Transportation Research Part B 161 (2022) 1–12

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Spatially correlated nested logit model for spatial location choice

Jose-Benito Perez-Lopez [a,*], Margarita Novales [b], Alfonso Orro [b]

[a] Universidade da Coruña, Group of Railways and Transportation Engineering, Department of Economics, Facultad de Economía y Empresa, Campus de Elviña, 15071 A Coruña, Spain

[b] Universidade da Coruña, Group of Railways and Transportation Engineering, Department of Civil Engineering, ETS Ingenieros de Caminos, Canales y Puertos, Elviña, 15071 A Coruña, Spain

ARTICLE INFO

Keywords:
Discrete choice models
Transport land use interaction
Spatial models
Spatial correlation
Residential location choice
Microeconometric choice models

ABSTRACT

Residential location choice is a key component of the models for predicting land-use and transport demand in urban planning. In general, it requires to consider correlation between spatial alternatives. The approach of nested alternatives of the nested logit model has proved highly efficient in this context. This approach incorporates into the nested logit model both spatial and non-spatial correlations due to unobserved variables. The approach of metric extensions to the spatially correlated logit model specifies models for capturing spatial correlations between alternatives without having to design a nested structure. A model combining both approaches is proposed in this research. The spatially correlated nested logit model proposed herein models the correlation between alternatives of the nests of a nested logit model using a metric of spatial correlation between pairs of alternatives. The proposed model improves the properties of the nested logit model without the need of increasing the number of unknown parameters. Our model also improves the properties of a spatially correlated model with the same spatial metric. When needing to incorporate preference heterogeneity into the model, the proposed model is compatible with a mixed specification with random coefficients. The spatially correlated nested logit model was empirically applied to the real case of residential location choice in the city of Santander in Spain. In this empirical context, this model improved the explanatory and predictive power of the models that it combines.

1. Introduction

People are frequently faced with decisions requiring choosing between a discrete set of alternatives, such as decisions about purchasing, mode of transport and travel destinations, among others. As highlighted by Takahashi (2019), huge studies have been conducted to capture discrete purchasing behavior through discrete choice models. Spatial location choices, in which choice alternatives refer to geographical locations, are a key feature of advanced disaggregate models of travel and activity demand. Within these models, the most important aspects refer to residential location choice and to a lesser extent employment location choice. Spatial location choices can also appear in other types of models, such as travel destination or public transport boarding or alighting stop choice models.

In the models for predicting land-use and transportation demand in urban planning, currently, the most widely used approach consists of mathematical simulation models of the interaction between land-uses and transportation (LUTI, see Torrens, 2000). LUTI

* Corresponding author.
E-mail address: benito.perez@udc.es (J.-B. Perez-Lopez).

https://doi.org/10.1016/j.trb.2022.05.007
Received 11 June 2021; Received in revised form 28 April 2022; Accepted 8 May 2022
Available online 16 May 2022
0191-2615/

models use zoning of the spatial area under study and require a prediction model of the individual choice of the area of residence. The choice of area of residence is a crucial decision for many families for two key reasons. First, the area of residence will tremendously impact housing prices, whether for buying or renting. Second, the area of residence influences travel times to daily activities and the type of social life the family will have according to accessibility of services offered and their characteristics.

At a disaggregate level, Weiss et al. (2019) state that spatial location choice is typically modeled using the econometric approach based on random utility maximizing (RUM) hypothesis (Thurstone, 1927; McFadden, 1974). RUM discrete choice modeling is the most commonly used mathematical framework in prediction models of residential location choice in the LUTI context (Pagliara and Wilson, 2010). Various RUM models have been developed, but they are not all applicable with spatial alternatives. In these cases, some alternatives will be spatially correlated. Bahamonde-Birke (2021) presents a discussion of the different kinds of spatial correlation that affects multinomial discrete choice models and how they have been addressed in the discrete modeling literature. The modeling approaches considered in this paper deal with the spatial correlation among alternatives, that is common in transport and land-use models. This correlation refers to substitution preferences on the part of the decision maker and are due to unobserved spatial elements of the utility. As a result, the models that overlook correlation between alternatives, like multinomial logit, are not suitable in this context. In turn, spatial location choices usually entail a high number of alternatives, thereby preventing or hindering the use of some approaches to capture this correlation. Specifying a mixed logit model using an error component structure allows flexible patterns of correlation between alternatives (see Train, 2009). This approach is usually unfeasible for correlation between spatial alternatives because such correlations require specifying as many error components as pairs of correlated alternatives, which are usually too many for the estimation process. The same limitation occurs in the probit model (Daganzo, 1979). If constraints are not included in the correlation structure of the perturbations of this model, the number of parameters to estimate in the covariance matrix of the perturbations may be so high that the estimation process of this model becomes unfeasible (see applications of the probit model for spatial correlation in Bolduc, 1992; Garrido and Mahmassani, 2000).

The main goal of this research is to propose a new model for predicting land-use and transport demand in urban planning, based on RUM and focused in spatial location choice of residence. In this context, the choice alternatives are geographic areas. The new model must consider the spatial characteristics of these alternatives to improve the ability to explain and predict the behavior of decision-makers in comparison with other current RUM models. The new generalized extreme value (GEV) model proposed combines the two current GEV approaches compatible with spatial correlation between alternatives, by incorporating the spatially correlated logit approach into the nested logit model. This spatially correlated nested logit model considers correlation through pre-specified nests and uses spatial information on the alternatives, without the need of increasing the number of estimated parameters in relation to a nested logit approach. Thanks to this combination, the proposed model improves the explanatory and predictive power of the previous GEV models. This model is compatible with a mixed GEV specification, which makes it possible to incorporate variations in decision-makers' preferences. In the next section, we review the state of the art in GEV models compatible with spatial location choice modeling. In Section 3, we present the proposed GEV model. In Section 4, the new model is applied to an urban residential location choice empirical context and compared with the GEV models compatible with spatial location choice modeling analyzed in Section 2. Finally, Section 5 presents our conclusions.

2. GEV models with spatial correlation between alternatives

The most widespread and simple RUM-consistent discrete choice model is multinomial logit (MNL) (McFadden, 1974; Domencich and McFadden, 1975). The MNL model assumes that the stochastic components (ε_i) of the utility of alternative i (U_i) have a marginal type I extreme value distribution (Gumbel; Johnson and Kotz, 1970) independent and equally distributed. The MNL model assumes uncorrelation between alternatives and between observations, overlooking unobserved variations in preferences or tastes. The parameter of the perturbation scale is usually normalized to one; a similar approach has been used in all models considered in this article, without loss of generality (Abbe et al., 2007).

2.1. Nested logit

The hierarchical or nested logit model (NL; Williams, 1977; Daly and Zachary, 1978; McFadden, 1978) extends the MNL model to allow for specific structures of correlation between alternatives. The stochastic components of the NL model maintain homoscedasticity and have a joint extreme value distribution. This model clusters alternatives to assess the correlation between them. The clusters of alternatives, termed nests, must be designed by the analyst. To design the nests, the analyst must use variables not incorporated in the utility function. For example, in an urban residential location choice context, these variables may represent how attractive the area is to the decision maker for its prestige, prevailing architecture, views or accessibility of services, such as transport, schools, leisure or employment. In the NL model, each alternative belongs to a nest. The structure of the resulting variance-covariance matrix is a diagonal matrix by blocks, one per nest, unlike the scalar structure of the MNL model. The parameters $0 < \mu_k \leq 1$, termed dissimilarity parameters of each nest N_k, modulate the value of the correlation between pairs of alternatives. The correlation between the perturbations of two alternatives, $i$and$j$, is calculated using Eq. (1) if both alternatives belong to the same nest, N_k, and is null if they belong to different nests.

$$Corr(\varepsilon_i,\ \varepsilon_j) = (1-\mu_k^2),\ \forall i,j \in N_k,\ k \in \{1,\ \ldots,\ M\} \tag{1}$$

The NL model is compatible with spatial location choice modeling if the analyst designs a structure with a viable number of nests.

