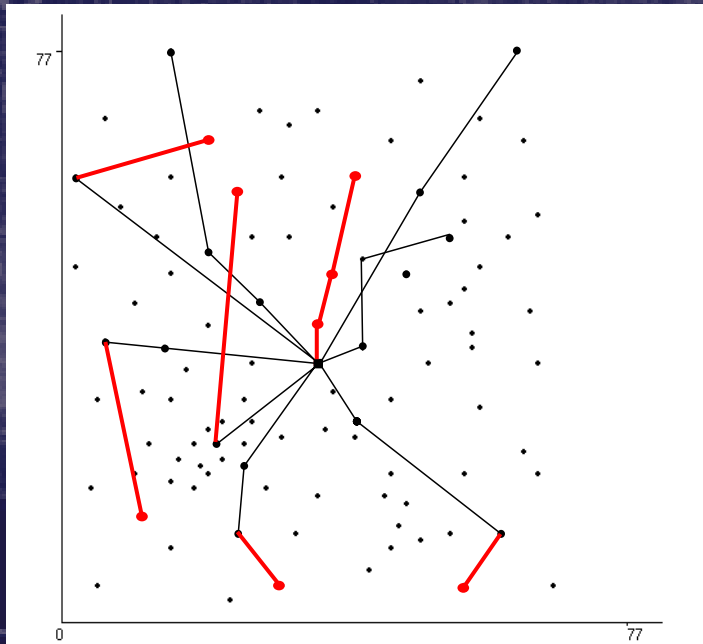


## Second and further assignments

R103

$R=8, \beta=1.2$  y  $\gamma=2$

$\min tc_i$



**Beginning of second and further phases**

One by one all nodes are assigned to the nearest vehicles, given that these arrive on time.

An aerial photograph of Orlando, Florida, overlaid with a 3D city model. The city model consists of numerous rectangular blocks of varying heights, representing buildings. The background is a dark blue-tinted aerial view of the city's street grid and landmarks. A white horizontal line is positioned near the top of the image, and a white crosshair is in the top right corner.

# 3. Results

## Comparison with other construction methods

Method	R1		R2		RC1		RC2		C1		C2	
	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV
Clarke and Wright Method	1499	16,60	-	-	-	-	-	-	976	11,70	-	-
Savings, waiting time limit	1517	15,10	-	-	-	-	-	-	987	10,70	-	-
Solomon I1	1437	13,60	1402	3,30	1597	13,50	1682	3,90	952	10,00	692	3,13
Solomon I2	1639	14,50	1471	3,30	1874	14,20	1798	4,10	1050	10,10	921	3,40
Solomon I3	1652	14,10	1475	3,40	1850	14,00	1816	4,00	1103	10,00	1073	3,50
Nearest Neighbour	1600	14,50	1472	3,40	1800	14,20	1755	3,90	1171	10,20	963	3,50
Gillet and Miller Method	1500	14,60	1449	3,20	1804	14,90	1736	4,00	941	10,00	712	3,00
Potvin and Rousseau (1993)	1509	13,30	1387	3,10	1724	13,40	1651	3,60	1343	10,67	797	3,38
Ioannu et al. (2001)	1370	12,67	1310	3,09	1512	12,50	1483	3,50	865	10,00	662	3,13
Guillén et al. (2004)	1955	26,00	1239	8,00	2247	21,00	1573	11,00	1955	25,00	1485	14,00