The increase in the number of nests increases not only the flexibility of the NL model in measuring the correlation between alternatives but also the number of dissimilarity parameters that the model will have to estimate. The handicap of the NL model lies in the need for the analyst to design the nest structure. Furthermore, the effectiveness of the NL model in collecting the correlation between alternatives will depend on the analyst's ability to design the nests. Eq. (2) shows the probability of each alternative i, where $P_{i|k}$ (3) is the conditional probability of the alternative i if nest N_k is selected and P_k (4) is the probability of choosing nest N_k. Eqs. (3) and (4) are modifications of that of Papola (2004) to facilitate the comparison between NL and the model proposed in this paper.

$$P_i = P_{i|k} \cdot P_k \tag{2}$$

$$P_{i|k} = \frac{\left(e^{V_i}\right)^{1/\mu_k}}{\sum_{j\in N_k}\left(e^{V_j}\right)^{1/\mu_k}} \tag{3}$$

$$P_k = \frac{\left(\sum_{j\in N_k}\left(e^{V_j}\right)^{1/\mu_k}\right)^{\mu_k}}{\sum_{l=1}^{M}\left(\sum_{r\in N_l}\left(e^{V_r}\right)^{1/\mu_l}\right)^{\mu_l}} \tag{4}$$

2.2. Spatially correlated logit

McFadden (1978) generalized the nest approach of the NL model in the class of generalized extreme value (GEV) models. The perturbations of the GEV models are homoscedastic, with a joint extreme value distribution. GEV models incorporate constraints in the covariance matrix from the nest structure, which, if relatively simple, maintain a closed structure. Probability is calculated based on a termed generating function $G(e^{V_1}, \ldots, e^{V_A})$, using Eq. (5). The generating function of a GEV model should meet a set of criteria established by McFadden (1978) and revised by Ben-Akiva and Francois (1983). Nest can assess both unobserved spatial correlation among alternatives and correlation due to unobserved non-spatial variables. As pointed out by Bahamonde-Birke (2021), GEV models cannot be used to capture spatial correlation among observations. GEV models can act as kernels of mixed logit specifications with random coefficients, termed mixed GEV (Bhat and Guo, 2004; Hess et al., 2005).

$$P_i = \frac{e^{V_i}\cdot\frac{\partial G\left(e^{V_1},\ \ldots, e^{V_A}\right)}{\partial e^{V_i}}}{G\left(e^{V_1},\ \ldots, e^{V_A}\right)},\ \forall i\in\{1,\ \ldots,\ A\} \tag{5}$$

Both MNL and NL are GEV models. The generating function of MNL model is shown in Eq. (6) and that of the NL model in Eq. (7), where μ_k is the dissimilarity parameter of the nest N_k. GEV extensions of the NL model are based on cross-nested logit (CNL; Small, 1987; Vovsha, 1997; Ben-Akiva and Bierlaire, 1999; Papola, 2004), which Wen and Koppleman (2001) formulated as generalized NL (GNL). The generating function of GNL model is shown in Eq. (8), where $\alpha_{ik}\geq 0$ is the allocation parameter of alternative i to nest N_k for all M nests and A alternatives, with zero value when the alternative does not belong to the nest. In CNL or GNL models, the alternatives can belong to more than one nest. For this reason, they incorporate the allocation parameters, which are interpreted as the level of membership of each alternative to each nest (Abbe et al., 2007). These models are nested with the two-level NL model that uses the same dissimilarity parameters if each alternative belongs to a single nest with an allocation parameter value of one.

$$G\left(e^{V_1},\ \ldots, e^{V_A}\right) = \sum_{i=1}^{A} e^{V_i} \tag{6}$$

$$G\left(e^{V_1},\ \ldots, e^{V_A}\right) = \sum_{k=1}^{M}\left(\sum_{i\in Nest_k}\left(e^{V_i}\right)^{1/\mu_k}\right)^{\mu_k} \tag{7}$$

$$G\left(e^{V_1},\ \cdots, e^{V_A}\right) = \sum_{k=1}^{M}\left(\sum_{i\in Nest_k}\left(\alpha_{ik}e^{V_i}\right)^{1/\mu_k}\right)^{\mu_k} \tag{8}$$

The GNL models include the constraint that the allocation parameters of each alternative add up to one, as expressed in Eq. (9) (see Abbe et al., 2007 to analyze normalization proposals in other CNL formulations). This normalization allows the allocation parameters of each alternative to represent the proportion of belonging to each nest.

$$\sum_{k=1}^{M}\alpha_{ik} = 1,\ \forall i = 1,\ \ldots,\ A \tag{9}$$

The unobserved correlation between pairs of alternatives of the CNL and GNL models is modulated by all structural parameters, that is, allocation and dissimilarity parameters. This correlation is calculated from the joint cumulative distribution function, by numerical integration. When the number of alternatives is high, the number of structural parameters of the GNL models increases considerably in relation to the NL model to the point that estimating all parameters is unfeasible. Under these conditions, it may be useful to calculate some parameters beforehand or incorporate constraints to reduce their number and then estimate the model only with the other parameters (Abbe et al., 2007).

The paired combinatorial logit (Chu, 1981; Chu, 1989; Koppelman and Wen, 2000) model proposes a GEV model with a nest structure not designed by the analyst but instead formed by each pair of alternatives. Therefore, this model has as many dissimilarity parameters as pairs of alternatives. Wen and Koppelman (2001) extended the paired combinatorial logit model with a GNL formulation termed paired generalized nested logit, which adds two allocation parameters for each pair of nests with respect to the paired combinatorial logit model.

The paired combinatorial logit and paired generalized nested logit models are not viable in the spatial location choice modeling context, except when previously calculating a significant number of structural parameters or incorporating constraints to reduce their number. As clearly shown, the number of structural parameters in the paired generalized nested logit model is much higher than in any other GNL specification based on a nest structure designed by the analyst. Using both possibilities, Bhat and Guo (2004) proposed, for the context of spatial location choice, a reduced specification of the paired generalized nested logit model, the spatially correlated logit (SCL) model, with a GEV generating function (10). On the one hand, the SCL model adds to paired generalized nested logit model the constraint that all pairs of contiguous alternatives have the same dissimilarity parameter $0 < \mu \leq 1$. On the other hand, the SCL model proposes that the paired generalized nested logit model allocation parameters be calculated before estimating the model, using data on the contiguity of the alternatives. These parameters are calculated using Eq. (11), where the value of the dichotomous spatial variable ω_{ij} is 1 when the alternatives *i, j* partly share the border, and 0 otherwise. Therefore, regardless of the number of alternatives, the SCL model requires estimating only one more parameter than the MNL model (with which the model is nested), the dissimilarity parameter.