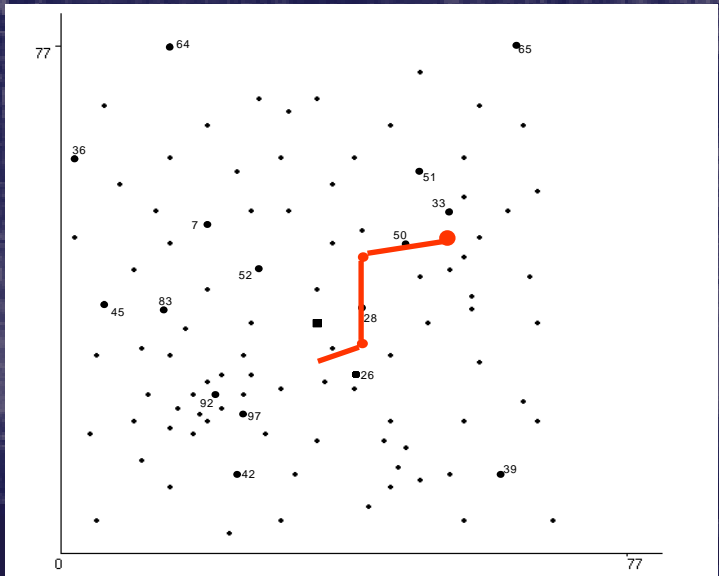
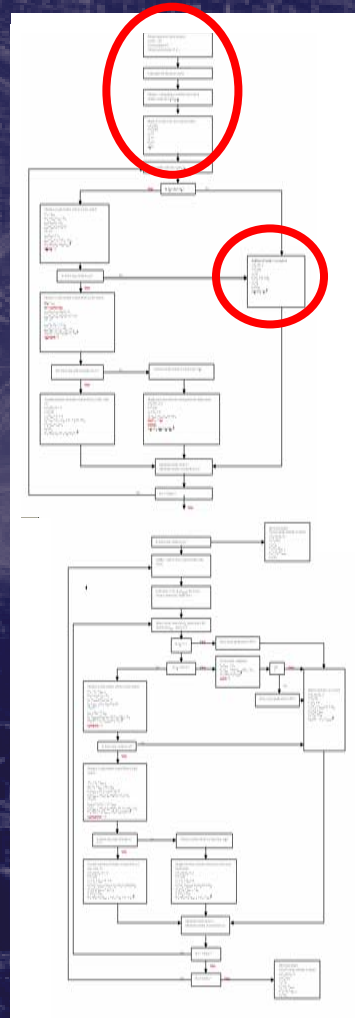


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# New Improvements

*Initial assignment rule: Nodes chose one by one*

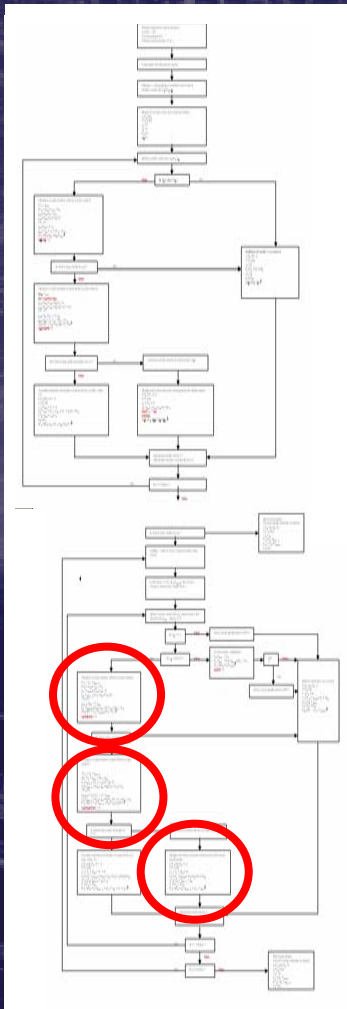
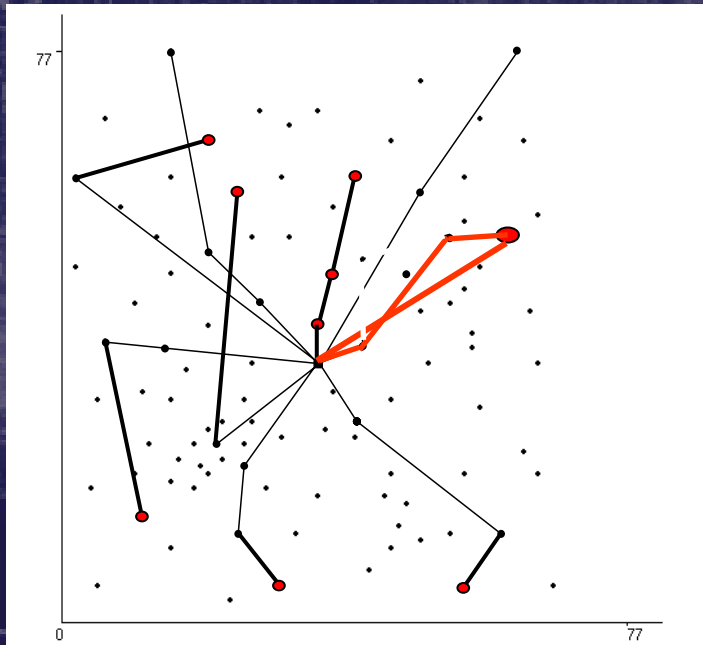
R103