$$G\left(e^{V_1},\ \dots,e^{V_A}\right) = \sum_{i=1}^{A-1}\sum_{j=i+1}^{A}\left(\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu} + \left(\alpha_{j,ij}e^{V_j}\right)^{1/\mu}\right)^{\mu} \tag{10}$$

$$\alpha_{i,ij} = \frac{\omega_{ij}}{\sum_{l=1}^{A}\omega_{il}},\ \forall i,j \in \{1,\ \cdots,\ A\} \tag{11}$$

The spatial approach of the SCL model was extended with new, spatially correlated GNL models. These SCL-based models use metrics of the spatial similarity of the alternatives to calculate allocation parameters between pairs of alternatives according to Eq. (12), where $f(i, j)$ is the value of a spatial metric f in each pair of alternatives *i, j*, whose values are non-negative, and which meets $f(i, i) = 0$, $\forall i$ (see Pérez-López et al., 2020).

$$\alpha_{i,ij} = \frac{f(i,\ j)}{\sum_{l=1}^{A}f(i,\ l)},\ \forall i,j \in \{1,\ \cdots,\ A\} \tag{12}$$

The distance-based SCL model (Sener et al., 2011) is an SCL-based model that uses a distance-based spatial metric. Both the contiguity of the alternatives and the distance-based metrics are efficient in a context of alternatives with a regular shape, such as some type of grid. However, residential location models commonly use zoning based on administrative areas, which tends to have irregular shapes, especially in cities with historic areas. Pérez-López et al. (2020) propose in this context an SCL-based model which uses the common border length between pairs of contiguous alternatives as a spatial metric to calculate allocation parameters (BSCL). Eq. (13) shows the probability of choosing each alternative *i* in BSCL model, where $P_{i|ij}$ (14) is the conditional probability of alternative *i* if the pair *i, j* is selected and P_{ij} (15) is the probability for the pair*i, j*.

$$P_i = \sum_{\substack{j=1 \\ j\neq i}}^{A} P_{i|ij}\cdot P_{ij},\ \forall i \in \{1,\ \dots,\ A\} \tag{13}$$

$$P_{i|ij} = \frac{\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu}}{\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu} + \left(\alpha_{j,ij}e^{V_j}\right)^{1/\mu}} \tag{14}$$

$$P_{ij} = \frac{\left(\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu} + \left(\alpha_{j,ij}e^{V_j}\right)^{1/\mu}\right)^{\mu}}{\sum_{r=1}^{A-1}\sum_{l=r+1}^{A}\left(\left(\alpha_{r,rl}e^{V_r}\right)^{1/\mu} + \left(\alpha_{l,rl}e^{V_l}\right)^{1/\mu}\right)^{\mu}} \tag{15}$$

3. Spatially correlated nested logit

In residential location choice context, alternatives are usually high in number and spatially correlated. The models with correlation between alternatives which we have considered viable or more appropriate for this context are GEV models with two different approaches. One approach is the NL model, with nested structures designed by the analyst for the application environment. The other approach corresponds to models based on spatially correlated logit model that use spatial correlation metrics between alternatives, which must be appropriate to the empirical context, such as the BSCL model, when the alternatives are built from irregularly shaped administrative geographic areas. This research postulates that both approaches are compatible and that their combination can improve the fit and predictive capability of the models specified with those approaches. The resulting GEV model has been termed spatially correlated nested logit (SCNL).

The SCNL model makes the NL model more flexible in the spatial location choice modeling context, without adding parameters to the estimation process. The new model makes it possible to model the spatial correlation between alternatives of the same NL nest. The alternatives still belong to a single nest and are not correlated with alternatives from different nests. However, the pairs of alternatives in the same nest do not have the same correlation. The spatial correlation between pairs of alternatives of the same nest is modeled from a metric of the spatial correlation between alternatives, following the approach based on spatially correlated logit model.

The SCNL model proposed in this research has been formulated from a paired generalized nested logit specification, starting from a NL-type nest structure (each alternative belongs to a single nest) in a spatial location choice modeling context, and incorporating spatial correlation between the alternatives of the same nest. The GEV generating function of the SCNL model is Eq. (16). The allocation parameters of each pair of alternatives are calculated from a spatial metric between alternatives *f*, as shown in Eq. (12). The dissimilarity parameters of the pairs of alternatives are assessed using Eq. (17), where $\delta_k(i,j)$ is a Boolean function, which is 1 if both alternatives belong to the same nest, and null otherwise. Thus, the dissimilarity parameters of the pairs of alternatives of the same nest are equal, and their μ_k values are estimated with sample data, reaching the value 1 in pairs of different nest alternatives. As a GNL model, the condition that $\mu_1,\ldots,\mu_M \in (0, 1]$ ensures that the SCNL model is consistent with RUM (Wen and Koppelman, 2001).

$$G\left(e^{V_1},\ \ldots, e^{V_A}\ \right) = \sum_{i=1}^{A-1}\sum_{j=i+1}^{A}\left[\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu_{ij}} + \left(\alpha_{j,ij}e^{V_j}\right)^{1/\mu_{ij}}\right]^{\mu_{ij}} \tag{16}$$

$$\mu_{ij} := \sum_{k=1}^{M}\mu_k\delta_k(i,j) + \prod_{k=1}^{M}[1-\delta_k(i,j)],\ \forall i,j \in \{1,\ \ldots,\ A\} \tag{17}$$

This SCNL model collapses on the SCL-based model specified with the same spatial metric when there is only one nest to which all alternatives belong. The allocation parameters of the SCNL model are independent of the unit of measure used in the spatial metric, as shown below.

Demonstration:

Let *f* the spatial metric of the model and $\alpha_{i,ij}$ the dissimilarity parameter of each alternative *i* with each other alternative *j, i, j* ∈ {1, …, *A*}. If we now have the same spatial metric but measured with other metric unit, f', then there is a non-zero number $a \in$ R, such that $f'(i, j) = a \bullet f(i, j)$, for all *i, j*∈*{1, …, A}*. The dissimilarity parameter calculated now with the new metric unit is:

$$\alpha'_{i,ij} := \frac{f'(i,j)}{\sum_{l=1}^{A} f'(i,l)} = \frac{a\cdot f(i,j)}{\sum_{l=1}^{A} a\cdot f(i,l)} = \frac{a}{a}\cdot\frac{f(i,j)}{\sum_{l=1}^{A} f(i,l)} = \alpha_{i,ij}$$

The spatial location choice models have a high number of alternatives and, for this reason, typically do not include a full set of alternative specific constants; therefore, the expectations of perturbations between alternatives would not be constant and therefore the model would be artificially biased. In the SCNL model, as in CNL models, normalizing the allocation parameters to one suffices to avoid this (Abbe et al., 2007). This normalization is demonstrated in Eq. (18).

$$\sum_{j=1}^{A}\alpha_{i,ij} = \sum_{j=1}^{A}\frac{f(i,\ j)}{\sum_{l=1}^{A}f(i,\ l)} = \frac{\sum_{j=1}^{A}f(i,\ j)}{\sum_{l=1}^{A}f(i,\ l)} = 1,\ \forall i \in \{1,\ \ldots,\ A\} \tag{18}$$

The probability function of the SCNL model (Eqs. (19), (20), and (21)) is the same as for paired generalized nested logit model (Wen and Koppelman, 2001), albeit with a different definition of the parameters μ_{ij}, and makes it possible to calculate the probability of each individual choosing the alternative *i* without integrations. The parameters of the model are estimated using maximum likelihood. The cumulative extreme-value distribution of the vector of perturbations of the utility equations of an SCNL model ($\varepsilon_1,\ldots,\varepsilon_A$) is expressed in Eq. (22). The marginal cumulative distribution function of each perturbation ε_i is a univariant extreme value. As confirmed in Eq. (23), the function is the Gumbel standard if the allocation parameters of each alternative are normalized to one, a requirement met in the SCNL model.

$$P_i := \sum_{\substack{j=1\\ j\neq i}}^{A} P_{i|ij}P_{ij},\ \forall i \in \{1,\ \ldots,\ A\} \tag{19}$$

$$P_{i|ij} = \frac{\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu_{ij}}}{\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu_{ij}} + \left(\alpha_{j,ij}e^{V_j}\right)^{1/\mu_{ij}}} \tag{20}$$

$$P_{ij} = \frac{\left(\left(\alpha_{i,ij}e^{V_i}\right)^{1/\mu_{ij}} + \left(\alpha_{j,ij}e^{V_j}\right)^{1/\mu_{ij}}\right)^{\mu_{ij}}}{\sum_{r=1}^{A-1}\sum_{l=r+1}^{A}\left(\left(\alpha_{r,rl}e^{V_r}\right)^{1/\mu_{rl}} + \left(\alpha_{l,rl}e^{V_l}\right)^{1/\mu_{rl}}\right)^{\mu_{rl}}} \tag{21}$$

$$F(\varepsilon_1,\ \ldots,\ \varepsilon_A) = \exp\left\{-\sum_{i=1}^{A-1}\sum_{j=i+1}^{A}\left[\left(\alpha_{i,ij}e^{-\varepsilon_i}\right)^{1/\mu_{ij}} + \left(\alpha_{j,ij}e^{-\varepsilon_j}\right)^{1/\mu_{ij}}\right]^{\mu_{ij}}\right\} \tag{22}$$

Table 1
Direct elasticities of each alternative iϵ{1,…,A}.

Model	Direct elasticity
SCNL	• If i is in root nest $(1-P_i)\beta_m X_{im}$ • If i is in N_k nest, $k\epsilon\{1,\ \dots,\ M\}$
SCL-based	$\dfrac{\sum_{j=1, j\neq i}^{A} P_{i\mid ij}P_{ij}[(1-P_i)+(\mu^{-1}-1)(1-P_{i\mid ij})]}{P_i}\beta_m X_{im}$
NL	• If i is in root nest $(1-P_i)\beta_m X_{im}$ • If i is in N_k nest, $k\epsilon\{1,\ \dots,\ M\}$ $[(1-P_i)+(\mu_k^{-1}-1)(1-P_{i\mid k})]\beta_m X_{im},\ \forall i\epsilon\{1,\ \dots,\ A\}$
MNL	$(1-P_i)\beta_m X_{im}$

Table 2
Cross-elasticities of each pair of alternatives i,jϵ{1,…,A},j≠i.

Model	Cross-elasticity
SCNL	$-\left[P_i+(\mu_{ij}^{-1}-1)\dfrac{P_{i\mid ij}P_{ij}P_{j\mid ij}}{P_j}\right]\beta_m X_{im}$ • If i, j are not in the same nest $-P_i\beta_m X_{im}$ • If i, j are in N_k nest, $k\in\{1,\ \dots,\ M\}$ $-\left[P_i+(\mu_k^{-1}-1)\dfrac{P_{i\mid ij}P_{ij}P_{j\mid ij}}{P_j}\right]\beta_m X_{im}$
SCL-based	$-\left[P_i+(\mu^{-1}-1)\dfrac{P_{i\mid ij}P_{ij}P_{j\mid ij}}{P_j}\right]\beta_m X_{im}$
NL	• If i, $j$are not in the same nest $-P_i\beta_m X_{im}$ • If i, j are in N_k nest, $k\in\{1,\ \dots,\ M\}$* $-[P_i+(\mu_k^{-1}-1)P_{i\mid k}]\beta_m X_{im}$ (*) Modified from Papola (2004) for easier comparison
MNL	$-P_i\beta_m X_{im}$

$$F(\varepsilon_i)=\exp\left(-\sum_{\substack{j=1\\ j\neq i}}^{A}\alpha_{i,ij}e^{-\varepsilon_i}\right)=\exp(-e^{-\varepsilon_i}),\ \forall i\in\{1,\ \dots,\ A\} \tag{23}$$

The correlation between each pair of alternatives (i, j) is calculated by numerical integration from the marginal bivariant cumulative distribution function of the perturbations of the alternatives Abbe et al., 2007), which is expressed in Eq. (24). Eqs. (22) to ((24) have been deduced from the SCL equations (Bhat and Guo, 2004) by incorporating different μ_{ij} parameters in each nest of the NL structure.

$$H(\varepsilon_i,\varepsilon_j)=\exp\left\{-\left[(1-\alpha_{i,ij})e^{-\varepsilon_i}+(1-\alpha_{j,ij})e^{-\varepsilon_j}\right]-\left[(\alpha_{i,ij}e^{-\varepsilon_i})^{1/\mu_{ij}}+(\alpha_{j,ij}e^{-\varepsilon_j})^{1/\mu_{ij}}\right]^{\mu_{ij}}\right\}\forall i,j\in\{1,\ \dots,\ A\},\ j\neq i \tag{24}$$

From the approach proposed by Papola (2004), the unobserved correlation between alternatives can be approximated by Eq. (25). It is null when the alternatives are not in the same nest (like NL), and it is $\alpha_{i,ij}^{1/2}\alpha_{j,ij}^{1/2}(1-\mu_k^2)$ when both alternatives are in a N_k nest. In comparison with NL, the correlation between alternatives of the same nest is not constant and depends on the allocation parameters and, therefore, on the spatial metric used. With respect to SCL-based models, the correlation between alternatives depends on the nest to which both of them belong.

$$\widehat{Corr}(\varepsilon_i,\ \varepsilon_j)=\alpha_{i,ij}^{1/2}\alpha_{j,ij}^{1/2}\left(1-\mu_{ij}^2\right),\ \forall i,j\in\{1,\ \dots,\ A\} \tag{25}$$

Considering a linear observed utility in the parameters, with coefficients β_m, the direct elasticity of the m-th regressor of alternative i, X_{im}, measures the expected percentage change in P_i for an increase of one percentage point of X_{im}. The cross-elasticity of X_{im} in P_j measures the expected percentage variation in P_j for an increase of one percentage point in X_{im}. Tables 1 and 2 show a comparison of direct and cross-elasticity of SCNL model with other GEV models.

Direct and cross-elasticity of the SCNL have the same formulation as MNL and NL models in the alternatives of the root nest. In comparison with SCL-based models, in the SCNL both formulations depend on the dissimilarity parameters of the nest for alternatives in the same nest. SCNL elasticities are equivalent to that of SCL-based models in the case that all alternatives are in the same nest. If spatial metric is based on contiguity, the cross-elasticity of non-contiguous alternatives for SCNL and SCL-based specification is equal

Table 3
Explanatory variables of the sample.