## Insertion with elimination of previously inserted node

R103

$R=1, \beta=1.2$  y  $\gamma=2$



## Comparison with other construction methods

Construction Methods	R1		R2		RC1		RC2		C1		C2	
	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV
Clarke and Wright Method	1499	16,60	-	-	-	-	-	-	976	11,70	-	-
Savings, waiting time limit	1517	15,10	-	-	-	-	-	-	987	10,70	-	-
Solomon I1	1437	13,60	1402	3,30	1597	13,50	1682	3,90	952	10,00	692	3,13
Solomon I2	1639	14,50	1471	3,30	1874	14,20	1798	4,10	1050	10,10	921	3,40
Solomon I3	1652	14,10	1475	3,40	1850	14,00	1816	4,00	1103	10,00	1073	3,50
Nearest Neighbour	1600	14,50	1472	3,40	1800	14,20	1755	3,90	1171	10,20	963	3,50
Gillet and Miller Method	1500	14,60	1449	3,20	1804	14,90	1736	4,00	941	10,00	712	3,00
Potvin and Rousseau (1993)	1509	13,30	1387	3,10	1724	13,40	1651	3,60	1343	10,67	797	3,38
Ioannu et al. (2001)	1370	12,67	1310	3,09	1512	12,50	1483	3,50	865	10,00	662	3,13
Guillén et al. (2006)	1955	26,00	1239	8,00	2247	21,00	1573	11,00	1955	25,00	1485	14,00
Guillén et al. (2007)	1386	14,25	1096	5,27	1607	14,12	1293	5,87	925	10,66	705	3,50

## Comparison with other improvement methods

Improvement methods	R1		R2		RC1		RC2		C1		C2	
	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV
Thompson y Psarftis (1993)	1367	13,1	1299	3,1	1534	13	1672	3,7	916	10	644	3
Potvin et al. (1995)	1381	13,33	1293	3,27	1545	13,25	1595	3,88	902	10	653	3,13
Russell (1995)	1317	12,7	1167	2,9	1523	12,4	1398	3,4	930	10	681	3
Antes y Derigs (1995)	1386	12,8	1367	3,1	1546	12,5	1598	3,4	955	10	717	3
Cordone et al. (1998)	1241	12,5	995	2,91	1408	12,3	1139	3,38	834	10	591	3
Zhu y Lee (1999) 2-int	1469		1330		1680		1700		965		780	
Guillén et al. (2006)	1955	26,00	1239	8,00	2247	21,00	1573	11,0	1955	25,00	1485	14,0
Guillén et al. (2007)	1386	14,25	1096	5,27	1607	14,12	1293	5,87	925	10,66	705	3,50