Name	Description	Type	Mean/ Distribution	Standard deviation
JT	Journey time in minutes between residential zone and employment zone.	Alternative –Specific of individual	7.57	3.93
FO	Number of non-EU foreigners in the residential zone (in thousands of people).	Alternative	0.461	0.224
HO	Natural log of the number of housing in the residential zone.	Alternative	7.858	0.228
PS	Dichotomous factor indicating that the residential zone has special prestige (subjective).	Alternative	NO: 95.51% YES: 4.49%	
PR	Average price of housing in the residential zone (in millions of €).	Alternative	0.28761	0.12670
SC	Number of primary and secondary education centers at a maximum distance of one km from residential zone centroid.	Alternative	2.22	1.70
WT	Average waiting time in minutes at public transport stops in the residential zone.	Alternative	10.51	0.78
H	Dichotomous factor indicating decider's high monthly net family incomes (more than 2500 €).	Individual	NO: 76.97% YES: 23.03%	

to that of MNL. SCNL and NL have the same cross-elasticity in alternatives from different nests (and equal to that of the MNL model). In alternatives belonging to the same nest, the second formulation in Table 2 of the cross-elasticity of the SCNL model looks quite similar to that of the NL model. However, this similarity is misleading. Unlike the NL model, the conditional probability of the SCNL model depends on the implicit effect of the assignment parameters.

The SCNL model requires a more complex design and estimation process than the NL and SCL-based models. Regarding the design, in comparison with NL, the SCNL model requires selecting a spatial metric appropriate to the zoning of the geographic area, and calculating the values of the metric in the zoning normally using a GIS; with respect to SCL-based models, SCNL requires designing a nested structure appropriate to the empirical context of application. Regarding estimation, although the SCNL model has the same unknown parameters as the NL model, its estimation process is more complex because the SCNL model is a reduced specification of the paired combinatorial nested logit model; in comparison with SCL-based models, the nested design increases the number of unknown parameters by a number equal to the number of nests minus one. When a SCNL model uses spatial metrics like the original SCL contiguity or the BSCL metric, the number of parameters of the SCNL is the same as that of the NL. The SCNL model is compatible with SCL-based specifications that require estimating additional parameters, as in the case of generalized spatially correlated logit (Sener et al., 2011).

SCNL model is compatible with a mixed GEV specification (MSCNL), which makes it possible to incorporate variations in decision-makers' preferences through an overlapping structure of random coefficients.

4. Empirical application of SCNL

This application of the SCNL model in the city of Santander (Spain) focuses on comparing the capacity to collect the spatial correlation between alternatives of this model, against the previous GEV models that are described in Section 2. To compare the explanatory power of the estimated models, we will use the statistical techniques of goodness-of-fit (GoF) (Hilbe, 2009). To compare how well the estimated models keep their predictive accuracy in a different sample, we will use statistical validation techniques as recommended by Parady et al. (2021). This application also shows the results of a proof of concept of MSCNL.

To avoid design bias, we use data and spatial elements designed to be applied with the models that are described in Section 2 (with the previous GEV approach) from research projects INTERLAND (see Ibeas et al., 2013) and TRANSPACE (Dell'Olio et al., 2016). The sample, the zoning and the nests structure are the same of Ibeas et al. (2013). Also, the spatial metric of the correlation between alternatives and the utility function, both the kernel GEV and its mixed specification are from Pérez-López et al. (2020).

Endogeneity has been established as a relevant issue in residential location choice models (Guevara and Ben-Akiva, 2006) and in other discrete choice models (Guevara and Ben-Akiva, 2012; Guerrero et al., 2021a; Guerrero et al., 2021b). When the alternatives are the specific dwelling to live, it is usually due to the omission of attributes of the dwelling that are correlated with the price and influence the choice. This misspecification will suppose that the impact of price in the choice process will not be correctly established and the estimators of the model parameters may be biased and inconsistent. It would be a serious problem for policy analysis. An indicator of the problem may be that the dwelling-unit price coefficient is non-significant, small or even positive. This problem can be addressed with the control function method (see Guevara and Ben-Akiva, 2012 for forecasting issues with that method and Guevara, 2015, for a critical assessment of several methods). This method requires to select adequate instrumental variables that are correlated with the price but are uncorrelated with the error term. For the kind of choice presented, those variables can be constructed as an average of the prices of other dwellings with similar observed attributes (other than price) and locating within certain vicinity (Guevara, 2010; Guevara and Ben-Akiva, 2012). In the application presented herein, the alternatives are not specific dwellings but areas, and the variable of the utility function is the mean price of dwellings in the area. In that situation, endogeneity due to omitted attributes of a specific dwelling that are correlated with price is not expected, although other sources of endogeneity cannot be discarded at all. As can be seen later, the results do not show indications of endogeneity, but it is advisable to carefully analyze this issue in models estimated for policy analysis (see Guerrero et al., 2021a; Guerrero et al., 2021b).

Fig. 1. Map of the scheme of the alternative residential zones in Santander.

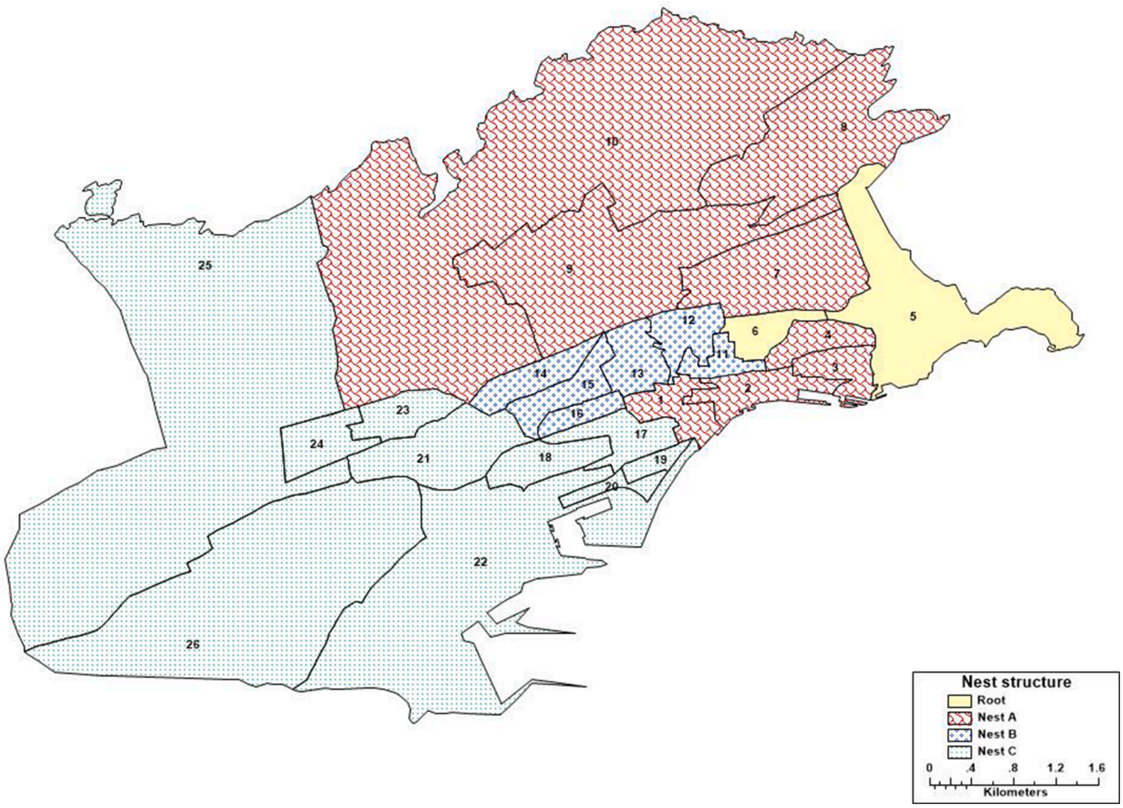

Fig. 2. Nest structure of alternatives.

4.1. Data and spatial elements

The sample contains 534 individual choices of deciders who live and work in the city of Santander. Table 3 includes descriptive statistics of the explanatory variables in the sample. The observed component of the utility function is a linear form which does not include alternative-specific constants. The regressors are the journey time (*JT*), the number of non-EU residents in the area (*FO*), the number of homes available in the area (*HO*) and the average house price in the area (*PR*), as well as the interactions between the high-

Table 4
Results of the estimation of the models NL, BSCL and BSCNL. LRT significant code: "**" if the test is significant at 1% significance level.

	Parameter	BSCL Value	SE	StC	NL Value	SE	StC	BSCNL Value	SE	StC
Estimation	β_{JT}	−0.18	0.0518	−0.708	−0.104	0.0271	−0.409	−0.104	0.0271	−0.409
	β_{FO}	−1.12	0.447	−0.251	−1.00	0.300	−0.224	−0.892	0.284	−0.200
	β_{HO}	2.05	0.417	0.467	1.55	0.305	0.353	1.29	0.283	0.294
	β_{PR}	−2.63	0.596	−0.333	−2.17	0.426	−0.275	−1.99	0.399	−0.252
	$\beta_{PS\bullet H}$	1.58	0.396	0.136	1.22	0.262	0.105	1.02	0.260	0.088
	$\beta_{SC\bullet H}$	0.302	0.0752	0.453	0.210	0.0460	0.315	0.173	0.0469	0.259
	μ^{-1}	1.74	0.0979		1			1		
	μ_A^{-1}				1.25	0.147		3.11	1.42	
	μ_B^{-1}				1.26	0.128		2.27	0.773	
	μ_C^{-1}				1.05	0.089		1.49	0.430	
	No. est. par.	7			9			9		
GoF	*LL*	−1663.183			−1661.270			−1659.038		
	ρ^2	0.0441			0.0452			0.0464		
	$\bar{\rho}_H^2$	0.0420			0.0426			0.0438		
	AIC	0.04003			0.03998			0.0413		
	LRT-MNL	9.570	**		13.396	**		17.860	**	
Val.	*PG-CV*	0.0437			0.0438			0.0440		

income level (*H*) with the prestige of the area (*PS*) and with the number of primary and secondary education centers near the centroid of the area (*SC*). The theoretically expected sign of estimated coefficients is negative in the case of *JT, FO* and *PR*, and positive in the case of *HO* and the interactions *PS* • *H* and *SC* • *H*. The mixed GEV specifies the coefficient of the regressor *SC* • *H* like a random variable with a normal distribution.

The zoning used in this section is based on the map of administrative areas of the city and resulted in 26 alternatives with very irregular shapes, as shown in Fig. 1. The spatial metric based on the length of the common border between pairs of alternatives is more efficient than the previous metrics in the context of alternatives with irregular shapes (Pérez-López et al., 2020). We are using this spatial metric in this application on a specification of a SCL-based model (BSCL) and on a specification of the SCNL model proposed in this paper (BSCNL).

The nests structure shown in Fig. 2 consists of three nests (A, B and C), leaving two alternatives in the root nest, that is, uncorrelated with each other or with other alternatives. Nest A consist of two different areas. This nested structure has a strong spatial component in order to capture the spatial correlation patterns between alternatives and those resulting from non-spatial characteristics, for the NL model. Thus, this design will test the ability of the SCNL model to capture spatial correlations between alternatives that have not already been identified by the nested structure, thereby verifying the ability of the SCNL model to complement the NL model.

4.2. Results analysis

All GEV models are estimated in this section by maximum likelihood (maximum simulated likelihood with 1000 iterations in the case of mixed specifications) using the Biogeme program (Bierlaire, 2003), applying the same DONLP2 (Spellucci, 1993) optimization algorithm in all estimates. In the models, the coherence of the signs of the estimated coefficients with those theoretically expected (described in the previous subsection) was verified. The relevance of the regressors was also checked using the asymptotic *t*-test at 5% of significant level of the corresponding estimated coefficients. The relative influence of the regressors was ordered using standardized coefficients, even though they are measured on different scales (in fact, in this case there are continuous, qualitative, and even dichotomous regressors). Different statistics are used to standardize the estimated coefficients (see Menard, 2004; Menard, 2011). In this case, we will use the statistic proposed by Menard (1995) and Agresti (1996) which is obtained by multiplying every estimated coefficient and the sample standard deviation of its regressor. The higher the absolute standardized value of the regressors is, the stronger their relative influence on the decision will be.

The GoF statistics calculated in every estimated model are the following likelihood ratio indexes: McFadden (ρ^2,1974), Horowitz ($\bar{\rho}_H^2$, 1983), and Akaike Information Criterion (AIC; Ben-Akiva and Swait, 1986). The last two penalize the number of parameters that have been estimated; thus, they are useful for comparing models that estimate different numbers of parameters. The AIC penalizes more the incorporation of parameters, which favors more parsimonious models. To compare the GoF of two of the estimated models, we will use the following procedure. If the two models estimate the same parameters, they will be compared using ρ^2. If the two models estimate different parameters, different criteria are used, depending on the situation. If the two models being compared are nested (where one model can be determined through linear constraints of the parameters of the other model) we use the likelihood ratio test (LRT). If the two models being compared are not nested we use the following criteria described in Horowitz (1983). First, the LRT of each model is performed with respect to a model with which both are nested (in this case, the null model or the MNL model with the same utility function). If one of the LRTs is significant and the other is not, the model with the significant LRT will be chosen. If both are significant, GoF statistics will be used to penalize the incorporation of additional parameters. Horowitz proposed $\bar{\rho}_H^2$, in this research, we will also analyze the AIC. If different conclusions are reached with each parameter, we will consider the result inconclusive.

A cross-validation process with $K = 10$ groups has been conducted, randomly partitioning the sample. The accuracy of the