## Comparison with other Tabu Search methods

Tabu Search	R1		R2		RC1		RC2		C1		C2	
	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV
Garcia et al. (1994)	1320	12,9	1229	3,1	1483	12,9	1551	3,9	877	10	602	3
Potvin et al. (1996a)	1295	12,6	1186	3,1	1465	12,6	1476	3,4	850	10	594	3
Taillard et al. (1997)	1220	12,3	1013	3	1381	12,9	1199	3,4	828	10	589	3
De Backer et al. (1997)	1214	14,17	930	5,27	1385	14,25	1099	6,25	829	10	604	3,25
Schulze y Fahle (1999)	1268	12,5	1056	3,1	1396	12,3	1308	3,4	828	10	589	3
Tan et al. (2000)	1266	13,83	1080	3,82	1458	13,63	1293	4,25	870	10	634	3,25
Guillén et al. (2006)	1955	26,00	1239	8,00	2247	21,00	1573	11,0	1955	25,00	1485	14,0
Guillén et al. (2007)	1386	14,25	1096	5,27	1607	14,12	1293	5,87	925	10,6	705	3,50

## Comparison with other Simulated Annealing methods

Simulated Annealing	R1		R2		RC1		RC2		C1		C2	
	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV
Thangiah et al. (1994)	1227	12,33	1005	3	1391	12	1173	3,38	830	10	640	3
Chiang et al. (1996)	1308	12,5	1166	2,91	1473	12,38	1393	3,38	909	10	666	3
Tan et al. (2000)	1420	14,5	1278	3,64	1648	14,75	1641	4,25	958	10,11	766	3
Zhu y Lee (1999) SA	1422		1279		1657		1642		943		766	
Guillén et al. (2006)	1955	26,00	1239	8,00	2247	21,00	1573	11,00	1955	25,00	1485	14,00
Guillén et al. (2007)	1386	14,25	1096	5,27	1607	14,12	1293	5,87	925	10,6	705	3,50

## Comparison with other Genetic Algorithms

Genetic Algorithms	R1		R2		RC1		RC2		C1		C2	
	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV	Dist	NV
Thangiah (1995a)	1300	12,75	1124	3,18	1474	12,5	1411	3,38	892	10	749	3
Potvin et al. (1996)	1296	12,58	1117	3	1446	12,13	1368	3,38	838	10	590	3
Berger et al. (1998)	1261	12,58	1030	3,09	1441	12,13	1284	3,5	834	10	594	3
Bräysy (1999b)	1272	12,58	1053	3,09	1417	12,13	1256	3,38	857	10	624	3
Homberger y Gehring (1999)	1226	12	1034	2,7	1407	11,5	1176	3,3	828,38	10	589	3,25
Gehring et al. (2001)	1217	12	961	2,73	1395	11,5	1139	3,25	828,63	10	590,3	3,25
Berger et al. (2000)	1221	11,92	975	2,73	1389	11,5	1159	3,25	828	10	590	3
Tan et al. (2000)	1314	14,42	1093	5,64	1512	14,63	1282	7	860	10,11	589	
Wee Kit et al. (2001)	1203	12,58	951	3,18	1382	12,75	1132	3,75	833	10	593	
Rahoual et al. (2001)	1362	12,92			1487	12,63			887	10		
Guillén et al. (2006)	1955	26,00	1239	8,00	2247	21,00	1573	11,00	1955	25,00	1485	14,00
Guillén et al. (2007)	1386	14,25	1096	5,27	1607	14,12	1293	5,87	925	10,66	705	3,50



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4. Adapting research  
method to real life  
problems

1. Problem Analysis

2. Model development

3. Results

4. **Conclusions**

## *Issues*

- We need to adapt the distance matrix
- It is necessary to link the algorithm to GIS databases
- We need to locate every customer and link it to the nearest road milestone
- We need to coordinate the demands to the capacity of the vehicles
- These modifications allow the algorithm to solve real life problems
  
- We are now working with a business partner in order to validate the results
- The obtained results are pretty close to those provided by their software
- **NEXT STEP:** Improve the interfave and the interactivity with the user

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# Thank you !

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