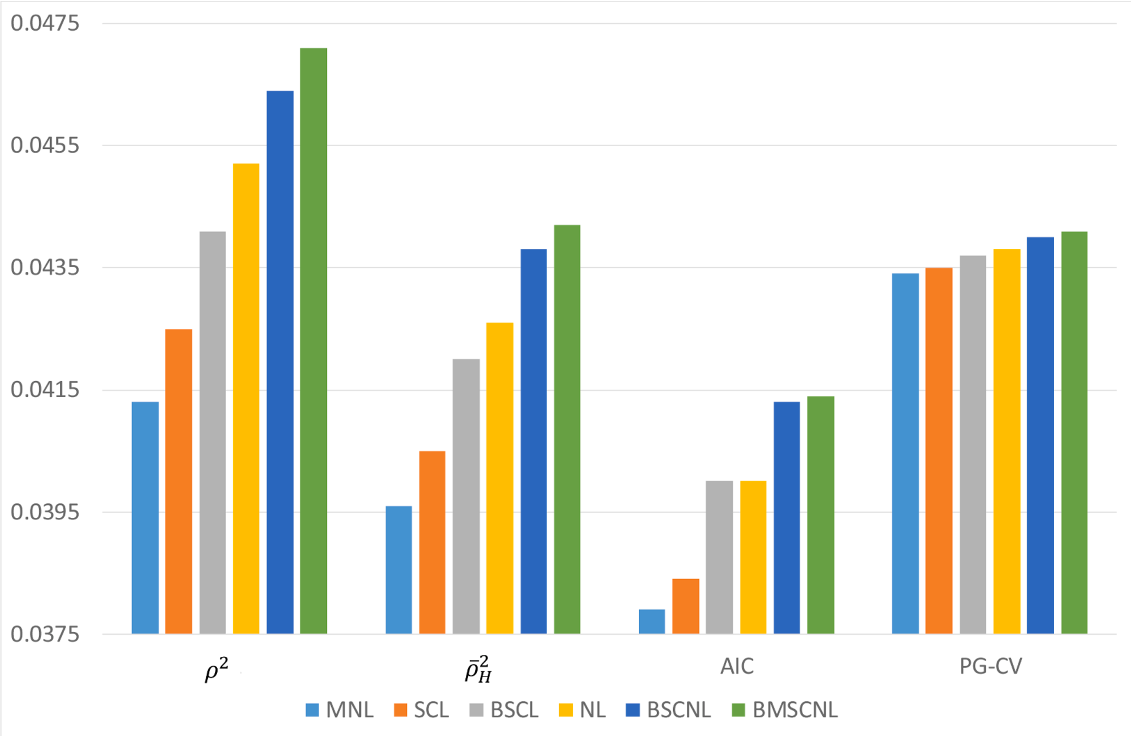

Fig. 3. GoF and validation statistics of estimated models.

predictions has been measured using *PG-CV* statistic, which is the geometric mean value of the ten values of the Predicting Geometric statistics. The Predicting Geometric values are obtained in each iteration of the cross-validation process with the geometric mean value of the correct probabilities (probability according to the model of the alternative chosen by the respondent) in the test sample, albeit using the model estimated with the training sample (Başar and Bhat, 2004; de Luca and Cantarella, 2009; Martínez-Pardo et al., 2020). In this research, we use the geometric mean, instead of the arithmetic mean that is commonly used. The geometric mean has better properties when using probability data.

Table 4 shows the estimation, GoF and validation results to compare NL, BSCL and BSCNL models. The estimation results of each model include, for each estimated parameter, the estimated value (Value), its standard error (SE) and its standardized coefficient (StC). The results table of each estimated model will also show the number of estimated parameters (No. est. par.) and the GoF and Validation results. In the three models, all the estimated parameters are significant, with signs coherent with the theoretically expected and with the same order from standardized coefficients. The most influential explanatory variable in the decision is *JT*, followed by *HO, SC* • *H, PR, FO* and finally *PS* • *H*. These results are similar to those obtained with the MNL and SCL models (Pérez-López et al., 2020).

The three models improve the GoF and the validation results of the MNL and SCL models. The NL model improves the validation results of the BSCL model, and some of the GoF results, but is not totally conclusive. They are not nested between them and have a different number of estimated parameters, therefore to compare their GoF, we use $\bar{\rho}_H^2$ and *AIC*. The NL model has a higher $\bar{\rho}_H^2$ value than BSCL but a lower *AIC* value. However, the proposed BSCNL model significantly improves the GoF and the validation results of both of them.

The mixed specification BMSCNL has one more parameter to estimate than its kernel BSCNL. BMSCNL improves the GoF and validation statistics results of BSCNL (but the LRT is not significant). The results are inconclusive, and for this reason it is not included in the table. Fig. 3 compares the values of GoF and validation measures assessed in the different models estimated, including MNL, SCL and BMSCNL that are not shown in table 4. The results of the BSCNL and BMSCNL specifications of the SCNL model improve those of the rest of the models in all the concepts considered.

5. Conclusions

The spatially correlated nested logit (SCNL) model is proposed in this research for spatial location choice modeling. The SCNL model makes it possible to combine the approach of nested alternatives of the nested logit model (NL) with that of extensions based on spatially correlated logit (SCL) model using metrics of spatial correlation between alternatives. Thanks to this combination, the SCNL model improves the explanatory and predictive power of the models that use the previous approaches.

The SCNL model can improve the explanatory and predictive power of the NL, even when the nested structure of the NL model has a strong spatial component, with the same unknown parameters (if the spatial metric does not require additional parameters, as the one employed in the application). This improvement occurs when the spatial metric selected in the SCNL model is able to capture the spatial correlation between alternatives that the nested structure designed by the analyst for the NL model was unable to capture. The SCNL model achieves this improvement over the NL model by modeling the correlation between pairs of alternatives belonging to the same nest using spatial metrics.

The SCNL model can also improve the explanatory and predictive power of SCL-based models with the same metric of spatial correlation between alternatives. This improvement occurs when non-spatial correlation between alternatives is captured by the nested structure or when this nested structure is capable of detecting spatial correlation between alternatives in addition to that captured by the spatial metric. The SCNL model achieves this improvement over these models thanks to the flexibility of the dissimilarity parameter between nests, which makes it possible to model the correlation between pairs of alternatives with greater flexibility than SCL-based models. Furthermore, unlike SCL-based models, SCNL models do so considering not only spatial but also other correlation factors.

In addition, the SCNL model proposed in this research is compatible with mixed specifications of random coefficients to incorporate heterogeneity into decision-makers' preferences. The mixed SCNL model thus built may have better properties than the kernel SCNL in the presence of heterogeneous preferences.

The application of the different models analyzed empirically confirmed that the proposed SCNL model has good properties. The BSCNL model (using the common spatial border correlation metric) provided better empirical results than the SCL-based model using the same spatial metric and NL models, both in goodness-of-fit and in validation.

CRediT authorship contribution statement

Jose-Benito Perez-Lopez: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Writing – original draft. **Margarita Novales:** Conceptualization, Methodology, Validation, Resources, Writing – review & editing, Supervision, Project administration, Funding acquisition. **Alfonso Orro:** Conceptualization, Methodology, Validation, Resources, Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declarations of competing interest

None.

Acknowledgments

The authors acknowledge the financial support provided by the Government of Spain under the projects TRA2012–37659 and RTI2018–097924-B-I00, funded by MCIN/AEI/10.13039/501100011033 and by "ERDFA way of making Europe". Funding for open access charge: Universidade da Coruña/CISUG.

References

Abbe, E., Bierlaire, M., Toledo, T., 2007. Normalization and correlation of cross-nested logit models. Transp. Res. B 41, 795–808.
Agresti, A., 1996. An Introduction to Categorical Data Analysis. Wiley. P. 129, New York.
Bahamonde-Birke, F.J., 2021. A brief discussion on the treatment of spatial correlation in multinomial discrete models. J Transp. Land Use 14 (1), 521–535.
Başar, G., Bhat, C., 2004. A parameterized consideration set model for airport choice: an application to the San Francisco Bay Area. Transp. Res. Part B 38, 889–904.
Ben-Akiva, M., Bierlaire, M., 1999. Discrete choice methods and their applications to short-term travel decisions. In Hall R (Eds.). Handbook Transp. Sci. 5–34.
Ben-Akiva, M., Francois, B., 1983. Mu-homogenous Generalized Extreme Value Model. Working Paper.
Ben-Akiva, M., Swait, J., 1986. The Akaike likelihood ratio index. Trans. Sci. 20 (2), 133–136.
Bhat, C.R., Guo, J., 2004. A mixed spatially correlated logit model: formulation and application to residential choice modeling. Transp. Res. B Methodol. 38 (2), 147–168.
Bierlaire, M., 2003. BIOGEME: a free package for the estimation of discrete choice models. Ascona, Switzerland. In: 3rd Swiss transportation research conference.
Bolduc, D., 1992. Generalized autoregressive errors: the multinomial probit model. Transp. Res. B 26, 155–170.
Chu, C., 1981. *Structural Issues and Sources of Bias in Residential Location and Travel Mode Choice Models*. ***Unpublished Ph.D. Dissertation***. Department of Civil Engineering, Northwestern University, USA.
Chu, C., 1989. A paired combinatorial logit model for travel demand analysis. In: Proceedings of the Fifth World Conference on Transportation Research. Ventura, CA, pp. 295–309 vol. 4.
Daganzo, C., 1979. Multinomial Probit: The Theory and Its Application to Demand Forecasting. Academic Press, New York.
Daly, A.J., Zachary, S., 1978. Improved multiple choice models. In: Hensher, DA, Dalvi, MQ (Eds.), Determinants of Travel Choice. Westmead, Saxon House, pp. 335–357.
de Luca, S., Cantarella, G.E., 2009. Validation and comparison of choice models. In: Saleh, W., Sammer, G. (Eds.), Travel Demand Management and Road User Pricing: Success, Failure and Feasibility. Ashgate publications, pp. 37–58.
Dell'Olio, L., Cordera, R., Ibeas, A., (Eds) Alonso, A., Alonso, B., Barreda, R., Comi, A., Coppola, R., González, E., Monzón, A., Moura, J., Nogués, S., Nuzzolo, A., Orro, A., Papa, E., Pérez-López, J.-B., Reques, P., Sañudo, R., Wang, Y., 2016. *Land Use - transport interaction models. The TRANSPACE Model*. 1st edn. Santander: GIST.
Domencich, T.A., McFadden, D., 1975. Urban Travel demand: a Behavioural analysis. American Elsevier, New York.
Garrido, R.A., Mahmassani, H.S., 2000. Forecasting freight transportation demand with the space-time multinomial probit model. Trans. Res. B 34, 403–418.
Guerrero, T.E., Guevara, C.A., Cherchi, E., Ortúzar, J.D.D., 2021a. Forecasting with strategic transport models corrected for endogeneity. Transportmetrica A 1–28. https://doi.org/10.1080/23249935.2021.1891154.
Guerrero, T.E., Guevara, C.A., Cherchi, E., Ortúzar, J.D.D., 2021b. Addressing endogeneity in strategic urban mode choice models. Transportation, 48, 2081–2102.
Guevara, C.A., Ben-Akiva, M., 2006. Endogeneity in residential location choice models. Transp. Res. Rec. (1) 197760-66.
Guevara, C.A., Ben-Akiva, M., 2012. Change of scale and forecasting with the control-function method in logit models. Transp. Sci. 46 (3), 425–437.
Guevara, C.A., 2010. Endogeneity and Sampling of Alternatives in Spatial Choice Models (Doctoral dissertation. Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA. Ph. D. Thesis.

Guevara, C.A., 2015. Critical assessment of five methods to correct for endogeneity in discrete-choice models. Transp. Res. Part A 82, 240–254.
Hess, S., Bierlaire, M., Polak, J., 2005. Capturing taste heterogeneity and correlation structure with Mixed GEV models. In: Scarpa, R, Alberini, A (Eds.), Applications of Simulation Methods in Environmental and Resource Economics. Springer Publisher, Dordrecht, The Netherlands, pp. 55–76 chapter 4.
Hilbe, J.M., 2009. *Logistic Regression Models*. CRC press. ISBN 978-1-138-10671-0.
Horowitz, J., 1983. Statistical comparison of non-nested probabilistic discrete choice models. Trans. Sci. 17, 319–350.
Ibeas, A., Cordera, R., Dell'Olio, L., Coppola, P., 2013. Modeling the spatial interactions between workplace and residential location. Transp Res A 49, 110–122.
Johnson, N., Kotz, S., 1970. Distributions in Statistics: Constinuous Univariate Distributions. John Wiley, New York. Chapter 21.
Koppelman, F.S., Wen, C.H., 2000. The paired combinatorial logit model: properties, estimation and application. Transp. Res. Part B 34 (2), 75–89.
Martínez-Pardo, A., Orro, A., Garcia-Alonso, L., 2020. Analysis of port choice: a methodological proposal adjusted with public data. Transp. Res. Part A 136, 178–193.
McFadden, D., 1974. Conditional logit analysis of qualitative choice behavior. In: Zarembka, P (Ed.), Conditional logit analysis of qualitative choice behavior. Frontiers in Econometrics 105–142.
McFadden, D., 1978. Modelling the choice of residential location. In: Karlqvist, A, Jundqvist, L, Snickars, F, Weibull, J (Eds.), Spatial Interaction Theory and Planning Models. North Holland, Amsterdam, pp. 75–96.
Menard, S., 1995. Applied Logistic Regression Analysis. Thousand Oaks, CA: Sage, p. 48. P.
Menard, S., 2004. Six Approaches to Calculating Standardized Logistic Regression Coefficients. Am Stat 58 (3), 218–223.
Menard, S., 2011. Standards for standardized logistic regression coefficients. Soc. Forces 89 (4), 1409–1428.
Pagliara, F., Wilson, A., 2010. The State-of-the-Art in Building Residential Location Models. Residential Location Choice. Springer.
Papola, A., 2004. Some developments on the cross-nested logit model. Transp. Res. B Methodol. 38, 833–851.
Parady, G., Ory, D., Walker, J., 2021. The overreliance on statistical goodness-of-fit and under-reliance on model validation in discrete choice models: a review of validation practices in the transportation academic literature. J. Choice Modell. 38, 100257.
Pérez-López, J.-B., Novales, M., Varela-Garcia, F.-.A., Orro, A., 2020. Residential location econometric choice modeling with irregular zoning: common border spatial correlation metric. Netw. Spatial Econ. 20, 785–802.
Sener, I.N., Pendyala, R.M., Bhat, C.R., 2011. Accommodating spatial correlation across choice alternatives in discrete choice models: an application to modeling residential location choice behavior. J. Transp. Geogr. 19, 294–303.
Small, K.A., 1987. A discrete choice model for ordered alternatives. Econometrica 55 (2), 409–424.
Spellucci, P., 1993. *DONLP2 Users Guide*, Dept. of Mathematics, Technical University at Darmstadt, 64289 Darmstadt, Germany.
Takahashi, K., 2019. Local relaxation of constraints on dissimilarity parameters in the generalized nested logit model. Int. J. Japan Assoc. Manag. Syst. 11 (1), 73–80. December 2019.
Torrens, P.M., 2000. How Land-Use Transportation Models Work. Centre for Advanced Spatial Analysis, London.
Train, K.E., 2009. Discrete Choice Methods With Simulation. Cambridge University Press, New York, New York, USA.
Thurstone, L.L., 1927. A law of comparative judgment. Psychol Rev 34, 273–286.
Vovsha, P., 1997. The cross-nested logit model: application to ode choice in the Tel-Aviv metropolitan area. Transp. Res. Rec. 1607, 6–15.
Weiss, A., Hasnine, S., Habib, K.N., 2019. A comparative study of alternative methods for capturing spatial correlations in discrete choice models through an empirical application on school choice location modelling. In: Paper presented at the 98th Annual Meeting of the Transportation Research Board. Washington, DC, January 13–17 (No. 19-05270).
Wen, C.H., Koppelman, F.S., 2001. The generalized nested logit model. Transp. Res. Part B 35 (7), 627–641.
Williams, H.C.W.L., 1977. On the formation of travel demand models and economic evaluation measures of user benefit. Environ. Plann. 9A, 285–344.